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# Accelerating the Numerical Generation of Aerodynamic Models for Flight Simulation

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**The generation of a tabular aerodynamic model for design-related flight dynamics studies, based on simulation generated data, is considered. The framework described accommodates two design scenarios. The first emphasizes the representation of the aerodynamic nonlinearities and is based on sampling. The second assumes an incremental change from an initial geometry, for which a high-fidelity model from the first scenario is available. In this case, data fusion is used to update the model. In both cases, Kriging is used to interpolate the samples computed using simulation. A commercial jet test case, using DATCOM as a source of data, is computed to illustrate the sampling and fusion. Future application using computational fluid dynamics as the source of data is considered.**

## Nomenclature

| | | |
|---|---|---|
| $C_L$, $C_D$, $C_m$ | = | lift, drag, and pitching moment coefficients |
| $C_{L\max}$ | = | maximum lift coefficient |
| $C_{L\max 0}$ | = | maximum lift coefficient of original geometry |
| $C_{L\alpha}$ | = | lift slope with respect to angle of attack |
| $C_{L\alpha 0}$ | = | lift slope of the original geometry |
| $C_{m\alpha}$ | = | pitching moment coefficient slope with respect to angle of attack |
| $C_{m\alpha 0}$ | = | pitching moment slope of the original geometry |
| $Cp$ | = | pressure coefficient |
| $C_{\mathrm{roll}}$, $C_n$ | = | rolling and yawing moment coefficients |
| $C_{y\beta}$ | = | side force coefficient slope with respect to angle of sideslip |
| $C_{y\beta 0}$ | = | side force slope of the original geometry |
| $d$ | = | distance function to define the correlation matrix |
| $E[\,]$ | = | expected value |
| $F$ | = | matrix of regression function evaluations at samples |
| $f$ | = | vector of regression function evaluations |
| $f_i(x)$ | = | regression functions |
| $n$ | = | number of samples |
| $p_q$ | = | parameters in the distance function definition |
| $R$ | = | correlation matrix in the Kriging formulation |
| $\mathbf{r}$ | = | vector of correlations in the Kriging formulation |
| $s^2$ | = | mean squared error of the Kriging predictor |
| $x/c$ | = | distance normalized by chord length |
| $x^i$ | = | sample location ($k$-dimensional vector with $j$th component $x^i_j$) |
| $\hat{y}$ | = | Kriging estimator of $y$ |
| $y_{\min}$ | = | minimum of the function values at sample points |
| $y^i$ | = | observation at sample point $x^i$ |
| $\beta$ | = | vector of regression coefficients |
| $\beta_i$ | = | regression coefficients |
| $\epsilon$ | = | error term (difference of target function from the mean) |
| $\eta$ | = | cheap observation in cokriging |
| $\mu$ | = | mean value |
| $\Theta_q$ | = | parameter in distance function definition |
| $\xi$ | = | cheap sample location in cokriging |
| $\rho$ | = | weighting factor for cheap samples in cokriging |
| $\sigma^2$ | = | variance for Kriging error term |
| $\Phi$, $\phi$ | = | normal distribution and density functions |

## Introduction

THERE is increasing interest in using physics-based modelling in aircraft design. For example, the design of unstable aircraft configurations would be helped by the early availability of high-quality models to allow control laws to be defined. One component of the required model arises from the aerodynamics. Stability and control analysis over the flight envelope traditionally uses a lookup table of forces and moments, measured from wind-tunnel tests relatively late in the design cycle. An alternative is to use simulation based data early in the design cycle. Computational fluid dynamics (CFD) has the predictive capability to generate the raw data for the lookup table. A problem is the computational cost involved, particularly if this is viewed as a brute-force calculation for every entry in the table.

Fortunately, methods are available that can reduce the computational cost. There are essentially three issues. First, a range of modelling levels are available, from Reynolds-averaged Navier–Stokes down to potential flow models and semi-empirical methods. Each of the levels has a range of validity and cost. Data fusion can be used for data from different methods, with low-cost data indicating trends and a small number of high-cost simulations correcting values. Second, reconstruction methods can significantly reduce the number of data points that actually need to be computed to fill the table. Third, the identification of parameter regions in which the aerodynamics are nonlinear and, hence, in which data are urgently needed is a sampling problem. The current paper considers these three elements.

Some studies [1–3] using Kriging for the generation of aerodynamic data have been published. The application of an expert system to reduce the required number of CFD simulations required was studied [1]. Different fidelity results were fused to generate the aerodynamic database for a crew transfer vehicle. The results showed that a reduction in the number of Navier–Stokes simulations from 90 to 20 was possible, saving 30% of the computational time without sacrificing accuracy. The fusion of results from different low-fidelity methods (DATCOM and vortex lattice methods) for a small unmanned vehicle was studied [2], along with the benefits of using these data to reduce the number of CFD computations. The effects of sampling methods and modeling techniques for building a response surface of an aerodynamic function of a deforming aerofoil was studied in [3].

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The current paper aims to develop methodology based on Kriging and sampling for the generation of aerodynamic tables for flight simulation. The logical progression is from initial sampling (which provides a first view of the aerodynamic forces and moments), to Kriging (which interpolates the data), to sampling for finding nonlinearities and, finally, to methods of data fusion (combining data from low- and high-fidelity sources). Following the discussion of these stages, the requirements for aerodynamic models are considered and an approach for model generation in two different scenarios is proposed. Results are then computed using DATCOM, which is used because the function evaluations are cheap but representative of the relevant aerodynamics, allowing focus to be kept on the performance of the Kriging and the sampling. Finally, conclusions are drawn. The overall aim is to demonstrate a methodology that can be applied to CFD-generated data.

## Initial Sampling

Ideally, observations of a function (in this case, aerodynamic forces and moments) would be available at many points, but in practice the number can be limited by computational cost. This is particularly true if computational fluid dynamics is used to generate observations of the function. The observed locations are referred to as samples.

