# Accepting Performance Degradation in Fault-Tolerant Control System Design

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***Abstract*—A novel fault-tolerant control system design technique has been proposed in this paper, which blends the multiple-model principle with the unavoidable performance degradation due to faults in actuators, sensors or system dynamics. The number of models employed depends on the characteristics of the system, the nature of the failures considered, and the physical limits of system variables. The achievable performance under various component failures are represented in the form of reference models, known as performance reduced reference models. These models are used to synthesize a set of controllers. Under a specific fault condition, proper controller and revised control system command input are selected automatically to achieve desired performance. A simulation example of an aircraft subject to different type of failures has been used to illustrate the design process and to demonstrate the effectiveness of the method.**

***Index Terms*—Fault detection and diagnosis (FDD), fault-tolerant control systems (FTCS), multiple-model, performance degradation, reconfigurable control.**

## I. Introduction

To improve safety and reliability of safety-critical systems, the importance of fault-tolerant control systems (FTCS) becomes increasingly apparent, and significant amount of research has already been done in this area [1]–[5], [8]. However, there are still many open issues yet to be resolved satisfactorily [8]. This paper examines one of these issues as how to deal with different levels of achievable performance in the presence of various faults under given potential system performance limitations. This is a challenging problem, as different faults may affect the system differently. Consequently, different levels of performance have to be considered in different fault scenarios.

Even though it is a common sense to accept a certain degree of performance degradation in the presence of system component failures, the fault-tolerant control system design which considers the fault-inflicted physical constraints for maintaining achievable performance has mostly been ignored until recently [9]. In this recent work, two reference models are used: one for the normal system operation and the other for the system under contingencies with actuator failures, respectively, where the magnitude of the fault is estimated and controller is reconfigured accordingly. Although a very important concept has been presented therein, it soon becomes evident that a twin model approach is not comprehensive enough to represent all potential system malfunctions. Different faults in a system can exhibit distinctive characteristics; a single performance reduced model cannot simply represent all of them. Naturally, a multiple-model approach offers a logical extension to the concept in dealing with multiple type of faults in actuators, sensors and system dynamics, which could not easily be done under the framework of [9] because of the difficulty faced when estimating on-line all fault parameters associated to different types of failures in actuators, sensors and system dynamics. By representing each fault type with a separate model, it has been shown that the overall fault handling capability for different types of fault occurred in the control system can be enhanced considerably. The control system performance also becomes less conservative, because each controller only needs to deal with a single fault scenario. Furthermore, the same failure type but at different severities may be represented by different performance reduced models under the very same framework.

The objective of this brief paper is to present an approach to incorporate performance limitations under different fault conditions using multiple-model technique. The current work differs significantly from that of [9] as a completely different control structure is used, in which the controller for each failure scenario is designed individually. Under the assumption that all potential faults in the system can be represented in terms of a finite set of models, a performance reduced reference model is synthesized for each failure scenario with due consideration of system performance limitations. There are three unique advantages associated with the current approach: a) it can handle multiple type of faults; b) it is able to isolate faults quickly by performing a simple statistical test on the multiple-model residuals; and c) it results in a less conservative control system for a specific fault situation by using the corresponding performance reduced reference model.

To determine the multiple reference models, one has to have the knowledge of the system performance requirements, availability of redundancies, and underlying physical limitations of the actuators. These quantities impose the ultimate performance limit for systems under component failures. It is crucial to embed these limits into the performance reduced reference models and also to modify the system input command accordingly so that the physical limits of the system are not violated either during transients or at steady-state. Based on multiple reference models, the corresponding fault-tolerant control system is synthesized using model reference approach. In this paper, the performance degradation is reflected in the reduction of the stability margins, the reduced level of dynamic performance and scaling back of the operational magnitude at the steady-state.

The main contribution of this paper is to introduce a new methodology to deal with potential performance limits in a system with different type of component failures. An aircraft example is used to illustrate the concepts, design procedures, and simulation studies.

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The paper is organized as follows: The basic concept and overall structure of the proposed FTCS based on multiple-model for handling different type of faults are presented in Section II. The procedures for designing performance reduced reference models and input command adjustment are also covered in this section. In Section III, the synthesis of a set of reconfigurable controllers is examined with the use of the above reference models to deal with the system performance limitations for each failure mode. Simulation results are presented in Section IV to illustrate the proposed scheme, followed by conclusions in Section V.

## II. Basic Concept and Overall Structure of the Proposed Scheme

### A. Faults and Their Models

Faults are those system malfunctions, which could lead to undesirable consequences if left unattended. In practice, faults may occur in actuators, sensors and system dynamics. Therefore, all three type of faults have been considered in this paper. As mentioned in the previous section, a natural way to represent different fault conditions is to employ a multiple-model approach. Each fault type can be represented by one or more models depending on the nature and severity of the fault.

Assume that a finite set of $N$ models is used to represent the system under the normal and the $(N-1)$ failure modes. Thus, the system can be represented as:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= (A+\Delta A_j)\mathbf{x}(t) + (B+\Delta B_j)\mathbf{u}(t) + \mathbf{w}(t)\\ &= A_j\mathbf{x}(t) + B_j\mathbf{u}(t) + \mathbf{w}(t) \quad (1)\\ \mathbf{z}(t) &= (C+\Delta C_j)\mathbf{x}(t) + \mathbf{v}(t)\\ &= C_j\mathbf{x}(t) + \mathbf{v}(t) \quad j=0,\ldots,N-1 \quad (2)\end{aligned}$$

where $\mathbf{x} \in \mathcal{R}^n$ is the state vector; $\mathbf{z} \in \mathcal{R}^m$ is the measurement vector; $\mathbf{u} \in \mathcal{R}^l$ is the control input vector, and $\mathbf{w} \in \mathcal{R}^n$ and $\mathbf{v} \in \mathcal{R}^m$ are the independent random processes with means $\bar{\mathbf{w}}$ and $\bar{\mathbf{v}}$ and covariances $Q$ and $R$, respectively. The initial state is assumed to have mean $\bar{\mathbf{x}}_0$ and covariance $\bar{P}_0$, and to be independent from $\mathbf{w}$ and $\mathbf{v}$. Furthermore, $\Delta A_j$, $\Delta B_j$ and $\Delta C_j$ $(j=1,\ldots,N-1)$ represent the fault-induced changes in the system dynamics, actuators and sensors, respectively. They are null matrices for $j=0$, which represents the normal condition. The subscript $j$ denotes quantities pertaining to the model, $m_j \in \mathcal{M}$. $\mathcal{M} = [m_0, m_1, \ldots, m_{N-1}]$ is a set containing system models for all the conditions. Matrices $A_j$, $B_j$, and $C_j$ $(j=1,\ldots,N-1)$ correspond to the $j$th post-fault model of the system.

