# Absolute Theory of Relativity \& Parameterized Special Theory of Relativity \& Noninertial Multirelativity 

Florentin Smarandache<br>University of New Mexico, smarand@unm.edu

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## Florentin Smarandache

## ABSOLUTE THEORY OF RELATIVITY \& <br> PARAMETERIZED SPECIAL THEORY OF RELATIVITY <br> \& <br> NONINERTIAL MULTIRELATIVITY



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## Preface

## ( Many Relativities )

In the first chapter of this book we discuss Einstein's thought experiment with atomic clocks:

A rocket travels at a constant speed $v$ with respect to the earth. In the rocket, a light pulse is emitted by a source from $A$ to a mirror $B$ that reflects it back to $A$ where it is detected. The rocket's movement and the light pulse's movement are orthogonal. There is an observer in the rocket (the astronaut) and an observer on the earth. The trajectory of light pulse (and implicitly the distance traveled by the light pulse), the elapsed time it needs to travel this distance, and the speed of the light pulse at which is travels are perceived differently by the two observers \{depending on the theories used in this book\}.


Fig. (p1)
The Special Theory of Relativity (STR) works fine in a spacetime $S_{c}$ where the speed of light $c$ is the ultimate speed and the relativistic addition of velocities applies:
$v_{1} \oplus_{c} v_{2}=\frac{v_{1}+v_{2}}{1+\frac{v_{1} v_{2}}{c^{2}}}$
(p1)

Several counter-intuitive examples of formula (pl) are presented. Also, in our opinion there is neither a real time dilation nor a length contraction, but an apparent time dilation and an apparent length contraction.

Yet, we extend this formula, by induction, using a recurrent proof, for the relativistic addition of $n \geq 2$ velocities at once.

We show that formula (p1) and its extension can be straightforwardly extended to a spacetime $S_{\underline{K}}$ where the ultimate speed is $K>0$, which can be smaller or greater than $c$. Therefore, the relativistic addition of two velocities in $S_{K}$ becomes:

$$
\begin{equation*}
v_{1} \oplus_{K} v_{2}=\frac{v_{1}+v_{2}}{1+\frac{v_{1} v_{2}}{K^{2}}} \tag{p2}
\end{equation*}
$$

and correspondingly all related formulas from STR get translated into the $S_{K}$ spacetime by simply substituting $c$ with $K$ : whence we obtain the Generalized Lorentz Factor, Generalized Minkowski Norm, Generalized Time Dilation, Generalized Length Contraction, Generalized Relativistic Momentum, Generalized Energy, Generalized Total Energy, and Generalized Kinetic Energy.

In the second chapter we present our hypothesis that there is no speed barrier in the universe and one can construct arbitrary speeds from zero to infinity (Smarandache -1972), thus refuting the speed of light postulate. We consider that the superluminal phenomena do not violate the causality principle, do not produce time traveling, and do not necessitate infinite energy for particles traveling at speeds greater than the speed of light.

While Einstein considered a relative space and relative time but ultimate speed of light, we do the opposite in the third chapter: we redo Einstein's experiment with atomic clocks by considering an absolute time and absolute space but no ultimate speed (according to our previous hypothesis). That's why we call our theory Absolute Theory of Relativity (ATR).

According to this theory, the speed of photon in the rocket, with respect to the observer on earth, is:

$$
\begin{equation*}
x=\sqrt{v^{2}+c^{2}} \tag{p3}
\end{equation*}
$$

which corresponds to the magnitude of the vectorial addition, i.e. $x=|\vec{v}+\vec{c}|$, since $\vec{c}$ and $\vec{v}$ are orthogonal:

or


Fig. (p2)
and thus $x>c$.
Therefore in the ATR we replace Einstein's Relativistic Addition of Velocities (p1) with ( $p 3$ ), allowing for superluminal speeds. Since in Absolute Theory of Relativity there is no time dilation, in consequence there is no length contraction, and no relative simultaneity. Lorentz Factor becomes equal to 1 , so it is useless.
As a consequence, many relativistic paradoxes are discarded: such as Ehrenfest Paradox (1909), the Twin Paradox (1911), Bell's Spaceship Paradox (1959), W. Rindler's Paradox about a man falling into a grate (1961), etc.

The famous physics formula $E_{0}=m c^{2}$ is questionable: why should it depend on the light speed?
Similarly:
a) the Michelson-Morley Experiment (1881, 1883-1887) of not having detected the ether, which implied the STR, might have been erroneous due to imprecise measurement instruments or construction. Maybe the ether is very little dense and its flow hardly perceptible;
b) and the constancy of the speed of light in vacuum could be flaw, since the speed of light might depend on the emitting source - the stronger the emitting source the faster its light pulse;
because if an inconsistent theory resulted from some ideas, those ideas must have been inconsistent too.

Not all physics laws might be the same in all inertial systems.
As a counter-example let's consider as physics law the Addition of Velocities in a given inertial system $S_{i}$. Let's take the collinear and in the same direction velocities $v_{1}=0.8 c$ and $v_{2}=0.9 c$. In STR we get $v_{1}+v_{2}=0.988372 c$ while in ATR we get a different result:
$v_{1}+v_{2}=1.7 c$.

ATR generalizes the relativistic addition of velocities in the following way: instead of a photon we consider any particle traveling from $A$ to $B$ with a speed $\mathrm{w}>0$ such that the angle between the particle's direction vector $w$ and the rocket's direction vector $v$ is $\theta$, with $0 \leq \theta \leq \pi$ :

Fig. (p3)

$x=\sqrt{v^{2}+w^{2}+2 v w \cdot \cos \theta}$
or
$x=|\vec{v}+\vec{w}|$
which is a generalization of ( $p 3$ ).
A computer simulation would better describe the resultant curve $A B^{\prime}$ '[from Fig. (p3)] as seen by the observer on earth. Herein we approximate it by a line.

In Chapter 4, we study the general case of Einstein's thought experiment with atomic clocks, when we suppose nothing is known about if the space and time are relative or absolute or if the speed of light is ultimate speed or not.

Therefore, we obtain the Parameterized Special Theory of Relativity (PSTR). Its equation is:

$$
x=\sqrt{v^{2}+c^{2}\left(\frac{\Delta t^{\prime}}{\Delta t}\right)^{2}}
$$

where $x$ is the speed of the photon as measured by the observer on earth, $\Delta t^{\prime}$ is the elapsed time as measured by the astronaut, and $\Delta t$ the elapsed time as measured by the observer on earth.

And the PSTR's parameter is

$$
\tau=\frac{\Delta t^{\prime}}{\Delta t}
$$

where $\tau_{\epsilon(0,+\infty)}$.
Our PSTR generalizes not only Einstein's STR, but also the previous ATR, and three other possible Relativities [3,4, and 5 from below] that readers can study, as follows:

1. If $\tau=\sqrt{1-\frac{v^{2}}{c^{2}}}$ we get the Special Theory of Relativity.
2. If $\tau=1$, we get our Absolute Theory of Relativity in the particular case when the two trajectory vectors are perpendicular.
3. If $0<\tau<\sqrt{1-\frac{v^{2}}{c^{2}}}$, the time dilation is increased with respect to that of the STR, therefore the speed $x$ as seen by the observer on earth is decreased (becomes subluminal) while in STR it is $c$.
4. If $\sqrt{1-\frac{v^{2}}{c^{2}}}<\tau<1_{\text {there }}$ is still time dilation, but less than the time dilation of the STR, but the speed $x$ as seen by the observer on earth becomes superluminal (yet less than in our Absolute Theory of Relativity).
5. If $\tau>1$, we get an opposite time dilation (i.e. $\Delta t^{\prime}>\Delta t$ ) with respect to the STR (instead of $\Delta t^{\prime}<\Delta t$ ), and the speed $x$ as seen by the observer on earth increases even more than in our ATR.

Then the above definitions and classification of Relativities are extended in the $S_{K}$ space (space where the ultimate speed if $K$ ) to $K$-Relativities.

In Chapter 5, we repeat Einstein's thought experiment introducing the acceleration. We consider the particle in the rocket moving at constant acceleration, while the rocket moving either at constant velocity or at a constant acceleration. The trajectory vectors of the particle and the rocket are either orthogonal or oblique to each other.

We used only linear trajectories for both observers, but at the end we propose, as research problems, the nonlinear arbitrary $3 D$-curves with nonconstant accelerations.

Chapter 6 is the most general, yet to be investigated.

Noninertial Multirelativity means that the particle $P_{0}$ in the reference frame $F_{1}$ travels on an arbitrary $3 D$-curve $C_{0}$ with a non-constant acceleration $a_{0}$ with initial velocity $v_{0}$. Then the reference frame $F_{l}$ moves with respect to another reference frame $F_{2}$ on an arbitrary $3 D$ curve $C_{l}$ with a non-constant acceleration $a_{l}$ with initial velocity $v_{l}$. And so on, the reference frame $F_{n-1}$ moves with respect to another reference frame $F_{n}$ (for $n \geq 2$ ) on an arbitrary $3 D$-curve $C_{n-1}$ with a non-constant acceleration $a_{n-1}$ with initial velocity $v_{n-1}$.

The following research problems are proposed with respect to the Noninertial Multirelativity:

1. How would the particle's trajectory curve $C_{0}$ be seen by an observer in the reference frame $F_{n}$ ?
2. What would be the particle's speed (acceleration) as measured by the observer from the reference frame $F_{n}$ ?
3. What would be the elapsed time of the particle as seen by the observer in the reference frame $F_{n}$ ?
4. What are the transformation equations from a reference frame to another?
5. Similar questions for rotating reference systems.