The task of the initial sampling [4] is to provide an informative picture of the function at minimum cost. A good initial sampling tends to fill the parameter space [5]. Monte Carlo sampling is the most widely used method for a computer design of experiments. For random sampling, each sample is generated independently, but samples may not provide significant additional information [6].

Latin hypercube sampling (LHS) is a modification of Monte Carlo sampling for the generation of random samples. To generate Latin hypercube samples, the range of each design variable is divided into bins of equal probability. Then, samples are randomly selected with the following restrictions: a) each sample is randomly placed inside a bin, and b) for all one-dimensional projections of the samples onto bins, there will be one and only one sample in each bin. The technique was first described by McKay et al. [7] in 1979 and further elaborated by Iman et al. [8]. In the current paper, LHS is used to give an initial view of the aerodynamic forces and moments, which is then refined by other sampling techniques that exploit the observations that have already been generated. All initial samples inside the parameter space are generated using the LHS method.

## Kriging Predictor

Assume that $n$ samples are available for a function of $k$ independent variables. Each sample is denoted as $x^{(i)} = (x_1^{(i)}, \ldots, x_k^{(i)})$ with corresponding observations (responses) $y^{(i)} = y(x^{(i)})$ for $i = 1, \ldots, n$. A Kriging function is used to approximate the target function as

$$\hat{y}(x^*) = \mu + \epsilon \tag{1}$$

where $\mu$ is the mean value, and $\epsilon$ is the normally distributed error term with zero mean and variance $\sigma^2$. Different Kriging methods have been introduced by Goovaerts [9]. Universal Kriging, which is used in this paper, assumes that the mean value $\mu$ is a linear combination of known functions $f_0(x), \ldots, f_k(x)$ [10]. In the current paper, the linear functions are used where $f_0(x) = 1$ and $f_j(x) = x_j$ for $j = 1, \ldots, k$. Thus, a universal Kriging model with linear regression functions is written as

$$\hat{y}(x^*) = \sum_{h=0}^{k} \beta_h f_h(x^*) + \epsilon \tag{2}$$

To estimate the correlation for the error term, define a spatially weighted distance formula between samples $x^{(i)}$ and $x^{(j)}$ as

$$d(x^{(i)}, x^{(j)}) = \sum_{q=1}^{k} \theta_q |x_q^{(i)} - x_q^{(j)}|^{p_q} (\theta_q \geq 0, p_q \epsilon [1, 2]) \tag{3}$$

The parameter $\theta_q$ expresses the importance of the $q$th component, and the exponent $p_q$ is related to the smoothness of the function in coordinate direction $q$. A correlation matrix $R$ is then defined by

$$R = \begin{bmatrix} \exp[-d(x^1, x^1)] & \exp[-d(x^1, x^2)] & \ldots & \exp[-d(x^1, x^n)] \\ \exp[-d(x^2, x^1)] & \exp[-d(x^2, x^2)] & \ldots & \exp[-d(x^2, x^n)] \\ \vdots & \vdots & & \vdots \\ \exp[-d(x^n, x^1)] & \exp[-d(x^n, x^2)] & \ldots & \exp[-d(x^n, x^n)] \end{bmatrix}$$

To compute the Kriging model, values must be estimated for $\beta' s$ and $\sigma$ in conjunction with $\theta_1, \ldots, \theta_k$ and $p_1, \ldots, p_k$. This results in $3k + 2$ parameters to be calculated for a linear regression model. The parameters can be quantified using the maximum likelihood estimator, as described by Jones et al. [11].

The predictions at an unsampled location, $x^*$,can then be obtained from Eq. (1). Let $\mathbf{r}$ denote the $n$ vector of correlations between the new point $x^*$ and the previous $n$ sample points, based on the distance formula, giving

$$\mathbf{r} = \begin{bmatrix} \exp[-d(x^*, x^1)] \\ \exp[-d(x^*, x^2)] \\ \vdots \\ \exp[-d(x^*, x^n)] \end{bmatrix}$$

Then the Kriging estimate is given by

$$\hat{y}(x^*) = \sum_{h=0}^{k} \beta_h f_h(x^*) + \mathbf{r}^T R^{-1} (y - F\beta) \tag{4}$$

where $\beta = (\beta_0, \beta_1, \ldots, \beta_k)$ is the $k + 1$ dimensional vector of regression coefficients and

$$F = \begin{bmatrix} f_0(x_1^{(i)}) & f_1(x_1^{(i)}) & \ldots & f_k(x_1^{(i)}) \\ f_0(x_2^{(i)}) & f_1(x_2^{(i)}) & \ldots & f_k(x_2^{(i)}) \\ \vdots & \vdots & & \vdots \\ f_0(x_n^{(i)}) & f_1(x_n^{(i)}) & \ldots & f_k(x_n^{(i)}) \end{bmatrix}$$

A confidence interval can be calculated for this prediction. If $x^*$ is close to sample points, there is a high level of confidence in the prediction. This is reflected by the expression for the mean squared error (MSE) of the predictor [5]:

$$s^2(x^*) = \sigma^2 - \mathbf{r}^T R^{-1} \mathbf{r} + (f(x^*) - \mathbf{r}^T R^{-1} F)(F^T R^{-1} F)^{-1} \times (f(x^*) - \mathbf{r}^T R^{-1} F)^T \tag{5}$$

where $f(x^*) = (f_0(x^*), f_1(x^*), \ldots, f_k(x^*))$ is the vector of regression coefficients corresponding to $x^*$.

## Sampling for Improved Kriging Predictor

The sampling quality depends both on the number and distribution of the samples. An effective pattern puts samples in regions of nonlinear behavior. In general, we need a systematic method for the generation of samples for Kriging that ensures that the uncertainty in the prediction of the target function is minimized. To minimize this uncertainty, we exploit for sample generation the Kriging predicted mean squared error. A second function, the expected improvement function, is defined to generate dense samples at the global minimum or maximum. In both cases, the information available in previous samples is exploited to drive the sampling.

