Accelerating NTRU based Homomorphic Encryption using GPUs

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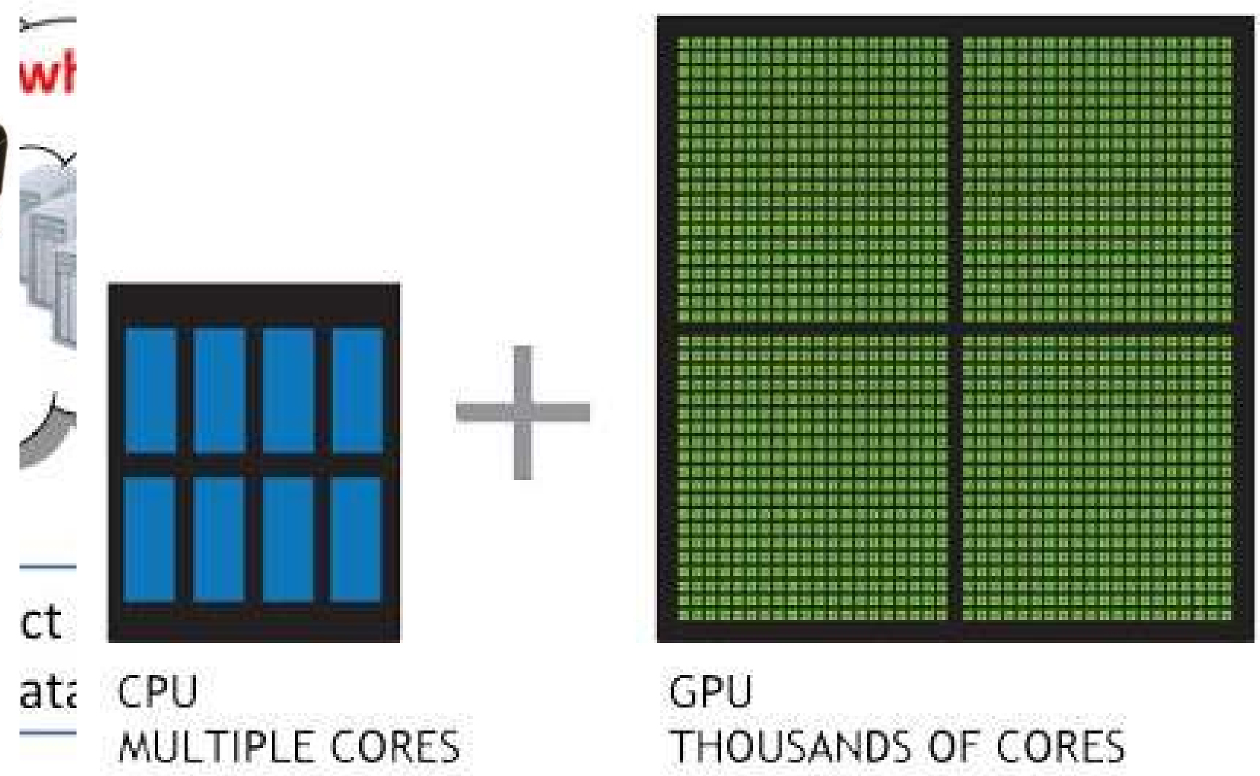
Outline:

- Motivations
- NTRU
- GPU NTRU
- Evaluation Results

Motivation

- Homomorphic Encryption is not efficient
- Speedup computations with GPUs



NTRU

- Keys, cipher texts are polynomials
  - n : polynomial degree
  - q : prime modulus
  - l : length of q
- Operations are performed in $R_q = \mathbb{Z}_q[x]/\langle x^n + 1\rangle$
- Previously, implemented with NTL Library in C++

GPU NTRU

- Core problem: *polynomial multiplications*
- How to create parallelism?
  - Chinese Remainder Theorem (CRT)
  - Number Theory Transform (NTT)
- Fast algorithms
- Memory access

GPU NTRU: Multiplication

- AES: 32768 degree, 1271-bit coefficients
- PRINCE: 16384 degree, 575-bit coefficients
- The Strassen's NTT based integer multiplication algorithm

Algorithm 1 Polynomial Multiplication

Input: Polynomials a, b with $(n, \log(q))$
Output: Polynomial c with $(2n, \log(nq^2))$