The author

## Chapter 1.

On Einstein's Thought Experiment with the Light Clocks

### 1.1. Einstein's Thought Experiment with the Light Clocks.

Let's consider the Einstein's thought experiment with the light clocks.

There are two identical clocks, one is placed aboard of a rocket, which travels at a constant speed $v$ relative to the earth, and the second one is on earth. In the rocket, a light pulse is emitted by a source from $A$ to a mirror $B$ that reflects it back to $A$ where it is detected. The rocket's movement and the light pulse's movement are orthogonal. There is an observer in the rocket (the astronaut) and an observer on the earth. The trajectory of light pulse (and implicitly the distance traveled by the light pulse), the elapsed time it needs to travel this distance, and the speed of the light pulse at which is travels are perceived differently by the two observers \{depending on the theories used see below in this book $\}$.

According to the astronaut:
Fig. 1


$$
\begin{equation*}
\Delta t^{\prime}=\frac{2 d}{c} \tag{1}
\end{equation*}
$$

where:
$\Delta t^{\prime}=$ time interval, as measured by the astronaut, for the light to follow the path of distance $2 d$;
$d=$ distance;
$c=$ speed of light.
According to the observer on earth:
Fig. 2


$$
\begin{align*}
2 l & =v \cdot \Delta t \\
s & =|A B|=\left|B A^{\prime}\right| \\
d & =\left|B B^{\prime}\right|  \tag{2}\\
l & =\left|A B^{\prime}\right|=\left|B^{\prime} A^{\prime}\right|
\end{align*}
$$

where $\Delta t=$ time interval as measured by the observer on earth.

And using the Pythagoras' Theorem in the right triangle $\triangle A B B^{\prime}$, one has

$$
\begin{equation*}
2 s=2 \sqrt{d^{2}+l^{2}}=2 \sqrt{d^{2}+\left(\frac{v \cdot \Delta t}{2}\right)^{2}} \tag{3}
\end{equation*}
$$

but $2 s=c \cdot \Delta t$, whence

$$
\begin{equation*}
c \cdot \Delta t=2 \sqrt{d^{2}+\left(\frac{v \cdot \Delta t}{2}\right)^{2}} \tag{4}
\end{equation*}
$$

Squaring and computing for $\Delta t$ one gets:

$$
\begin{equation*}
\Delta t=\frac{2 d}{c} \cdot \frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} . \tag{5}
\end{equation*}
$$

Whence Einstein gets the following time dilation:

$$
\begin{equation*}
\Delta t=\frac{\Delta t^{\prime}}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \tag{6}
\end{equation*}
$$

where $\Delta t>\Delta t^{\prime}$.

### 1.2. Relativistic Addition of Velocities.

According to the Relativistic Addition of velocities:

$$
\begin{equation*}
v=\frac{v_{1}+v_{2}}{1+\frac{v_{1} v_{2}}{c^{2}}} \tag{7}
\end{equation*}
$$

where:
$v_{1}=$ velocity of the particle inside the rocket as measured by the astronaut;
$v_{2}=$ velocity of the rocket as measured by the observer on earth;
$v=$ velocity of the particle as measured by the observer on earth.

### 1.3. Counter-Intuitive Results of the Relativistic Addition of Velocities.

Some curious additions, in our opinion:

$$
\begin{equation*}
c-0.999 c=c \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
c-c=\frac{0}{0}=\text { undefined } . \tag{9}
\end{equation*}
$$

### 1.4. Contradictory Results of the Relativistic Addition of Velocities.

One has not only $c+v=c$ (for $v<c$ ), but also $c-v=c$, or $c+v=c-v$, so $v=-v$ ? (10)

$$
\begin{align*}
& \text { Also } c+c=c \text {, and in general } \\
& \underbrace{c+c+\ldots+c}_{n \text { times }}=c ; \text { therefore } n \cdot c=c \tag{11}
\end{align*}
$$

for any integer $n \geq 1$.
More general, $r \cdot c=c$ for any real number $r \geq 1$,
since if

$$
r=n+a
$$

where

$$
n=[r], \text { integer part of } r,
$$

and

$$
a=\{r\}, \text { the fractional part of } r
$$

then

$$
r \cdot c=(n+a) c=n c+a c=c+a c=c .
$$

but, having $r \cdot c=c$ for any real number $r \geq 1$ looks unrealistic.

If we compute

$$
\begin{equation*}
\lim _{n \rightarrow \infty}(n \cdot c)=\infty \cdot c=\infty, \tag{13}
\end{equation*}
$$

but on the other side, because $n \cdot c=c$, we have

$$
\begin{equation*}
\lim _{n \rightarrow \infty}(n \cdot c)=\lim _{n \rightarrow \infty}(c)=c, \tag{14}
\end{equation*}
$$

which is a contradiction.
Similarly, if $0<v<c$, then

$$
\begin{align*}
c+n \cdot v & =c+\underbrace{v+v+\ldots+v}_{n \text { times }}=(c+v)+\underbrace{v+\ldots+v}_{n-1 \text { times }}= \\
& =c+\underbrace{v+\ldots+v}_{n-1}=\ldots=c . \tag{15}
\end{align*}
$$

Then

$$
\begin{equation*}
\lim _{n \rightarrow \infty}(c+n \cdot v)=c+\infty \cdot v=\infty \tag{16}
\end{equation*}
$$

but, also

$$
\begin{equation*}
\lim _{n \rightarrow \infty}(c+n \cdot v)=c=c, \tag{17}
\end{equation*}
$$

therefore, again, a contradiction.

### 1.5. Relativistic Addition of Superluminal Velocities.

Of course, for superluminal velocities Einstein's Relativistic Addition of velocities does not work. See the following counterexample:

$$
\begin{align*}
& \text { If } v_{1}=2 c \text { and } v_{2}=3 c \text {, then: } \\
& v_{1} \oplus v_{2}=\frac{2 c+3 c}{1+\frac{(2 c)(3 c)}{c^{2}}}=\frac{5 c}{1+6}=\frac{5}{7} c<c \tag{18}
\end{align*}
$$

but it does not make sense to adding superluminal speeds (in the same linear direction) and the result be subluminal.

Actually, the Relativistic Addition of Velocities for any $v_{1}, v_{2}>c$ gives:

$$
\begin{equation*}
\frac{v_{1}+v_{2}}{1+\frac{v_{1} v_{2}}{c^{2}}}<c \tag{19}
\end{equation*}
$$

while for $0<v_{1}<c<v_{2}$ gives

$$
\frac{v_{1}+v_{2}}{1+\frac{v_{1} v_{2}}{c^{2}}}<v_{2}
$$

which doesn't make sense either.
For example:
$0.2 c \oplus 3 c=\frac{0.2 c+3 c}{1+\frac{(0.2 c)(3 c)}{c^{2}}}=\frac{3.2 c}{1+0.6}=2 c<3 c$

Therefore a new formula is needed for addition of superluminal velocities.
1.6. Extension of Relativistic Formula for the Addition of Many Subluminal Velocities.

Let's use the notation $v_{1} \oplus v_{2}$ in order to denote the relativistic addition of the speeds,
and $v_{1}+v_{2}$ in order to denote the Newtonian (classical) addition of speeds. According to Einstein, we have:

$$
\begin{equation*}
v_{1} \oplus v_{2}=\frac{v_{1}+v_{2}}{1+\frac{v_{1} v_{2}}{c^{2}}}=\frac{\frac{s_{1}^{2}}{c^{0}}}{s_{0}^{2}+\frac{s_{2}^{2}}{c^{2}}} \tag{22}
\end{equation*}
$$

where

$$
\begin{align*}
& S_{1}^{2}=v_{1}+v_{2}  \tag{23}\\
& S_{2}^{2}=v_{1} v_{2}  \tag{24}\\
& S_{0}^{2}=1 \tag{25}
\end{align*}
$$

(where the superscript means the number of speeds, i.e. $n=2$ ).

We extend the relativistic addition of speeds for three or more speeds following Einstein's formula.

Therefore, for $n=3$, we get:

$$
\begin{equation*}
v_{1} \oplus v_{2} \oplus v_{3}=\frac{\frac{s_{1}^{3}}{c^{0}}+\frac{s_{3}^{3}}{c^{2}}}{\frac{s_{0}^{3}}{c^{0}}+\frac{s_{2}^{3}}{c^{2}}} \tag{26}
\end{equation*}
$$

Let's prove it by simple algebraic calculation:

$$
\begin{aligned}
& v_{1} \oplus v_{2} \oplus v_{3}=\left(v_{1} \oplus v_{2}\right) \oplus v_{3}= \\
& =\frac{\frac{v_{1}+v_{2}}{1+\frac{v_{1} v_{2}}{c^{2}}}+v_{3}=\frac{c^{2}\left(v_{1}+v_{2}\right)}{c^{2}+v_{1} v_{2}}+v_{3}}{\frac{v_{1}+v_{2}}{1+\frac{v_{1} v_{2}}{c^{2}}}=v_{3}} \frac{\frac{c^{2}\left(v_{1}+v_{2}\right) \cdot v_{3}}{c^{2}+v_{1} v_{2}}}{1+\frac{c^{2}}{1}} \\
& =\frac{\frac{c^{2}\left(v_{1}+v_{2}+v_{3}\right)+v_{1} v_{2} v_{3}}{c^{2}+v_{1} v_{2}} \cdot\left(c^{2}+v_{1} v_{2}\right)}{\left(c^{2}+v_{1} v_{2}\right)+\frac{\left(v_{1}+v_{2}\right) v_{3}}{c^{2}+v_{1} v_{2}} \cdot\left(e^{2}+v_{1} v_{2}\right)}=
\end{aligned}
$$

$$
\begin{align*}
& =\frac{c^{2}\left(v_{1}+v_{2}+v_{3}\right)+v_{1} v_{2} v_{3}}{c^{2}+v_{1} v_{2}+\left(v_{1}+v_{2}\right) v_{3}} \\
& =\frac{c^{2}\left(v_{1}+v_{2}+v_{3}\right)+v_{1} v_{2} v_{3}}{c^{2}+v_{1} v_{2}+v_{2} v_{3}+v_{1} v_{3}} \\
& =\frac{v_{1}+v_{2}+v_{3}+\frac{v_{1} v_{2} v_{3}}{c^{2}}}{1+\frac{v_{1} v_{2}+v_{2} v_{3}+v_{1} v_{3}}{c^{2}}}=\frac{\frac{s_{1}^{3}}{c^{0}}+\frac{s_{3}^{3}}{c^{2}}}{\frac{s_{0}^{3}}{c^{0}}+\frac{s_{2}^{3}}{c^{2}}} \tag{27}
\end{align*}
$$