The Kriging predictor provides an estimation of the MSE, given in Eq. (5). The MSE is zero at observed points, increases as the distance between samples increases, and can be used as the criterion for sample generation. A sample is generated at the location where the MSE is maximum.

To illustrate this approach, the pressure distribution over an aerofoil is used as the unknown function. In this case, the independent variable is the normalized distance from the leading edge $(x/c)$ and is in the range of [0,1]. Starting with three samples, two are located at the borders of the parameter space and the third is at a random location in the range. Samples at the borders eliminate the need for extrapolation. Kriging is applied to these three samples, and the location of the MSE is maximized to locate a new sample. This procedure is repeated until the maximum error falls below a specified tolerance, and then the sampling stops. The Kriging function and the MSE are shown in Fig. 1 for varying numbers of samples. This shows that, by increasing the number of samples from three to 15, with new samples positioned at the location of maximum MSE, Kriging produces an accurate representation of the target function. The samples are spread over the parameter space (in contrast to being concentrated around maxima or minima when using the expected improvement function, as discussed next).

In applications in which the maximum and minimum values of the function are needed, new samples must be introduced in nonlinear regions. Sampling based on the MSE puts all of the effort on a global search of the function, driven by the weighted distance correlation for the error terms. In practice, a combined global and local search is needed [11]. To illustrate, an estimate of the minimum value of the function is first based on the available samples, that is, $y_{\min} = \min(y^{(1)}, y^{(2)}, \ldots, y^{(n)})$. The Kriging predictor at any point can be regarded as a random variable with mean given by the predictor and variance given by the mean standard error. Viewed in this way, a probability can be computed that the value at any point will fall below the current minimum. The expected improvement function (EIF) is obtained by weighting the possible improvements by these probability densities and is written as

$$E[I(x)] = \begin{cases} (y_{\min} - \hat{y})\Phi(\frac{y_{\min}-\hat{y}}{s}) + s\phi(\frac{y_{\min}-\hat{y}}{s}) & s > 0 \\ 0 & s = 0 \end{cases} \tag{6}$$

where $\phi$ and $\Phi$ are the normal density and distribution functions. The new samples are located where the expected improvement function has a maximum. A similar formulation can be used to search for global maxima. Note that local maxima, local minima, and inflection points will not be found by this sampling method.

To illustrate the performance of this sampling technique, the aerofoil test case is again considered. The example shown aims to capture the global minimum point using EIF sampling. The Kriging approximation and the EIF are both shown in Fig. 2 for varying

