Abstention in two-candidate and three-candidate elections when voters use mixed strategies

BERNARD GROFMAN*

In a recent important (and controversial) paper, Ferejohn and Fiorina (1974) examine two models of voter choice in two-candidate and three-candidate elections.[1] The first model they consider is the expected utility maximizing model of decision under risk standard in the rational choice literature.[2] The second is a minimax-regret model of decision under uncertainty first proposed by Leonard Savage.[3]

For both two-candidate elections and three-candidate elections they find (1) that neither expected utility maximizers nor minimax regretters should ever vote for their least preferred candidate; (2) that minimax regretters can be expected to be less likely to abstain than expected utility maximizers; and, (3) that for both minimax regretters and expected utility maximizers the higher the cost of voting, *ceteris paribus*, the more likely are voters to abstain.

For three-candidate elections they also find (4) that expected utility maximizers in a three-candidate election may sometimes vote for their second choice; minimax regretters never should.

The Ferejohn-Fiorina analysis is based on voters choosing exactly one alternative from the set: A (abstention); V_1 (vote for one's first choice); V_2 (vote for one's second choice); and, in the case of a three-candidate election, V_3 (vote for one's third choice).

We shall extend the Ferejohn-Fiorina analysis by allowing voters to use mixed strategies rather than the pure strategies to which Ferejohn-Fiorina restrict them. By a mixed strategy we mean a probability vector over the available pure strategies. For example, in a two-candidate election with choices V_1, V_2, and A, the mixed strategy vector $(^1/_2, ^1/_3, ^1/_6)$ denotes choice of V_1 with probability $^1/_2$, choice of V_2 with probability $^1/_3$, and a choice of A with probability $^1/_6$. Of course, the sum of the probabilities in such a mixed strategy vector must be one. Any pure strategy may be represented as a vector with exactly one entry of a one; e.g. (0,0,1) denotes the pure strategy 'always abstain'. Extending the range of voter choice to

*Associate Professor of Political Science and Social Sciences at the University of California, Irvine.

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mixed strategies permits voters to make probabilistic rather than deterministic choices among alternatives. Such mixed strategies can, we believe, be defended on the grounds that their use leads to an increase in a player's long-run security level. The voter game may be thought of as a single shot game or it may also be seen as one in which voters must choose a strategy which they will use *each* election, since each election they face a similar pattern of choice. Empirical data can be used to support this view of voters as 'rule' rather than 'act' utilitarians (Wuffle, 1979).

If a citizen is an expected utility maximizer, we assume he has certain beliefs about what his fellow citizens are likely to do. Those may be summarized as a vector of subjective probabilities $(p_1, p_2, \ldots p_n)$ over the n possible states of nature $(s_1, s_2, \ldots, s_n)$, where p_i refers to $p(S_i)$, i.e. the probability that S_i will occur. The assumption of expected utility maximizers postulates that a citizen chooses the act whose associated expected utility is greatest.

The Savage minimax regret criterion does not operate directly on a matrix of outcomes. It operates on a 'regret matrix' defined as follows: the regret, r_{ij} accruing to an act a_i if state s_j comes to pass is defined as the difference between what the decision maker could have attained had he known the true state of nature before he chose his action and what he actually gets by choosing a_i. Let V_{ij} denote the outcome which occurs when the decision maker chooses a_i and nature is in state s_j. Thus:

$$r_{ij} = (\max_i V_{ij}) - V_{ij} \tag{1}$$

1. Two-candidate contests

Assume that each voter has preferences over the set of possible candidate-party-issue packages represented by a utility function, U. Let C_1 and C_2 be the two candidates (who may be viewed as bundles of personal, party and issue characteristics). The gain to a voter if his candidate wins is $|U(C_1) - U(C_2)|$. For notational simplicity assume that the voter prefers candidate 1 to candidate 2 and that his utility function is normalized so that $U(C_1) = 1$, $U(C_2) = 0$. Let c be the cost of voting expressed on the same utility scale. In the case of tie votes, assume a fair coin is flipped so that each voter expects to receive:

$$\tfrac{1}{2}U(C_1) + \tfrac{1}{2}U(C_2) = \tfrac{1}{2} \tag{2}$$

The decision problem of the citizen takes the form of a $3 \times 3^{(N-1)}$ table, where the voter has a choice of voting for C_1, C_2, or abstaining (strategies V_1, V_2, A), and the states of nature are configurations of the same 3 choices by the other $(N-1)$ citizens. This table reduces to a 3×5 table by collapsing all of the identical columns. The five mutually exclusive and collectively exhaustive states of nature (where n_i equals the number of votes for C_i *exclusive* of the citizen under consideration) are given in Table 1.