### B. System Performance Limitations in the Presence of Faults

In practice, the expected system performance in the presence of a fault can be significantly different from that under normal operation. With limited system redundancies, the performance generally has to be scaled back to avoid reaching the physical limits of some system components. Generally speaking, there are two types of performance in a control system corresponding to transients and steady-state conditions, respectively. Using an aircraft as an example, the transient performance may include items such as the maneuvering capabilities, the achievable acceleration, or the radius of a turn. In other words, they are associated with the dynamic aspects of the aircraft. On the other hand, the steady-state performance relates largely to the equilibrium points in different flight conditions, such as the cruising speed and altitude, the weight of the payload, etc. The steady-state operating condition is often related to the system command input. Consequently, both the dynamic properties and the command input should be considered in the design of FTCS in the presence of failures in the system. In the current work, the achievable performance under these conditions includes both aspects: 1) the dynamic part is dealt with through the use of performance reduced reference models and the model reference control approach, and 2) the steady-state part relies on the adjustment of the level of the control command inputs.

### C. Determination of Multiple Performance Reduced Reference Models

To capture and specify the characteristics of the handicapped system under each fault scenario, a corresponding "performance reduced reference model" needs to be synthesized. These models will represent the desirable dynamic behaviors of the closed-loop system under specific fault conditions. To handle different type of faults, different models are often needed. Several models may even be needed for a single failure type if the characteristics of the system changes significantly at different fault severities. In particular, the dynamic behavior of the post-fault system is governed by the characteristics of the designed reference model, which takes into consideration of the allowable performance limits under a given fault condition without violating the physical constraints in any system variables.

Assume that a reference model of the system under the normal condition is represented by:

$$\begin{aligned}\dot{\mathbf{x}}_0^m &= A_0^m\mathbf{x}_0^m + B_0^m\mathbf{r}'_0\\ \mathbf{y}_0^m &= C_0^m\mathbf{x}_0^m \end{aligned}\quad (3)$$

where $\mathbf{x}_0^m \in \Re^{n^m}$ is the state vector of the reference model; $\mathbf{y}_0^m \in \Re^{p^m}$ is the output vector; and $\mathbf{r}'_0 \in \Re^{l^m}$ is the command input vector. The above model, known as the desired reference model, specifies the desired dynamic characteristics of the system under the normal condition.

Let's assume that the eigenvalues of this system are represented as:

$$\Lambda_0 = \mathrm{diag}\left[\lambda_1^0, \lambda_2^0, \ldots, \lambda_{n^m}^0\right] \quad (4)$$

In the presence of a fault, based on failure models represented in (1) and (2), it is expected that the eigenvalues of the performance reduced reference models would shift toward the imaginary axis to reflect the loss of system dynamic performance. This can be achieved by simply selecting a mode degradation matrix, $\Psi_j, j \in \{1,\ldots,N-1\}$, for each fault condition. Suppose that the eigenvalues of the performance reduced reference model under each fault condition are related to those under normal condition by:

$$\Lambda_j = \Psi_j^{-1}\Lambda_0, \quad j=1,\ldots,N-1 \quad (5)$$

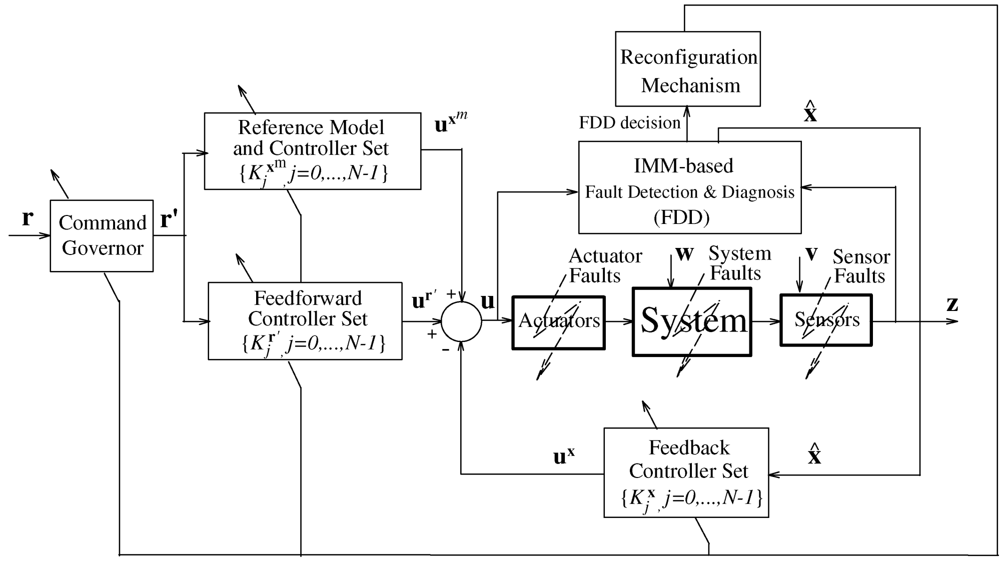


Fig. 1. Overall structure of the proposed FTCS.

where

$$\Psi_j = \text{diag}\left[\psi_1^j, \psi_2^j, \ldots, \psi_{n^m}^j\right] \tag{6}$$

and $\psi_i^j \geq 1, \forall i = 1, \ldots, n^m; j = 1, \ldots, N-1$.

The transfer function matrix of the reference model for the system in the failure mode $j$ can then be obtained as:

$$\begin{aligned} T_j(s) &= C_0^m \left(Is\Psi_j - A_0^m\right)^{-1} B_0^m \\ &= C_0^m \left(Is - \Psi_j^{-1} A_0^m\right)^{-1} \Psi_j^{-1} B_0^m \\ &= C_j^m \left(Is - A_j^m\right)^{-1} B_j^m, \quad j = 1, \ldots, N-1 \end{aligned} \tag{7}$$

Hence, a set of performance reduced reference models can be obtained as:

$$\begin{aligned} \dot{\mathbf{x}}_j^m &= A_j^m \mathbf{x}_j^m + B_j^m \mathbf{r}'_j, \\ \mathbf{y}_j^m &= C_j^m \mathbf{x}_j^m, \quad j = 1, \ldots, N-1 \end{aligned} \tag{8}$$

where $A_j^m = \Psi_j^{-1} A_0^m$, $B_j^m = \Psi_j^{-1} B_0^m$, $C_j^m = C_0^m$, $j = 1, \ldots, N-1$.

