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Acceleration of cosmic ray electrons by ion-excited waves at quasi-perpendicular shocks

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ABSTRACT

The standard model of cosmic ray electron acceleration requires electrons to be pre-accelerated to mildly relativistic energies. It has been suggested that energy transfer from waves, generated by gyrotropic ions reflected from quasi-perpendicular shocks, could provide the necessary pre-acceleration. The distribution of shock-reflected ions at any upstream point is more likely to consist of two beams rather than a gyrotropic ring. Wave excitation in the presence of both types of ion distribution is studied. It is shown that gyrotropic or beam ions, reflected from shocks associated with supernova remnants, can excite waves capable of accelerating electrons to beyond the required injection energies. The wave group velocity along the shock normal can be approximately equal to the shock velocity: such waves are not rapidly convected away from the shock, and can thus grow to a high level. Moreover, waves satisfying this condition which also have phase velocities parallel to the magnetic field ranging from the electron thermal speed to relativistic speeds are excited in high Mach number shocks with a low ratio of electron plasma frequency to cyclotron frequency. Bulk electrons can then be accelerated to the required energies within the region in which shock-reflected ions are present. This is consistent with a suggestion, based on a comparison between Wolf–Rayet stars and radio supernovae, that there exists a threshold perpendicular shock speed (between 1 and 3 per cent of the speed of light) above which the efficiency of electron injection increases by several orders of magnitude.

Key words: plasmas – shock waves – waves – supernovae: general – cosmic rays – ISM: general.

1 INTRODUCTION

The 'standard' astrophysical particle acceleration mechanism, diffusive shock acceleration (also referred to as Fermi acceleration at shocks), is only capable of accelerating electrons if they are already sufficiently energetic to pass freely through any electrostatic potential barriers associated with the shock front and also have magnetic rigidities sufficiently high to be scattered by the quasi-static magnetic structures invoked to scatter and accelerate the ions (Bell 1978). The first condition is satisfied if the electron energy exceeds $m_p v_s^2/2$, where m_p is the proton mass and v_s is the shock speed (assumed here to be non-relativistic). The second condition is much more stringent, and is satisfied if the electron momentum is greater than $m_p v_s$. For interstellar shock velocities of the order of a few thousand km s^{-1}, this suggests that the conventional diffusive shock mechanism can accelerate electrons with initial energies of the order of 10 MeV. However, some additional process is needed to bridge the gap between the thermal electron population and the relativistic region. The need for electron pre-acceleration, often referred to as electron injection, arises physically from the fact that there is a significant interval of energy or velocity space in which the electrons are magnetized and the ions are not, with the consequence that the collective interactions of the two species with the plasma must be quite different.

This injection problem has been studied in the context of acceleration at parallel collisionless shocks by several authors, notably Levinson (1994) and Bykov & Uvarov

(1993), but electron injection at quasi-perpendicular shocks has received less attention. Galeev (1984) proposed a mechanism whereby electrons could be accelerated to ultra-relativistic energies, involving the excitation of waves with frequencies between the ion and electron cyclotron frequencies, immediately upstream of high Mach number quasi-perpendicular shocks. Wave excitation is driven by ions reflected by a potential barrier in the shock: it has been suggested that such a mechanism is responsible for the acceleration of electrons at the quasi-perpendicular bow shock of the Earth (Papadopoulos 1981), at interplanetary shocks (Vaĭsberg et al. 1983), at shocks associated with type II solar radio bursts (Krasnosel'skikh et al. 1985; Thejappa 1987), and in solar flares (McClements et al. 1990, 1993). Recently, Galeev, Malkov & Völk (1995) have suggested that the flux of accelerated electrons could be greatly enhanced by a macroscopic electric field required to maintain quasi-neutrality. Krasnosel'skikh (1992) has argued that instabilities driven by shock-reflected ions are unlikely to play a significant role in the acceleration of electrons at high Mach number shocks, on the grounds that the waves have group velocities which are much smaller than the shock speed, and are thus convected away from the region in which reflected ions are present before they can grow to a significant level.

In this paper we re-examine the question of whether or not waves excited by shock-reflected ions are capable of accelerating electrons out of a thermal pool up to the required injection energies. Levinson (1996) has noted that the distribution function of the reflected ions, upon which the acceleration efficiency partly depends, is still an open issue. Accordingly, we consider the linear stability of electromagnetic waves using two alternative representations of the reflected ion distribution, the parameters of which are determined self-consistently by the shock Mach number. On the basis of a comparative study of radio emissions from supernova remnants and Wolf–Rayet stars, and various model assumptions, Biermann & Cassinelli (1993) have suggested that the injection efficiency of electrons at perpendicular shocks may increase by several orders of magnitude when the shock speed exceeds a threshold, which they estimate to lie between 1 and 3 per cent of the light. This result is strongly model-dependent, and should thus be treated with caution. However, the possible existence of such a threshold indicates the importance of assessing the extent to which the efficiency of any proposed acceleration mechanism depends on shock speed. This is particularly so in the present context, in view of the suggestion by Krasnosel'skikh (1992) that electron acceleration by waves associated with shock-reflected ions may become less effective as the Mach number increases.

2 SHOCK-REFLECTED ION DISTRIBUTION

Ion reflection from a perpendicular shock is said to be 'specular' if kinetic energy is conserved in the shock frame. The trajectory of a specularly reflected ion can be represented by (Phillips & Robson 1972)

$$x(t)=\frac{v_s}{\Omega_i}[\Omega_i t-2\sin\Omega_i t], \tag{1}$$

$$y(t)=2\frac{v_s}{\Omega_i}[1-\cos\Omega_i t] \tag{2}$$

(see Fig. 1, discussed below). Here, x (<0) denotes the distance of a reflected ion from the shock (in the rest frame of the shock), the magnetic field lies along the z direction, Ω_i is the upstream ion cyclotron frequency, and t is the time which has elapsed since reflection. Thus, the x-direction defines the direction of plasma flow into the shock. We have assumed that the upstream plasma is cold, so that the velocity of an incoming upstream ion relative to the shock is unique and equal to v_s. The assumption of specular reflection means that $\dot{x}(0)=-v_s$, $\dot{y}(0)=0$. Recent analytical studies by Gedalin (1996) have shown that, in practice, the reflection process is always non-specular, in the sense that it results in a net loss of kinetic energy in the shock frame. This result is consistent with particle simulations carried out by Leroy et al. (1981) and Burgess, Wilkinson & Schwartz (1989), indicating that ions are invariably reflected back into the upstream plasma with velocities less than v_s. If $\dot{x}(0)=-\epsilon v_s$, where $0<\epsilon<1$, and $\dot{y}(0)=0$, the trajectory of a reflected ion is defined by