Table 1. Possible outcomes for two-candidate plurality election.

S_1	$n_1 > n_2 + 1$	C_1 wins by more than one vote regardless of the citizen's vote
S_2	$n_1 = n_2 + 1$	C_1 wins by exactly one vote without the citizen's vote
S_3	$n_1 = n_2$	C_1 ties C_2 without the citizen's vote
S_4	$n_1 = n_2 - 1$	C_1 loses by exactly one vote without the citizen's vote
S_5	$n_1 < n_2 - 1$	C_1 loses by more than one vote without the citizen's vote

Thus, the decision problem appears as Table 2, where the cell entries represent the payoffs forthcoming from choice of a particular action under a particular state of nature.

Table 2. Payoff matrix for two-candidate plurality election.

Acts	S_1	S_2	S_3	S_4	S_5
V_1	$1 - c$	$1 - c$	$1 - c$	$1/2 - c$	$-c$
V_2	$1 - c$	$1/2 - c$	$-c$	$-c$	$-c$
A	1	1	$1/2$	0	0

Ferejohn and Fiorina (1974) show that:

(a) strategy V_2 is dominated by both V_1 and A and should, therefore, never be chosen;
(b) voters who are expected utility maximizers will vote rather than abstain if:

$$p_3 + p_4 > 2c \tag{3}$$

Only if $c > 1/2$, will expected utility maximizers always abstain.

The regret matrix for a two-candidate plurality election is shown in Table 3.

Table 3. Regret matrix for two-candidate election.

Acts	S_1	S_2	S_3	S_4	S_5
V_1	c	c	0	0	c
V_2	c	$1/2 + c$	1	$1/2$	c
A	0	0	$1/2 - c$	$1/2 - c$	0

$1/2 > c$.

The minimax regret criterion requires that the citizen choose the act which minimizes his maximum regret.

Ferejohn and Fiorina show that:

(c) voters who are minimax regretters will vote rather than abstain if:

$$x < 1/4 \tag{4}$$

They note that the inequality of expression (4) is probably satisfied for many voters while that of expression (3) is probably not. Thus:

(d) minimax regretter voters are less likely to abstain than expected utility maximizers.

We shall extend this analysis of two-candidate plurality elections by allowing for mixed strategies. We may readily show that, for expected utility maximizers, no mixed strategy is ever optimal.[5] Thus, admitting mixed strategies does not affect the decision analysis of expected utility maximizers and the Ferejohn-Fiorina results remain unchanged. On the other hand, admitting mixed strategies does significantly affect the decision analysis for minimax regretters.

Because V_2 is a dominated strategy, we need only consider the reduced regret matrix (derived from that in Table 3) which is shown in Table 4.

The optimal mixed strategy for (V_1, V_2, A) is given by:[6]

$$(1 - 2c, 0, 2c) \tag{5}$$

Thus:

Table 4. Reduced regret matrix for two-candidate election.

	States	
Acts	S_1, S_2, S_5	S_3, S_4
V_1	c	0
A	0	$1/2 - c$

Assumption: $1/2 > c$.

(c)′ voters who are minimax regretter voters will sometimes vote for their first choice as long as:

$$c < 1/2 \tag{6}$$

Moreover, the probability that they will vote for their first choice rather than abstain equals $1 - 2c$. If $c < 1/4$, then the probability that a minimax regretter will vote will exceed $1/2$.

Thus, for example, if $c = 3/8$, minimax regretters could be expected to vote for C_1 25% of the time; while if $c = 1/8$, they could be expected to vote for C_1 75% of the time.

Result (c)′ is considerably stronger than the result (c) obtained by Ferejohn and Fiorina for the pure strategy minimax regret case. *Thus, in a two-candidate election when voters are permitted mixed strategies,*

(d)′ *minimax regretter voters are considerably less likely to abstain than expected utility maximizers.*

This we believe to be an important and nonobvious result.

One other point: *if voters are maximiners they will always abstain in a two-candidate plurality election.* This result follows from inspection of Table 2. We see that 0 is saddlepoint; i.e. 0 is the maximum row minimum and the minimum column maximum. Thus, maximining voters will always choose to guarantee themselves at least a 0 payoff and will, therefore, always abstain.

