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Accelerating Dark Energy Models in Bianchi Type-V Space-Time with Time Dependent Deceleration Parameter

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Abstract: Some new exact solutions of Einstein's field equations have emerged in a spatially homogeneous and anisotropic Bianchi type-V space-time with minimally interacting perfect fluid and anisotropic dark energy (DE) components, which has dynamic equation of state (EoS). We consider Bianchi type-V space-time, introducing three different skewness parameters along spatial directions to quantify deviation of pressure from isotropy. To obtain the deterministic solution we choose the scale factor $a(t) = \sqrt{t^n e^t}$, which yields a time-dependent deceleration parameter (DP). We find that the time dependent value of deceleration parameter is reasonable for the present day universe which yields a transition of the universe from the early decelerating phase to the recent accelerating phase. For different values of n, we can generate a class of physically viable DE models. It is found that quintessence model is suitable for describing the present evolution of the universe. The physical and geometric properties of spatially homogeneous and anisotropic cosmological models are discussed.
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1. Introduction

Now-days, it is strongly believed that the universe is experiencing an accelerated expansion. Recent observations of type Ia supernova (SNIa) suggest that the expansion of the universe is accelerating and two-thirds of the total energy density exists in a dark energy component with negative pressure [1, 2] and recent observations of SNIa at high confidence level [3, 4] have further confirmed this (for a recent review, see Padmanabhan [5]; Copeland et al. [6]). In addition, measurements of the cosmic microwave background [7, 8], large scale structure [9, 10]. and the galaxy power spectrum [11] also indicate the existence of the dark energy. However, the observational data are far from being complete (for a recent review, see Perivolaropoulos [12]; Jassal et al. [13]). It is not even known what is the current value of the dark energy effective equation of state (EoS) parameter $\omega = p/\rho$ which lies close to -1: it could be equal to -1 (standard ΛCDM cosmology), a little bit upper than -1 (the quintessence dark energy) or less than -1 (phantom dark energy). While the possibility $\omega \ll -1$ is ruled out by current cosmological data from SN Ia (Supernovae Legacy Survey, Gold sample of Hubble Space Telescope) (Riess et al. [14]; Astier et al. [15]), CMB (WMAP, BOOMERANGE) (Eisentein et al. [16]; MacTavish et al. [17]) and large scale structure (Sloan Digital Sky Survey) (Komatsu et al. [18]) data, the dynamically evolving DE crossing the phantom divide line (PDL) ($\omega = -1$) is mildly favoured. The simplest candidate for the dark energy is a cosmological constant Λ, which has pressure $P_\Lambda = -\rho_\Lambda$. Specifically, a reliable model should explain why the present amount of the dark energy is so small compared with the fundamental scale (fine-tuning problem) and why it is comparable with the critical density today (coincidence problem).

The study of Bianchi type V cosmological models create more interest as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some instant of cosmic time. This property makes them suitable as model of our universe. The homogeneous and isotropic Friedman-Robertson-Walker (FRW) cosmological models, which are used to describe standard cosmological models, are particular case of Bianchi type I, V and IX universes, according to whether the constant curvature of the physical three-space, t = constant, is zero, negative or positive. These models will be interesting to construct cosmological models of the types which are of class one. Present cosmology is based on the FRW model which is completely homogeneous and isotropic. This is in agreement with observational data about the large scale structure of the universe. However, although homogeneous but anisotropic models are more restricted than the inhomogeneous models, they explain a number of observed phenomena quite satisfactorily. This stimulates the research for obtaining exact anisotropic solution for Einstein's field equations (EFEs) as a cosmologically accepted physical models for the universe (at least in the early stages). The anisotropy of DE within the frame work of Bianchi type is found to be useful in generating arbitrary ellipsoidality to the Universe, and to fine tune the observed CMBR anisotropies. Koivisto and Mota [19, 20] have investigated cosmological models with anisotropic EoS and have also shown that the present SN Ia data allows large anisotropy. Among different models Bianchi type-V universes are the natural generalization of the open FRW model, which eventually become isotropic. Thus, the Bianchi type-V models which remain anisotropic are of rather academic interest. Recently, Yadav [21] and Kumar & Yadav [22] have dealed with a spatially homogeneous and anisotropic Bianchi type-V DE models by considering constant deceleration parameter.

Due to lack of the observational evidence in making a distinction between constant and variable ω, particularly the EoS parameter (ω) is considered as a constant (Kujat et al. [23]; Bartelmann et al. [24]; ' Yadav [21], Kumar and Singh [25]) with phase wise value $-1, 0, -\frac{1}{3}$ and $+1$ for vacuum fluid, dust fluid, radiation and stiff dominated universe, respectively. But in general, ω is a function of time or redshift (Ratra and Peebles [26]; Jimenez [27]; Das et al. [28]). Some literature are also available on models with varying fields, such as cosmological models with variable EoS parameter in Kaluza-Klein metric and wormholes (Steinhardt et al. [29]; Rahaman et al. [30]). In recent years various form of time dependent ω have been used for variable Λ models by Mukhopadhyay et al. [31]. Setare [32]−[34] and Setare & Saridakis [35] have also studied the DE models in different contexts. Recently, dark energy models with variable EoS parameter have been studied by Ray et al. [36], Akarsu and Kilinc [37, 38], Yadav et al. [39], Yadav and Yadav [40], Pradhan and Amirhashchi [41], Pradhan et al. [42] and Amirhashchi et al. [43, 44]. In well-known reviews on modified gravity (Nojiri and Odintsov [45, 46], it is clearly indicated that any modified gravity may be represented as effective fluid with time dependent ω. The dark energy universe EoS with inhomogeneous, Hubble parameter dependent term is considered by Nojiri and Odintsov [47]. Later on, Nojiri and Odintsov [48] have also presented the late-time cosmological consequences of dark energy with time-dependent periodic EoS in oscillating universe.