Similarly, we get:

$$
\begin{equation*}
v_{1} \oplus v_{2} \oplus v_{3} \oplus v_{4}=\frac{\frac{s_{1}^{4}}{c^{0}}+\frac{s_{3}^{4}}{c^{2}}}{\frac{s_{0}^{4}}{c^{0}}+\frac{s_{2}^{4}}{c^{2}}+\frac{s_{4}^{4}}{c^{4}}} \tag{28}
\end{equation*}
$$

and

$$
v_{1} \oplus v_{2} \oplus v_{3} \oplus v_{4} \oplus v_{5}=\frac{\frac{s_{1}^{5}}{c^{0}}+\frac{s_{3}^{5}}{c^{2}}+\frac{s_{5}^{5}}{c^{4}}}{\frac{s_{0}^{5}}{c^{0}}+\frac{s_{2}^{5}}{c^{2}}+\frac{s_{4}^{5}}{c^{4}}}
$$

(29)

Let $S_{0}^{n} \stackrel{\text { def }}{=} 1$, for all integers $n \geq 2$, then

$$
\begin{equation*}
v_{1} \oplus v_{2} \oplus \ldots \oplus v_{n}=\frac{\sum_{i=0}^{\left.\frac{n-1}{2}\right]} \frac{S_{2 i+1}^{n}}{c^{2 i}}}{\left[\frac{n}{2}\right]} \sum_{i=0}^{c^{2 i}} \frac{s_{2 i}^{n}}{c^{2 i}} \tag{30}
\end{equation*}
$$

where for $0 \leq j \leq n$ we have

$$
\begin{align*}
& S_{j}^{n}=\sum_{\left(m_{1}, m_{2}, \ldots, m_{j}\right) \in \ominus_{n}^{j}} v_{m_{1}} \cdot v_{m_{2}} \cdot \ldots \cdot v_{m_{j}} \\
& =\sum_{1 \leq m_{1}<m_{2}<\ldots<m_{j} \leq n} v_{m_{1}} \cdot v_{m_{2}} \cdot \ldots \cdot v_{m_{j}} \tag{31}
\end{align*}
$$

with $\ominus_{n}^{j}$ being all the combinations of the elements $\{1,2, \ldots, n\}$ taken by groups of $j$ elements.

We note, for easier computation:

$$
\begin{equation*}
\alpha=\sum_{i=0}^{\left[\frac{n-1}{2}\right]} \frac{s_{2 i+1}^{n}}{c^{2 i}} \text { and } \beta=\sum_{i=0}^{\left[\frac{n}{2}\right]} \frac{s_{2 i}^{n}}{c^{2 i}} \tag{32}
\end{equation*}
$$

We prove it by induction upon $n \geq 2$.
For $n=2$ we get Einstein's relativistic addition formula.

Let's suppose that this general addition formula is true for all $h, 2 \leq h \leq n$. We need to prove it for $\mathrm{n}+1$.

$$
\begin{gather*}
v_{1} \oplus v_{2} \oplus \ldots \oplus v_{n} \oplus v_{n+1}=\left(v_{1} \oplus v_{2} \oplus \ldots \oplus v_{n}\right) \oplus v_{n+1} \\
=\frac{\alpha}{\beta} \oplus v_{n+1}=\frac{\frac{\alpha}{\beta}+v_{n+1}}{\frac{\alpha}{\beta} \cdot v_{n+1}}=\frac{\frac{\alpha+\beta \cdot v_{n+1}}{\beta}}{1+\frac{\beta}{c^{2}}}= \\
1+\frac{\frac{\alpha \cdot v_{n+1}}{\beta}}{\frac{c^{2}}{1}}  \tag{33}\\
=\frac{\frac{\alpha+\beta \cdot v_{n+1}}{\beta} \cdot \beta}{\beta \frac{1}{1}+\frac{\alpha \cdot v_{n+1}}{\beta c^{2}} \cdot \beta}=\frac{\alpha+\beta \cdot v_{n+1}}{\beta+\frac{\alpha \cdot v_{n+1}}{c^{2}}}
\end{gather*}
$$

Now, the numerator:

$$
\begin{aligned}
& \alpha+\beta \cdot v_{n+1}=\sum_{i=0}^{\left[\frac{n-1}{2}\right]} \frac{S_{2 i+1}^{n}}{c^{2 i}}+\sum_{i=0}^{\left[\frac{n}{2}\right]} \frac{S_{2 i}^{n}}{c^{2 i}} \cdot v_{n+1}= \\
& =\left(\frac{s_{1}^{n}}{c^{0}}+\frac{s_{3}^{n}}{c^{2}}+\frac{S_{5}^{n}}{c^{4}}+\ldots\right)+\left(\frac{s_{0}^{n}}{c^{0}} \cdot v_{n+1}+\frac{s_{2}^{n}}{c^{2}} \cdot v_{n+1}+\frac{s_{4}^{n}}{c^{4}} \cdot v_{n+1}+\ldots\right)=
\end{aligned}
$$

$$
\begin{align*}
& =\left(\frac{s_{1}^{n}}{c^{0}}+\frac{s_{0}^{n}}{c^{0}} \cdot v_{n+1}\right)+\left(\frac{s_{3}^{n}}{c^{2}}+\frac{s_{2}^{n}}{c^{2}} \cdot v_{n+1}\right)+\left(\frac{s_{5}^{n}}{c^{4}}+\frac{s_{4}^{n}}{c^{4}} \cdot v_{n+1}\right)+\ldots= \\
& =\frac{s_{1}^{n+1}}{c^{0}}+\frac{s_{3}^{n+1}}{c^{2}}+\frac{s_{5}^{n+1}}{c^{4}}+\ldots=\sum_{i=0}^{\left[\frac{n}{2}\right]} \frac{s_{2 i+1}^{n+1}}{c^{2 i}} \tag{34}
\end{align*}
$$

For the denominator

$$
\begin{align*}
& \beta+\frac{\alpha \cdot v_{n+1}}{c^{2}}=\sum_{i=0}^{\left[\frac{n}{2}\right.} \frac{s_{2 i}^{n}}{c^{2 i}}+\sum_{i=0}^{\left[\frac{n-1}{2}\right]} \frac{s_{2 i+1}^{n}}{c^{2 i}} \cdot \frac{v_{n+1}}{c^{2}}= \\
& =\left(\frac{s_{0}^{n}}{c^{0}}+\frac{s_{2}^{n}}{c^{2}}+\frac{s_{4}^{n}}{c^{4}}+\ldots\right)+\left(\frac{s_{1}^{n}}{c^{n}} \cdot \frac{v_{n+1}}{c^{2}}+\frac{s_{3}^{n}}{c^{2}} \cdot \frac{v_{n+1}}{c^{2}}+\frac{s_{5}^{n}}{c^{4}} \cdot \frac{v_{n+1}}{c^{2}}+\ldots\right)= \\
& =\left(\frac{s_{0}^{n}}{c^{0}}\right)+\left(\frac{s_{2}^{n}}{c^{2}}+\frac{s_{1}^{n}}{c^{0}} \cdot \frac{v_{n+1}}{c^{2}}\right)+\left(\frac{s_{4}^{n}}{c^{4}}+\frac{s_{3}^{n}}{c^{2}} \cdot \frac{v_{n+1}}{c^{2}}\right)+\ldots= \\
& \left.=\frac{s_{1}^{n+1}}{c^{0}}+\frac{s_{2}^{n+1}}{c^{2}}+\frac{s_{4}^{n+1}}{c^{4}}+\ldots=\sum_{i=0}^{2}\right] \frac{s_{2 i}^{n+1}}{c^{2 i}} \tag{35}
\end{align*}
$$

and the formula is proved.