$$x(t)=\frac{v_s}{\Omega_i}[\Omega_i t-(1+\epsilon)\sin\Omega_i t], \tag{3}$$

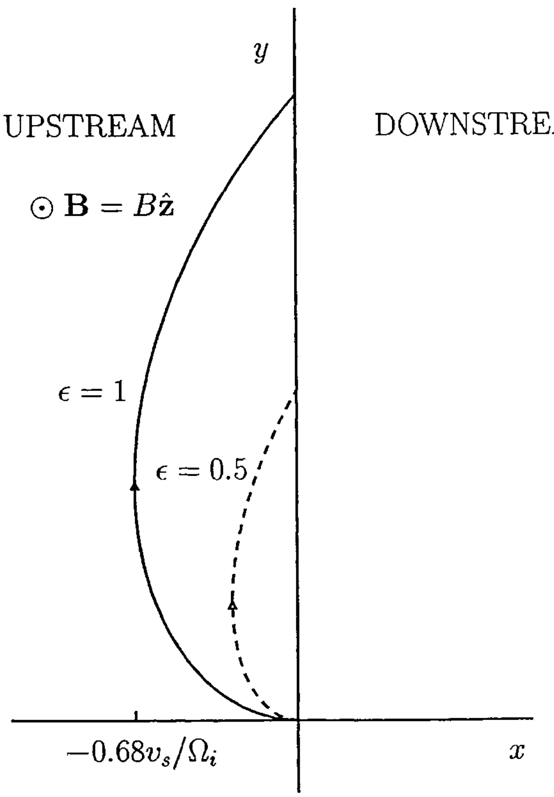

Figure 1. Upstream trajectories of ions reflected from a perpendicular shock with initial velocities $\dot{x}(0)=-\epsilon v_s$, $\dot{y}(0)=0$, where $\epsilon=1$ (solid curve) and 0.5 (broken curve). The shock normal lies in the $-x$ direction, and the magnetic field lies in the z direction. The maximum value of $|x|$, and thus the thickness of the shock foot, is given by equation (5).

$$y(t)=(1+\epsilon)\,\frac{v_s}{\Omega_i}\,[1-\cos\Omega_i t]. \qquad (4)$$

Regardless of the value of ϵ, the reflected ion re-encounters the shock after less than half of one cyclotron period. The upstream trajectories represented by equations (1)–(2) and (3)–(4) are indicated by the solid curve ($\epsilon=1$) and broken curve ($\epsilon=0.5$) in Fig. 1, respectively. In each case, the reflected ion first encounters the shock while moving at velocity v_s along the $+x$ axis. In the case of specular reflection ($\epsilon=1$), it is straightforward to show, using equation (1), that the maximum value of $|x|$ is approximately $0.68v_s/\Omega_i$. When $\epsilon<1$, equation (3) indicates that

$$|x|\le x_{\max}=\frac{v_s}{\Omega_i}\left[\epsilon^{1/2}(2+\epsilon)^{1/2}-\cos^{-1}\left(\frac{1}{1+\epsilon}\right)\right]<0.68\,\frac{v_s}{\Omega_i}. \qquad (5)$$

The quantity $x_{\max}$ defines the size of the 'shock foot', in which reflected gyrating ions are present. Direct observations of the quasi-perpendicular bow shock of the Earth (Scudder et al. 1986), hybrid simulations (Burgess et al. 1989), and the analysis of Gedalin (1996), all indicate foot lengths of about $(0.3\text{–}0.4)v_s/\Omega_i$. According to equation (5), the corresponding values of ϵ range from about 0.5 to about 0.65. However, ϵ is not uniquely determined for a particular shock: the reflected ions exhibit considerable velocity dispersion (Gedalin 1996). In the present paper, for simplicity, we assume that the upstream ions are cold, and that ϵ has a unique value. In these circumstances, it is apparent from Fig. 1 that the reflected ion velocity distribution in the upstream plasma frame f_r, at a particular point within the shock foot, consists of two monoenergetic beams: one moving away from the shock, the other moving towards it. Assuming that the two beams are of equal density, we can write

$$f_r(x,\boldsymbol{v})=\frac{1}{2}\,n_r\delta(v_z)\{\delta[v_y-(1+\epsilon)v_s\sin\phi_1]$$
$$\times\,\delta[v_x+(1+\epsilon)v_s\sin\phi_1]+\delta[v_y-(1+\epsilon)v_s\sin\phi_2]$$
$$\times\,\delta[v_x+(1+\epsilon)v_s\sin\phi_2]\}, \qquad (6)$$

where n_r is the total reflected ion density, δ is the delta function, $|x|\le x_{\max}$ is now a specified distance upstream, and $\phi_1(x)$, $\phi_2(x)$ are the solutions of the equation

$$x=\frac{v_s}{\Omega_i}\,[\phi-(1+\epsilon)\sin\phi], \qquad (7)$$

where $\phi=\Omega_i t$.

If, on the other hand, the reflected ion distribution were to be represented as a gyrotropic, monoenergetic velocity space ring, an appropriate expression for f_r would be

$$f_r(v_z,v_\perp)=\frac{n_r}{2\pi\epsilon v_\perp}\,\delta(v_z)\,\delta[v_\perp-(1+\epsilon)v_s], \qquad (8)$$

where $v_\perp=(v_x^2+v_y^2)^{1/2}$. In this case f_r is assumed to be independent of x. Gyrotropic distributions such as this, which describe the reflected ion distribution spatially averaged over the shock foot, have generally been invoked in the context of wave excitation at quasi-perpendicular shocks (e.g. Papadopoulos 1981; Galeev 1984). In the following section we will demonstrate that non-gyrotropic distributions of the type represented by equation (6), which provide a more accurate description of the local distribution at a particular point within the shock foot, can give rise to stronger instability.