Recently, Pradhan and Hassan [49] studied accelerating dark energy models in Bianchi type-V space-time. In this paper, we have revisited the solution [49] and obtained some physically realistic and totally anisotropic Bianchi-V models with anisotropic DE and perfect fluid which is different from the previous one. We have assumed time-dependent skewness parameter which modify EoS to study the anisotropic nature of DE. This provides the exact solution of the Einstein's field equations together with time dependent deceleration parameter which also yields time dependent scale factor. This paper is organized as follows: the metric and the field equations are presented in Sect. 2. Sect. 3 deals with the exact solutions of the field equations and the physical behaviour of the model. Discussions and concluding remarks are given in Section 4.

2. The Metric and the Field Equations

We consider the space time metric of the spatially homogeneous and anisotropic Bianchi type-V of the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{2\alpha x}\left[B^2dy^2 + C^2dz^2\right], \tag{1}$$

where the metric potentials A, B and C are functions of cosmic time t alone and α is a constant.

We define the following physical and geometric parameters to be used in formulating the law and further in solving the Einstein's field equations for the metric (1).

The average scale factor a of Bianchi type-V model (1) is defined as

$$a = (ABC)^{\frac{1}{3}}. \tag{2}$$

A volume scale factor V is given by

$$V = a^3 = ABC. \tag{3}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_x + H_y + H_z), \tag{4}$$

where $\mathrm{H_x} = \frac{\dot{\mathrm{A}}}{\mathrm{A}}$, $\mathrm{H_y} = \frac{\dot{\mathrm{B}}}{\mathrm{B}}$ and $\mathrm{H_z} = \frac{\dot{\mathrm{C}}}{\mathrm{C}}$ are the directional Hubble's parameters in the directions of x, y and z respectively. A dot stands for differentiation with respect to cosmic time t.

From Eqs. (2)-(4), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \tag{5}$$

The physical quantities of observational interest in cosmology i.e. the expansion scalar θ, the average anisotropy parameter Am and the shear scalar σ^2 are defined as

$$\theta = u^i_{;i} = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{6}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}\right] - \frac{\theta^2}{6}, \tag{7}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\triangle H_i}{H}\right)^2, \tag{8}$$

where $\triangle H_i = H_i - H (i = x, y, z)$ represents the directional Hubble parameter in the direction of x, y, z respectively. $A_m = 0$ corresponds to isotropic expansion.

The Einstein's field equations (in gravitational units $8\pi G = c = 1$) read as

$$R^i_j - \frac{1}{2}Rg^i_j = -T^{(m)i}_j - T^{(de)i}_j, \tag{9}$$

where $T_j^{(m)i}$ and $T_j^{(de)i}$ are the energy momentum tensors of perfect fluid and DE, respectively. These are given by

$$T_j^{(m)i} = \text{diag}[-\rho^{(m)}, p^{(m)}, p^{(m)}, p^{(m)}], \tag{10}$$

and

$$\begin{aligned} T_j^{(de)i} &= \text{diag}[-\rho^{(de)}, p_x^{(de)}, p_y^{(de)}, p_z^{(de)}], \\ &= \text{diag}[-1, \omega_x, \omega_y, \omega_z]\rho^{(de)}, \\ &= \text{diag}[-1, \omega+\delta, \omega+\gamma, \omega+\eta]\rho^{(de)}, \end{aligned} \tag{11}$$

where $\rho^{(m)}$ and $p^{(m)}$ are, respectively the energy density and pressure of the perfect fluid component; $\rho^{(de)}$ is the energy density of the DE component; $\delta(t)$, $\gamma(t)$ and $\eta(t)$ are skewness parameters, which modify EoS (hence pressure) of DE component and are functions of the cosmic time t; ω is the EoS parameter of DE; ω_x, ω_y and ω_z are the directional EoS parameters along x, y, and z coordinate axes, respectively. We assume the four velocity vector $u^i = (1, 0, 0, 0)$ satisfying $u^i u_j = -1$.

In a co-moving coordinate system ($u^i = \delta_0^i$), Einstein's field equations (9) with (10) and (11) for B-V metric (1) subsequently lead to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -p^{(m)} - (\omega+\delta)\rho^{(de)}, \tag{12}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{\alpha^2}{A^2} = -p^{(m)} - (\omega+\delta)\rho^{(de)}, \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -p^{(m)} - (\omega+\eta)\rho^{(de)}, \tag{14}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} = \rho^m + \rho^{de}, \tag{15}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{16}$$

We assume that the perfect fluid and DE components interact minimally. Therefore, the energy momentum tensors of the two sources may be conserved separately.