$\cancel{\{a_i\} = \mathsf{CRT}(a), \{b_i\} = \mathsf{CRT}(b)}$
$\{A_i\} = \mathsf{NTT}(\{a_i\}), \{B_i\} = \mathsf{NTT}(\{b_i\})$
$\{C_i\} = \{A_i\} \cdot \{B_i\}$
$\{c_i\} = \mathsf{INTT}(\{C_i\})$
$\cancel{c = \mathsf{ICRT}(\{c_i\})}$

GPU NTRU: CRT

$$\mathtt{CRT}: x \longrightarrow \{x \bmod p_0, x \bmod p_1, \cdots, x \bmod p_{l-1}\}$$

- Reduce coefficient size

  1271-bit $\longrightarrow$ 42-bit   575-bit $\longrightarrow$ 25-bit

- Size and number of p_i are decided automatically
  - in different circuit levels
  - only according to n and q
  - as level increases, computation goes faster
- Rules will be explained later

GPU NTRU: ICRT

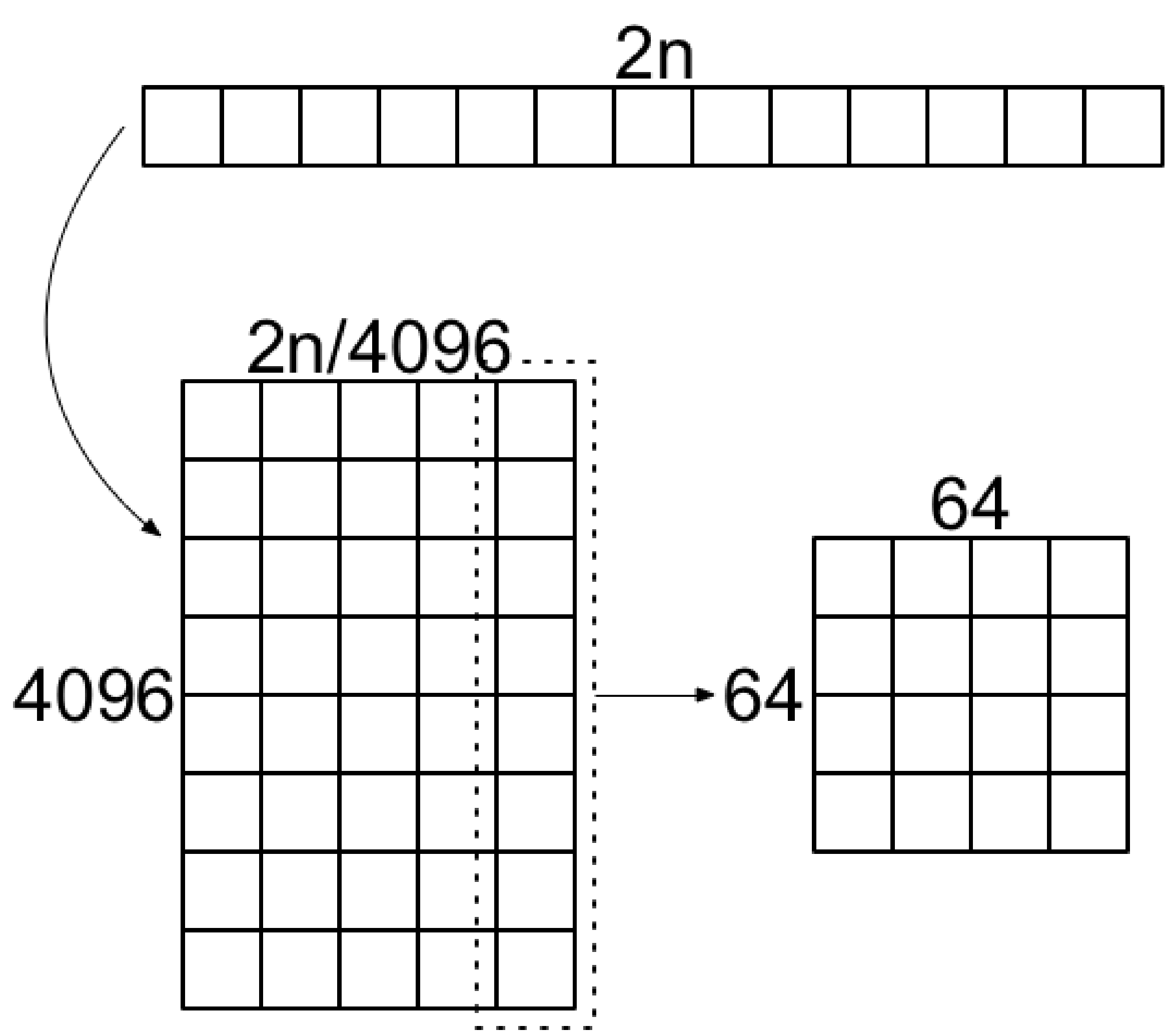
$$\texttt{ICRT}: x = \sum_{i=0}^{l-1} \left(\frac{M}{p_i}\right) \cdot \left(\left(\frac{M}{p_i}\right)^{-1} \cdot x_i \mod p_i \right) \mod M$$