### 1.7. Generalization of Einstein's Thought Experiment with the Light Clocks for Arbitrary Ultimate Velocity K.

We change Einstein's thought experiment since $c$ can be replaced by another speed, $K$, which can be smaller or greater than c, but consider the ultimate speed in a given space $S_{K}$, and repeat Einstein's experiment. Therefore:

$$
\begin{equation*}
\frac{2 d}{K}=\Delta t^{\prime} \tag{36}
\end{equation*}
$$

which is the proper time interval.
For the non-proper time interval we have the same calculations as in the Theory of Relativity:

$$
\begin{equation*}
2 s=2 \sqrt{d^{2}+l^{2}}=2 \sqrt{d^{2}+\left(\frac{v \cdot \Delta t}{2}\right)^{2}} \tag{37}
\end{equation*}
$$

But $2 s=K . \Delta t$, therefore:

$$
\begin{equation*}
K . \Delta t=2 \sqrt{d^{2}+\frac{v^{2}(\Delta t)^{2}}{4}} \tag{38}
\end{equation*}
$$

Raise to the second power both sides:

$$
\begin{align*}
& K^{2} \cdot(\Delta t)^{2}=4 d^{2}+v^{2} \cdot(\Delta t)^{2}  \tag{39}\\
& (\Delta t)^{2}\left(K^{2}-v^{2}\right)=4 D^{2}, \text { whence } K>v \tag{40}
\end{align*}
$$

Or:

$$
\begin{equation*}
\Delta t=\frac{2 d}{\sqrt{K^{2}-v^{2}}}=\frac{2 d}{K \sqrt{1-\frac{v^{2}}{K^{2}}}} \tag{41}
\end{equation*}
$$

where $K$ should be strictly greater than $v$.

Whence:

$$
\begin{equation*}
\Delta t=\frac{\Delta t^{\prime}}{\sqrt{1-\frac{v^{2}}{K^{2}}}} \tag{42}
\end{equation*}
$$

where $K>v$.

Therefore now the relativistic factor which makes the so-called time dilation depends on $v$ and $K$.

### 1.8. Generalization of the Relativistic Addition Formula for Arbitrary Ultimate Velocity K.

Actually, we can generalize Einstein's formula of relativistic addition of velocities in the following way:

Let's consider a constant $0<K<\infty$ and a physical imaginary space $\mathrm{S}_{\mathrm{K}}$, where the ultimate speed is $K$.
Postulating, similarly to Einstein, that no velocity overpass $K$ in a space denoted $S_{K}$, we define just by substituting " $c$ " by" $K$ " the addition velocities for an observer on Earth:

$$
\begin{equation*}
v=\frac{v_{1}+v_{2}}{1+\frac{v_{1} v_{2}}{K^{2}}} \tag{43}
\end{equation*}
$$

Then, same properties occur:

$$
\begin{equation*}
K+v=K \text { for } v \leq K ; \tag{4}
\end{equation*}
$$

$\max \left\{v_{1}, v_{2}\right\}<v_{1} \oplus_{k} v_{2} \leq K$
for any $v_{1}, v_{2} \leq K$, with equality in the case when at least one of $v_{1}$ or $v_{2}$ is equal to $K$.

In this imaginary space $S_{K}$, the ultimate velocity $K$ can be subluminal, for example the speed of sound, or superluminal speed (for example ten times the speed of light).

Multiple addition formula of speeds in $S_{K}$ will have a similar formula:

$$
\begin{equation*}
v_{1} \oplus_{k} v_{2} \oplus_{k} \ldots \oplus_{k} v_{n}=\frac{\sum_{i=0}^{\left.\frac{n-1}{2}\right]} \frac{S_{2 i+1}^{n}}{K^{2 i}}}{\sum_{i=0}^{\left[\frac{n}{2}\right]} \frac{S_{2 i}^{n}}{K^{2 i}}} \tag{45}
\end{equation*}
$$

We prove it by induction upon $n \geq 2$ as we did before, simply substituting " $c$ "(the speed of light) by " $K$ " in the previous proof.

### 1.9. Generalized Lorentz Factor.

Lorentz factor will also be generalized to

$$
\begin{equation*}
\frac{1}{\sqrt{1-\frac{v^{2}}{K^{2}}}} \in[1,+\infty) \tag{46}
\end{equation*}
$$

### 1.10. Generalized Minkowski Norm.

The norm of $(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})$ in the Minkowski spacetime also becomes:

$$
\begin{equation*}
\sqrt{x^{2}+y^{2}+z^{2}-K^{2} t^{2}} \tag{47}
\end{equation*}
$$

### 1.11. Generalized Time Dilation

 becomes:$$
\begin{equation*}
\Delta t=\frac{\Delta t^{\prime}}{\sqrt{1-\frac{v^{2}}{K^{2}}}} . \tag{48}
\end{equation*}
$$

### 1.12. Generalized Length Contraction

 becomes:$$
\begin{equation*}
\ell=\ell^{\prime} \cdot \sqrt{1-\frac{v^{2}}{K^{2}}} . \tag{49}
\end{equation*}
$$

1.13. Generalized Relativistic

Momentum of an object of mass $m$, moving with speed $v$, becomes:

$$
\begin{equation*}
p=\frac{m v}{\sqrt{1-\frac{v^{2}}{K^{2}}}} . \tag{50}
\end{equation*}
$$

1.14. Generalized Energy of an object at rest, with rest mass m , is $E_{0}=m K^{2}$.

### 1.15. The Generalized Total Energy of

 an object of mass m , moving at speed v , becomes:$$
\begin{equation*}
E=\frac{m K^{2}}{\sqrt{1-\frac{v^{2}}{K^{2}}}} . \tag{52}
\end{equation*}
$$

### 1.16. Generalized Kinetic Energy of

 an object of mass $m$, moving at speed $v$, becomes:$$
\begin{equation*}
E_{k}=m K^{2}\left(\frac{1}{\sqrt{1-\frac{v^{2}}{K^{2}}}}-1\right) \tag{53}
\end{equation*}
$$

## Chapter 2.

A Hypothesis:
There is No Speed Barrier in the Universe
and
One Can Construct Arbitrary Speeds

### 2.1. Introduction.

In this short chapter, as an extension and consequence of Einstein-Podolski-Rosen paradox and Bell's inequality, one promotes the hypothesis that: There is no speed barrier in the universe and one can construct any speed, from zero to infinite speed (instantaneous transmission).

Future research: to study the composition of faster-than-light velocities and what happens with the laws of physics at faster-than-light velocities?

### 2.2. Entangled Particles.

We recall the following:

- photon is a bit of light, the quantum of electromagnetic radiation (quantum is the smallest amount of energy that a system can gain or lose);
- polarization refers to the direction and characteristics of the light wave vibration;
- if one uses the entanglement phenomenon, in order to transfer the polarization between two photons, then: whatever happens to one is the opposite of what happens to the other; hence, their polarizations are opposite of each other;
- in quantum mechanics, objects such as subatomic particles do not have specific, fixed characteristic at any given instant in time until they are measured;
- suppose a certain physical process produces a pair of entangled particles $A$ and $B$ (having opposite or complementary characteristics), which fly off into space in the opposite direction and, when they are billions of miles apart, one measures particle $A$; because $B$ is the opposite, the act of measuring $A$ instantaneously tells $B$ what to be; therefore those instructions would somehow have to travel between $A$ and $B$ faster than the speed of light; hence, one can extend the Einstein-Podolsky-Rosen paradox and Bell's inequality and assert that the light speed is not a speed barrier in the universe;
- even more, one can construct any speed, even greater than the speed of light (c), by measuring particle $A$ at various intervals;
- also, the information from particle $A$ and $B$ is transmitted instantaneously (thus, there is no speed barrier in the universe).


### 2.3. Scientific Hypothesis.

We promote the hypothesis that: there is no speed barrier in the universe and one can construct any speed even infinite (instantaneous transmission), which would be theoretically proven by increasing, in the previous example, the distance between particles $A$ and $B$ as much as the universe allows it, and then measuring particle $A$.

We consider that the superluminal phenomena do not violate the causality principle, do not produce time traveling, and do not necessitate infinite energy for particles traveling at speeds greater than the speed of light.

### 2.4. Open Question.

Wouldn't it be possible to accelerate a photon (or another particle traveling at, let's say, $0.999 c$ ) and thus to get a speed greater than $c$ (where $c$ is the speed of light)? We don't think it is needed an infinite energy for this.

### 2.5. Future Possible Research.

It would be interesting to study the composition of two velocities $v$ and $w$ in the cases when:

$$
\begin{align*}
& v<c \text { and } w=c .  \tag{54}\\
& v=c \text { and } w=c .  \tag{55}\\
& v>c \text { and } w=c .  \tag{5}\\
& v>c \text { and } w>c .  \tag{57}\\
& v<c \text { and } w=\infty .  \tag{58}\\
& v=c \text { and } w=\infty .  \tag{59}\\
& v>c \text { and } w=\infty . \tag{60}
\end{align*}
$$

$$
\begin{equation*}
v=\infty \text { and } w=\infty . \tag{61}
\end{equation*}
$$

What happens with the laws of physics in each of these cases?

## Chapter 3.

Absolute Theory of Relativity (ATR)

### 3.1. Refuting Einstein's Speed of Light Postulate.

Again, we do the opposite of what Einstein did. Instead of considering the speeds of the clock light the same for both observers, while the time intervals different, we consider the time intervals are the same for both observers, while the speeds $c$ and respectively $v+c$ not equal. We refute Einstein's speed of light postulate according to our hypothesis that there is no speed limit in the universe and one can construct arbitrary speeds.

The classical formula

$$
\begin{equation*}
\text { Distance }=\text { Speed } \times \text { Time } \tag{62}
\end{equation*}
$$

was distorted in the Special Theory of Relativity in the following way: Time was increased (dilated), while Distance was decreased (contracted).

In order to still keep the validity of this formula as Einstein did, the Speed had to be extremely decreased in order to compensate both the increment of Time and the decrement of Distance. The Speed was automatically
decreased by the fact that it was not allowed to overpass the speed of light - this was the flaw of Relativity.

In our opinion $v+c$ should be strictly greater than $c$ for the observer on earth, since to the speed of light it is added the speed of the rocket.