2. Three-candidate contests

Let $U(C_1) = 1$, $U(C_2) = k$, $U(C_3) = 0$; i.e. normalize the citizen's utility function to give his most preferred choice a value of 1 and his least preferred choice a value of 0, with his second choice given a value of k, $0 < k < 1$.

Ferejohn and Fiorina (1974) show that:

(a) Strategy V_3 is dominated by both V_1 and A and should, therefore, never be chosen.
(b) Voters who are expected utility maximizers will sometimes prefer A to V_1 to V_2, will sometimes prefer V_1 to V_2 to A, and will sometimes even prefer V_2 to V_1 to A. For the exact conditions under which these situations will obtain, we refer the reader to the Ferejohn and Fiorina analysis. The states S_1 through S_{19} of their decision matrix are specified in Table 5.

As before, introducing the possibilities of mixed strategies into the expected utility maximizer's calculations does not affect his decision-making and thus the Ferejohn and Fiorina results remain unchanged for expected utility maximizing voters. On the other hand, as in the two-candidate case, admitting mixed strategies does affect the decision analysis of minimax regretter voters and again leads to results significantly different from those of Ferejohn and Fiorina in the minimax regret pure strategy case.

Fiorina and Ferejohn show that in the three-candidate case, for the minimax regretter voter, neither V_2 or V_3 can ever be optimal. Thus we need not consider the full 4 X 19 decision and regret matrices they present. (The interested reader is referred to the Ferejohn and Fiorina [1974] analysis.) Instead, we need only deal with three reduced regret matrices given in Tables 6 (a) through 6 (d).[7]

Using the standard technique for finding minimax solutions to 2 X 2 games, in the case where $^1/_2 > k/2 \geqslant c \geqslant {}^1/_4$ we obtain as minimax-regret optimal mixed strategy vector over V_1 and A:

$$(1 - k, k) \tag{7}$$

If voters use this strategy, they will hold their expected maximum regret to $k(1 - c)/2$. This regret is less than the regrets of $k/2$ or $(1 - c)/2$. They might consistently suffer if they used a pure strategy and one of the worst states of nature was consistently obtained. Thus, using a mixed strategy improves a voter's second level. In the case where $^1/_4 \geqslant k/2 \geqslant c$ we obtain as our minimax-regret optimal mixed strategy vector:

$$\left(\frac{4 - 5k}{4 - 2k}, \frac{3k}{4 - 2k}\right) \tag{8}$$

In the case where $^1/_2 > c > k/2 \geqslant {}^1/_4$, we obtain as our minimax-regret optimal mixed strategy:

$$(1 - 2c, 2c) \tag{9}$$

Finally, for the case where $^1/_4 > c > k/2$, we obtain an optimal minimax-regret mixed strategy vector given by:[8]

Table 5. Possible outcomes for the three-candidate election.*

State	Definition	Description
1	$n_1 = n_j + 1 \; j = 2, 3$	A clear winner exists; one vote cannot change the outcome
2	$n_2 = n_j + 1 \; j = 1, 3$	
3	$n_3 = n_j + 1 \; j = 1, 2$	
4	$n_1 = n_2 + 1 = n_3 + 1$	A narrow winner exists; one vote for just losing candidate produces a tie
5	$n_1 = n_3 + 1 = n_2 + 1$	
6	$n_1 = n_2 + 1 = n_3 + 1$	
7	$n_2 = n_1 + 1 = n_3 + 1$	Same, with C_2 being the narrow winner
8	$n_2 = n_3 + 1 = n_1 + 1$	
9	$n_2 = n_3 + 1 = n_1 + 1$	
10	$n_3 = n_1 + 1 = n_2 + 1$	Same, with C_3 being the narrow winner
11	$n_3 = n_2 + 1 = n_1 + 1$	
12	$n_3 = n_1 + 1 = n_2 + 1$	
13	$n_1 = n_2 = n_3 + 1$	A two-way tie for first place exists; one vote could break tie
14	$n_1 = n_3 = n_2 + 1$	
15	$n_2 = n_3 = n_1 + 1$	
16	$n_1 = n_2 = n_3$	A three-way tie exists; one vote could break tie
17	$n_1 = n_2 = n_3 + 1$	A two-way tie for first place exists, but now one vote could break this tie or produce a three-way tie
18	$n_1 = n_3 = n_2 + 1$	
19	$n_2 = n_3 = n_1 + 1$	

* Where n_i = number of votes for C_i *exclusive* of voter under consideration.