The matrix triplets $\{A_j^m, B_j^m, C_j^m, j = 1, \ldots, N-1\}$ specify the characteristics of the system with achievable performance under various fault conditions. By choosing different values in the diagonal elements of $\Psi_j$, various dynamic characteristics and different levels of performance reduction can be accommodated. The selection of each element in $\Psi_j$ is application dependent and needs certain engineering insights into system performance limitations under different fault conditions.

Once designed, these performance reduced reference models will be used as the reference models in FTCS design and implementation for achievable performance.

### D. Command Input Adjustment

To ensure that all system variables are within the safe operating range and that all of the control effectors are free from saturation in the event of failures, one has to make appropriate adjustments to the level of control commands as well. A command governor is used just for this purpose. Essentially, it performs two functions to determine: 1) which output variables the closed-loop system should follow; and 2) what is the appropriate reduced level of command inputs for a given fault scenario. A similar scheme as in [9] is tailored to the current control structure.

### E. Overall Structure of the Proposed FTCS

Based on the above description, the overall configuration of the proposed FTCS can be depicted in Fig. 1, which includes the following modules: 1) an interacting multiple-model (IMM) based fault detection and diagnosis (FDD) [10], 2) multiple performance reduced reference models and the associated controllers, 3) a reconfigurable control mechanism, and 4) a command governor.

A significant portion of the design involves the synthesis of reconfigurable controllers. These controllers depend on the system failure models (1), (2) and the corresponding performance reduced reference models (8). The details of the controller design is the subject of the next section.

## III. Controller Design With Achievable Performance

### A. Control System Synthesis

For the purpose of reconfigurable controller design, the system models (1), (2) are represented alternatively in the discrete domain as follows:

$$\begin{aligned} \mathbf{x}_j(k+1) &= F_j \mathbf{x}_j(k) + G_j \mathbf{u}_j(k) + \mathbf{w}_j(k) \\ \mathbf{y}_j(k) &= H_j^{\mathbf{y}} \mathbf{x}_j(k) \\ \mathbf{z}(k) &= H_j \mathbf{x}_j(k) + \mathbf{v}_j(k), \quad j = 0, \ldots, N-1 \end{aligned} \tag{9}$$

where an additional equation is added to represent those elements in the system output $\mathbf{y}_j(k)$ which are being regulated. Under the normal operation, the system is represented by matrices $\{F_0, G_0, H_0\}$. Once a fault occurs, the system will be represented by one of the models in the set $\{F_j, G_j, H_j, j \in \{1, \ldots, N-1\}\}$.

TABLE I
FAULT MODES AND FAULT-INDUCED CHANGES

| Modes & Faults | $A_j/\Delta A_j$ | $B_j/\Delta B_j$ | $C_j/\Delta C_j$ |
|---|---|---|---|
| Normal (*j=0*) | $A_0=\begin{bmatrix} -3.5980 & 0.1968 & -35.18 & 0 \\ -0.0377 & -0.3576 & 5.8840 & 0 \\ 0.0688 & -0.9957 & -0.2163 & 0.0733 \\ 0.9947 & -0.1027 & 0 & 0 \end{bmatrix}$ | $B_0=\begin{bmatrix} 14.65 & 6.538 \\ 0.2179 & -3.087 \\ -0.0054 & 0.0516 \\ 0 & 0 \end{bmatrix}$ | $C_0=\begin{bmatrix} 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$ |
| Dynamic fault (*j=1*) 50% loss of rudder control surface | $\Delta A_1=\begin{bmatrix} \underline{0.7196} & \underline{-0.0394} & \underline{7.0360} & 0 \\ \underline{0.0075} & \underline{0.0715} & \underline{-1.1768} & 0 \\ \underline{-0.0138} & \underline{0.1991} & \underline{0.0433} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ | $\Delta B_1=\begin{bmatrix} 0 & \underline{-3.2690} \\ 0 & \underline{1.5435} \\ 0 & \underline{-0.0258} \\ 0 & \underline{0} \end{bmatrix}$ | $\Delta C_1 = [0]$ |
| Actuator #1 fault (*j=2*) 50% loss of effectiveness in aileron | $\Delta A_2 = [0]$ | $\Delta B_2=\begin{bmatrix} \underline{-7.3250} & 0 \\ \underline{-0.1090} & 0 \\ \underline{0.0027} & 0 \\ \underline{0} & 0 \end{bmatrix}$ | $\Delta C_2 = [0]$ |
| Actuator #2 fault (*j=3*) 50% loss of effectiveness in rudder | $\Delta A_3 = [0]$ | $\Delta B_3=\begin{bmatrix} 0 & \underline{-3.2690} \\ 0 & \underline{1.5435} \\ 0 & \underline{-0.0258} \\ 0 & \underline{0} \end{bmatrix}$ | $\Delta C_3 = [0]$ |
| Sensor fault (*j=4*) 50% loss of effectiveness in sideslip angle sensor | $\Delta A_4 = [0]$ | $\Delta B_4 = [0]$ | $\Delta C_4=\begin{bmatrix} 0 & 0 & \underline{-0.5} & 0 \\ 0 & 0 & \underline{0} & 0 \end{bmatrix}$ |

The corresponding discrete multiple reference models can be described by:

$$\begin{aligned} \mathbf{x}_j^m(k+1) &= F_j^m \mathbf{x}_j^m(k) + G_j^m \mathbf{r}'_j(k) \\ \mathbf{y}_j^m(k) &= H_j^m \mathbf{x}_j^m(k), \quad j = 0, \dots, N-1 \end{aligned} \tag{10}$$

Based on the system models (9) and the multiple reference models (10), the design objective of the above FTCS is to synthesize a set of control gains $\{K_j^{\mathbf{x}}, K_j^{\mathbf{x}^m}, K_j^{\mathbf{r}'}, j = 0, \dots, N-1\}$ to meet the design performance with the following control signals:

$$\mathbf{u}_j(k) = \underbrace{-K_j^{\mathbf{x}} \mathbf{x}_j(k)}_{\text{feedback}} + \underbrace{K_j^{\mathbf{x}^m} \mathbf{x}_j^m(k)}_{\text{reference model}} + \underbrace{K_j^{\mathbf{r}'} \mathbf{r}'_j(k)}_{\text{feedforward}} \tag{11}$$