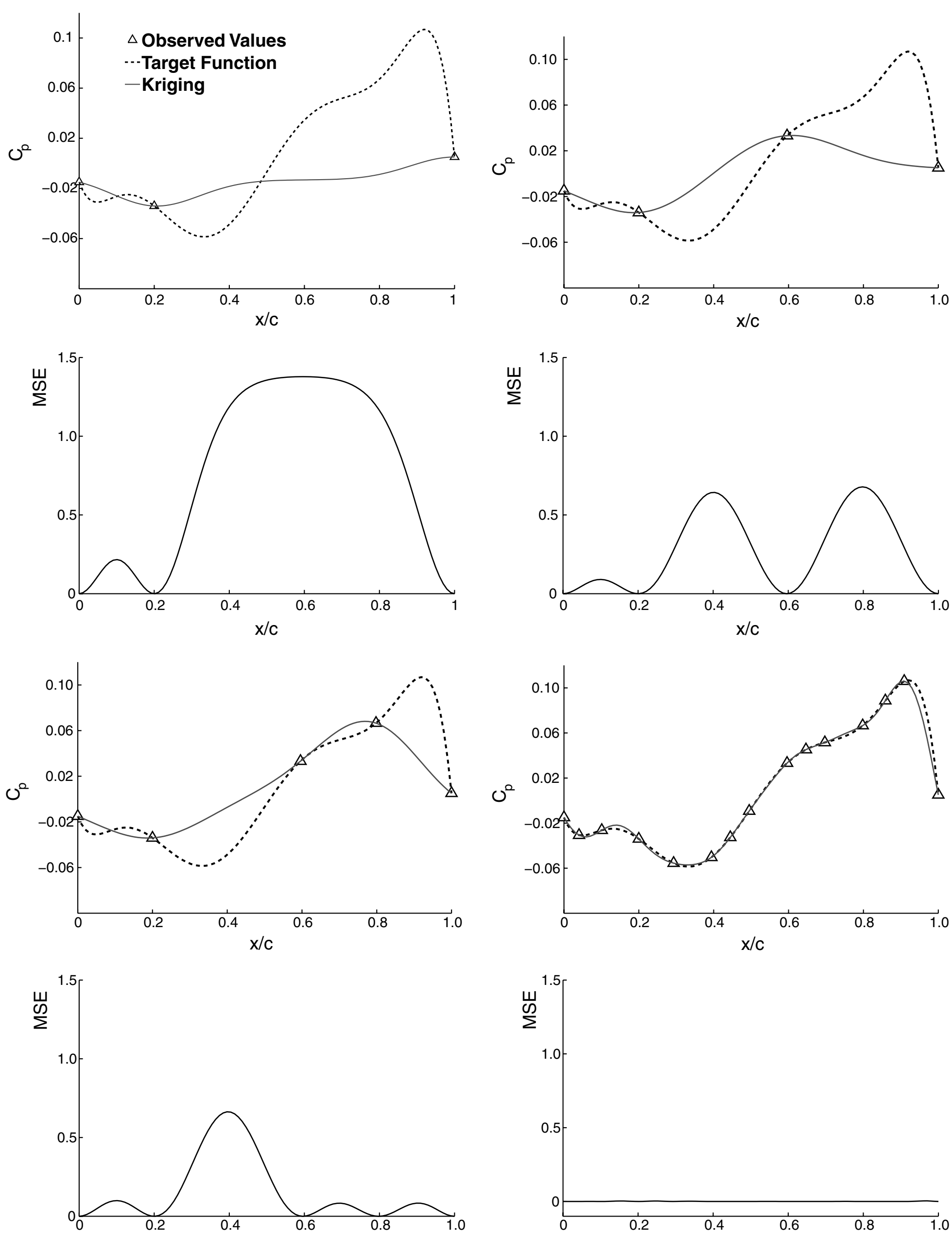


**Fig. 1 Kriging approximation of the aerofoil pressure distribution based on samples generated using the MSE.**

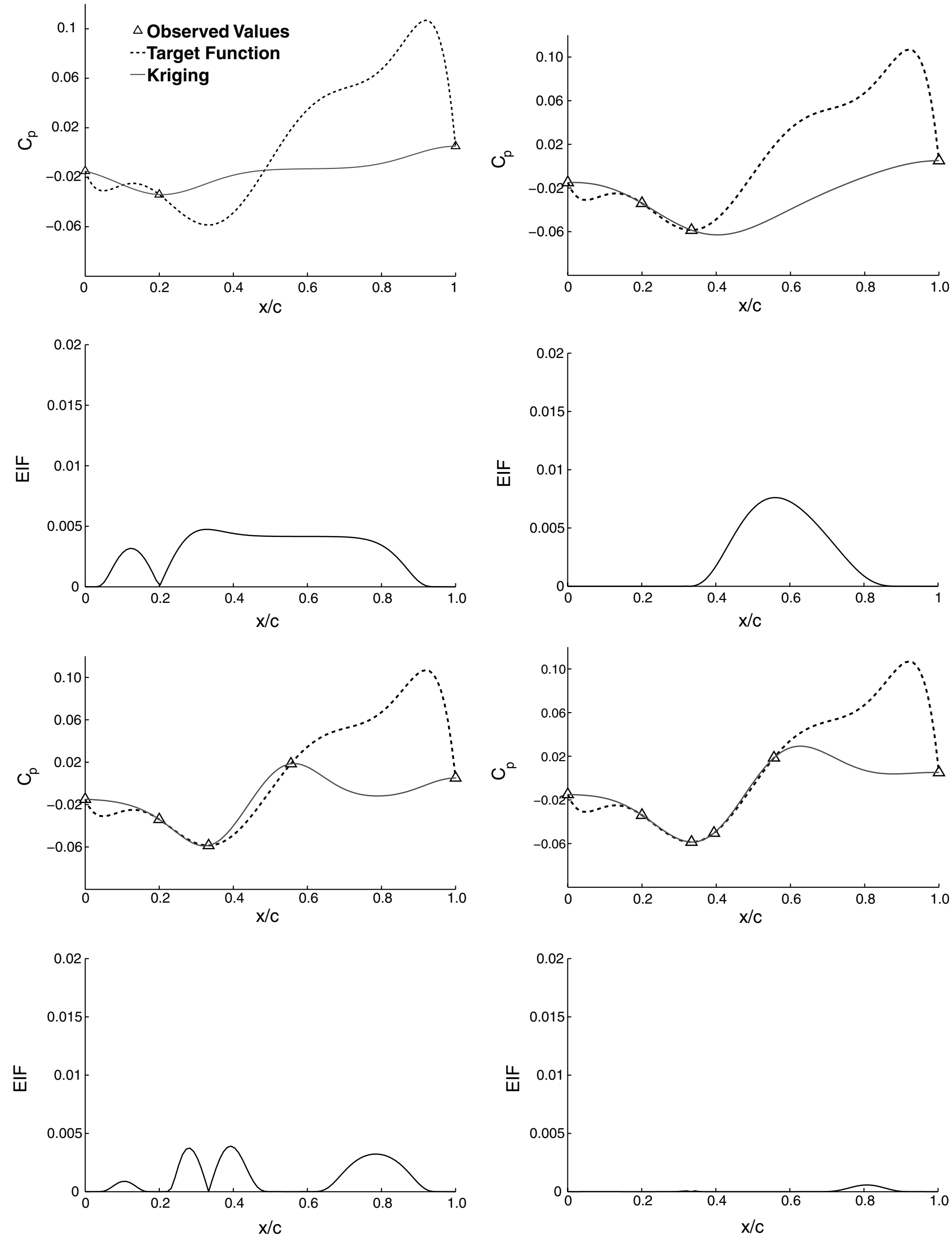


**Fig. 2 Kriging approximation of the aerofoil pressure distribution based on samples generated using the EIF.**

numbers of samples. The location of samples in Fig. 2 can be contrasted with those shown in Fig. 1. The MSE sample locations are strongly influenced by the consideration of reducing uncertainty generated by gaps in the parameter space (i.e., large distances between samples). In contrast, the EIF uses the emerging approximation of the function to locate samples where the global minimum may be reduced. The global minimum is sharply defined using a small number of EIF samples. A similar approach could be used to locate the global maximum. The magnitude of the MSE and EIF is used to terminate the sampling.

## Data Fusion

Data fusion combines samples from different sources. It is assumed that data are available from two sources that are, respectively, expensive and cheap to evaluate. The cheap samples are considered to provide information at least about the trend of the target function, whereas the expensive samples give quantitative information. The formulation used builds on the Kriging formulation described earlier and is called cokriging.

To explain the approach, first consider the cheap samples. A Kriging function $\hat{\eta}$ is calculated using just these samples. This Kriging function is then evaluated at the location of the expensive samples, $\hat{\eta}(x^i)$. As a second stage, the expensive samples are augmented by this evaluation of the Kriging function for the cheap samples, that is, the sample locations are now $k + 1$ dimensional vectors $x^{(i)} = (x_1^{(i)}, \ldots, x_k^{(i)}, \hat{\eta}(x^i))$ with corresponding observations $y^{(i)} = y(x^{(i)})$ for $i = 1, \ldots, n$. A Kriging function is calculated for these augmented samples, with the extra component $\hat{\eta}(x^i)$ bringing information to the correlation calculation from the cheap samples.