The law of energy-conservation equation ($T_{;j}^{(m)ij} = 0$) of the perfect fluid gives

$$\dot{\rho}^{(m)} + 3(\rho^{(m)} + p^{(m)})H = 0, \tag{17}$$

where as energy-conservation equation ($T_{;j}^{(de)ij} = 0$) of the DE component leads to

$$\dot{\rho}^{(de)} + 3\rho^{(de)}(\omega+1)H + \rho^{(de)}(\delta H_x + \gamma H_y + \eta H_z) = 0, \tag{18}$$

where we have used the equation of state $p^{(de)} = \omega\rho^{(de)}$.

The Raychaudhuri equation is found to be

$$\dot{\theta} = -\frac{1}{2}\left(\rho^{(de)} + 3p^{(de)}\right) - \frac{1}{3}\theta^2 - 2\sigma^2 - \frac{1}{2}(3\omega + \delta + \gamma + \eta + 1)\rho^{(de)}. \tag{19}$$

The field equations (12) − (15) can be reduced in terms of H, σ^2 and q as follows:

$$p^{(m)} + \frac{1}{3}(3\omega + \delta + \gamma + \eta)\rho^{(de)} = H^2(2q - 1) - \sigma^2 + \frac{\alpha^2}{A^2}, \tag{20}$$

$$\rho^{(m)} + \rho^{(de)} = 3H^2 - \sigma^2 - \frac{3\alpha^2}{A^2}. \tag{21}$$

3. Solution of the Field Equations and its Physical Significance

We have revisited the solution [49]. Following, Akarsu and Kilinc [37, 38, 50] and Yadav [21], we split the conservation of energy momentum tensor of the DE into two parts, one corresponds to deviations of EoS parameter and other is the deviation-free part of $T^{(de)ij}_{;j} = 0$:

$$\dot{\rho}^{(de)} + 3\rho^{(de)}(\omega + 1)H = 0, \tag{22}$$

and

$$\rho^{(de)}(\delta H_x + \gamma H_y + \eta H_z) = 0. \tag{23}$$

According to (22) and (23) the behaviour of $\rho^{(de)}$ is restrained by the deviation-free part of EoS parameter of DE but deviations will bear upon $\rho^{(de)}$ indirectly, since, as can be observed later, they pretend the value of EoS parameter. Naturally, the choice of skewness parameters are quite arbitrary but, since we are searching visually for a physically feasible models of the universe consistent with observation, we consider the skewness parameters δ, γ and η as to be function of cosmic time. We restrained δ, γ and η by assuming a special dynamics of skewness parameters on $x-$, $y-$ and $z-$ axes consistent with (23) as

$$\delta(t) = k(H_y + H_z)\frac{1}{\rho^{(de)}}, \tag{24}$$

$$\gamma(t) = -kH_x\frac{1}{\rho^{(de)}}, \tag{25}$$

$$\eta(t) = -kH_x\frac{1}{\rho^{(de)}}, \tag{26}$$

where k is an arbitrary constant, which parameterizes the anisotropy of DE.

Secondly, we assume that $\omega =$ constant, so that we can study different models related to the DE by choosing different values of ω, viz. phantom ($\omega < -1$), cosmological constant ($\omega = -1$) and quintessence ($\omega > -1$). In view of these assumptions (24)-(26) and $\omega =$ constant, Eq. (22) can be integrated to obtain

$$\rho^{(de)}(t) = \rho_0 a^{-3(\omega+1)}, \tag{27}$$

where ρ_0 is a positive constant of integration.

Integrating (16) and engrossing the constant of integration in B or C, without any loss of generality, we obtain

$$A^2 = BC. \tag{28}$$

To solve Einstein's field equations (12) − (15), subtracting (12) from (13), (12) from (14), and (13) from (15) and taking second integral of each, we obtain the following three relations respectively:

$$\frac{A}{B} = d_1 \exp\left[k_1 \int \frac{dt}{a^3} - \frac{\alpha\rho_0}{\omega} \int \frac{dt}{a^{3(\omega+1)}}\right], \tag{29}$$

$$\frac{A}{C} = d_2 \exp\left[k_2 \int \frac{dt}{a^3} - \frac{\alpha\rho_0}{\omega} \int \frac{dt}{a^{3(\omega+1)}}\right], \tag{30}$$

and

$$\frac{B}{C} = d_3 \exp\left(k_3 \int \frac{dt}{a^3}\right), \tag{31}$$

where d_1, d_2, d_3, k_1, k_2 and k_3 are constants of integration. From (29)−(31) and (28), the metric functions can be explicitly obtained as

$$A(t) = a \exp\left[-\frac{2\alpha\rho_0}{3\omega} \int \frac{dt}{a^{3(\omega+1)}}\right], \tag{32}$$

$$B(t) = ma \exp\left[\ell \int \frac{dt}{a^3} + \frac{\alpha\rho_0}{3\omega} \int \frac{dt}{a^{3(\omega+1)}}\right], \tag{33}$$

$$C(t) = \frac{a}{m} \exp\left[-\ell \int \frac{dt}{a^3} + \frac{\alpha\rho_0}{3\omega} \int \frac{dt}{a^{3(\omega+1)}}\right], \tag{34}$$

where

$$m = \sqrt[3]{(d_2 d_3)}, \quad \ell = \frac{(k_2 + k_3)}{3}, \quad d_2 = d_1^{-1}, \quad k_2 = -k_1. \tag{35}$$