Table 6 (a). Reduced regret matrix for three-candidate election for the case where $^1/_2 > k/2 \geqslant c \geqslant {}^1/_4$.

	States	
Acts	S_{11}	$S_{10}, S_{12}, S_{14}, S_{18}$
V_1	$k/2$	0
A	$k/2 - c$	$^1/_2 - c$

Table 6 (b). Reduced regret matrix for three-candidate election for the case where $1/4 \geqslant k/2 \geqslant c$.

Acts	States S_{11}	S_{16}
V_1	$k/2$	0
A	$k/2 - c$	$2/3 - k/3 - c$

Table 6 (c). Reduced regret matrix for three candidate election for the case where $1/2 > c > k/2 \geqslant 1/4$.

Acts	States $S_1, S_2, S_3, S_4, S_5, S_6, S_8, S_{11}, S_{15}$	$S_{10}, S_{12}, S_{14}, S_{18}$
V_1	c	0
A	0	$1/2 - c$

Table 6 (d). Reduced regret matrix for the three-candidate election for the case where $1/4 \geqslant c > k/2$.

Acts	States $S_1, S_2, S_3, S_4, S_5, S_6, S_8, S_{11}, S_{15}$	S_{16}
V_1	c	0
A	0	$2/3 - k/3 - c$

$$\left(\frac{2 - k - 3c}{2 - k}, \frac{3c}{2 - k}\right) \tag{10}$$

While these results, like those of Ferejohn and Fiorina in the minimax regret pure strategy case, are not easy to interpret in intuitive terms, we can say this much: *for minimax regretters permitted mixed strategies there will always be some probability of voting as long as $c < 1/2$ (otherwise A becomes a dominant strategy); furthermore, if $k/2 \geqslant c$, the greater k* (i.e. the greater one's preference for one's second choice), *the greater the likelihood of abstention; and if $c > k/2$, the greater c the greater the likelihood of abstention.* These results are very similar to, though stronger than, the Ferejohn-Fiorina conclusions for the three-candidate case that:

> Minimax regret decision-makers find it rational to vote for their most preferred candidate rather than abstain under relatively weak conditions . . . [and that] a high cost of voting . . . and a relatively high utility for one's second choice reduces the 'likelihood' that voting will be one's minimax regret strategy.

To facilitate direct comparison of our results with those of Ferejohn and Fiorina, we specify both sets of results in Table 7.

Table 7. Optimal strategies for minimax regret voters in three-candidate elections: pure and mixed strategies case.

	Pure strategy:* Vote for C_1 if:	Mixed strategy: Vote for C_1 with probability given by:
If $1/2 > k/2 \geqslant c \geqslant 1/4$	$c < 2/3 - 5k/6$	$1 - k$
If $1/4 \geqslant k/2 \geqslant c$	$c < 1/2 - k/2$	$(4 - 5k)/(4 - 2k)$
If $1/2 > c > k/2 \geqslant 1/4$	$c < 1/3 - k/6$	$1 - 2c$
If $1/4 > k/2 > c$	$c < 1/4$	$(2 - k - 3c)/(2 - k)$

* Fiorina and Ferejohn (1974).

In all but one case, it is easier to obtain some probability of voting for minimax regret voters when mixed strategies are admissible, as it is in the pure strategy case. Consider, for example, the case where $1/2 > c > k/2 \geqslant 1/4$. If, for example, $k = 45/50$, it will require $c \leqslant 11/60$ for a minimax regretter to vote for C_1 as his pure strategy choice. On the other hand, if $k = 45/60$, in the mixed strategy case, any $c \leqslant 1/2$ is sufficient to generate some probability of voting. In each of the other rows, the inequalities on the left hand side of Table 7 are always at least as restrictive as those on the right.[9]

One final point: *in the three-candidate election, as in the two-candidate election, maximiners always abstain.* In this game there is again a saddlepoint at the value 0. (This saddlepoint occurs when nature is in state s_3 and the citizen abstains. See the full 4 X 19 decision matrix in Ferejohn and Fiorina [1974].)

3. Conclusions

In re-analyzing the Ferejohn and Fiorina (1974) examination of 'rational' abstention we found that admitting mixed strategies does not affect findings as to the optimal behavior of voters who are expected utility maximizers but does significantly affect our expectations as to behavior of voters using a minimax regret rule. *We found that minimax regretters with admissible mixed strategies would always have some probability of voting rather than abstaining, except under the quite restrictive condition that* $c > 1/2$. Thus, to the extent that *some* voters can be seen as operating from a minimax regret perspective, a decision to vote on their part can be understood without recourse to ideas like the 'psychic benefits of voting' or 'citizen duty'.