To illustrate cokriging, the aerofoil pressure distribution example is again used. Three samples of the expensive function are generated (see Fig. 3). Two samples are positioned at the borders of the parameter space to avoid extrapolation. The third sample is located randomly in the interior. Many cheap predictions were generated that had the correct trend of the target function, but with erroneous values. The cheap samples were actually based on the expensive samples with an applied (known) error. The cokriging of the cheap and expensive data produces an excellent approximation of the target

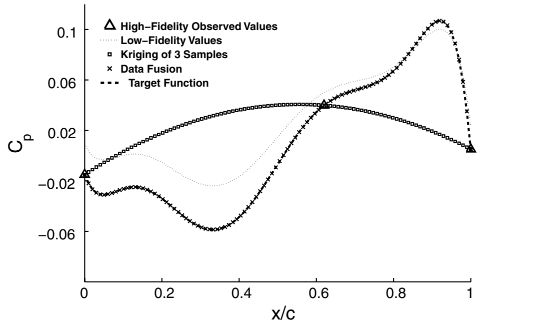


Fig. 3 Cokriging approximation of the aerofoil pressure distribution based on many cheap samples and three expensive samples.

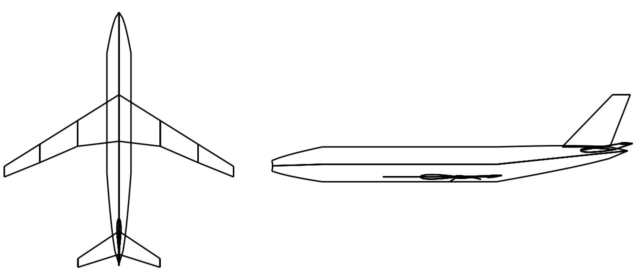

Fig. 4 Passenger jet aircraft model.

function using only three expensive samples. The role of these samples is to correct the values of the cheap samples, which nonetheless provide valuable information about the trend of the target function. In contrast, Kriging applied to the three expensive samples alone provides a poor representation of the target function.

## Aerodynamic Model Requirements

In this paper, the aerodynamic inputs to conceptual design and flight simulation are considered together to cater to a requirement to include improved stability and control analysis in design. For flight simulation, a tabular model is commonly used. To illustrate this, twelve variables are included in the tables: 1) the state variables of angle of attack, Mach number, sideslip angle, pitch rate, yaw rate, roll rate, time rates of angles of attack, and sideslip; and 2) the control variables of the deflection angles of the elevator, aileron, rudder, and wing inboard flaps.

With three forces and three moments depending on these parameters, a very large table is required for the general case (the size of the table rising with a power of 12). This is unfeasible from storage and calculation viewpoints (to illustrate, if five values are used to provide a very coarse resolution for each parameter in the table, the total number of entries in the table would be $5^{12}$, which is already of magnitude $100 \times 10^6$). It is also probably unnecessary. The main aerodynamic variables are taken to be the angle of attack and the Mach number. All forces and moments are assumed to depend on these variables in combination with each of the remaining variables separately. Hence, there are ten tables for each force and moment, which are then each three dimensional. The sampling and fusion framework is applied next to generate tables of this type. However, the framework is also applicable to higher-dimension tables, if this is required.

Two modes of data generation are considered. These are based on the following scenarios. First, it is assumed that the requirement is for a model based on high-fidelity CFD data, and that this can be generated offline (i.e., the calculation can perhaps be done overnight without a user waiting for the model during an interactive session). In this scenario, the emphasis is on finding nonlinearities in the forces and moments. The second scenario is when a designer is involved in an interactive session. It is assumed that the aircraft geometry is incremented from an initial design, perhaps selected from a library of designs, and that a high-fidelity model is available for the initial design from the first scenario. Data fusion is then used to update this initial model, based on a small number of calculations at an acceptable cost. In this scenario, it is assumed that the flow topology resulting from the initial geometry does not change due to the geometry increments. If this is not the case (e.g., the wing sweep angle increases so that vortical flow starts to dominate at high angles of attack), then either a new initial geometry/table needs to be selected or the interactive session needs to be suspended so that a new high-fidelity model can be generated under the first scenario.

## Results

### Overview

The application of the methods described herein to the initial generation and incrementing of aerodynamic tables is considered in

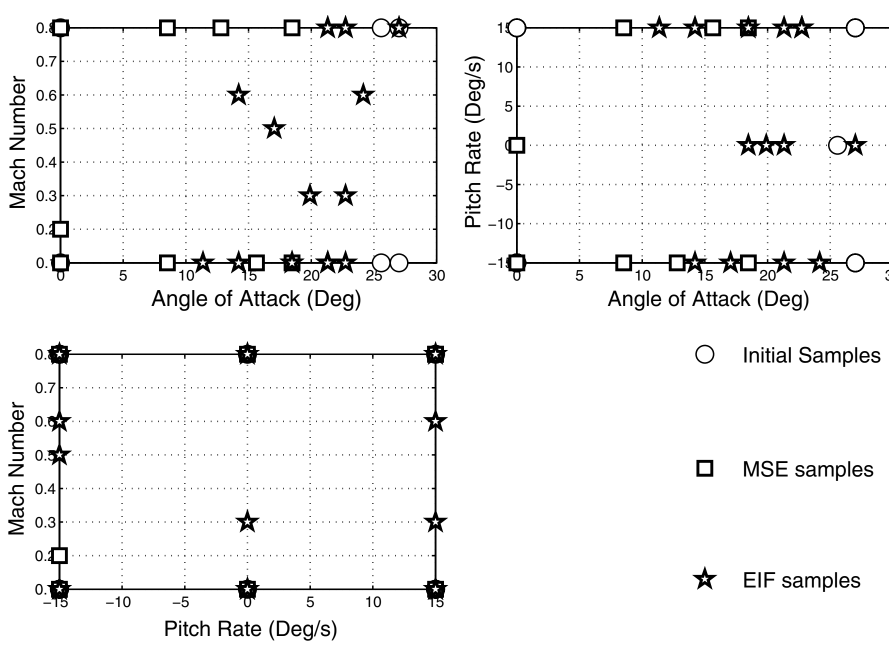


Fig. 5 Final location of the samples for the Mach number/angle of attack/pitch rate table.

