# Access Point Selection for Hybrid Li-Fi and Wi-Fi Networks

**Citation for published version:**
Wu, X, Safari, M & Haas, H 2017, 'Access Point Selection for Hybrid Li-Fi and Wi-Fi Networks', *IEEE Transactions on Communications*. https://doi.org/10.1109/TCOMM.2017.2740211

**Digital Object Identifier (DOI):**
10.1109/TCOMM.2017.2740211

**Link:**
Link to publication record in Edinburgh Research Explorer

**Document Version:**
Publisher's PDF, also known as Version of record

**Published In:**
IEEE Transactions on Communications

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2740211, IEEE Transactions on Communications



# Access Point Selection for Hybrid Li-Fi and Wi-Fi Networks

Xiping Wu, *Member, IEEE*, Majid Safari, *Member, IEEE*, and Harald Haas, *Senior Member, IEEE*

*Li-Fi Research and Development Centre, Institute for Digital Communications*
*School of Engineering, The University of Edinburgh*
*EH9 3JL, Edinburgh, UK*
{xiping.wu, majid.safari, h.haas}@ed.ac.uk

***Abstract*—Hybrid light fidelity (Li-Fi) and wireless fidelity (Wi-Fi) networks are an emerging technology for future indoor wireless communications. This hybrid network combines the high-speed data transmission offered by visible light communication (VLC) and the ubiquitous coverage of radio-frequency (RF) techniques. While a hybrid network can improve the system throughput and users' experience, it also challenges the process of access point selection (APS) due to the mixture of heterogeneous access points (APs). In this paper, the differences between homogeneous and heterogeneous networks regarding APS are discussed, and a two-stage APS method is proposed for hybrid Li-Fi/Wi-Fi networks. In the first stage, a fuzzy logic system is developed to determine the users that should be connected to Wi-Fi. In the second stage, the remaining users are assigned in the environment of a homogeneous Li-Fi network. Compared with the optimisation method, the proposed method achieves a close-to-optimal throughput at significantly reduced complexity. Simulation results also show that our method greatly improves the system throughput over the conventional methods such as the signal strength strategy (SSS) and load balancing (LB), at slightly increased complexity.**

***Index Terms*—Hybrid network, access point selection, light fidelity (Li-Fi), wireless fidelity (Wi-Fi)**

## I. Introduction

Mobile communication has been technically challenged by exponentially increasing demands for data traffic. The Cisco visual networking index (VNI) [2] reports that global mobile data traffic reached 2.5 exabytes per month at the end of 2014, which is 69% more than the traffic at the end of 2013. During the same period, the average cellular network connection speed increased by 20% only. One solution to relieve the pressure on existing base stations is offloading traffic to wireless fidelity (Wi-Fi), based on the fact that over 80% of mobile data traffic comes from indoor locations. However, the dense deployment of Wi-Fi hotspots becomes the bottleneck of improving the system capacity. An alternative short-range wireless communication technology is visible light communication (VLC) [3] and its networking variant, light fidelity (Li-Fi) [4]. In Li-Fi, light-emitting diode (LED) lamps act as access points (APs), and light is used as a medium to carry information bits via intensity modulation and direct detection (IM/DD). At the receiver, a photon diode (PD) is employed to collect photons and convert them into electric current. Unlike the radio-frequency (RF) techniques including Wi-Fi, Li-Fi does not experience interference from other sources because it is contained within a specific area, and light is not transferred through opaque objects such walls. In addition, Li-Fi offers a much wider spectrum than RF, and is licence-free. Furthermore, Li-Fi can be used in RF-restricted areas such as hospitals and underwater. Recent research shows that by using a single LED, Li-Fi is capable of offering high-speed data transmission in the Gbps range [5].

A Li-Fi AP has a smaller coverage area than Wi-Fi, of approximately 2-3 m diameter [6]. In order to provide enhanced coverage, a hybrid Li-Fi and Wi-Fi network, which combines the high-speed data transmission of Li-Fi and the relatively large coverage of Wi-Fi, is envisioned for indoor wireless communications [7]. In [8], it was shown that such a hybrid network can achieve a greater throughput than stand-alone Wi-Fi or Li-Fi networks. In that study, all of the users are first connected to the Li-Fi network, and then those of low achievable data rates are switched to Wi-Fi. This access point selection (APS) method fails to take into account the fact that the required data rates might vary with users. Also, due to the limited Wi-Fi resource, switching a Li-Fi user that achieves a low data rate to Wi-Fi does not necessarily benefit the overall network performance. An apparent example is that user receives a very weak Wi-Fi signal and could drain the Wi-Fi resource to meet its demand for data rate.