An important difference between the Ferejohn and Fiorina (1974) analysis and our own is that their models all have step-function threshold effects, while ours is probabilistic in nature and behavior is gradient-like. In the usual rational choice modeling of voter behavior (including the minimax regret model restricted to the pure strategy case) voters behave deterministically: e.g. if $p_3 + p_4 > 2c$ then expected utility maximizing voters *always* abstain in two-candidate plurality elections; if $p_3 + p_4 < 2c$ then they *always* vote for their first choice. In the mixed strategy minimax regret model, on the other hand, raising c (or k) does not operate in a dichotomous fashion – instantaneously shifting the voter from voting to abstention once a critical threshold is passed. Rather, as c (or k) increases, the *probability* of voting decreases.

We believe that probabilistic choice mechanisms are more descriptive of human choice behavor (including voter behavior) than are deterministic ones. Thus, while the mixed strategy minimax-regret model we propose may only account for the behavior of some voters, we believe it is desirable to pursue other models for voter choice which, unlike those now current in the literature, do not postulate sharp 'on-off' effects but make use instead of response-gradient notions.[10]

Notes

1.
The Ferejohn and Fiorina analysis has been vehemently attacked by a number of authors (see, e.g. Mayer and Good, 1975; Stephens, 1975; and rejoinder by Ferejohn and Fiorina, 1975). The principal complaint is that minimax regret is not a reasonable standard for voter choice since it requires a voter to pay no heed to the probability of various outcomes occurring and instead asks him to calculate regrets based on outcomes which may have zilch probability of occurrence. Ferejohn and Fiorina's reply that 'the available data provide no empirical basis for rejecting the minimax regret model as a *descriptive model of the turnout decision*, regardless of its weaknesses as a prescriptive theory of decision-making' (Ferejohn and Fiorina, 1975, p. 925). I share Ferejohn and Fiorina's view of the importance of distinguishing between prescriptive and

descriptive aspects of models and I hold the view minimax regret is not appealing as a *prescriptive* model for rational choice. My aim in this paper, however, is the quite limited one of extending the analysis of minimax regret, the case where mixed strategies are allowed and show some interesting and nonobvious result of this extension. These new results (see especially concluding discussion) have important consequences for the likelihood of abstention. For a comprehensive and up to date survey of the standard political science analysis of the demographic characteristics of voters and nonvoters, see Wolfinger and Rosenstone (1977). A number of articles dealing with voter abstention have appeared in *Public Choice* in recent years including Brody and Page (1973), Barzel and Silberberg (1973), Tollison and Willett (1973), Smith (1975), Settle and Abrams (1976).

2.

See, for example, Downs (1957) and Riker and Ordeshook (1973).

3.

See Luce and Raiffa (1957, Chapter 13), for a discussion of Savage's work.

4.

The use of mixed strategies in games played only once is controversial. See the useful discussion in Luce and Raiffa, *op cit*., pp. 74-76. Even for games played only once, most scholars who have applied game theory to military and political decision-making have made use of mixed strategies in their analysis (for examples see Brams, 1975, Chapter 2).

5.

See Grofman (1976).

6.

For a 2 × 2 game where there is no saddlepoint, the optimal strategy for row is given by

$$\frac{|p_{21} - p_{22}|}{|p_{21} - p_{22}| + |p_{11} - p_{12}|}, \quad \frac{|p_{11} - p_{12}|}{|p_{21} - p_{22}| + |p_{11} - p_{12}|}$$

(Luce and Raiffa, 1957).

7.

See discussion in Fiorina and Ferejohn (1974), for a statement of the inequalities which give rise to these reduced regret matrices.

8.

If $c < 1/4$, then the probability that a minimax regretter will vote will exceed $1/2$.

9.

We have the verification of this claim as an exercise to the reader. For the case where $1/4 \geqslant c > k/2$, for $c < 1/4$ there is a probability of voting in the pure strategy case.

10.

There has been considerable controversy in the mathematical psychology literatures in signal detection and models of choice over threshold vs. gradient models (see, e.g. Luce, 1959). While we do not claim to have mastered the niceties of these debates, in both cases it appears to us that stochastic models are in the ascendant.

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