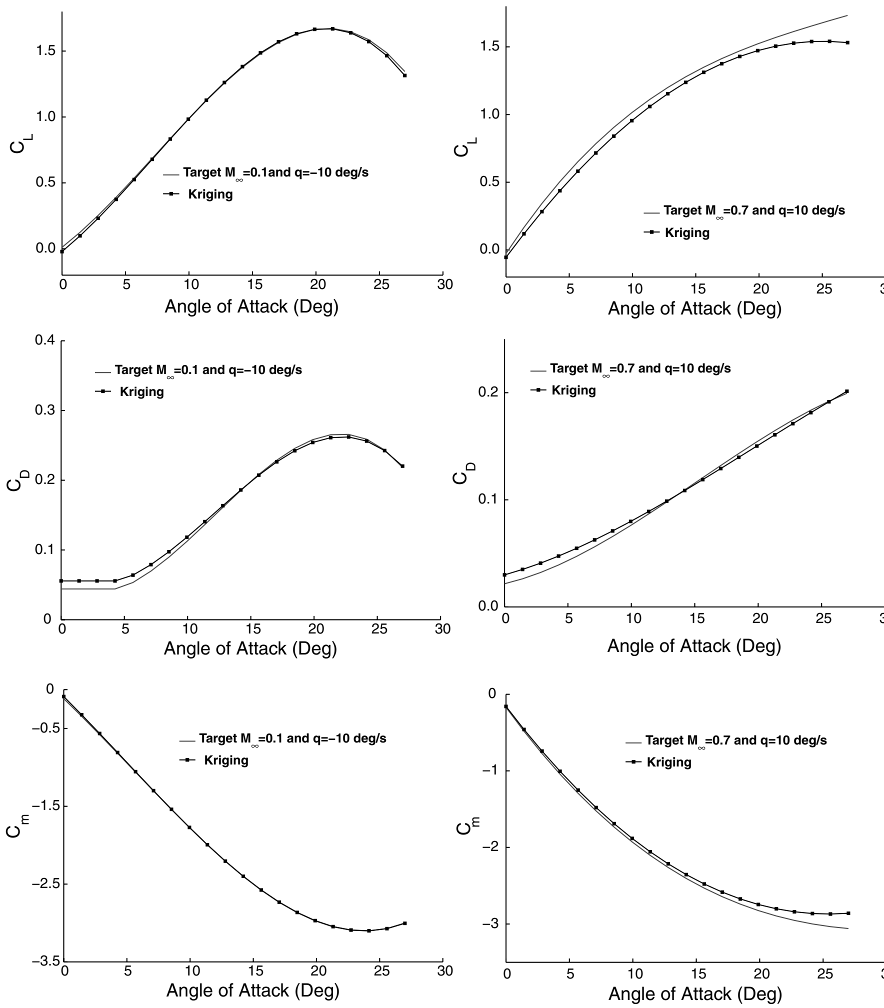


**Fig. 6 Comparison of brute force (marked "target") and Kriging predictions of force and moment coefficients for the Mach number/angle of attack/ pitch rate table. The label "q" is the pitch rate and is in degrees per second.**

this section. The methods have been implemented in MATLAB®.[§] The geometry used is based on a passenger jet wind-tunnel model, shown in Fig. 4, and is representative of a civil transport. DATCOM [12] is used to calculate the aerodynamic forces and moments to allow a comprehensive testing of the sampling, Kriging, and data fusion. It is expected that DATCOM will give representative aerodynamic behavior at a low cost. The objective of the exercise is to establish an approach that can be practically used with CFD-generated data. This is evaluated in terms of the number of samples required to accurately resolve the forces and moments.

### High-Fidelity Model

The tables are generated for Mach number, angle of attack, and one other variable at a time. For the purposes of illustration, results are shown for sideslip and the pitch, roll and yaw rates. Each table has 2080 entries to resolve the force and moment variations. The first step was to generate all entries using brute-force calculation. The objective of the generation based on sampling and Kriging is to reproduce this table with a small number of samples. Note that the sampling is terminated in the examples presented when the magnitude of the EIF is driven down 6 orders of magnitude from the initial value.

The approach is illustrated using the Mach number/angle of attack/ pitch rate table, because the longitudinal forces and moments in DATCOM are a function of all three parameters. First, 10 samples are generated along the borders of the parameter space to avoid extrapolation by the Kriging function and inside the parameter space using the LHS method. The initial samples are shown by circles in Fig. 5. To build up a better picture of the behavior of the forces and moments, MSE sampling was used to define the next nine samples (shown by squares in Fig. 5). MSE sampling looks for samples at the maximum of the mean squared error. For each added sample, Kriging is again used to approximate the function with all available samples and the MSE is updated. The results show that the additional samples are located along the border of the parameter space with respect to the angle of attack and fill in the gaps between the initial samples. For Mach number and pitch rate, the generated samples are mainly toward the minimum and maximum values (i.e., 0.1 and 0.8 for Mach number and $-15$ and 15 deg/s for pitch rate). In the target function, the main nonlinearity is from the angle of attack and the EIF sampling places samples around the stall angle over the range of Mach numbers. The influence of pitch rate is weak, and the samples are placed at the extrema and the zero value.