3 ION-EXCITED WAVE GROWTH RATES

For a plasma with an energetic ion population, the dispersion relation for waves propagating at large angles (>45°) with respect to the magnetic field with frequencies between the ion and electron cyclotron frequencies can be written in the form (Papadopoulos 1981)

$$1+\frac{\omega_{pe}^2}{\Omega_e^2}\frac{k_\perp^2}{k^2}\left(1+\frac{\omega_{pe}^2}{k^2c^2}\frac{1}{1+\beta_e}\right)-\frac{\omega_{pe}^2}{\omega^2}\frac{k_\parallel^2}{k^2}$$
$$\times\left(1+\frac{\omega_{pe}^2}{k^2c^2}\right)^{-1}-\frac{\omega_{pi}^2}{\omega^2}-D_r=0, \qquad (9)$$

where: ω_{pe}, Ω_e are the electron plasma and cyclotron frequencies; k is the magnitude of the wavevector $\boldsymbol{k}$, with components parallel and perpendicular to the magnetic field $k_\parallel$, $k_\perp$ ($k_\perp>k_\parallel$); c is the speed of light; β_e is the electron plasma beta, i.e. $2\mu_0 n_e T_e/B^2$, where μ_0 is the free space permeability, n_e is the electron density, T_e is the electron temperature in energy units and B is the magnetic field; ω_{pi} is the plasma frequency of the bulk ions (i.e. the ions which have not been reflected by the shock); and D_r is a function of ω and $\boldsymbol{k}$ which depends on the energetic ion distribution. In the case of a reflected ion distribution given by equation (6) we have (McBride et al. 1972)

$$D_r=\frac{1}{2}\left[\frac{\omega_{pr}^2}{(\omega-\boldsymbol{k}\cdot\boldsymbol{v}_1)^2}+\frac{\omega_{pr}^2}{(\omega-\boldsymbol{k}\cdot\boldsymbol{v}_2)^2}\right], \qquad (10)$$

where $\boldsymbol{v}_1=[-(1+\epsilon)v_s\cos\phi_1,\ (1+\epsilon)v_s\sin\phi_1,\ 0]$ and $\boldsymbol{v}_2=[-(1+\epsilon)v_s\cos\phi_2,(1+\epsilon)v_s\sin\phi_2,0]$. The corresponding result for a monoenergetic gyrotropic ion distribution is (Papadopoulos 1981).

$$D_r=\frac{\omega_{pr}^2\omega}{[\omega^2-(1+\epsilon)^2k_\perp^2v_s^2]^{3/2}}. \qquad (11)$$

In the absence of energetic ions, equation (9) trivially yields

$$\omega^2=\frac{\omega_{pe}^2\dfrac{k_\parallel^2}{k^2}\left(1+\dfrac{\omega_{pe}^2}{k^2c^2}\right)^{-1}+\omega_{pi}^2}{1+\dfrac{\omega_{pe}^2}{\Omega_e^2}\dfrac{k_\perp^2}{k^2}\left(1+\dfrac{\omega_{pe}^2}{k^2c^2}\dfrac{1}{1+\beta_e}\right)}. \qquad (12)$$

For $k^2c^2\gg\omega_{pe}^2$ (the electrostatic limit) and $k_\parallel\to0$, equation (12) reduces to $\omega=\omega_{pi}/(1+\omega_{pe}^2/\Omega_e^2)^{1/2}$, which is the lower hybrid frequency ω_{LH}. More generally, waves described by equations (9)–(12) are often referred to as lower hybrid waves (see, for example, McBride et al. 1972). They are characterized by $k_\perp\gg k_\parallel$, and hence $\omega/k_\parallel\gg\omega/k_\perp$. If the parallel phase velocity $\omega/k_\parallel$ is less than c, it is possible for wave energy to be resonantly transferred to electrons via the process of Landau damping. This provides an efficient method of electron heating and current drive in laboratory plasmas (e.g. Cairns 1993), and the waves are also believed to be an important source of electron energization in various space plasmas, including the Earth's auroral zones (Bryant et al. 1991; Dendy et al. 1995) and bow shock (Papadopoulos 1981). Most recently, X-ray emission detected by the *ROSAT* spacecraft from comet Hyakutake (Ganz 1996) has been attributed to electrons accelerated by lower hybrid wave turbulence (Bingham et al. 1997).

We now present numerical and analytical solutions of the dispersion relation equation (9), comparing and contrasting results obtained for the two classes of energetic ion distribution represented by equations (10) and (11). We begin with equation (11), since it contains fewer free parameters than equation (10), and it represents the case most frequently considered by previous authors.

3.1 Instability driven by gyrotropic reflected ions

In the gyrotropic case f_r is determined by the two parameters ϵ and n_r. As discussed above, $0.5 \lesssim \epsilon \lesssim 0.65$: here, we set $\epsilon = 0.5$. The concentration of shock-reflected ions n_r/n_i (n_i being the bulk ion density) is a function of the fast magnetosonic Mach number $M_{ms} \equiv v_s/(c_A^2 + c_s^2)^{1/2}$, where c_A, c_s are the upstream Alfvén and sound speeds. Paschmann & Sckopke (1983) used a simple analytical model to compute n_r/n_i for Mach numbers in the range 1–10: the results are almost independent of the plasma beta, and are broadly consistent with both numerical simulations (Leroy et al. 1982) and spacecraft observations of the Earth's bow shock. The reflected ion concentration is typically about 5 per cent when $M_{ma} = 3$, rising to about 20 per cent when $M_{ms} \gtrsim 8$. Here, we consider perpendicular shocks with Mach numbers in the range 2–9, and set the electron and ion plasma betas equal to 0.1. In that case, $M_{ms} \simeq v_s/c_A \equiv M_A$, the Alfvén Mach number. For each Mach number, we set n_r/n_i equal to the value obtained by Paschmann & Sckopke (1983). The only other dimensionless plasma parameter appearing in equation (9) is ω_{pe}/Ω_e, which we set equal to 200: this is appropriate for certain regions of the interstellar medium (Spitzer 1978).