Fig. 3


We think the time intervals are the same for both observers in accordance with the common experience: an event, which occurs in an inertial reference frame (in this case: the observer in the rocket) and has a time interval $\Delta t$, lasts the same time interval $\Delta t$ if it is regarded from another inertial reference frame (in this case: the observed on earth); we use the real (absolute) time interval, not the apparent time interval.

We agree with Einstein that the trajectories of the clock light are different for the observers, i.e.

Fig. 4
for the observer in the rocket,
and
Fig. 5
for the observer on earth, and we also agree with the mathematics used in the Special Theory of Relativity to compute their length.

In our opinion there is neither a real time dilation nor a real length contraction, but an apparent time dilation and an apparent length contraction. Surely, we can consider in a metaphoric way that: time passes faster when we enjoy it, and slower when we endure it (for example in prison).

Or, under certain environmental conditions our biological or psychological processes could run faster or slower. We have moments when we can age more or less than normal. Therefore, our interior clock does not run constantly. Biologically, it is a chance that the more active you are (i.e. moving fast), the less you age (because the brain is more active) so "time dilation," but with respect to the absolute time you have the same age as somebody less active but simultaneously born with you.

However, time dilation could produce nice science fiction stories, but it is not fact.

Einstein did not prove that the speed of light cannot be surpassed, he only postulated it. Therefore we have the right to question this. He did a thought not lab experiment. We mean we don't believe that $v+c=c$ for the observer on earth as Einstein asserted, but we think that $v+c>c$ for $0<v \leq c$. We prove below that there is no anomaly alike "time dilation", but the speeds are different: for the observer in the rocket the speed of the clock light is c , while for the observer on earth the speed of the clock light is $c+v$, which should be greater than $c$ in order to avoid time dilation anomaly.

Let's note by $x$ the speed of the clock light as seen by the observer on earth. We compute it mathematically:

$$
\begin{equation*}
2 l=v \Delta t \tag{63}
\end{equation*}
$$

and:

$$
\begin{equation*}
2 s=2 \sqrt{d^{2}+l^{2}}=2 \sqrt{d^{2}+\left(\frac{v \Delta t}{2}\right)^{2}}=x . \Delta t . \tag{64}
\end{equation*}
$$

We need to solve for $x$ the last equality:

$$
\begin{align*}
& (x \Delta t)^{2}=\left[2 \sqrt{d^{2}+\frac{v^{2}(\Delta t)^{2}}{4}}\right]^{2}  \tag{65}\\
& x^{2}(\Delta t)^{2}=4 d^{2}+v^{2}(\Delta t)^{2} . \tag{66}
\end{align*}
$$

Dividing both sides with $(\Delta t)^{2}$, we get:

$$
\begin{equation*}
x^{2}=\left(\frac{2 d}{\Delta t}\right)^{2}+v^{2} . \tag{67}
\end{equation*}
$$

We know for the observer in the rocket, that $\frac{2 d}{c}=\Delta t$, and thus $\frac{2 d}{\Delta t}=c$. Therefore:

$$
\begin{equation*}
x^{2}=c^{2}+v^{2} \tag{68}
\end{equation*}
$$

Whence the speed of photon in the rocket, with respect to the observer on earth, is:

$$
\begin{equation*}
x=\sqrt{v^{2}+c^{2}}>c \tag{69}
\end{equation*}
$$

which corresponds to the magnitude of the vectorial addition, i.e. $x=|\vec{v}+\vec{c}|$, since $\vec{c}$ and $\vec{v}$ are orthogonal:


Fig. 6

### 3.2. About Inconsistent Theories.

When a theory is based on a set of axioms (or propositions) and one or more axioms are inconsistent, the theory produces as consequences new inconsistencies, and so on... Therefore, an anomaly gives birth to other anomalies...

Reciprocally, if an inconsistent theory results from some assumptions or propositions, then those propositions on the whole must be inconsistent too.
a) Therefore, the Michelson-Morley Experiment (1881, 1883-1887) of not having detected the ether, which implied the STR, might have been erroneous due to imprecise measurement instruments or
construction. Maybe the ether is very little dense and its flow hardly perceptible.
b) Similarly the constancy of the speed of light in vacuum could be flaw, since the speed of light might depend on the emitting source - the stronger the emitting source the faster its light pulse.

### 3.3. No Relativistic Paradoxes in ATR.

Since in Absolute Theory of Relativity there is no time dilation, in consequence there is no length contraction, and no relative simultaneity.

Therefore, many relativistic paradoxes are discarded:
a) Ehrenfest Paradox (1909) - since there is no length contraction.
b) Twin Paradox (1911) - since there is no time dilation and no gravitational time dilation.
c) Bell's Spaceship Paradox (1959) since there is no length contraction.
d) W. Rindler's Paradox about a man falling into a grate (1961) - since there is no length contraction.
Etc.

### 3.4. Removing Lorentz Factor in ATR.

Lorentz Factor
$\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$
becomes equal to 1 in ATR, because in the equality

$$
\begin{equation*}
\Delta t=\frac{\Delta t^{\prime}}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \tag{71}
\end{equation*}
$$

we replace
$\Delta t=\Delta t^{\prime}$
Therefore Lorentz Factor has no effect in our ATR.
As a consequence in ATR we get:

- No time dilation, since $\Delta t=\frac{\Delta t}{1}$
- No Lorentz-FitzGerald Length Contraction, since $l=l \cdot 1$
- The Relativistic Momentum of an object of mass $m$, moving with speed $v$, becomes classical:

$$
\begin{equation*}
p=\frac{m v}{1} . \tag{74}
\end{equation*}
$$

- The Total Energy (upon Einstein) of an object of mass $m$, moving at speed $v$, becomes:

$$
\begin{equation*}
E=\frac{m c^{2}}{1} \tag{75}
\end{equation*}
$$

- The Rest Energy (upon Einstein) of an abject of mass $m$ is

$$
\begin{equation*}
E_{0}=m c^{2} \tag{76}
\end{equation*}
$$

- Whence we obtain the Kinetic Energy of an object of mass $m$, moving at speed $v$, becoming:

$$
\begin{equation*}
E_{k}=m c^{2}\left(\frac{1}{1}-1\right)=0 \tag{77}
\end{equation*}
$$

which doesn't make sense.
Therefore, in our opinion, the famous physics formula $E_{0}=m c^{2}$ is questionable. We understand that light is electromagnetic energy, but we don't understand why the energy of an object should depend on the light speed? We mean why on the speed?

### 3.5. Physics Laws might not be the same in all Inertial Systems.

If there exist superluminal velocities, there might be possible that not all physics laws are the same in all inertial systems.
As a counter example let's consider as physics law the Addition of Velocities in a given inertial system $S_{i}$. Let's take the collinear and in the same direction velocities $v_{l}=0.8 c$ and $v_{2}=0.9 c$.
If we add them in STR we get:
$v_{1}+\quad v_{2}=\frac{0.8 c+0.9 c}{1+\frac{0.8 c \cdot 0.9 c}{c^{2}}}=\frac{1.70 c}{1.72} \simeq 0.988372 c$
while in ART we simply get
$v_{1}+v_{2}=0.8 \mathrm{c}+0.9 \mathrm{c}=1.7 \mathrm{c}$.
The results are different.

### 3.6. Linear Trajectories for Both Observers.

We consider the case when the trajectories seen by both observers are linear.
3.6.1. Orthogonal Trajectory Vectors and Arbitrary Velocity K.

We can generalize this relationship, replacing " $c$ " by any speed $K>0$ such that $\vec{v}$ and $\vec{K}$ are orthogonal. Then:

$$
\begin{equation*}
x=\sqrt{v^{2}+K^{2}}=|\vec{v}+\vec{K}| . \tag{80}
\end{equation*}
$$

### 3.6.2. Non-orthogonal Trajectory

 Vectors and $c$ as Ultimate Velocity.Let's change Einstein's theoretical experiment, and consider $d$ making an angle $\theta$, $0 \leq \theta \leq \pi$, with the motion direction (rocket's). Similarly, as before:

$$
\begin{equation*}
\frac{2 d}{c}=\Delta t \quad \text { whence } \quad \frac{2 d}{\Delta t}=c \tag{81}
\end{equation*}
$$

Fig. $7 \quad C$


In the triangle $\triangle A O C$ we apply the Theorem of Cosine (which is a generalization of Pythagorean Theorem used in the Special Theory of Relativity):

$$
\begin{align*}
s_{1}^{2} & =l^{2}+d^{2}-2 l d \cdot \cos (\pi-\theta) \\
& =\left(\frac{v \Delta t}{2}\right)^{2}+d^{2}-2 \frac{v \Delta t}{2} d(-\cos (\theta) \\
& =v^{2} \frac{(\Delta t)^{2}}{4}+d^{2}+v \Delta t \cdot D \cdot \cos \theta \tag{82}
\end{align*}
$$

Similarly, in the triangle $\triangle O B C$ we apply the Theorem of Cosine, and we get:

$$
\begin{aligned}
s_{2}^{2} & =l^{2}+d^{2}-2 l d \cdot \cos \theta \\
& =v^{2} \frac{(\Delta t)^{2}}{4}+d^{2}-v \cdot d \cdot \Delta t \cdot c \cos (\theta)
\end{aligned}
$$

(83)

But $x \cdot \Delta t=s_{1}+s_{2}$, then:

$$
\begin{align*}
& x \Delta t=\sqrt{v^{2} \frac{(\Delta t)^{2}}{4}+d^{2}+v \cdot \Delta t \cdot d \cdot \cos \theta}+ \\
& \sqrt{v^{2} \frac{(\Delta t)^{2}}{4}+d^{2}-v \cdot \Delta t \cdot d \cdot \cos \theta} \tag{84}
\end{align*}
$$