The EIF sampling was then used to refine the Kriging approximation. The version to locate the maxima and minima were used in turn. The locations of these samples are shown by stars in Fig. 5, and they cluster in the interior of the parameter space, toward higher angles of attack, where, for example, the lift curve has a maximum. In total, 35 samples were generated to approximate the behavior of the forces and moments, with the magnitude of the EIF used to terminate the sampling.

Next, a selection of predicted forces and moments are compared with the brute-force-generated tables in Fig. 6. This shows that all the features in the lift, drag, and pitching moment curves are essentially captured with the Kriging based on 35 samples.

A similar sampling approach was applied to the table of Mach number/angle of attack/roll rate and the results are shown in Fig. 7.

[§]Data about MATLAB® V2008a from The MathWorks, Inc., is available at http://www.mathworks.com [retrieved 27 February 2009].

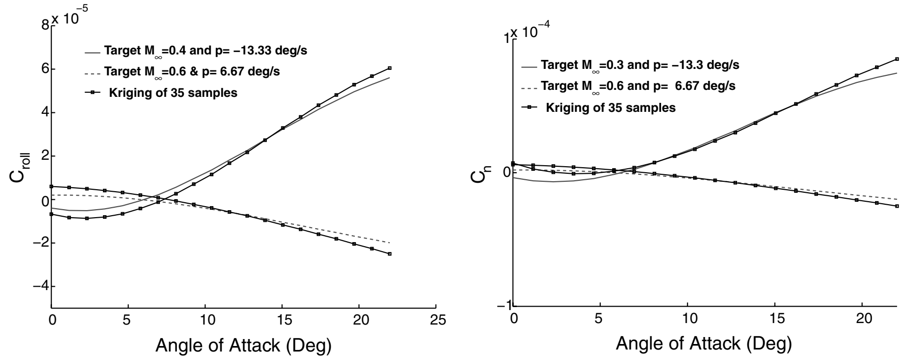


**Fig. 7 Comparison of brute force (marked "target") and Kriging predictions of force and moment coefficients for the Mach number/angle of attack/roll rate table. The label "p" is the roll rate.**

Again, a small number of samples can reproduce the brute-force results.

The conclusion from this study is that we can generate the tables to good accuracy based on 35 samples generated using MSE and EIF sampling. This would allow CFD to be used as a source of the data.

### Model Increment

Next, we illustrate how the aerodynamic tables can be derived, for an incremented design, from an initial high-fidelity table. The geometry increments are assumed not to change the flow regimes captured in the initial table, and the task is to exploit the trends in this table while updating values. The aim is to replicate the brute-force-generated table for the new geometry using a very small number of additional samples. To test the approach, a number of geometry increments for the passenger jet were defined and are shown in Fig. 8.

The high-fidelity aerodynamic model for the original geometry was generated using the sampling techniques, as described in the previous section. Assuming that the trend of aerodynamic forces and moments remains unchanged for each incremented geometry, we

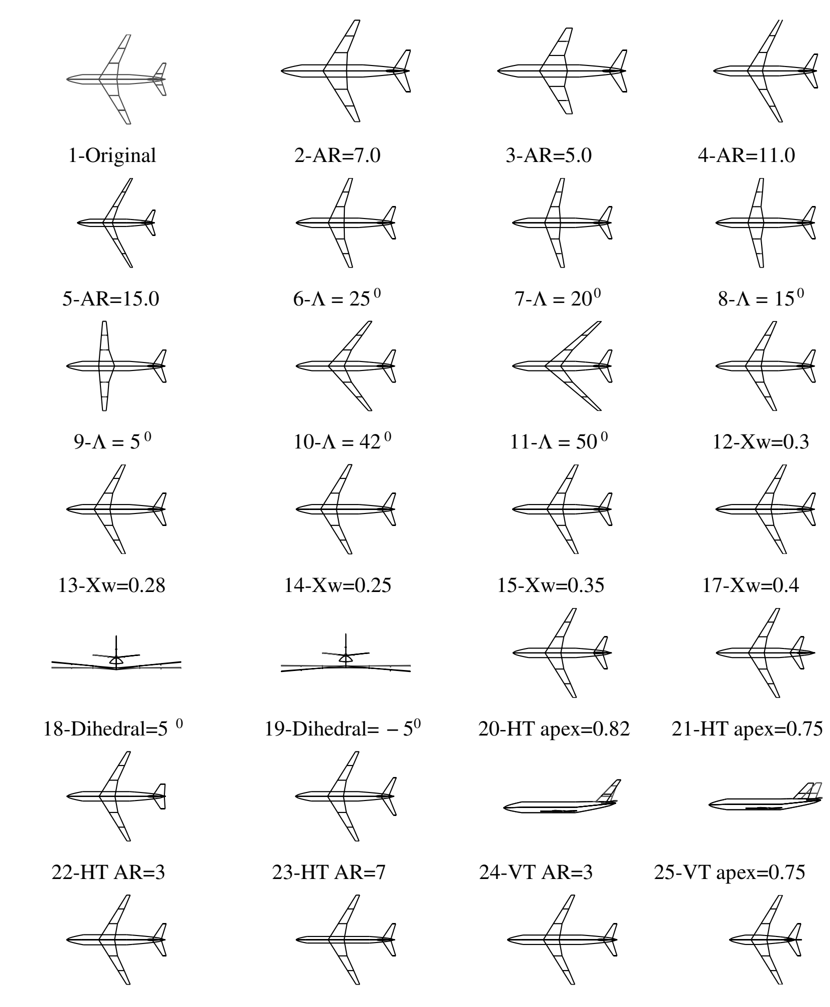


**Fig. 8 Geometry increments used to test the data fusion approach. AR: aspect ratio; Λ: wing sweep; *Xw*: location of the wing down the fuselage; Dihedral: dihedral of the wing; VT AR: aspect ratio of the vertical tail; VT apex: apex of the vertical tail; HT AR: aspect ration of the horizontal tail; HT apex: apex of the horizontal tail; Fus. diameter: diameter of fuselage; Fus. length: fuselage length. Note that in cases 18, 19, 24, and 25 the outline of the original geometry is included to emphasize the geometry increment involved.**