Solving equations (9) and (11) numerically for the complex frequency $\omega \equiv \omega_0 + i\gamma$, we vary k and $\theta \equiv \cos^{-1}(k_{\parallel}/k)$ to obtain growth rates γ for the parameters listed above. We find that strong instability, such that $\gamma \gtrsim \Omega_i$, only occurs at wave numbers $k > \omega_{pe}/c$ and at propagation angles $\theta < 88.66°$, such that $k_{\parallel}/k_{\perp} > (m_e/m_p)^{1/2}$, where m_e is the electron mass. In these limits, with $\beta_e \ll 1$ and $\omega_{pe}^2 \gg \Omega_e^2$, equation (12) yields $\omega_0 \simeq \Omega_e k_{\parallel}/k_{\perp}$. Strongly growing waves satisfy the wave–particle resonance condition implied by equation (11), namely $\omega_0 \simeq (1+\epsilon)k_{\perp}v_s$. To accelerate electrons out of the bulk of the distribution, it is necessary that the parallel phase velocity $\omega_0/k_{\parallel}$ be of the order of, or greater than, the electron thermal speed $v_e = (2T_e/m_e)^{1/2}$. Equating these two speeds, and writing $\omega_0 = (1+\epsilon)k_{\perp}v_s$, we find that the corresponding propagation angle θ_{crit} is given in terms of dimensionless quantities by $\cot\theta_{crit} = (1+\epsilon)(m_e/m_p)^{1/2}\beta_e^{-1/2}M_A$. At $\theta > \theta_{crit}$, the parallel phase velocity exceeds the electron thermal speed. The solid circles in Fig. 2 represent the maximum growth rate at $\theta = \theta_{crit}$ for M_A ranging from 2 to 9.

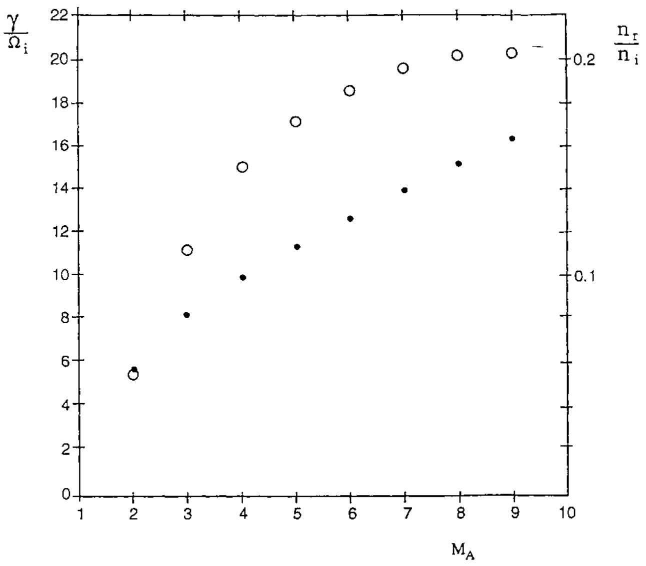

Figure 2. Open circles: concentration of shock-reflected ions n_r/n_i as a function of Alfvén Mach number M_A (from Paschmann & Sckopke 1983). Solid circles: maximum growth rate of waves in the lower hybrid range of frequencies propagating at an angle θ_{crit} (defined in the text) with respect to the magnetic field. The gyrotropic reflected ion distribution is assumed to be given by equation (8), with $\epsilon = 0.5$. The other parameters are: $\omega_{pe}/\Omega_e = 100$, $\beta_e = 0.1$.

The open circles show the values of n_r/n_i corresponding to particular values of M_A, according to Paschmann & Sckopke (1983). It is apparent that the maximum growth rate of waves with $\omega_0/k_{\parallel} = v_e$ increases steadily with Mach number.

To accelerate electrons to mildly relativistic injection energies, waves with parallel phase speeds $\omega/k_{\parallel} \simeq c$ must be present. For the parameter set assumed above, this is only possible if $k_{\parallel}$ is extremely small, in which case high growth rates do not occur. The reason for this is that equation (12), in this limit, reduces to the fast Alfvén wave dispersion relation $\omega = kc_A$: since $v_s > c_A$ and $k_{\perp} \simeq k$, the exact resonance condition $\omega \simeq (1+\epsilon)k_{\perp}v_s$ is impossible to satisfy. Numerically, we find that instability persists because of the finite effective width of the resonance, but the maximum growth rate is typically at least two orders of magnitude lower than Ω_i. In those circumstances, significant electron acceleration is unlikely to occur.

The situation changes if the Mach number is very high, or if ω_{pe}/Ω_e is significantly lower than the figure of 200 assumed above. The propagation angle at which $\omega_0/k_{\parallel}$ equals an arbitrary speed v_0 is given by

$$\cot\theta = (1+\epsilon)\left(\frac{m_e}{m_p}\right)^{1/2}\frac{\Omega_e}{\omega_{pe}}\frac{c}{v_0}M_A. \tag{13}$$

Thus, for θ to be less than $\cos^{-1}(m_e/m_p)^{1/2}$ when $v_0 \simeq c$, it is necessary that either M_A or Ω_e/ω_{pe} be sufficiently large. The resonance condition $\omega_0 \simeq (1+\epsilon)k_{\perp}v_s$ is then satisfied for waves with $v_e \lesssim \omega_0/k_{\parallel} \lesssim v_0$: this is a necessary condition for electrons to be accelerated to the injection energy. It is not, however, a sufficient condition. To be amplified to a significant level, the waves must remain in contact with shock-reflected ions for at least several growth times. In the limit $k_\parallel/k_\perp < (m_e/m_p)^{1/2}$, the perpendicular group velocity $\partial\omega/\partial k_\perp$ has an upper bound equal to the Alfvén speed c_A (Galeev 1984). Thus, if $M_A \gg 1$, such waves will be convected into the shock at a speed approximately equal to $v_s \simeq M_A c_A$, and will only remain in contact with shock-reflected ions for a time $t_c \simeq x_{max}/v_s \simeq (0.3\text{–}0.4)/\Omega_i$ (see Fig. 1). Growth rates of many times Ω_i would therefore be required: as discussed above, the growth rate of waves with $k_\parallel/k_\perp < (m_e/m_p)^{1/2}$, driven by gyrotropic shock-reflected ions, is typically $10^{-2}\Omega_i$. If, on the other hand, $k_\parallel/k_\perp > (m_e/m_p)$, the perpendicular group velocity can be much higher than c_A: using $\omega \simeq \Omega_e k_\parallel/k_\perp$ and $v_s \simeq M_A c_A$, it is straightforward to show that

$$\left|\frac{1}{v_s}\frac{\partial\omega}{\partial k_\perp}\right| = \left(\frac{m_p}{m_e}\right)^{1/2} \frac{\cos\theta}{\sin^2\theta}\frac{\omega_{pe}}{M_A ck}. \tag{14}$$