Divide by $\Delta t$ :

$$
\begin{aligned}
& x=\sqrt{\frac{v^{2}}{4}+\left(\frac{d}{\Delta t}\right)^{2}+\frac{d}{\Delta t} \cdot v \cdot \cos \theta}+ \\
& \sqrt{\frac{v^{2}}{4}+\left(\frac{d}{\Delta t}\right)^{2}-\frac{d}{\Delta t} \cdot v \cdot \cos \theta}
\end{aligned}
$$

(85)

$$
\begin{align*}
& x=\sqrt{\frac{v^{2}}{4}+\frac{c^{2}}{4}+\frac{c}{2} \cdot v \cdot \cos \theta}+ \\
& \sqrt{\frac{v^{2}}{4}+\frac{c^{2}}{4}-\frac{c}{2} \cdot v \cdot \cos \theta}  \tag{86}\\
& x=\frac{1}{2} \sqrt{v^{2}+c^{2}+2 v c \cdot \cos \theta}+ \\
& \frac{1}{2} \sqrt{v^{2}+c^{2}-2 v c \cdot \cos \theta} \tag{87}
\end{align*}
$$

Distance $s_{1}$ is traveled with the speed $\sqrt{v^{2}+c^{2}+2 v c \cdot \cos \theta}$, while the distance $s_{2}$ is traveled with the speed $\sqrt{v^{2}+c^{2}-2 v c \cdot \cos \theta}$, each of them in the same time interval $\frac{\Delta t}{2}$.

If $\theta=\frac{\pi}{2}$ when $\vec{v}$ and $\vec{c}$ are perpendicular, then $x=\sqrt{v^{2}+c^{2}}$, therefore we get the same result as in our previous work [section 2.1].

If $\theta=0$, then $x=\frac{1}{2}|v+c|+\frac{1}{2}|v-c|$ which means that $\vec{v}$ and $\vec{c}$ are collinear. For $s_{l}$ the
speed is $v+c$ since $\vec{v}$ and $\vec{c}$ are in the same direction, while for $s_{2}$ the speed is $v-c$ since $\vec{v}$ and $\vec{c}$ are in opposite directions (like in Galilean Relativity).

$$
\text { If } \theta=\pi \text {, then } x=\frac{1}{2}|v-c|+\frac{1}{2}|v+c| \text { since }
$$

$\vec{v}$ and $\vec{c}$ are collinear, with opposite directions on $s_{1}$ and with the same direction on $s_{2}$ (again similarly as in Galilean Relativity).

### 3.6.3. Non-Orthogonal Trajectory

 Vectors and Arbitrary Ultimate Velocity.We can extend this thought experiment by substituting " $c$ " for any speed $K$ (negative, positive, or zero, which can be smaller or greater than $c$ ). Then the speed as measured by the observer on earth is:

$$
\begin{align*}
& x=\frac{1}{2} \sqrt{v^{2}+K^{2}+2 v K \cdot \cos \theta}+ \\
& \frac{1}{2} \sqrt{v^{2}+K^{2}-2 v K \cdot \cos \theta} \tag{88}
\end{align*}
$$

### 3.6.4. Non-Orthogonal Trajectory

 Vectors and Arbitrary Velocities.We can again generalize the previous thought experiment by substituting " $c$ " for any speed $w$ (negative, zero, or positive smaller or greater than $c$ ). Then, the speed as measured by the observer on earth is:

$$
\begin{align*}
& x=\frac{1}{2} \sqrt{v^{2}+w^{2}+2 v w \cdot \cos \theta}+ \\
& \frac{1}{2} \sqrt{v^{2}+w^{2}-2 v w \cdot \cos \theta} \tag{89}
\end{align*}
$$

## Chapter 4.

## Parameterized Special Theory of Relativity

 (PSTR)
### 4.1. PSTR Equation.

In a more general case when we don't know the speed $x$ nor the relationship between $\Delta t^{\prime}$ and $\Delta t$, we get:

$$
\begin{equation*}
x \Delta t=2 \sqrt{d^{2}+\left(\frac{v \Delta t}{2}\right)^{2}} \tag{90}
\end{equation*}
$$

But $d=\frac{c \Delta t^{\prime}}{2}$, therefore:

$$
\begin{equation*}
x \Delta t=2 \sqrt{\left(\frac{c \Delta t^{\prime}}{2}\right)^{2}+\left(\frac{v \Delta t}{2}\right)^{2}} . \tag{91}
\end{equation*}
$$

Or: $\quad x \Delta t=\sqrt{c^{2}\left(\Delta t^{\prime}\right)^{2}+v^{2}(\Delta t)^{2}}$.
Dividing the whole equality by $\Delta t$ we obtain:

$$
\begin{equation*}
x=\sqrt{v^{2}+c^{2}\left(\frac{\Delta t^{\prime}}{\Delta t}\right)^{2}} \tag{93}
\end{equation*}
$$

which is the PSTR Equation.

### 4.2. PSTR Elapsed Time Ratio $\tau$ (Parameter).

We now substitute in a general case

$$
\begin{equation*}
\frac{\Delta t^{\prime}}{\Delta t}=\tau \in(0,+\infty) \tag{94}
\end{equation*}
$$

where $\tau$ is the PSTR Elapsed Time Ratio.
therefore we get another extension of the Special Theory of Relativity (STR), i.e.:
4.3. PSTR Extends STR, ATR, and Introduces Three More Relativities.

1. If $\tau=\sqrt{1-\frac{v^{2}}{c^{2}}}$ we get the STR, since
replacing $x$ by $c$, one has

$$
\begin{aligned}
& c^{2}=v^{2}+c^{2}\left(\frac{\Delta t^{\prime}}{\Delta t}\right)^{2} \\
& \frac{c^{2}}{c^{2}}-\frac{v^{2}}{c^{2}}=\left(\frac{\Delta t^{\prime}}{\Delta t}\right)^{2}
\end{aligned}
$$

or

$$
\frac{\Delta t^{\prime}}{\Delta t}=\sqrt{1-\frac{v^{2}}{c^{2}}} \in[0,1] \text { as in the STR. }
$$

2. If $\tau=1$, we get our Absolute Theory of Relativity [see Chapter 3] in the particular case when the two trajectory vectors are perpendicular, i.e.

$$
x=\sqrt{v^{2}+c^{2}}=|\vec{v}+\vec{c}|
$$

3. If $0<\tau<\sqrt{1-\frac{v^{2}}{c^{2}}}$, the time dilation is increased with respect to that of the STR, therefore the speed $x$ as seen by the observer on earth is decreased (becomes subluminal) while in STR it is $c$.
4. If $\sqrt{1-\frac{v^{2}}{c^{2}}}<\tau<1$ there is still time dilation, but less than the time dilation of the STR, but the speed $x$ as seen by the observer on earth becomes superluminal (yet less than in our Absolute Theory of Relativity).
5. If $\tau>1$, we get an opposite time dilation (i.e. $\Delta t^{\prime}>\Delta t$ ) with respect to the STR (instead
of $\Delta t^{\prime}<\Delta t$ ), and the speed $x$ as seen by the observer on earth increases even more than in our ATR.

The reader might be interested in studying these new Relativities mathematically resulted from the above 3,4 , and 5 cases.

### 4.4. PSTR $_{K}$ Equation and Elapsed Time Ratio (Parameter) in $S_{K}$ Space

The PSTR $_{K}$ Equation becomes be a simple substitution of $c$ with $K$ :
$x=\sqrt{v^{2}+K^{2}\left(\frac{\Delta t^{\prime}}{\Delta t}\right)^{2}}$
while the PSTR $_{K}$ Elapsed Time Ratio (Parameter) is the same $\tau$ :

$$
\begin{equation*}
\frac{\Delta t^{\prime}}{\Delta t}=\tau \in(0,+\infty) \tag{96}
\end{equation*}
$$

4.5. PSTR $_{K}$ Extends $\mathbf{S T R}_{K}$, ATR $_{K}$, and Introduces Three More $\boldsymbol{K}$-Relativities
6. If $\tau=\sqrt{1-\frac{v^{2}}{K^{2}}}$ we get the $\operatorname{STR}_{\mathrm{K}}$ (Special Theory of Relativity with $K$ as Ultimate Speed).
7. If $\tau=1$, we get the ASTR $_{K}$ (Absolute Special Theory of Relativity as derived from $\mathrm{STR}_{K}$, although there is no distinction between ASTR and $\mathrm{ASTR}_{\mathrm{K}}$ ) in the particular case when the two trajectory vectors are perpendicular, i.e.

$$
\begin{equation*}
x=\sqrt{v^{2}+K^{2}}=|\vec{v}+\vec{K}| \tag{97}
\end{equation*}
$$

8. If $0<\tau<\sqrt{1-\frac{v^{2}}{K^{2}}}$, the time dilation is increased with respect to that of the $\mathrm{STR}_{\mathrm{K}}$, therefore the speed $x$ as seen by the observer on earth is decreased (becomes subluminal) while in $\mathrm{STR}_{\mathrm{K}}$ it is $c$.
9. If $\sqrt{1-\frac{v^{2}}{K^{2}}}<\tau<1_{\text {there }}$ is still time dilation, but less than the time dilation of the $\mathrm{STR}_{\mathrm{K}}$, but the speed $x$ as seen by the observer on earth becomes superluminal (yet less than in our Absolute Theory of Relativity).
10.If $\tau>1$, we get an opposite time dilation (i.e. $\Delta t^{\prime}>\Delta t$ ) with respect to the $\mathrm{STR}_{\mathrm{K}}$ (instead of $\Delta t^{\prime}<\Delta t$ ), and the speed $x$ as seen by the observer on earth increases even more than in our ATR $_{K}$.