Finally, following Saha et al. [51] and Pradhan & Hassan [49], we take following *ansatz* for the scale factor, where increase in term of time evolution is

$$a(t) = \sqrt{t^n e^t}, \tag{36}$$

where n is a positive constant. This *ansatz* generalized the one proposed by Amirhashchi et al. [52]. In literature it is common to use a constant deceleration parameter [37, 38, 41, 53, 21, 22] as it duly gives a power law for metric function or corresponding quantity. The motivation to choose such time dependent DP is behind the fact that the universe is accelerated expansion at present as observed in recent observations of Type Ia supernova [1]−[4] and CMB anisotropies [54]−[56] and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a Universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping [57]−[59]. So, there is no scope for a constant DP at the present epoch. So, in general, the DP is not a constant but time variable. The motivation to choose such scale factor (36) yields a time dependent DP.

Using (36) into (32)-(34), we get the following expressions for scale factors:

$$A(t) = \sqrt{t^n e^t} \exp\left[-\frac{2\alpha\rho_0}{3\omega}\int (t^n e^t)^{-\frac{3}{2}(\omega+1)}dt\right], \tag{37}$$

$$B(t) = m\sqrt{t^n e^t} \exp\left[\ell\int (t^n e^t)^{-\frac{3}{2}}dt + \frac{\alpha\rho_0}{3\omega}\int (t^n e^t)^{-\frac{3}{2}(\omega+1)}dt\right], \tag{38}$$

$$C(t) = m^{-1}\sqrt{t^n e^t} \exp\left[-\ell\int (t^n e^t)^{-\frac{3}{2}}dt + \frac{\alpha\rho_0}{3\omega}\int (t^n e^t)^{-\frac{3}{2}(\omega+1)}dt\right]. \tag{39}$$

The expressions for physical parameters such as directional Hubble parameters (H_x, H_y,

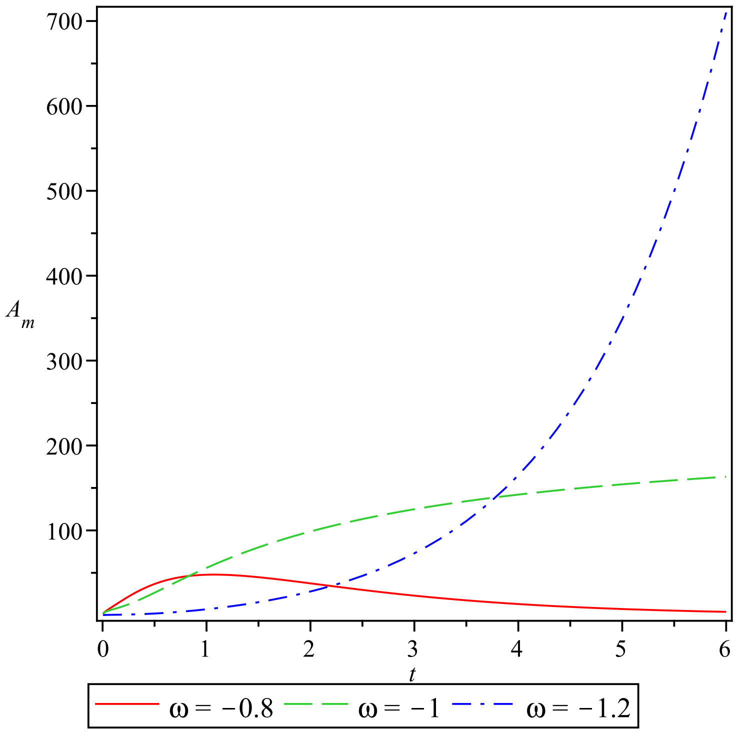

Fig. 1 The anisotropic parameter A_m versus t. Here $\rho_0 = 5$, $\alpha = \ell = 1$, $n = 0.5$.

H_z), the Hubble parameter (H), scalar of expansion (θ), shear scalar (σ), spatial volume V and the anisotropy parameter (A_m) are, respectively, given by

$$H_x = \frac{1}{2}\left(\frac{n}{t}+1\right) - \frac{2\alpha\rho_0}{3\omega}(t^n e^t)^{-\frac{3}{2}(\omega+1)}, \tag{40}$$

$$H_y = \frac{1}{2}\left(\frac{n}{t}+1\right) + \ell(t^n e^t)^{-\frac{3}{2}} + \frac{\alpha\rho_0}{3\omega}(t^n e^t)^{-\frac{3}{2}(\omega+1)}, \tag{41}$$

$$H_z = \frac{1}{2}\left(\frac{n}{t}+1\right) - \ell(t^n e^t)^{-\frac{3}{2}} + \frac{\alpha\rho_0}{3\omega}(t^n e^t)^{-\frac{3}{2}(\omega+1)}, \tag{42}$$

$$\theta = 3H = \frac{3}{2}\left(\frac{n}{t} + 1\right), \tag{43}$$

$$\sigma^2 = \ell^2 (t^n e^t)^{-3} + \frac{\alpha^2 \rho_0^2}{3\omega^2} (t^n e^t)^{-3(\omega+1)}, \tag{44}$$

$$V = (t^n e^t)^{\frac{3}{2}} \exp(2\alpha x) \tag{45}$$

$$A_m = \frac{4}{3}\left(\frac{t}{t+1}\right)^2 \left[2\ell^2 (t^n e^t)^{-3} + \frac{2\alpha^2 \rho_0^2}{3\omega^2} (t^n e^t)^{-3(\omega+1)}\right]. \tag{46}$$