Since the instability in the gyrotropic case is azimuthally symmetric, and $0 \le |\partial\omega/\partial k_x| \le |\partial\omega/\partial k_\perp|$, the occurrence of an instability such that $|\partial\omega/\partial k_\perp| \ge v_s$ implies the presence of growing waves whose group velocity component towards the shock is zero (see Fig. 1): if the shock is strictly perpendicular, and of infinite spatial extent in the (y,z) plane, such waves can remain in contact with shock-reflected ions indefinitely, and can then be amplified to a level determined essentially by the requirement that the intrinsic growth rate of the instability be equal to the rate of Landau damping on electrons (Galeev 1984). In the case of the results presented in Fig. 2, the right-hand side of equation (14) is always greater than unity. Indeed, using the fact that $\omega \simeq \Omega_e k_\parallel/k_\perp \simeq (1+\epsilon)k_\perp v_s$, we find that

$$\left|\frac{1}{v_s}\frac{\partial\omega}{\partial k_\perp}\right| \simeq 1+\epsilon > 1. \tag{15}$$

Thus, the result $|\partial\omega/\partial k_\perp| > v_s$ is independent of Mach number. On the other hand, the equally important condition that $\cos\theta > (m_e/m_p)^{1/2}$ for the appropriate range of parallel phase velocities becomes easier to satisfy as M_A is increased.

3.2 Instability driven by reflected ion beams

We now proceed to solve equations (9) and (10), adopting the same parameters as those used to obtain Fig. 2. The additional beam parameters ϕ_1, ϕ_2 are obtained from equation (7), with x arbitrarily set equal to $-0.2v_s/\Omega_i$ (putting $\epsilon = 0.5$, we find from equation 5 that the shock foot extends to $x \simeq -0.28v_s/\Omega_i$). Solutions now depend on the values of both k_x and k_y, rather than $k_\perp$ alone: we define an azimuthal wavevector angle ψ by writing $k_x = k\cos\psi\sin\theta$, $k_y = k\sin\psi\sin\theta$. Thus, $\psi = 0$ defines the direction of plasma flow into the shock.

Strong instability is found to occur for $k_\parallel \to 0$ and $k^2 < \omega_{pe}^2/c^2$. In this limit, as previously mentioned, equation (12) reduces to $\omega_0 \simeq kc_A$: this is consistent with the resonance condition $\omega = \boldsymbol{k}\cdot\boldsymbol{v}_1$ if there is a sufficiently large angle between $\boldsymbol{k}$ and $\boldsymbol{v}_1$ (clearly the waves can also resonate with the second ion beam). With $\beta_e \ll 1$ and $\omega_{pe}^2 \gg \Omega_e^2$, equation (9) yields

$$\frac{\gamma}{\Omega_i} = \frac{1}{\sqrt{2}}\left(\frac{m_p}{m_e}\right)^{1/2}\left(\frac{n_r}{n_i}\right)^{1/2}\frac{kc}{\omega_{pe}}. \tag{16}$$

This expression only holds for $k^2 < \omega_{pe}^2/c^2$: for $k_\parallel \to 0$, the maximum growth rate occurs for $kc/\omega_{pe} \sim 1$. In the appropriate limit, numerical solutions of equation (9) agree with equation (16). High growth rates are found to occur for $\omega/k_\parallel$ ranging all the way from v_e up to c. Thus, in contrast to the gyrotropic case, a beam distribution of shock-reflected ions can excite waves in the lower hybrid range with $k_\parallel \to 0$. As we noted above, waves with group velocities equal to or less than the Alfvén speed will be rapidly convected away from the interaction region, and are thus unlikely to play any significant role in electron acceleration unless the growth rate is many times Ω_i. For a strong quasi-perpendicular shock with $n_r/n_i \simeq 0.2$, equation (16) gives $\gamma \sim 10\Omega_i$. However, the condition $\omega = \boldsymbol{k}\cdot\boldsymbol{v}_1$ is only strictly satisfied at one upstream value of x. It is not necessary for the resonance condition to be exactly satisfied for strong wave growth to occur, but nevertheless the fact that f_r varies rapidly with x makes it unlikely that strong wave growth will occur unless, as in the gyrotropic case, there exist strongly unstable waves with perpendicular group velocities of the order of the shock speed.

In fact, such waves do exist. The beam-driven instability extends to $k_\parallel/k_\perp > (m_e/m_p)^{1/2}$ and $k^2 > \omega_{pe}^2/c^2$. Thus, we again have the possibility of $\partial\omega/\partial k_x \simeq -v_s$, which allows the unstable waves to remain in contact with shock-reflected ions. To illustrate this point, Fig. 3 shows γ as a function of ψ at a perpendicular shock with $M_A = 2$, $n_r/n_i = 0.055$. The plasma parameters are identical to those used to obtain Fig. 2 and $\theta = 81.6°$: k was set equal to $3.2\omega_{pe}/c$. The growth rate peaks at azimuthal angles which are such that the projection of one of the reflected ion beam velocity vectors on to the direction of $\boldsymbol{k}$ is approximately equal to the phase speed implied by equation (12). Fig. 4 shows a similar set of results for $M_A = 9$, $n_r/n_i = 0.203$, $\theta = 56.4°$, $k = 3.8\omega_{pe}/c$. The azimuthal angular range of the instability is similar to the low Mach number case, but the maximum growth rate is considerably higher. In fact, the maximum growth rate is a monotonic increasing function of Mach number. In both Figs 3 and 4, θ and k were chosen such that $\partial\omega/\partial k_x \simeq -v_s$ and $\omega/k_\parallel \simeq v_e$ (for each set of plasma parameters, the values of θ and k satisfying these criteria are unique). As one might expect, the maximum growth rates produced by beam ions are higher than those produced by gyrotropic energetic ions. Setting $\omega = \boldsymbol{k}\cdot\boldsymbol{v}_1 + i\gamma$ and assuming $k_\parallel/k_\perp > (m_e/m_p)^{1/2}$, $k^2 > \omega_{pe}^2/c^2$, we obtain from equation (9) a growth rate

$$\frac{\gamma}{\Omega_i} = \frac{3^{1/2}}{2^{5/3}}\frac{\cos^{1/3}\theta}{\sin\theta}\left(\frac{m_p}{m_e}\right)^{2/3}\left(\frac{n_r}{n_i}\right)^{1/3}. \tag{17}$$

In this case $\gamma \to 0$ as $\theta \to 90°$, but very slowly. It is straightforward to verify that the maximum growth rates in Figs 3 and 4 lie close to the values indicated by equation (17).