The reader might be interested in studying these new $K$-Relativities mathematically resulted from the above 8,9 , and 10 cases (relativities in the $S_{K}$ space, where the ultimate speed is $K$ ).

## Chapter 5.

Accelerated Reference Frames.

### 5.1. Formulas of Kinematics for Constant Acceleration.

We use the following Formulas of Kinematics for Constant Acceleration:

$$
\begin{align*}
& v=v_{0}+a t  \tag{98}\\
& s=\frac{1}{2}\left(v_{0}+v\right) t  \tag{99}\\
& s=v_{0} t+\frac{1}{2} a t^{2}  \tag{100}\\
& v^{2}=v_{0}^{2}+2 a s \tag{101}
\end{align*}
$$

where:
$a=$ constant acceleration;
$v_{0}=$ initial velocity at time $t_{0}=0$;
$v=$ final velocity at time $t$;
$t=$ time elapsed since $t_{0}=0$;
$s=$ distance (displacement).

### 5.2. Gravitation and Constant Velocity of the Rocket.

Let's modify again Einstein's thought experiment. In the rocket let's consider a rock in a free fall on a distance $d$ :

| Fig. 8 | $A$ |
| :---: | :---: |
|  | $d$ |
|  |  |
|  | $B$ |$|$

Under the earth's acceleration:

$$
\begin{gathered}
a=g \\
v_{0}=0
\end{gathered}
$$

Then $s=v_{0} t+\frac{1}{2} a t^{2}$ becomes:

$$
\begin{align*}
d & =0 \cdot\left(\Delta t^{\prime}\right)+\frac{1}{2} g\left(\Delta t^{\prime}\right)^{2}, \\
\text { or } \quad d & =\frac{1}{2} g\left(\Delta t^{\prime}\right)^{2} . \tag{102}
\end{align*}
$$

The rocket is moving at a constant speed $v$.
For the observer on earth, the trajectory of the rock is $A B^{\prime}$ (not $A B$ as seen by the astronaut).


$$
\begin{equation*}
|A B|=d=\frac{1}{2} g\left(\Delta t^{\prime}\right)^{2} \tag{103}
\end{equation*}
$$

But the distance $B B^{\prime}$, traveled by the rocket in the elapsed time $\Delta t$, is:

$$
\begin{equation*}
\left|B B^{\prime}\right|=l=v \Delta t \tag{104}
\end{equation*}
$$

Using the Pythagorean Theorem in the triangle $\triangle A B B^{\prime}$ we get:

$$
\begin{aligned}
\left|A B^{\prime}\right| & =\sqrt{d^{2}+l^{2}} \\
& =\sqrt{\left[\frac{1}{2} g\left(\Delta t^{\prime}\right)^{2}\right]^{2}+(v \Delta t)^{2}}
\end{aligned}
$$

$$
\begin{gather*}
=\sqrt{\frac{g^{2}}{4}\left(\Delta t^{\prime}\right)^{4}+v^{2}(\Delta t)^{2}}  \tag{105}\\
\text { But } \sqrt{\frac{g^{2}}{4}\left(\Delta t^{\prime}\right)^{4}+v^{2}(\Delta t)^{2}}=0 \cdot \Delta t+\frac{1}{2} x_{a}(\Delta t)^{2}
\end{gather*}
$$

where $x_{a}=$ constant acceleration as seen by the observer on earth.

We get:

$$
\begin{equation*}
\sqrt{\frac{g^{2}}{4}\left(\Delta t^{\prime}\right)^{4}+v^{2}(\Delta t)^{2}}=\frac{1}{2} x_{a}(\Delta t)^{2} \tag{106}
\end{equation*}
$$ (92)

If we consider an absolute time as in our ATR, then we take $\Delta t^{\prime}=\Delta t$, and it results:

$$
\begin{equation*}
\frac{g^{2}}{4}(\Delta t)^{4}+v^{2}(\Delta t)^{2}=\frac{1}{4} x_{a}^{2}(\Delta t)^{4} \tag{107}
\end{equation*}
$$

Divide by $(\Delta t)^{4}$ :

$$
\frac{g^{2}}{4}+\left(\frac{v}{\Delta t}\right)^{2}=\frac{x_{a}{ }^{2}}{4}
$$

or

$$
\begin{equation*}
g^{2}+4\left(\frac{v}{\Delta t}\right)^{2}=x_{a}^{2} \tag{108}
\end{equation*}
$$

whence:

$$
\begin{equation*}
x_{a}=\sqrt{g^{2}+\left(\frac{2 v}{\Delta t}\right)^{2}}>g . \tag{109}
\end{equation*}
$$

The observer on earth perceives a greater constant acceleration of the rock than the astronaut does.

It is remarkable to know that $\frac{2 v}{\Delta t}$ is equal to the constant acceleration of a rocket that would start to move in the same moment when the rock starts to freely fall and whose final velocity at time $\Delta t$ (when the rock reaches the rocket floor) is $2 v$.

### 5.3. Gravitation with Constant Acceleration of the Rocket.

We consider the previous thought experiment: a rock in fall free in the rocket, but the rocket having an initial speed $v$, and at the moment when the rock begins to fall the rocket starts to accelerate with constant acceleration $a$.

Similarly: $|A B|=d=\frac{1}{2} g\left(\Delta t^{\prime}\right)^{2}$


The new trajectory $A B^{\prime \prime}$ is greater than the previous trajectory $A B^{\prime}$.

The distance $B B^{\prime \prime}$ is:

$$
\begin{align*}
& \left|B B^{\prime \prime}\right|=l=v \Delta t+\frac{a}{2}(\Delta t)^{2} .  \tag{110}\\
& \left|A B^{\prime \prime}\right|=\sqrt{d^{2}+l^{2}}
\end{align*}
$$

$$
=\sqrt{\left[\frac{1}{2} g\left(\Delta t^{\prime}\right)^{2}\right]^{2}+\left[v \Delta t+\frac{a}{2}(\Delta t)^{2}\right]^{2}}
$$

(111)

But supposing that $x_{a}$ is the constant acceleration of the rock travelling on the trajectory $A B^{\prime \prime}$, as seen by the observer on earth, and using the distance formula with respect to the acceleration, we also have:

$$
\begin{equation*}
\left|A B^{\prime \prime}\right|=0 \cdot \Delta t+\frac{1}{2} x_{a}(\Delta t)^{2}, \tag{112}
\end{equation*}
$$

whence:

$$
\begin{equation*}
\sqrt{\frac{1}{4} g^{2}\left(\Delta t^{\prime}\right)^{4}+\left[v+\frac{a}{2}(\Delta t)\right]^{2}(\Delta t)^{2}}=\frac{1}{2} x_{a}(\Delta t)^{2} \tag{113}
\end{equation*}
$$

Multiplying with $\frac{2}{(\Delta t)^{2}}$ we get:

$$
\begin{equation*}
x_{a}=\sqrt{g^{2}+\left(\frac{2 v}{\Delta t}+a\right)^{2}}>g \tag{114}
\end{equation*}
$$

and also $x_{a}>a$.
The observer on earth sees the rock falling with a constant acceleration greater than both the gravitation $g$ and the rocket's constant acceleration $a$.

Instead of a rocket, let's suppose a train with initial velocity $v=0$. Then

$$
\begin{equation*}
x_{a}=\sqrt{g^{2}+a^{2}} . \tag{115}
\end{equation*}
$$

### 5.4. Constant Accelerations and Oblique Direction Vectors.

In the rocket a particle travels from $A$ to $B$ with a constant acceleration $a_{1}$ starting at initial speed $v_{l}$ in the elapsed time $\Delta t_{l}$ with respect to the observer in the rocket.


Fig. 11

$$
\begin{equation*}
d=v_{1}\left(\Delta t_{1}\right)+\frac{1}{2} a_{1}\left(\Delta t_{1}\right)^{2} \tag{116}
\end{equation*}
$$

The rocket travels with a constant acceleration $a_{2}$ starting at initial speed $v_{2}$ in the elapsed time $\Delta t_{2}$ with respect to the observer on earth. The angle between the two direction vectors is $\theta$, $0 \leq \theta \leq \pi$.

$$
\begin{equation*}
\left|B B^{\prime \prime}\right|=v_{2} \cdot\left(\Delta t_{2}\right)+\frac{1}{2} a_{2} \cdot\left(\Delta t_{2}\right)^{2} \tag{117}
\end{equation*}
$$

Using the Cosine Theorem in the triangle $\triangle A B B$ "' we get:

$$
\begin{equation*}
\left|A B^{\prime \prime \prime}\right|^{2}=|A B|^{2}+\left|B B^{\prime \prime \prime}\right|^{2}-2|A B| \cdot\left|B B^{\prime \prime \prime}\right| \cdot \cos \theta \tag{118}
\end{equation*}
$$

$$
\begin{align*}
& =\left(\Delta t_{1}\right)^{2}\left[v_{1}+\frac{a_{1} \cdot \Delta t_{1}}{2}\right]^{2}+ \\
& \left(\Delta t_{2}\right)^{2}\left[v_{2}+\frac{a_{2} \cdot \Delta t_{2}}{2}\right]^{2}- \\
& 2 .\left(\Delta t_{1}\right)\left(\Delta t_{2}\right)\left[v_{1}+\frac{a_{1} \cdot \Delta t_{1}}{2}\right]\left[v_{2}+\frac{a_{2} \cdot \Delta t_{2}}{2}\right] \cos \theta \\
& \quad=\left[x_{v_{3}}\left(\Delta t_{2}\right)+\frac{1}{2} x_{a_{3}}\left(\Delta t_{2}\right)^{2}\right]^{2} \tag{119}
\end{align*}
$$

where $x_{v_{3}}$ is the initial speed and $x_{a_{3}}$ is the constant acceleration of the particle on trajectory $A B^{\prime \prime}$ ' as seen by the observer on earth (gravitation not included).