It is observed that at $t = 0$, the spatial volume vanishes and other parameters θ, σ, H diverge. Hence the model starts with a big bang singularity at $t = 0$. This is a Point Type singularity [60] since directional scale factor $A(t)$, $B(t)$ and $C(t)$ vanish at initial time. Figure 1 depicts the variation of anisotropic parameter (A_m) versus cosmic time t. The figure suggests that for $\omega \leq -1$ (cosmological constant and phantom scenario), A_m increases with time but for $\omega > -1$ (quintessence region), A_m decreases with time and tends to zero as $t \to \infty$. Thus, the observed isotropy of the universe can be achieved in the quintessence model. The shear tensor also tends to zero in this model. Thus, in our analysis, the quintessence model is turning out as a suitable model for describing the present evolution of the universe. The other models, viz., phantom and cosmological models possibly represent relatively earlier epoch of the universe.

We define the deceleration parameter q as usual, i.e.

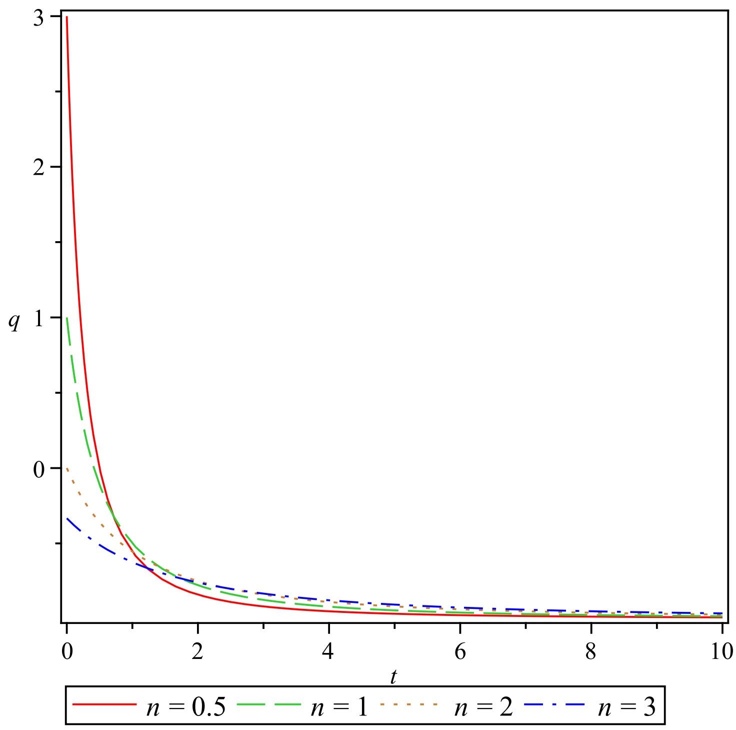

Fig. 2 The deceleration parameter q versus t

.

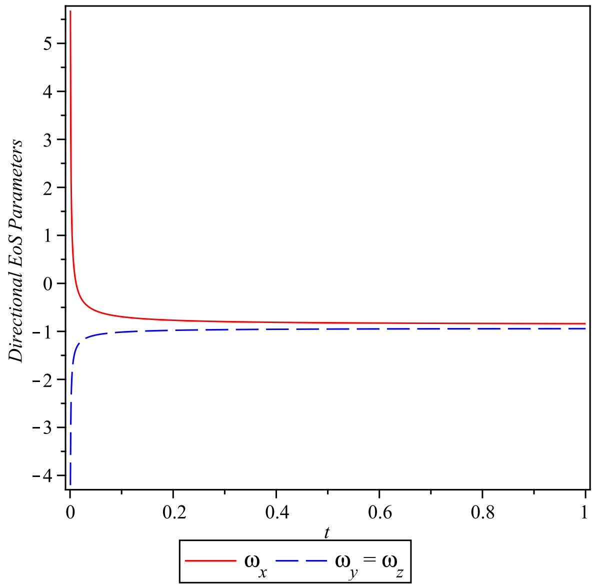

Fig. 3 The plot of directional EoS parameters $(\omega_x, \omega_y, \omega_z)$ versus t. Here $\rho_0 = 5$, $n = 2$, $\alpha = 0.1$, $\omega = -0.8$
.