### B. Determination of Reconfigurable Control Gains

To implement the above control system, one has to determine the control gains for each system mode systematically. The principle of the multiple-model based FTCS is to make the selected system variables follow the outputs of the respective reference models during the normal and under the fault conditions, i.e. to force the error $\mathbf{e}_j(k)$, $j = 0, \dots, N-1$, to be zero at the steady-state under each condition. The error signal is defined as

$$\mathbf{e}_j(k) = \mathbf{y}_j(k) - \mathbf{y}_j^m(k) = H_j^{\mathbf{y}} \mathbf{x}_j(k) - H_j^m \mathbf{x}_j^m(k) \tag{12}$$

Based on the similar derivation as in [9] for a single performance reduced reference model case, the multiple-model reconfigurable control gains specified in (11) can be determined as follows:

$$\begin{aligned} K_j^{\mathbf{x}} &= \text{stabilizing feedback controller} \\ K_j^{\mathbf{x}^m} &= S_j^{21} + K_j^{\mathbf{x}} S_j^{11} \\ K_j^{\mathbf{r}'} &= S_j^{22} + K_j^{\mathbf{x}} S_j^{12} \end{aligned} \tag{13}$$

where the control gains $K_j^{\mathbf{x}^m}$ and $K_j^{\mathbf{r}'}$ are functions of the feedback control gains $K_j^{\mathbf{x}}$, and the constant gain matrices $S_j^{kl}$, $k, l = 1, 2$; $j = 0, \dots, N-1$, are calculated by

$$S_j^{11} = \Phi_j^{11} S_j^{11} \left(F_j^m - I\right) + \Phi_j^{12} H_j^m \tag{14}$$
$$S_j^{12} = \Phi_j^{11} S_j^{11} G_j^m \tag{15}$$
$$S_j^{21} = \Phi_j^{21} S_j^{11} \left(F_j^m - I\right) + \Phi_j^{22} H_j^m \tag{16}$$
$$S_j^{22} = \Phi_j^{21} S_j^{11} G_j^m \tag{17}$$

and gain matrices $\Phi_j^{kl}$, $k, l = 1, 2$; $j = 0, \dots, N-1$, are given by

$$\Phi_j = \begin{bmatrix} \Phi_j^{11} & \Phi_j^{12} \\ \Phi_j^{21} & \Phi_j^{22} \end{bmatrix} = \begin{bmatrix} F_j - I & G_j \\ H_j^{\mathbf{y}} & 0 \end{bmatrix}^{-1} \tag{18}$$

It should be noted that the stability and dynamic performance of the designed FTCS are mainly governed by the feedback control gains, $K_j^{\mathbf{x}}$, $j = 0, \dots, N-1$. In principle, any control system design technique can be used to determine these gains so long as the synthesized controllers can stabilize the post-fault systems to achieve satisfactory dynamic performance. In view of the advantages offered by eigenstructure assignment (EA) technique, a feedback controller can be designed to achieve the desired stability and dynamic performance with a specified eigenstructure. Hence, the EA technique as developed in [7] is used herein. For the interest of space and also to avoid repetition, details on the EA technique are omitted.

Furthermore, it should be pointed out that if switching occurs rapidly among multiple controllers, even for a bank of stable systems, the overall stability of the closed-loop system will not be guaranteed. Investigation of such stability issue within the stochastic framework of the IMM structure is beyond the scope of this brief paper and it remains an open problem for future research. However, the fault occurrence is usually a rare event in a well-engineered system. Therefore, one would not expect frequent switching any way in a FTCS.

During the system operation, the most appropriate controller will automatically be selected based on the decision of the FDD scheme. Furthermore, for on-line implementation of (11), the

TABLE II
REFERENCE MODELS UNDER THE NORMAL AND THE FAULT CONDITIONS

| | Reference Models | $A_j^m$ | $B_j^m$ | Eigenvalues of $A_j^m$ |
|---|---|---|---|---|
| Mode 0 | Normal (RM0: $j=0$) | $\begin{bmatrix} -10.0 & 0 & -10.0 & 0 \\ 0 & -0.7 & 4.5 & 0 \\ 0 & -0.5 & -0.7 & 0 \\ 1 & 0 & 0 & -0.5 \end{bmatrix}$ | $\begin{bmatrix} 10.0 & 5.0 \\ -5.48 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} -0.5 \\ -10.0 \\ -0.7 \pm i1.5 \end{bmatrix}$ |
| Mode 1 | Dynamic fault (RM1: $j=1$) | $\begin{bmatrix} -2.0 & 0 & -2.0 & 0 \\ 0 & -0.14 & 0.9 & 0 \\ 0 & -0.1 & -0.14 & 0 \\ 0.2 & 0 & 0 & -0.1 \end{bmatrix}$ | $\begin{bmatrix} 2.0 & 1.0 \\ -1.0960 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} -0.1 \\ -2.0 \\ -0.14 \pm i0.30 \end{bmatrix}$ |
| Mode 2 | Aileron fault (RM2: $j=2$) | $\begin{bmatrix} -5.0 & 0 & -5.0 & 0 \\ 0 & -0.1167 & 0.75 & 0 \\ 0 & -0.1667 & -0.2333 & 0 \\ 0.3333 & 0 & 0 & -0.1667 \end{bmatrix}$ | $\begin{bmatrix} 5.0 & 2.5 \\ -0.9133 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} -0.167 \\ -5.0 \\ -0.175 \pm i0.349 \end{bmatrix}$ |
| Mode 3 | Rudder fault (RM3: $j=3$) | $\begin{bmatrix} -3.3333 & 0 & -3.3333 & 0 \\ 0 & -0.7 & 4.5 & 0 \\ 0 & -0.125 & -0.175 & 0 \\ 0.25 & 0 & 0 & -0.125 \end{bmatrix}$ | $\begin{bmatrix} 3.3333 & 1.6667 \\ -5.48 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} -0.125 \\ -3.333 \\ -0.437 \pm i0.703 \end{bmatrix}$ |
| Mode 4 | Sensor fault (RM4: $j=4$) | $\begin{bmatrix} -10.0 & 0 & -10.0 & 0 \\ 0 & -0.7 & 4.5 & 0 \\ 0 & -0.5 & -0.7 & 0 \\ 1 & 0 & 0 & -0.5 \end{bmatrix}$ | $\begin{bmatrix} 10.0 & 5.0 \\ -5.48 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} -0.5 \\ -10.0 \\ -0.7 \pm i1.5 \end{bmatrix}$ |