$$M = \prod_{i=0}^{l} p_i$$

- Modified ICRT scheme
  - avoid large integer multiplication
  - avoid large integer modular reduction
- NVIDIA GPU Constant memory

GPU NTRU: NTT

- Emmart and Weems' approach
- 2n-point coefficient-wise NTT (padding with 0)
- Four-step Cooley-Tukey NTT iterations:



GPU NTRU: NTT

- Over the finite field $\mathbb{Z}/P\mathbb{Z}$, $P = \texttt{0xffffffff00000001}$
- Prime numbers:

$$P > n \cdot p_i^2 \qquad \prod_{i=0}^{l-1} p_i > n \cdot q^2$$

- Memory arrangement:
  - coalesced global memory access
  - shared memory as buffers
  - registers for arithmetic operations

GPU NTRU: Relinearization

- Input: cipher text $c(x)$, evaluation keys $\{EK_i(x)\}$
- Take the *i*-th bit of coefficients in $c(x)$
- Binary polynomials $\tilde{c}_i(x)$
- Output:

$$\tilde{c}(x) = \sum_{i=0}^{l-1} \tilde{c}_i(x) \cdot EK_i(x)$$

- Thousands of polynomial multiplications

GPU NTRU: Relinearization

- Evaluation keys are stored in NTT domain
- Computations are mainly in NTT domain

Algorithm 2 Relinearization

Input: Polynomial c with $(n, \log(q))$

Output: Polynomial d with $(2n, \log(nq\log(q)))$

1: $\{\tilde{C}_\tau\} = \mathsf{NTT}(\{\tilde{c}_\tau\})$

2: **for** $i = 0 \ldots, l-1$ **do**

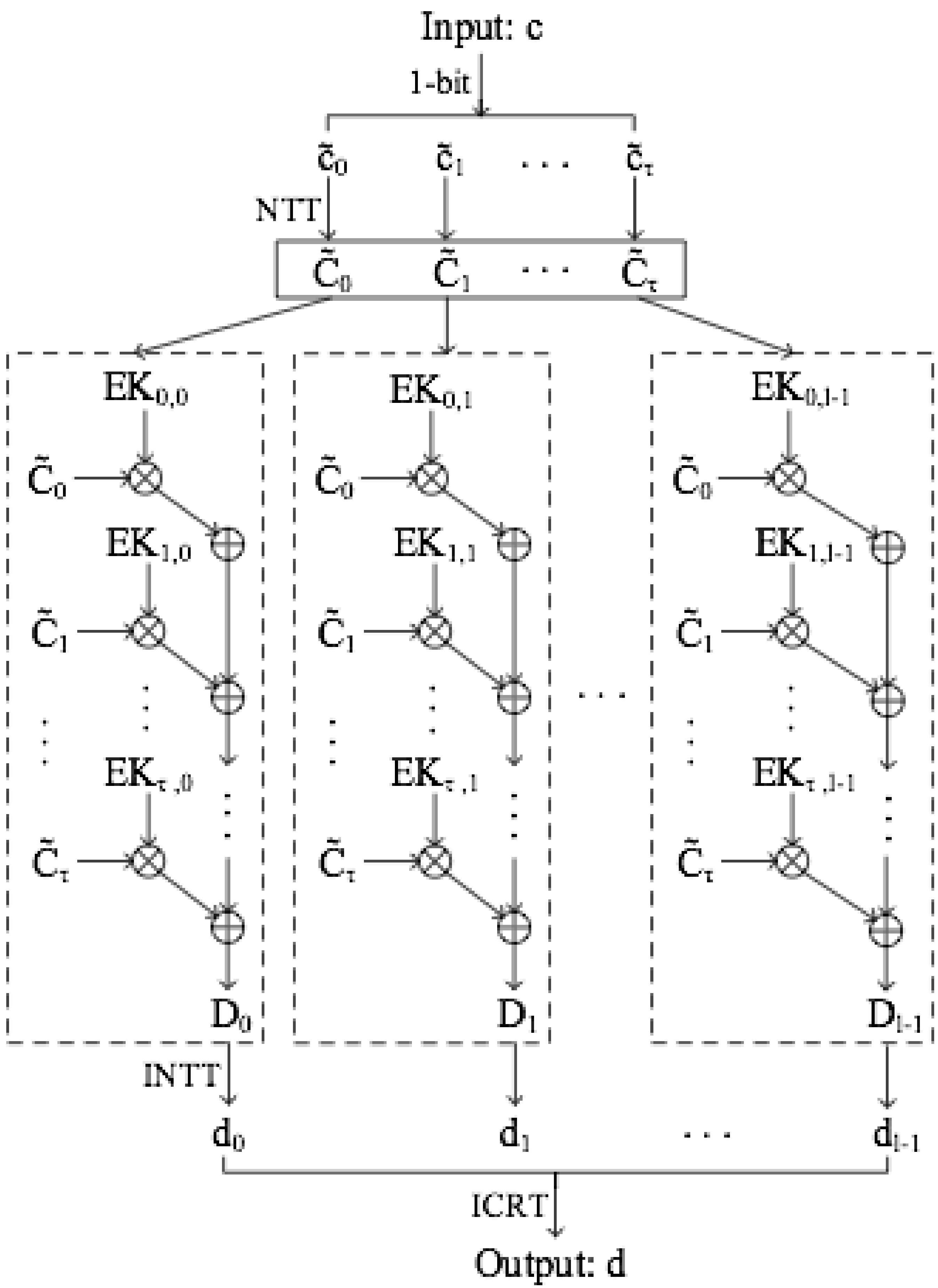
3: load $EK_{i,0}, EK_{i,1}, \cdots, EK_{i,\lceil\log(q)\rceil-1}$