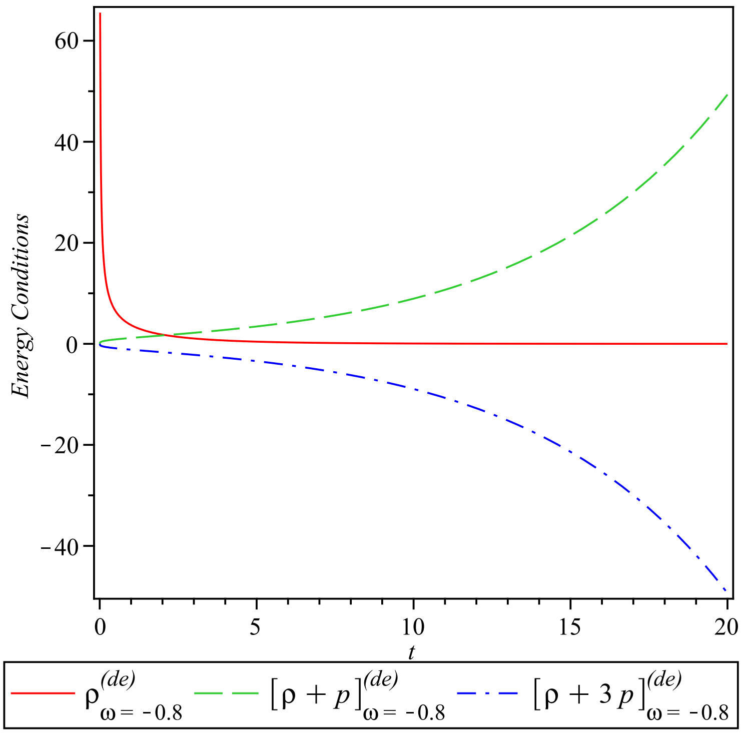

Fig. 4 The plot of energy conditions versus t in quintessence region. Here $\rho_0 = 5$, $n = 2$, $\omega = -0.8$
.

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}. \tag{47}$$

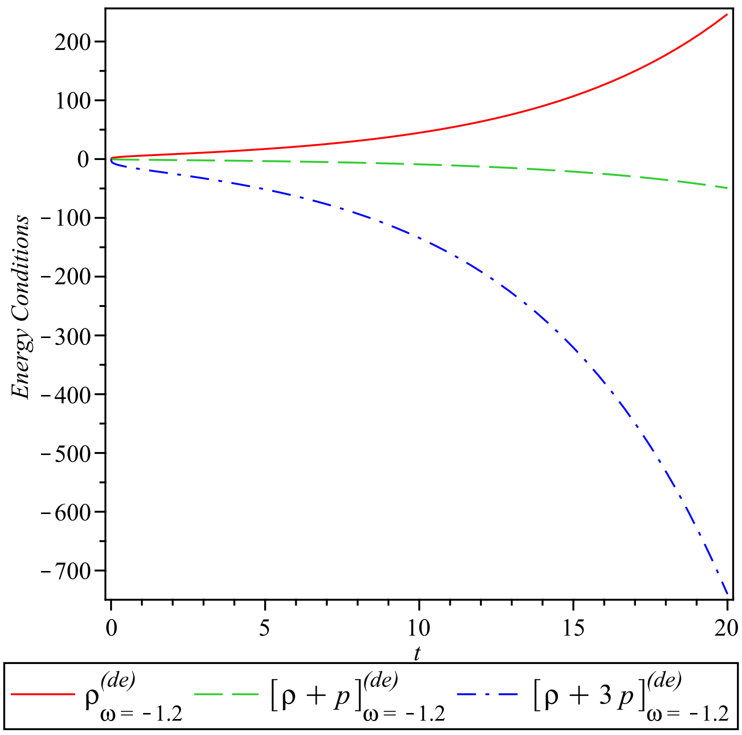

Fig. 5 The plot of energy conditions versus t in phantom region. Here $\rho_0 = 5$, $n = 2$, $\omega = -1.2$
.

Using Eq. (36) into Eq. (47), we find

$$q = \frac{2n}{(n+t)^2} - 1. \tag{48}$$

From Eq. (48), we observe that $q > 0$ for $t < \sqrt{2n} - n$ and $q < 0$ for $t > \sqrt{2n} - n$. It is observed that for $0 < n < 2$, our model is evolving from deceleration phase to acceleration phase. Also, recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range $-1 < q < 0$. It follows that in our derived model, one can choose the value of DP consistent with the observation. Figure 2 graphs the deceleration parameter (q) versus time which gives the behaviour of q from decelerating to accelerating phase for different values of n.

The skewness parameters of DE are found to be as

$$\delta(t) = \frac{\alpha}{\rho_0}\left[\left(\frac{n}{t}+1\right)(t^n e^t)^{\frac{3}{2}(\omega+1)} + \frac{2\alpha\rho_0}{3\omega}\right], \tag{49}$$

$$\gamma(t) = \eta(t) = -\frac{\alpha}{\rho_0}\left[\frac{1}{2}\left(\frac{n}{t}+1\right)(t^n e^t)^{\frac{3}{2}(\omega+1)} - \frac{2\alpha\rho_0}{3\omega}\right]. \tag{50}$$

In view of (10), the directional EoS parameter of DE are obtained as

$$\omega_x = \omega + \frac{\alpha}{\rho_0}\left[\left(\frac{n}{t}+1\right)(t^n e^t)^{\frac{3}{2}(\omega+1)} + \frac{2\alpha\rho_0}{3\omega}\right], \tag{51}$$