state variables of the system and those of the reference models, and the command inputs are all needed. In cases where only a subset of the system state variables are measurable, the estimated state variables from the IMM estimator in the FDD module can be used. If this is the case, (11) can be alternatively written as:

$$\mathbf{u}_j(k) = -K_j^{\mathbf{x}}\hat{\mathbf{x}}(k|k) + K_j^{\mathbf{x}^m}\mathbf{x}_j^m(k) + K_j^{\mathbf{r}}\mathbf{r}_j'(k) \tag{19}$$

where the state variable $\mathbf{x}_j(k)$ in (11) is now replaced by its estimate $\hat{\mathbf{x}}(k|k) = \sum_{j=0}^{N-1}\hat{\mathbf{x}}_j(k|k)\cdot\mu_j(k)$ from the IMM estimator, where $\mu_j(k)$ denotes the mode probability for the $j$th model. Details on the IMM state estimator can be found in [10].

## IV. AN ILLUSTRATIVE EXAMPLE

To demonstrate the effectiveness of the proposed approach over that in [9], the same illustrative example is chosen. The system involved is an F-8 aircraft model initially used in [6].

### *A. Aircraft Model*

The linearized model of the aircraft under the normal condition can be described as

$$\begin{aligned}\dot{\mathbf{x}}(t) &= A_0\mathbf{x}(t) + B_0\mathbf{u}(t) \\ \mathbf{z}(t) &= C_0\mathbf{x}(t)\end{aligned} \tag{20}$$

where the state and the input vectors are $\mathbf{x} = [p\ r\ \beta\ \phi]^T$ and $\mathbf{u} = [\delta_a\ \delta_r]^T$, respectively, with $p$ representing the roll rate, $r$ the yaw rate, $\beta$ the sideslip angle, $\phi$ the bank angle, $\delta_a$ the aileron deflection, and $\delta_r$ the rudder deflection.

The discrete version of the system takes on the form of (9) with parameters given in Table I. It should be pointed out that only two out of four state variables, sideslip and bank angle, are measurable. For simplicity, these two variables will be designated as the controlled variables that will follow the command inputs under all simulations considered. Hence the output matrices, $H_j^{\mathbf{y}}, j = 0, \ldots, N-1$, become

$$H_j^{\mathbf{y}} = H_0 = C_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad j = 0, \ldots, N-1$$

TABLE III
COMMAND INPUTS UNDER THE NORMAL AND THE FAULT CONDITIONS

| Inputs\Modes | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Sideslip angle | 3.0 | 0.75 | 0.3 | 0.6 | 1.5 |
| Bank angle | 8.0 | 2.0 | 4.0 | 1.0 | 4.0 |

### *B. Fault Scenarios, Performance Reduced Reference Models and Adjusted Command Inputs*

*1) Fault Scenarios:* In this section, it is assumed that there is a single failure among actuators, sensors or aircraft dynamics. All faults are simulated to occur at $t_F = 8$ sec. The specific faults are: 1) a system dynamic fault as a result of a partial loss of the rudder control surface, 2) a fault in either one of the two actuators, and 3) a fault in sideslip angle sensor. Therefore, there are total of 5 possible operating modes. In practice, if additional fault scenarios or the same fault type but with different severities need to be considered, more fault modes would have to be included in the model set.

The above considered fault modes and the fault-induced changes in the system matrices are listed in Table I, with the fault-induced changes $\{\Delta A_j, \Delta B_j, \Delta C_j, j = 1, \ldots, 4\}$ being highlighted by under bar with respect to the normal system matrices $\{A_0, B_0, C_0\}$.

As can be seen from Table I, the actuator faults result in reduced values in the corresponding columns of the control matrix $B$, the sensor fault is represented also by a reduction in the corresponding row of the measurement matrix $C$, and the loss of control surface is reflected as the changes in both $A$ and $B$ matrices.

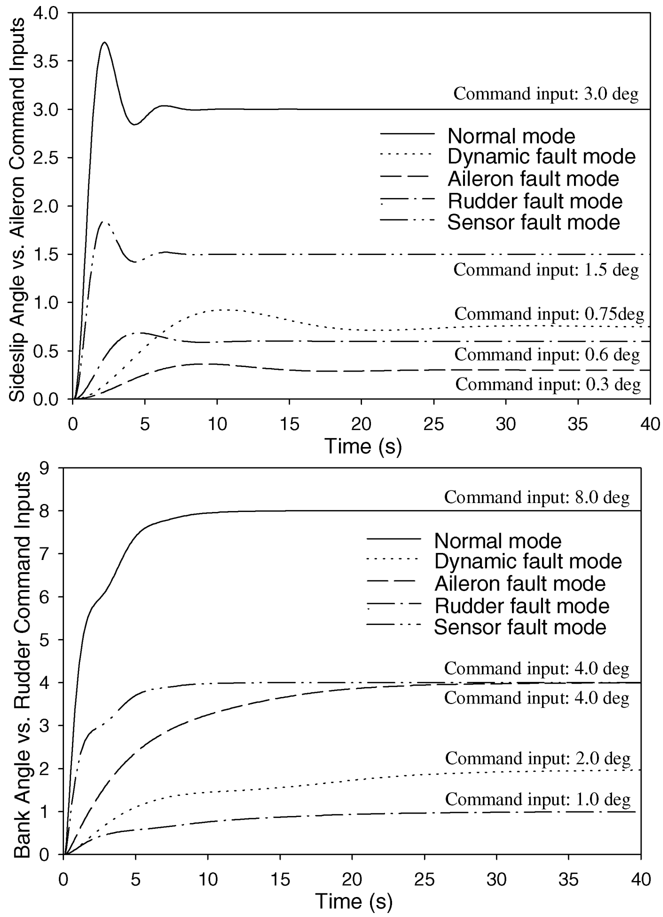


Fig. 2. Step responses of the reference models with different input levels.