This is the general equation, without any assumption about the relationships between the elapsed times $\Delta t_{1}$ and $\Delta t_{2}$, nor about the
acceleration $x_{a_{3}}\left(\right.$ with initial velocity $x_{v_{3}}$ ) as seen by the observer on earth.

Now, in particular case when we consider the absolute time (therefore $\Delta t_{1}=\Delta t_{2}$ ) we get, after dividing by $(\Delta t)^{2}=\left(\Delta t_{1}\right)^{2}=\left(\Delta t_{2}\right)^{2}$ :
$\left[v_{1}+\frac{a_{1} \cdot \Delta t}{2}\right]^{2}+\left[v_{2}+\frac{a_{2} \cdot \Delta t}{2}\right]^{2}-$
2. $\left[v_{1}+\frac{a_{1} \cdot \Delta t}{2}\right]\left[v_{2}+\frac{a_{2} \cdot \Delta t}{2}\right] \cos \theta=\left[x_{v_{3}}+\frac{1}{2} x_{a_{3}}(\Delta t)\right]^{2}$

Divide again by $(\Delta t)^{2}$ :
$\left[\frac{v_{1}}{\Delta t}+\frac{a_{1}}{2}\right]^{2}+\left[\frac{v_{2}}{\Delta t}+\frac{a_{2}}{2}\right]^{2}-2 \cdot\left[\frac{v_{1}}{\Delta t}+\frac{a_{1}}{2}\right]\left[\frac{v_{2}}{\Delta t}+\frac{a_{2}}{2}\right] \cos \theta$
$=\left[\frac{x_{v_{3}}}{\Delta t}+\frac{1}{2} x_{a_{3}}\right]^{2}$
5.5. Constant Accelerations with Zero Initial Velocity and Oblique Direction Vectors.

In the previous case, we set:

$$
v_{1}=v_{2}=x_{v_{3}}=0
$$

Therefore:

$$
\frac{a_{1}^{2}}{4}+\frac{a_{2}^{2}}{4}-\frac{2 a_{1} a_{2}}{4} \cos \theta=\frac{x_{a_{3}}^{2}}{4}
$$

whence the constant acceleration of the particle as seen by the observer on earth on the path $A B^{\prime \prime}$ ' is:

$$
\begin{equation*}
x_{a_{3}}=\sqrt{a_{1}^{2}+a_{2}^{2}-2 a_{1} a_{2} \cdot \cos \theta} \tag{121}
\end{equation*}
$$

## Chapter 6.

 Noninertial Multirelativity
### 6.1. Multirelativity with Nonconstant

## Acceleration and 3D-Curves.

In a $3 D$-Euclidean space for location and in an $1 D$-oriented Euclidean space for time we consider a reference frame $F_{1}$ with respect to which a particle $P_{0}$ travels with a nonconstant acceleration $a_{0}$ on a $3 D$ curve $C_{0}$ in an elapsed time $\Delta t_{0}$.

Then we suppose the reference frame $F_{1}$ is moving with nonconstant acceleration $a_{l}$ on a $3 D$ curve $C_{1}$ with respect to another reference frame $F_{2}$. Similarly, the reference frame $F_{2}$ is moving with a nonconstant acceleration $a_{2}$ on a $3 D$ curve $C_{2}$ with respect to another reference frame $F_{3}$.

And so on: the reference frame $F_{n-1}$ is moving with a nonconstant acceleration $a_{n-1}$ on a $3 D$ curve $C_{n-1}$ with respect to another reference frame $F_{n}$ (where $n \geq 2$ ).

We call this a Noninertial Multirelativity, i.e. the most general case.

### 6.2. Research Problems.

1. How would the particle's trajectory curve $C_{0}$ be seen by an observer in the reference frame $F_{n}$ ?
2. What would be the particle's speed (acceleration) as measured by the observer from the reference frame $F_{n}$ ?
3. What would be the elapsed time of the particle as seen by the observer in the reference frame $F_{n}$ ?
4. What are the transformation equations from a reference frame to another?
5. Similar questions for rotating reference systems.

Particular cases would be helpful in starting such research, for example studying particles or reference frames travelling on linear curves, or on special curves, with constant speeds or constant accelerations, in reference frames that have one, two, or three parallel coordinate axes. Then later trying to generalize the results.

### 6.3. Example of Nonlinear 3D-Trajectories of Particle and Reference Frames.

Since each constant speed $v$ can be considered a constant zero-acceleration with initial velocity $v$, we treat the general case (i.e. the constant acceleration).

Let's consider in the reference frame $F_{1}$ a particle $P_{0}$ traveling on a curve $C_{0}$ from $A$ to $B$ :

Fig. 12


A
with a constant acceleration $a_{0}$ and initial velocity $v_{0}$. Let's take into consideration the earth's gravity $g$ too that influences the trajectory.
$F_{1}$ (which has the Cartesian system $X_{l} Y_{l} Z_{l}$ ) is moving with a constant acceleration $a_{1}$ with initial velocity $v_{l}$ in the positive direction of the $X_{l}$-axis (the $O Y_{l^{-}}$and $O Z_{l^{-}}$ axes are parallel respectively with $O Y_{2}$ and $O Z_{2}$ ) with respect to the frame $F_{2}$ (whose Cartesian system is $X_{2} Y_{2} Z_{2}$ ).

The arclength of $A B$ is noted by $d$.
From an observer in $F_{2}$ the trajectory $\overrightarrow{A B}$ of the particle $P_{0}$ in $F_{l}$ is seen as a $2 D$ - or $3 D$-curve $\overrightarrow{A B}$.

The curve $A B^{\prime}$ is described in $F_{2}$ by a function

$$
\begin{equation*}
f\left(a_{0}, v_{0}, a_{1}, v_{1}, g, C_{0}, A, B, \theta\right)=\left(x_{2}(t), y_{2}(t), z_{2}(t)\right) \tag{122}
\end{equation*}
$$

i.e.

$$
\operatorname{ArcLength}\left(A B^{\prime}\right)=\int_{0}^{\Delta} \sqrt{\left[x_{2}^{\prime}(t)\right]^{2}+\left[y_{2}^{\prime}(t)\right]^{2}+\left[z_{2}^{\prime}(t)\right]^{2} d t} \equiv L\left(\Delta t^{\prime}, \Delta t\right)
$$

where $x_{2}{ }^{\prime}(t), y_{2}{ }^{\prime}(t), z_{2}{ }^{\prime}(t)$ are respectively the derivatives of $x_{2}(t), y_{2}(t), z_{2}(t)$ with respect to $t$, and $L\left(\Delta t^{\prime}, \Delta t\right)$ is a notation to mean that the arclength $L$, from $A$ to $B^{\prime}$, depends on $\Delta t$ and also on $d$, but $d$ depends on $\Delta t^{\prime}$.

The distance traveled by the reference frame $F_{1}$ in $\Delta t$ elapsed time is

$$
\begin{equation*}
s_{1}=v_{1}(\Delta t)+\frac{1}{2} a_{1}(\Delta t)^{2} \tag{124}
\end{equation*}
$$

Supposing that particle's traveling is seen as a constant acceleration by the observer in $F_{2}$, then we have:

$$
\begin{equation*}
L\left(\Delta t^{\prime}, \Delta t\right)=x_{v_{0}}(\Delta t)+\frac{1}{2} x_{a_{0}}(\Delta t)^{2} \tag{125}
\end{equation*}
$$

where $x_{v_{0}}=$ the initial particle's velocity as seen by the observer in $\mathrm{F}_{2}$,
and $x_{a_{0}}=$ the particle's constant acceleration as seen by the observer in $F_{2}$.

$$
\begin{align*}
& \text { We know that in } \\
& |A B|=d=v_{0}\left(\Delta t^{\prime}\right)+\frac{1}{2} a_{0}\left(\Delta t^{\prime}\right)^{2} . \tag{126}
\end{align*}
$$

Depending on the suppositions regarding he connections between $\Delta t^{\prime}$ and $\Delta t$ (in an absolute time reference frame they are equal), or on the supposition about the acceleration of the particle $x_{a_{0}}$ and $x_{v_{0}}$ we get particular cases in formula (125).

The reader can repeat this thought experiment for the case when the accelerations $a_{0}$ and $a_{1}$ are not constant, and the reference frame $F_{l}$ is moving with respect to the reference frame $F_{2}$ on an arbitrary $3 D$-curve.

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In this book we present the hypothesis that there is no speed barrier in the universe thus refuting the speed of light postulate.

While Einstein considered a relative space and relative time but the ultimate speed of light, we do the opposite: we consider an absolute time and absolute space but no ultimate speed, and we call it the Absolute Theory of Relativity (ATR).

We then parameterize Einstein's thought experiment with atomic clocks, supposing that we know neither if the space and time are relative or absolute, nor if the speed of light is ultimate speed or not. We obtain a Parameterized Special Theory of Relativity (PSTR). Our PSTR generalizes not only Einstein's Special Theory of Relativity, but also our ATR, and introduces three more possible Relativities to be studied in the future.

Afterwards, we extend our research considering not only constant velocities but constant accelerations too.

Eventually we propose a Noninertial Multirelativity for the same thought experiment, i.e. considering non-constant accelerations and arbitrary 3D-curves.