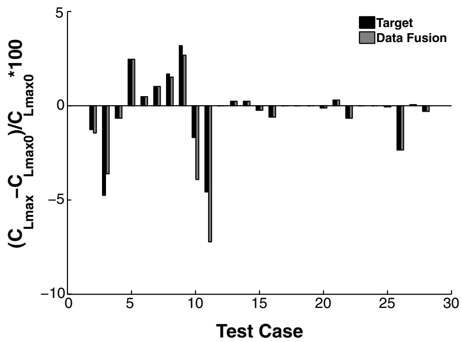


**Fig. 9 Comparison of cokriging-predicted maximum lift coefficient from original case with target values.**

attempt to generate new tables using cokriging with evaluations from the original table as the cheap observation and a very small number of simulations as the expensive evaluations.

The first example considers the high-fidelity table including angle of attack, Mach number, and angle of sideslip. This table again has 2080 entries. For data fusion, we limit the number of simulations to only 10 cases. The samples are located at the high-angle-of-attack border and the angles of attack in the original table that produce the maximum lift coefficient. These samples are expected to be located in the regions of most nonlinearity in the updated tables.

The quantitative comparison of maximum lift coefficient at $M_\infty = 0.1$ is shown in Fig. 9. There is a sharper stall at low speed within the range of angles of attack; hence, the low speed was chosen for the comparison. The bar chart shows the percentage of variation from the original geometry value of $C_{L\max}$ from both the target and fusion values for all test cases. Here the target values are evaluated from the full brute-force table. Figure 9 shows that there is a close agreement between the predicted values from cokriging and those simulated from brute-force calculation. Predicted values are closer for small increments or those parameters that have little influence on the dependent variable as expected. Note that all cokriging predictions have the same sign for the change as the target.

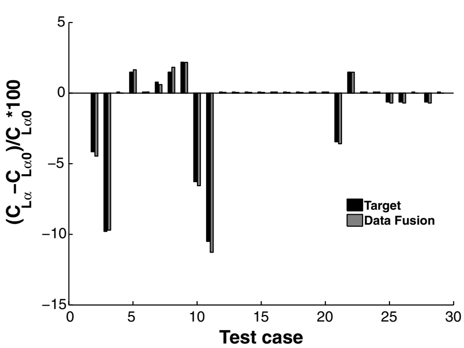


**Fig. 10 Comparison of cokriging-predicted lift curve slop from original case with target values.**

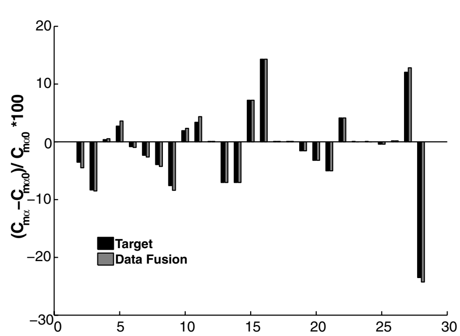


**Fig. 11 Comparison of cokriging-predicted pitch moment curve slope from original case with target values.**

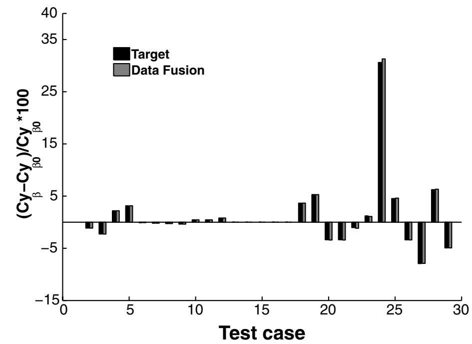


**Fig. 12 Comparison of cokriging-predicted $C_{y\beta}$ from original case with target values.**

Likewise, the plots of the lift curve slope at high speed are presented in Fig. 10. The figure shows that the predicted values from fusion match favorably with the target values. For the pitching moment comparison, the change in $C_{m\alpha}$ is illustrated in Fig. 11. This parameter is frequently used for aircraft stability and control. The results again show a close match of cokriging and target values. Results for the lateral force are presented in Fig. 12. The predictions are very close to the target values. The agreement of $C_{y\beta}$ is close for all cases. This is not strictly a hard case for the method because DATCOM provides a simple lateral side force correlation that only varies linearly with flight speed and does not vary with angles of attack and sideslip.

## Conclusions

A framework based on sampling and data fusion has been presented for the generation of aerodynamic tables for flight simulation. Two modes of use are anticipated. In the first, sampling is used to generate high-fidelity tables (in the sense that nonlinearities are represented). In the second, interactive geometry changes (such as those shown that might be applied during an interactive conceptual design session) are assumed from the initial geometry, and data fusion with the initial aerodynamic tables is used. The brute-force development of a three-dimensional table requires around 2000 evaluations of the aerodynamic forces and moments (in this case, using DATCOM). The sampling for the high-fidelity table allows the same result to be obtained using 35 evaluations. The fusion allows the table for the incremented geometry to be obtained in 10 samples. This level of performance makes it conceivable to apply CFD as the source of the aerodynamic data.

Future work will include the following: 1) the use of CFD-generated data, requiring sampling to be run on a desktop machine, with the calculation of the observations on remote clusters; 2) the investigation of data fusion for aerofoil section changes; and 3) the testing of the data fusion for different aircraft topologies, giving rise to different aerodynamic regimes.

The most important research topic is the development of a sampling approach that can locate nonlinearities away from the global extrema. Methods that exploit flowfield information are being investigated.

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