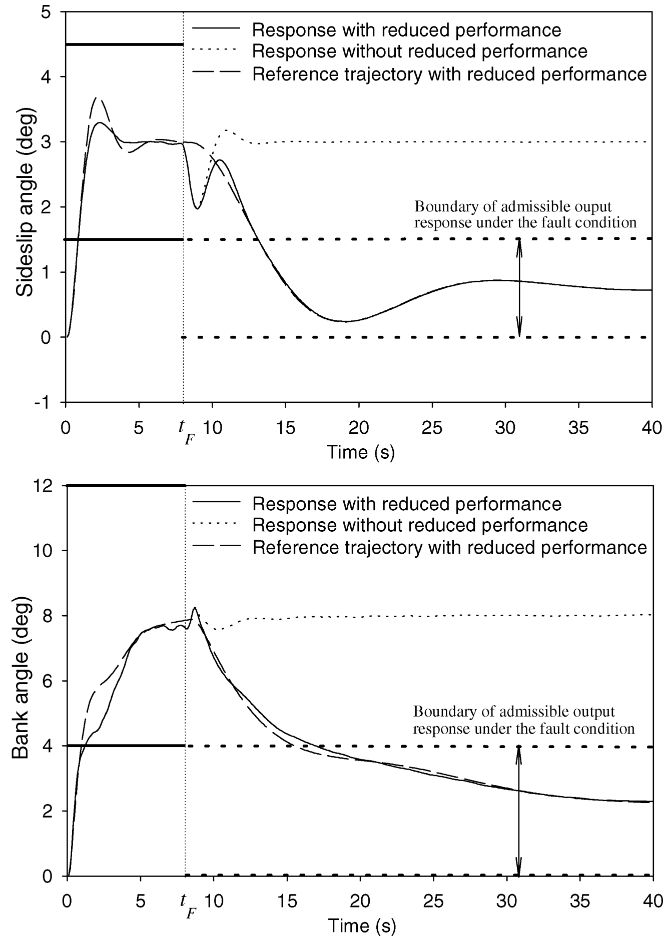


Fig. 3. Output responses under the dynamic fault.

*2) Synthesis of Performance Reduced Reference Models and Adjustment of Command Inputs:* In view of the above fault scenarios and the physical limits of the actuators, four reference models have been synthesized based on the techniques in Section II-C. The details of these models are presented in Table II. For completeness, the reference model for the normal operation has also been included.

By comparing the eigenvalues of the performance reduced models with those under the normal condition, it becomes evident that the system dynamics of the performance reduced models are much slower. This reflects the philosophy of the current approach, i.e. to reduce the performance demand accordingly whenever there is a loss of control effectiveness due to actuator or control surface fault. However, in the case of a sensor fault, an attempt has been made to maintain the original system performance by using the estimated states in feedback control.

Furthermore, to meet the steady-state performance specifications, the levels of the command inputs are also adjusted based on a command governor technique. In this example, the desired command inputs under different fault conditions are shown in Table III.

To further illustrate the characteristics of the performance reduced reference models with the revised command inputs, the step responses are shown in Fig. 2. These responses provide a visual illustration of the information in Tables II and III. Since there are 5 modes in total, there are 5 corresponding curves to represent the expected dynamic and steady-state performance. Different command input level is used in each mode. Since different reference model is designed for different fault condition, the shape of the response in each case is different. However, the mode shape under the normal (Mode 0) and sensor fault (Mode 4) conditions is the same since the same reference model has been used in these cases, except that the input level has been reduced in the sensor fault case.

## C. Simulation Results and Performance Evaluation

*1) System Performance Under Dynamic Faults:* In this case, the performance of the aircraft is examined under two conditions: with and without considering the performance limitations. The results are shown in Fig. 3. The reference trajectories for the normal and the fault conditions are also overlaid in the figures to demonstrate the command following capability. When the performance limitation is considered, the outputs of the aircraft can follow the desired reference trajectories satisfactorily in both the pre-fault and the post-fault intervals through an automatic controller switching. Before the occurrence of the fault, the two system outputs have followed the desired reference trajectories specified by the desired reference model (RM0), with

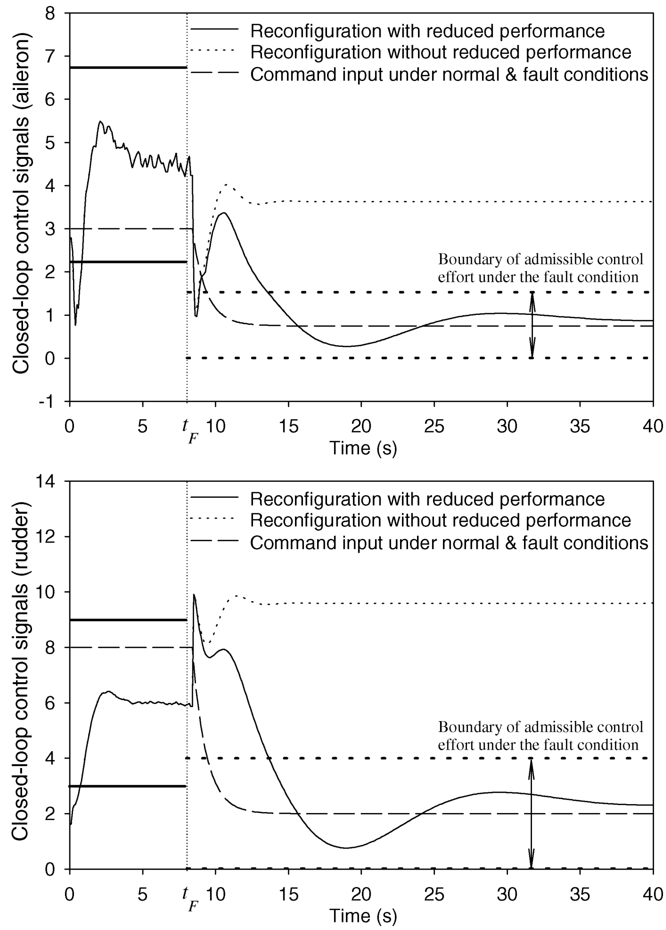


Fig. 4. Control signals under the dynamic fault.