Strong instability may also occur for waves with $\partial\omega/\partial k_x \simeq -v_s$ and $\omega/k_\parallel \simeq v_0$, where $v_0 \le c$. Using the fact that $\omega \simeq \Omega_e k_\parallel/k_\perp$, it is straightforward to show that the perpendicular group velocity is equal to the shock speed for θ given by

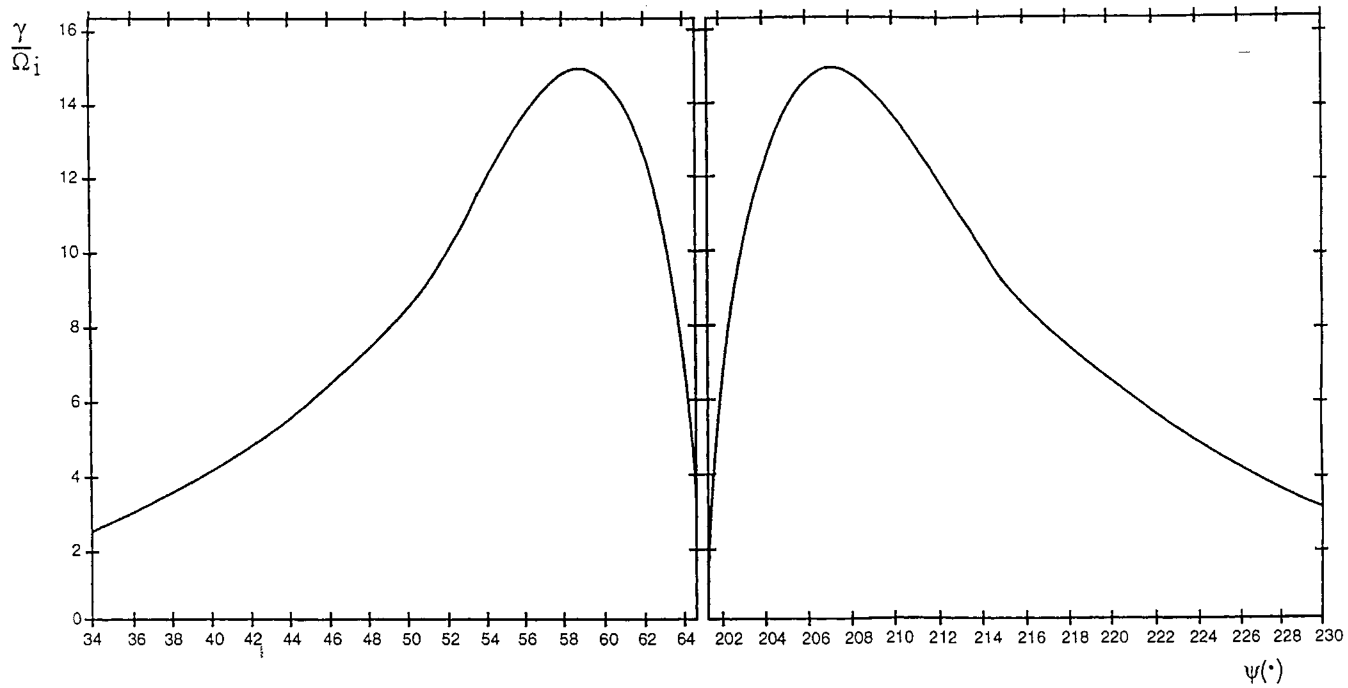

Figure 3. Growth rate of lower hybrid waves as a function of azimuthal angle ψ at a perpendicular shock with $M_A=2$, $n_r/n_i=0.055$. The direction of upstream plasma flow into the shock is defined by $\psi=0°$. The plasma parameters are identical to those used to obtain Fig. 2. The wave propagation angle relative to the magnetic field is $\theta=81.6°$ and $k=3.2\omega_{pe}/c$. The non-gyrotropic reflected ion distribution is given by equation (6), with $\epsilon=0.5$. The parameters ϕ_1, ϕ_2 were obtained from equation (7), with $x=-0.2v_s/\Omega_i$. The growth rate peaks at azimuthal angles such that the projection of one of the reflected ion beam velocity vectors on to the direction of $\boldsymbol{k}$ is approximately equal to the phase velocity associated with equation (12). The wavevector parameters θ and k have been chosen such that the x component of the group velocity is approximately equal to minus the shock speed, and the parallel component of the phase velocity is approximately equal to the electron thermal speed.