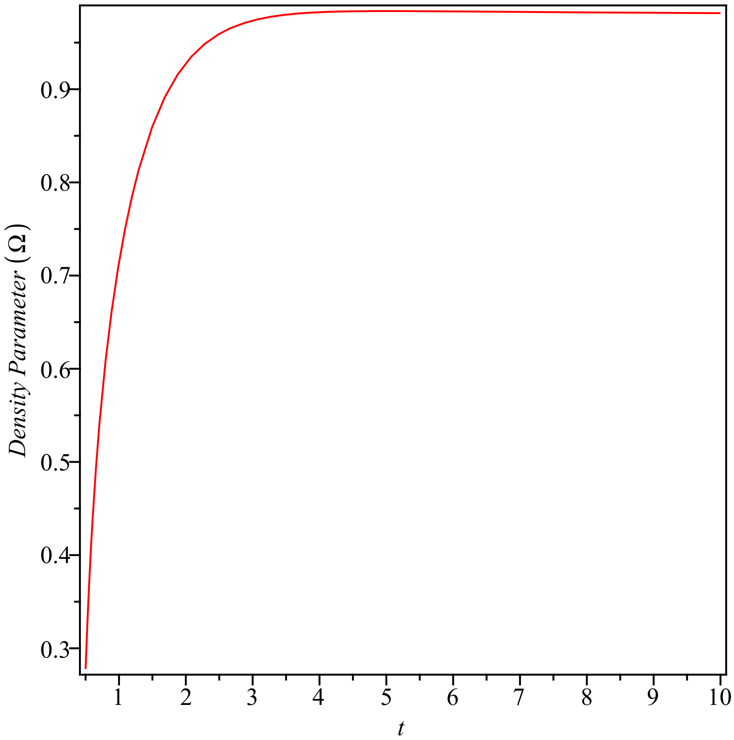

Fig. 6 The plot of total energy density parameter Ω versus t. Here $\rho_0 = 5$, $\alpha = \ell = 1$, $\omega = -1$
.

$$\omega_y = \omega_z = \omega - \frac{\alpha}{\rho_0}\left[\frac{1}{2}\left(\frac{n}{t}+1\right)(t^n e^t)^{\frac{3}{2}(\omega+1)} - \frac{2\alpha\rho_0}{3\omega}\right]. \tag{52}$$

Figure 3 is a plot of the variation of directional EoS parameter $(\omega_x, \omega_y, \omega_z)$ versus the cosmic time t in evolution of the universe, as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well known situations (parameters are given in Figure caption). From the Figure 3, it is clearly observed that directional EoS parameter along x-axis (ω_x) is a decreasing function of time whereas directional EoS parameter along y-axis (or z-axis) are found to be an increasing function of time. But it is worth mentioned here that all the directional EoS parameters approaches to -1 at the later stage of the evolution of the universe, as expected. The same is anticipated by recent observations.

The energy density and pressure of DE components are given by

$$\rho^{(de)} = \rho_0 (t^n e^t)^{-\frac{3}{2}(\omega+1)}, \tag{53}$$

$$p^{(de)} = \omega\rho_0 (t^n e^t)^{-\frac{3}{2}(\omega+1)}. \tag{54}$$

From Eqs. (20) and (21), the pressure and energy density of the perfect fluid are obtained as

$$p^{(m)} = \frac{n}{t^2} - \frac{3}{4}\left(\frac{n}{t}+1\right)^2 - \frac{\alpha^2\rho_0^2}{3\omega^2}(t^n e^t)^{-3(\omega+1)} - \ell^2 (t^n e^t)^{-3}$$

$$+\frac{\alpha^2}{(t^n e^t)\exp\left[-\frac{4\alpha\rho_0}{3\omega}\int (t^n e^t)^{-\frac{3}{2}(\omega+1)}dt\right]} - \left(\omega + \frac{2\alpha^2}{3\omega}\right)\rho_0(t^n e^t)^{-3(\omega+1)}, \tag{55}$$

$$\rho^{(m)} = \frac{3}{4}\left(\frac{n}{t}+1\right)^2 - \frac{\alpha^2\rho_0^2}{3\omega^2}(t^n e^t)^{-3(\omega+1)} - \ell^2(t^n e^t)^{-3}$$
$$-\rho_0(t^n e^t)^{-\frac{3}{2}(\omega+1)} - \frac{3\alpha^2}{(t^n e^t)\exp\left[-\frac{4\alpha\rho_0}{3\omega}\int (t^n e^t)^{-\frac{3}{2}(\omega+1)}dt\right]}. \tag{56}$$

The dark energy with $\omega < -1$, the phantom component of the universe, leads to uncommon cosmological scenarios as it was pointed out by Caldwell et al.[61]. First of all, there is a violation of the dominant energy condition (DEC), since $\rho + p < 0$. The energy density grows up to infinity in a finite time, which leads to a big rip, characterized by a scale factor blowing up in this finite time. These sudden future singularities are, nevertheless, not necessarily produced by a fluid violating DEC. Cosmological solutions for phantom matter which violates the weak energy condition were found by Dabrowski et al. [62]. Caldwell [63], Srivastava [64], Yadav [21] have investigated phantom models with $\omega < -1$ and also suggested that at late time, phantom energy has appeared as a potential DE candidate which violets the weak as well as strong energy condition. The left hand side of energy conditions have been depicted in Figures 4 and 5 for quintessence and phantom models respectively.