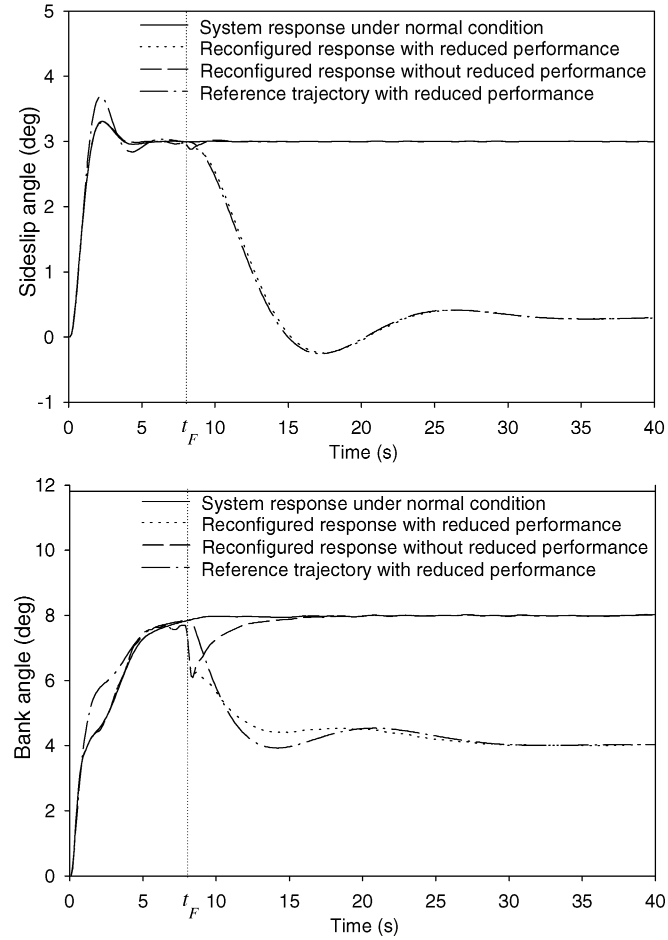


Fig. 5. Output responses under the aileron fault.

the command input set at $[\beta, \phi] = [3.0, 8.0]\,\mathrm{deg}$. After the fault is detected, the system outputs start to follow the revised reference trajectories governed by the performance reduced reference model (RM1) at the level of $[\beta, \phi] = [0.75, 2.0]\,\mathrm{deg}$. Smooth transitions in reconfigured output responses have been obtained. It is evident that the design objective with the specified performance reduction has been satisfactorily achieved. However, if no performance limitation is considered, the outputs simply cannot follow the specified reference trajectories after the fault occurrence as illustrated.

To show how the closed-loop control signals react to the fault by adapting to new values to satisfy the physical limits of the system, the closed-loop control signals and the associated command inputs are illustrated in Fig. 4. It can be seen that, compared with the closed-loop control signals under the normal condition, the control signals in both control channels have been reduced accordingly after the occurrence of the fault. However, significantly larger control signals would have been used if the performance limitation had not been considered. This is highly undesirable or even physically unrealizable, since the required control signals in both channels would have exceeded the maximal control limits, i.e. [0,1.5] for the aileron channel and [0,4] for the rudder channel. These limits are calculated based on a 100% variation around the magnitude of the control signal at the steady-state for each control channel.

*2) System Performance Under Aileron Faults:* The responses of the system in the presence of an aileron fault are illustrated in Fig. 5. For comparison purpose, system responses under the normal condition are also shown. The corresponding control signals are illustrated in Fig. 6. It can be seen that satisfactory responses have been obtained. Since the performance reduction has been considered, significantly reduced control signals in both channels are required for the aircraft to follow the performance reduced reference trajectories, as compared to the case where no performance limitations were imposed. Without due respect to the physical limitations, the magnitude of the control signals could exceed the allowable limits, or worse still, it could exceed the actuator saturation limits if a more severe aileron fault had occurred.

*3) Robustness Against System-Model Mismatch:* In practice, one of the concerns in using multiple-model approach is the robustness with respect to system-model mismatch (modeling error) since the actual fault may be different from any models in the pre-designed model set. To evaluate the robustness of the developed FTCS, different levels of mismatch have been considered for each fault mode. In this section, only the robustness analysis of the aileron fault with the mismatch ranging from $-50\%$ to $50\%$ is reported, for the interest of space. Behaviors of FDD and closed-loop tracking performance with respect to the desired reference trajectories are analyzed. For

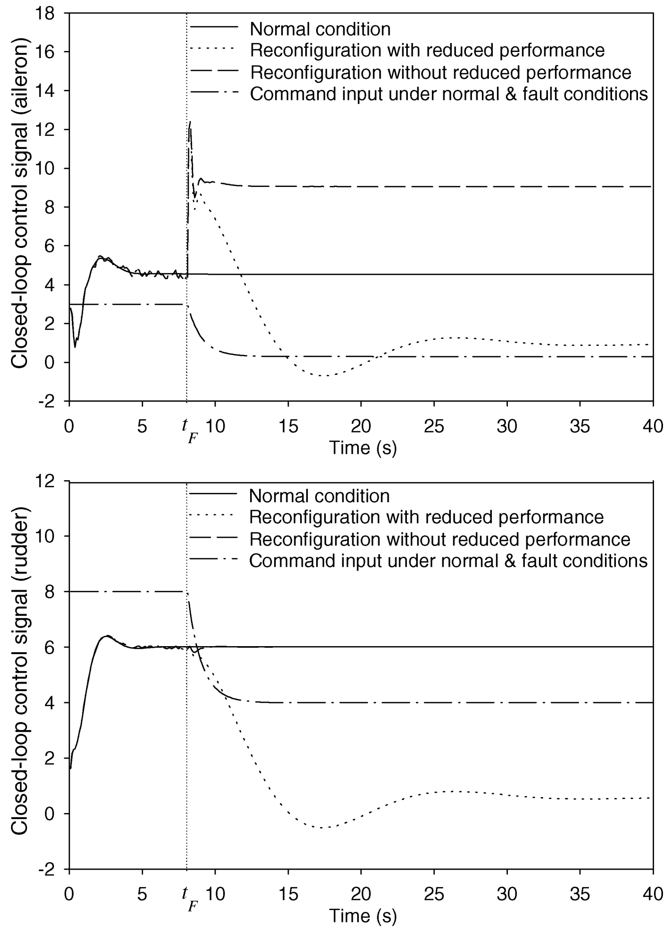


Fig. 6. Control signals under the aileron fault.