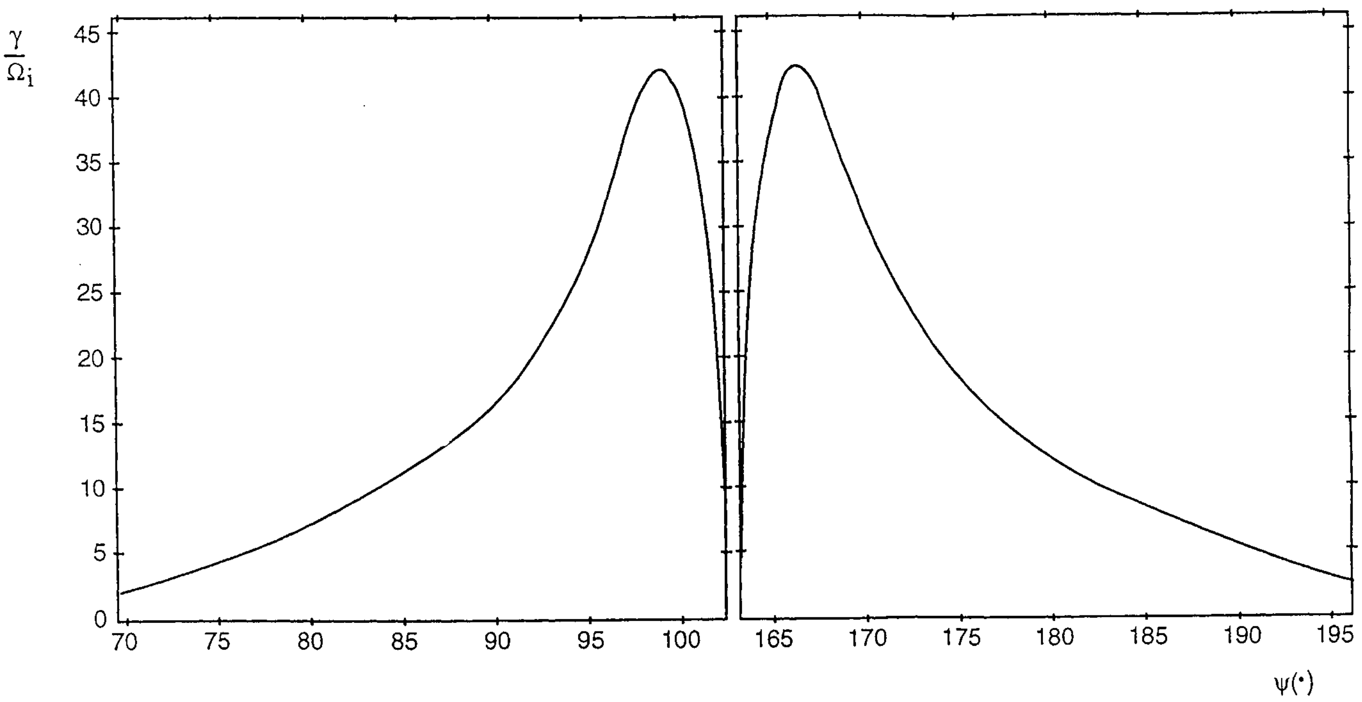

Figure 4. As Fig. 3 except that $M_A=9$, $n_r/n_i=0.203$, $\theta=56.4°$ and $k=3.8\omega_{pe}/c$.

$$\cot\theta = \left(\frac{m_e}{m_p}\right)^{1/2} \frac{\Omega_e}{\omega_{pe}} \frac{c}{v_0} M_A. \tag{18}$$

This differs from equation (13) only by a factor of $1+\epsilon$. As before, for θ to be less than $\cos^{-1}(m_e/m_p)^{1/2}$, so that super-Alfvénic group velocities are possible, it is necessary for M_A or Ω_e/ω_{pe} to be sufficiently large. If this condition is satisfied, waves which are capable of accelerating electrons to the required injection energies can be strongly amplified.

4 DISCUSSION

We have used two simple models of the distribution function of reflected ions at a quasi-perpendicular shock to study the possibility that electrons could be accelerated from thermal energies to MeV energies at such shocks. The basic mechanism is the excitation by the ions of waves with frequencies between the ion and electron cyclotron frequencies. The distribution function parameters depend essentially on the shock Mach number. Under certain conditions, shock-reflected gyrotropic or beam ions can excite waves which are capable of accelerating electrons up to MeV energies. The excited waves can have group velocities along the direction of the shock normal which are approximately equal to the shock velocity: such waves are not rapidly convected out of the interaction region, and can thus grow to a high level. Also, waves with phase velocities parallel to the magnetic field ranging from the electron thermal speed to the speed of light have been shown to be excited. In these circumstances, bulk electrons can be rapidly accelerated to relativistic energies. The mechanism is particularly effective in high Mach number shocks with a low ratio of electron plasma frequency to cyclotron frequency.

Following up on this last point, it is appropriate now to consider the possible existence of a threshold shock speed for electron injection. As we have seen, waves with group velocities comparable to the shock speed will always be excited, regardless of the Mach number. On the other hand, in order to satisfy the additional criteria $\omega/k_\parallel = v_0$ and $\theta < \cos^{-1}(m_e/m_p)^{1/2}$, which make it possible for electrons with parallel speed v_0 to resonate with such waves, it is necessary that

$$M_A \frac{\Omega_e}{\omega_{pe}} = \left(\frac{m_p}{m_e}\right)^{1/2} \frac{v_s}{c} > \frac{v_0}{c}. \tag{19}$$

This result follows from equation (18). As we discussed in Section 1, the electron momentum required for diffusive shock acceleration is equal to the proton mass times the shock speed. Obviously, the corresponding electron speed v_0 must also depend on v_s. Setting $m_p v_s = m_e v_0/(1-v_0^2/c^2)^{1/2}$, $(m_p/m_e)^{1/2} v_s = v_0$ (from equation 19), and eliminating v_s, we obtain $v_0 = c(1-m_e/m_p)^{1/2} \simeq c$. Using the exact expression for v_0, we deduce that the corresponding electron energy is approximately $(m_p m_e)^{1/2} c^2 \simeq 22$ MeV, and that the inequality in equation (19) reduces to

$$v_s > c\left(1-\frac{m_e}{m_p}\right)^{1/2} \left(\frac{m_e}{m_p}\right)^{1/2} \simeq c\left(\frac{m_e}{m_p}\right)^{1/2}. \tag{20}$$

Thus, to accelerate electrons up to the threshold energy for the diffusive shock process, the shock speed must exceed $(m_e/m_p)^{1/2} c \simeq 0.023c$. This result is independent of the plasma parameters, and is consistent with a threshold shock speed for electron injection $(0.02 \pm 0.01)c$ conjectured by Biermann & Cassinelli (1993). These authors invoked an injection threshold in order to reconcile observed radio luminosities of supernovae and Wolf–Rayet stars with an assumed model for the stellar wind associated with such objects. Biermann & Cassinelli suggested that the threshold might lie close to $(m_e/m_p)^{1/2} c$, on the grounds that electrons can reach relativistic speeds when they are thermalized downstream of shocks with speeds which exceed this value. In the present paper, $(m_e/m_p)^{1/2} c$ has been deduced as the injection threshold on the basis that the electrons are energized upstream of the shock by resonant energy transfer from ion-excited waves. Essentially, the requirements for efficient electron injection to occur are that the plasma can excite strongly growing waves with super-Alfvénic group velocities and parallel phase velocities ranging from the electron thermal speed to the speed of light: a shock with $v_s \gtrsim 0.023c$ satisfies these requirements.

An electron moving through a quasi-perpendicular shock remains in contact with shock-reflected ions, and hence with wave turbulence, over a distance which is approximately $(0.3\text{–}0.4)\,v_s/\Omega_i$: is this far enough for it to be accelerated to the required injection energy? The acceleration process is likely to be governed by a quasi-linear diffusion equation of the form (Spicer, Benz & Huba 1981; Galeev 1984)

$$v_s \frac{\partial f_e}{\partial x} = \frac{\partial}{\partial p_z} D \frac{\partial f_e}{\partial p_z}, \tag{21}$$

where f_e is the electron distribution, $p_z = \Gamma m_e v_z$ where Γ is the Lorentz factor, and D is proportional to the energy density W_{LH} of the excited waves. Thus, the distance required for an electron to be accelerated up to a speed v_0 is $L_{acc} \propto v_s v_0^2 \Gamma^2 / W_{LH}$, while the mechanism can only work if $L_{acc} \lesssim (0.3\text{–}0.4)\,v_s/\Omega_i$. This condition will be satisfied if W_{LH} exceeds a certain fraction of the reflected ion density $W_r \simeq m_p n_r (1+\epsilon)^2 v_s^2/2$. Using the expression for D given by Spicer, Benz & Huba (1981), we infer that electrons will be accelerated to the required energies if

$$\frac{W_{LH}}{W_r} \gtrsim \frac{3}{2\pi(1+\epsilon)^2} \left(\frac{m_e}{m_p}\right)^{3/2} \frac{n_i}{n_r} \frac{v_0^2}{c^2} \frac{\Gamma^2}{M_A^2}. \tag{22}$$