4: $\{D_i\} = \{\sum_{\tau=0}^{\lceil\log(q)\rceil-1} \tilde{C}_\tau \cdot EK_{i,\tau}\}$

5: **end for**

6: $\{d_i\} = \mathsf{INTT}(\{D_i\})$

7: $d = \mathsf{ICRT}(\{d_i\})$



- EKs are huge (23 GB)
- Memory copy takes most of time
- Page-locked host memory
- Prime numbers:

$$P > \lceil \log(q) \rceil \cdot n \cdot p_i$$

$$\prod_{i=0}^{l-1} p_i > \lceil \log(q) \rceil \cdot n \cdot q$$

GPU NTRU

- NTL Data ⟷ 1-D arrays
- Coefficient and polynomial reductions

Implementation

Implementation Parameters

	AES	PRINCE
Levels	40	24
Polynomial Size	(32768, 1271)	(16384, 575)
Maximum Size of Evaluation Keys	23 GBytes	2 GBytes

Server specs:

- Intel Xeon E5-2609
  @2.5 GHz, 64 GB (1 thread)
- NVIDIA GeForce GTX690
  @915 MHz, 3072 CUDA cores, 4 GB (1536 cores, 2 GB)

GPU NTRU

TABLE II. TIMING COMPARISON BETWEEN THE CPU AND GPU IMPLEMENTATIONS FOR THE OPERATIONS.

	Prince			AES		
	GPU	CPU	SPEEDUP	GPU	CPU	SPEEDUP
MULTIPLICATION	0.063	0.18	× **2.8**	0.34	0.97	×**2.8**
RELINEARIZATION	0.89	10.9	×**12.2**	8.97	103.37	×**11.5**

TABLE III. PERFORMANCE COMPARISON OF PRINCE AND AES IMPLEMENTATIONS.

		TOTAL TIME	#BLOCKS	PER BLOCK	Speedup
AES	SIMD Xeon [9]	36 h	54	2400 sec	×1
	Byte Xeon [9]	65 h	720	300 sec	×8
	NTRU Xeon [10]	31 h	2048	55 sec	×43
	Ours (GPU)	**4.15 h**	**2048**	**7.3 sec**	×**328**
Prince	Prince [32]	57 min	1024	3.3 sec	×1
	Ours (GPU)	**22 min**	**1024**	**1.28 sec**	×**2.57**

Questions?

Thank you.