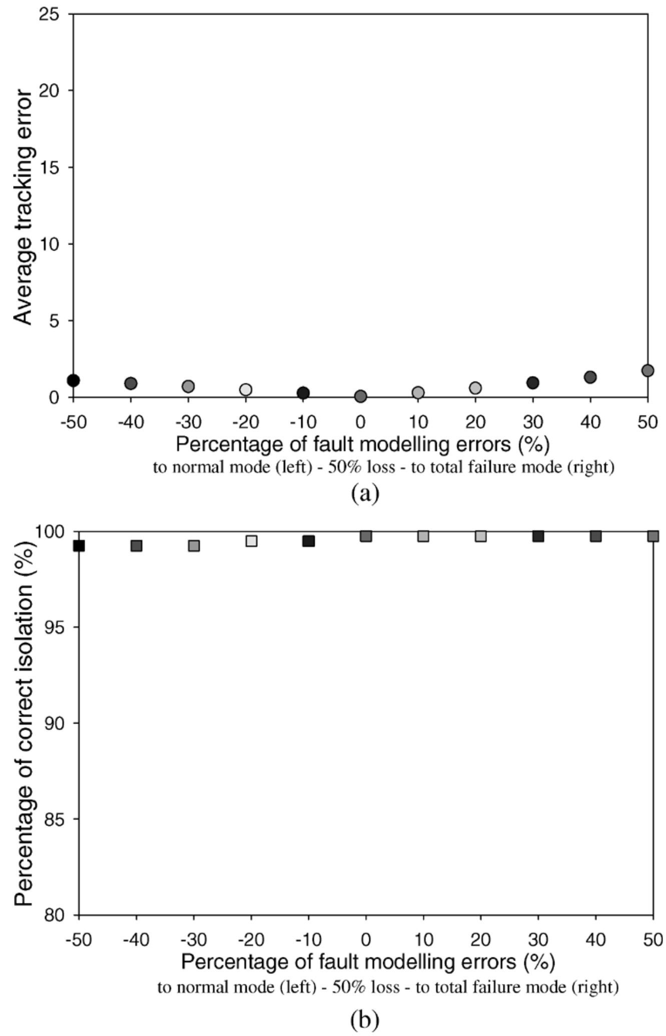


Fig. 7. Robustness analysis of modeling errors under the aileron fault.

the FDD performance, only the performance index of correct isolation is shown.

The average output tracking error and the percentage of correct fault isolation versus different levels of fault modeling errors are shown in Fig. 7(a) and (b), respectively. The average output tracking error is defined based on the following performance index

$$\bar{e} = \frac{1}{M}\sum_{k=1}^{M} e(k) \tag{21}$$

where the output tracking error at each time instant is calculated as:

$$e(k) = \begin{cases} \frac{\left\|\mathbf{y}_{desired}^{ref}(k) - \mathbf{y}^{plant}(k)\right\|}{\left\|\mathbf{y}_{desired}^{ref}(k)\right\|} & k < k_F \\ \frac{\left\|\mathbf{y}_{degraded}^{ref}(k) - \mathbf{y}^{plant}(k)\right\|}{\left\|\mathbf{y}_{degraded}^{ref}(k)\right\|} & k \geq k_F \end{cases} \tag{22}$$

where $\mathbf{y}_{desired}^{ref}(k)$ denotes the desired reference response generated on-line by the corresponding reference model associated with the command input for the normal operating condition at time $k$, and $\mathbf{y}_{degraded}^{ref}(k)$ denotes the output of the degraded reference model corresponding to one of the fault conditions with the corresponding reduced command input. $\mathbf{y}^{plant}(k)$ denotes the output of the closed-loop system before and after the fault occurrence. $M$ denotes the total data point used in the simulation.

To demonstrate further the transient behavior of the reconfigured system with respect to the robustness against system-model mismatch under different levels of mismatch, time responses of the two worst cases (−50% and +50% mismatch) versus that with no system-model mismatch are plotted in Fig. 8.

As expected, in general, the tracking performance deteriorates as the system-model mismatch increases. However, as can be seen from Fig. 7(a), the average tracking errors are less than 2 in the range of −50% to +50% mismatch. Furthermore, as can be seen from Fig. 7(b), the correct fault detection and isolation have been achieved for all cases in the considered range of the mismatch. The effects of mismatch between faults and pre-designed model can also be shown in time domain as in Fig. 8, where system outputs are compared. As can be seen, mismatch does lead to more transients, but nevertheless, the effect is relatively small at the steady-state.

## V. Conclusion

A new design method for fault-tolerant control systems with explicit consideration of achievable performance has been pro-

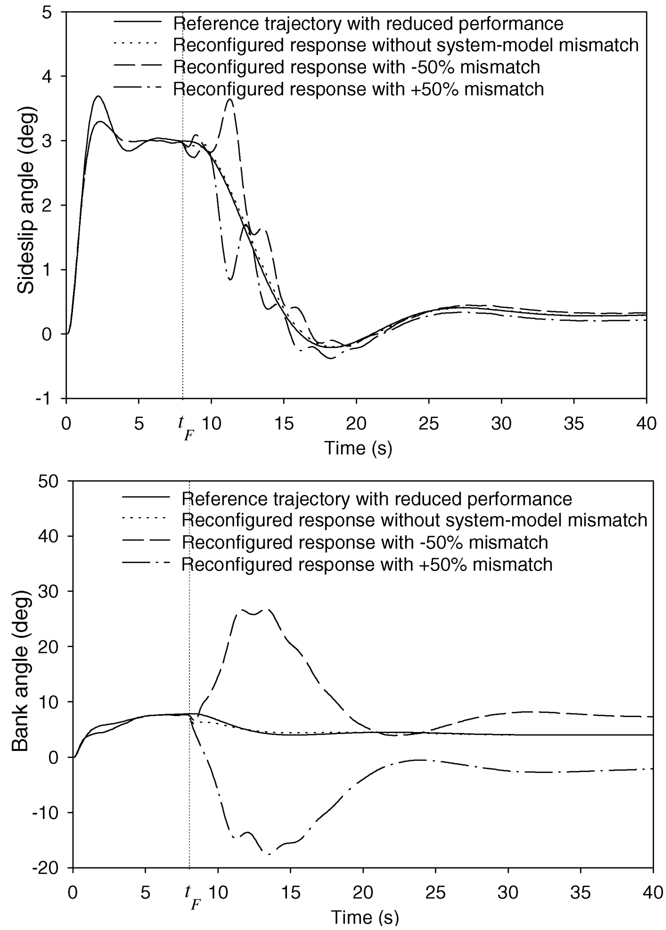


Fig. 8. Output responses with different levels of system-model mismatch.

posed in this paper based on the multiple-model approach. Associated with each fault mode is a performance reduced reference model that is synthesized by taking into consideration of the physical limits of the handicapped system. Fault-tolerant control is designed using the model reference approach with support of these reference models. Furthermore, the control system command inputs are also adjusted accordingly to avoid potential actuator saturation at the steady-state condition. Simulation results have demonstrated the effectiveness of the proposed methodology using an aircraft example and shown that if the performance limitations had not been considered, actuator saturation would have occurred.

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