Particle simulations of lower hybrid wave excitation in the presence of beam-like or ring-like ion distributions indicate that the saturation value of W_{LH}/W_r is typically of the order of 10^{-2} (McBride et al. 1972; McClements et al. 1993). Setting $v_0 = c$, $M_A \geq 2$ and $n_r/n_i \geq 0.05$ (cf. Fig. 2), we find that the right-hand side of equation (22) is $\lesssim 10^{-5}\Gamma^2$. Thus, the intensity of wave turbulence which is likely to be generated is easily sufficient to accelerate electrons to mildly relativistic energies. Since n_r/n_i is a monotonic increasing function of M_A (Paschmann & Sckopke 1983), the right-hand side of equation (22) is a rapidly decreasing function of M_A, and therefore the inequality is most likely to be satisfied in high Mach number shocks. This is further evidence that the acceleration mechanism proposed here becomes more effective as M_A is increased, and is consistent with the existence of a threshold shock speed for electron injection suggested by Biermann & Cassinelli (1993).

Equation (22) indicates that resonant acceleration by ion-excited waves is capable of producing electrons with energies which are much higher than the threshold required for diffusive shock acceleration. In principle, there is no upper limit to the energies which can result from the interaction between electrons and waves with $\omega/k_{\parallel} \leq c$. Thus, the principal conclusion of this paper, that the acceleration of electrons to ultrarelativistic energies is possible in shocks with speeds exceeding $(m_e/m_p)^{1/2}c$, is not contingent on the efficacy of diffusive shock acceleration.

Our analysis is based on a number of approximations and assumptions which we now discuss. The delta function representation of f_r is undoubtedly artificial, and leads to an overestimate in the wave growth rate γ. In practice, as we discussed in Section 2, the velocity distribution of reflected gyrating ions has a finite width. It would be straightforward to modify equations (10) and (11) to take this into account: numerical solutions of the dispersion relation would then yield growth rates somewhat lower than those in Figs 2–4, but still high enough for the waves to grow to saturation levels. The crucial parameters of the model are the group and phase velocities of the unperturbed waves, rather than γ. Thus, our analysis is not strongly dependent on the velocity-space width of the reflected ion distribution.

Strictly speaking, equations (9) and (12) are only valid if $\omega/k_{\parallel} \gg v_e$: if $\omega/k_{\parallel}$ is comparable to v_e, as in Figs 2–4, the net growth rate of the instability is significantly affected by electron Landau damping. This, in fact, is the process which causes electrons to be accelerated out of the thermal pool (e.g. Kirk, Melrose & Priest 1994). It is necessary to ensure, however, that the Landau damping rate γ_L is less than the intrinsic growth rate γ. If ω is not much larger than $k_{\parallel}v_e$, equation (9) is replaced with a more general dispersion relation given by equation (4) in McBride et al. (1972). Setting $\omega = \omega_0 + i\gamma_L$ in this equation, assuming $|\gamma_L| \ll \omega_0$, and neglecting the beam ion term, we obtain

$$\frac{\gamma_L}{\Omega_i} = -2\sqrt{\pi}\cot\theta \frac{m_p}{m_e}\frac{\omega_0^3}{k_{\parallel}^3 v_e^3}\exp\left[-\frac{\omega_0^2}{k_{\parallel}^2 v_0^2}\right]. \tag{23}$$

The exponential factor in this expression means that $|\gamma_L|$ is typically less than the growth rate given by equation (17) whenever $\omega > k_{\parallel}v_e$. For waves of a given perpendicular group velocity, this condition will be satisfied for the full range of propagation angles exceeding those used to obtain Figs 2–4. Thus, although electron Landau damping is essentially for the transfer of energy to electrons, its magnitude is typically small compared to the intrinsic growth rate of the instability.

Another of the assumptions of our model is that n_r/n_i increases with Mach number. This is certainly true for the range of Mach numbers occurring at planetary bow shocks ($M_{ms} \lesssim 10$), but, as Levinson (1996) has noted, the distribution of reflected gyrating ions at very high Mach number quasi-perpendicular shocks is uncertain. However, there is no reason to suppose that a high proportion of the ions encountering such shocks will not be reflected, and we stress again that the effectiveness of the wave–particle acceleration process is not strongly dependent on the velocity–space width of f_r.

It should be noted that Landau damping of ion-excited waves leads to electron acceleration along the magnetic field: the perpendicular component of the electron velocity is unaffected, and therefore the accelerated electrons have small pitch angles. This would appear to be an impediment to diffusive shock acceleration, which requires electrons and ions to have comparable Larmor radii. In response to this point, we make three observations. First, the electron Larmor radius increases with Γ, large values of which can result from acceleration by ion-excited waves. Secondly, electron distributions with a high degree of anisotropy are likely to be unstable to the generation of waves which cause pitch angle scattering (e.g. Moghaddam-Taaheri et al. 1985), thus increasing the energetic electron Larmor radii still further. Finally, we stress again that the process considered in this paper makes it possible for electrons to acquire ultrarelativistic energies even in the absence of diffusive shock acceleration.

Direct observational evidence for the process discussed in this paper may be difficult to obtain. One possible observational test of the model is suggested by simulations of lower hybrid wave acceleration in solar flares, which reveal the excitation of coherent narrow-band electromagnetic radiation at frequencies of around Ω_e (McClements et al. 1993). The physical origin of this emission is not entirely clear: it appears to be driven by the strong anisotropy of the electron distribution (see remarks above). The circular electron cyclotron frequency is $2.8B$ MHz, where B is the upstream field in gauss, and radio waves with frequencies below about 10 MHz are reflected by the Earth's ionosphere. Thus, ground-based telescopes cannot be used to detect coherent emission at $\omega \simeq \Omega_e$ from astrophysical shocks, except for those with upstream magnetic fields of at least several gauss.

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