From Figure 4, for $\omega = -0.5$ (i.e. quintessence model), we observe that

$$(i)\ \ \rho^{(de)} \geq 0, \quad (ii)\ \ \rho^{(de)} + p^{(de)} \geq 0, \quad (iii)\ \ \rho^{(de)} + 3p^{(de)} < 0.$$

Thus, from above expressions, we observe that the quintessence model violates the strong energy conditions, as expected.

Further, from Figure 5, for $\omega = -1.5$ (i.e. phantom model), we observe that

$$(i)\ \ \rho^{(de)} \geq 0, \quad (ii)\ \ \rho^{(de)} + p^{(de)} < 0, \quad (iii)\ \ \rho^{(de)} + 3p^{(de)} < 0.$$

Thus the derived phantom model violates the weak as well as strong energy conditions as the same is predicted by current astronomical observations.

The perfect fluid density parameter ($\Omega^{(m)}$) and DE density parameter ($\Omega^{(de)}$) are given by

$$\Omega^{(m)} = 1 - \frac{4}{3}\left(\frac{n}{t}+1\right)^{-2} \times \left[\frac{\alpha^2\rho_0^2}{3\omega^2}(t^n e^t)^{-6(\omega+1)} + \ell^2(t^n e^t)^{-3} + \rho_0(t^n e^t)^{-\frac{3}{2}(\omega+1)}\right.$$
$$\left. + \frac{3\alpha^2}{(t^n e^t)\exp\left\{-\frac{4\alpha\rho_0}{3\omega}\int (t^n e^t)^{-\frac{3}{2}(\omega+1)}dt\right\}}\right], \tag{57}$$

$$\Omega^{(de)} = \frac{4}{3}\left(\frac{n}{t}+1\right)^{-2}\rho_0(t^n e^t)^{-\frac{3}{2}(\omega+1)}. \tag{58}$$

Thus the over all density parameter (Ω) is obtained as

$$\Omega = \Omega^{(m)} + \Omega^{(de)}$$

$$= 1 - \frac{4}{3}\left(\frac{n}{t}+1\right)^{-2} \times \left[\frac{\alpha^2\rho_0^2}{3\omega^2}(t^n e^t)^{-6(\omega+1)} + \ell^2(t^n e^t)^{-3} + \frac{3\alpha^2}{(t^n e^t)\exp\left\{-\frac{4\alpha\rho_0}{3\omega}\int (t^n e^t)^{-\frac{3}{2}(\omega+1)}dt\right\}}\right]. \tag{59}$$

We observe that the last expression of Eq. (59) is easily integrable if we choose $\omega = -1$. Figure 6 depicts the variation of the density parameter (Ω) versus cosmic time t for $\omega = -1$ during the evolution of the universe. From the Figure 6, it can be seen that the total energy density Ω tends to 1 for sufficiently large time which is supported by the current observations. It is also worth mentioned here that whatever be the value of ω in Eq. (59), we find $\Omega = 1$ for large values of time.

Discussions and Concluding Remarks

This paper is an extension of the recent work of Pradhan & Amirhashchi [49]. In this paper, we have studied a spatially homogeneous and anisotropic Bianchi type-V space time filled with perfect fluid and anisotropic DE possessing dynamic EoS. The field equations in this paper have been solved exactly with suitable physical assumptions but in a different manner. The solutions satisfy the Raychaudhuri Eq. (19) and the energy conservation Eq. (18) identically as earlier. Therefore, exact and physically viable Bianchi type-V model stands. It is to be noted that our procedure of solving the field equations is altogether different from what Yadav [21] and Kumar & Yadav [22]. Yadav [21] and Kumar & Yadav [22] have solved the field equations by considering the constant DP whereas we have considered time dependent DP. As we have already discussed in previous Sect. 3 that for a Universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping (see the Refs. Padmanabhan and Roychowdhury [57], Amendola [58], Riess et al. [59]) and hence, scope for a constant DP gets dispensed with. The main features of the model are as follows:

- The directional EoS parameters (ω_x, ω_y or ω_z) evolve within the range predicted by observations. It is worth mention here that for all the directional EoS parameters approach to -1 at the later stage of the evolution of the universe (see, Figure 3). The same is predicted by recent observations.

• The present DE model has a transition of the universe from the early deceleration phase to the recent acceleration phase (see, Figure 2) which agrees with recent observations (Caldwell et al. 2006).

• In the present study we find that the quintessence model is consistent with present and expected future evolution of the universe. The quintessence model reaches isotropy at late time (see, Figure 1). The other models, viz., phantom and cosmological constant models possibly represent relatively earlier epoch of the universe.

• The derived quintessence model is found to violate the strong energy condition whereas the phantom model violates the strong and weak energy conditions both (see, Figures 4 & 5).

• The total density parameter (Ω) approaches 1 for sufficiently large time (see, Figure 6) which is sound agreement with current observations.

• For different choice of n, a class of DE models in Bianchi type-V space-time which are found to be in good harmony with recent observations may be generated. Thus, the present solutions may be of immense help in better understanding of the characteristic of anisotropic DE in the evolution of the universe within the framework of Bianchi type-V space-time.

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