Absorbing Boundary Conditions for Hyperbolic Systems

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Abstract

This talk deals with absorbing boundary conditions (ABCs) for hyperbolic systems in one and two space dimensions. We prove the strict well–posedness of the resulting initial boundary value problem in 1D. Afterwards we establish the GKS–stability of the corresponding Lax-Wendroff–type finite difference scheme. Hereby, we have to extend the classical proofs, since the (discretized) ABCs do not fit the standard form of boundary conditions for hyperbolic systems.

In the second part we present the approach of deriving so-called discrete absorbing boundary conditions, i.e. ABCs constructed on a purely discrete level. These discrete ABCs are better adapted to the interior scheme: they lead to less unphysical reflections and the resulting overall scheme has better stability properties.

Finally, we sketch briefly how ABCs can be derived for nonlinear hyperbolic systems.

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