The APS issue in a hybrid network is more complicated than in a homogeneous network. For homogeneous networks, a straightforward APS method is to select the AP from which the user can receive the strongest signal. This method is called the signal strength strategy (SSS), which is widely used in the current wireless networks when unbalanced load is not considered. In a homogeneous network, APs are deployed in a way with little coverage overlap to avoid inter-cell interference (ICI). In this situation, unbalanced load occurs when the number of users or their required data rates are unequally distributed among the coverage areas of APs. In a hybrid network, however, the coverage areas of different networks overlap each other. Therefore even if the traffic demands are equally distributed in geography, Wi-Fi still faces more traffic load than Li-Fi due to a larger coverage area. This makes the SSS infeasible for a hybrid network, and load balancing (LB) is of vital importance.

Considerable research has been undertaken on load balancing in a homogeneous wireless network, such as [9] and [10].

[1]This paper was presented in part at IEEE PIMRC 2016 [1].

On the contrary, few studies have been done with respect to a hybrid Li-Fi and Wi-Fi network, and those few treat heterogeneous APs in the same way as in a homogeneous network. In [11], a distributed load balancing method was proposed based on game theory, which requires quantities of iterations to reach a steady state. With the aim of maximising the overall throughput, [12] reported a centralised optimisation method, which requires extensive computational complexity. Those methods were developed from the APS solutions in a homogeneous network, and they fail to exploit the distinguishing characteristics between Li-Fi and Wi-Fi. In general, a hybrid Li-Fi and Wi-Fi network challenges the APS in two aspects: i) a Wi-Fi AP dominantly attracts the users close to it, leading to an inefficient use of nearby Li-Fi APs; and ii) a Wi-Fi AP has a larger coverage area but less capacity than a Li-Fi AP, and thus is more susceptible to overload. To the authors' best acknowledge, so far there has been no APS method specially tailored for a hybrid Li-Fi and Wi-Fi network.

Motivated by this, we propose a novel APS method based on fuzzy logic for a hybrid Li-Fi and Wi-Fi network. Fuzzy logic, which was first introduced by Lotfi A. Zadeh in 1965 [13], is an approach to computing based on "degrees of truth" rather than the usual "true or false" Boolean logic. This approach can readily handle a complicated problem by transforming it into a checklist of rules, and thus has been widely used in control. In [14], fuzzy logic was applied to the APS and resource allocation for wireless networks. The major advantage of this heuristic and near-optimal method is that it can achieve low computational complexity relative to numerically involved optimisation techniques which are often required to solve complex problems such as resource allocation in wireless networks. However, this method was developed in the context of homogeneous networks and does not address the noted APS issues in hybrid networks. The APS method proposed in this paper has two stages. In the first stage, a fuzzy logic system is developed to determine the users that are connected to the Wi-Fi network. Then in the second stage, the remaining users are assigned as if they are in a homogeneous Li-Fi network. The proposed method is a centralised algorithm, and unlike distributed methods it does not need iterations to reach a steady state. In contrast to the centralised optimisation method, the proposed method can significantly reduce processing power thanks to the use of fuzzy logic. Unlike most research in this field and our previous work in [1], in this paper a generalised indoor scenario of multiple compartments is considered. Also, the optimality and complexity of the proposed method are analysed against the optimisation method. Results show that the proposed method achieves a near-optimal solution at significantly reduced complexity. Comparisons between the proposed method and the conventional APS methods are also carried out.

The remainder of this paper is organised as follows. The system model of the hybrid Li-Fi and Wi-Fi network is described in Section II, including the network deployment and channel model. Three conventional APS methods are introduced in Section III, including the SSS, the LB and the optimisation method. Following a discussion on the key issues when conducting the conventional APS methods in a hybrid network, the novel two-stage method is proposed in Section IV. An analysis of optimality and complexity for the proposed method is given in Section IV. Simulation results are presented in Section IV. Finally, conclusions are drawn in Section VII.

## II. System Model

### A. Hybrid Li-Fi and Wi-Fi Network

Consider a generalised hybrid Li-Fi and Wi-Fi network for indoor downlink communications, where a number of rooms or compartments are taken into account, as shown in Fig. 1. Each room has a number of ceiling LED lamps, and each lamp is enabled as a Li-Fi AP covering a confined area. Also, a Wi-Fi AP is fitted in each room, providing coverage for the entire room. Though the APs might be irregularly placed in practice, we assume Li-Fi APs to be arranged in a rectangular shape and Wi-Fi APs in the room centres for the purpose of simplicity. Carrier sense multiple access with collision avoidance (CSMA/CA) is used in the Wi-Fi system [15], and therefore no interference occurs among Wi-Fi APs. Regarding the Li-Fi system, all of the Li-Fi APs reuse the same bandwidth. Since light does not penetrate walls, interference only exists between those Li-Fi APs in the same room. At each Li-Fi AP, time-division multiple access (TDMA) is adopted to serve multiple users.

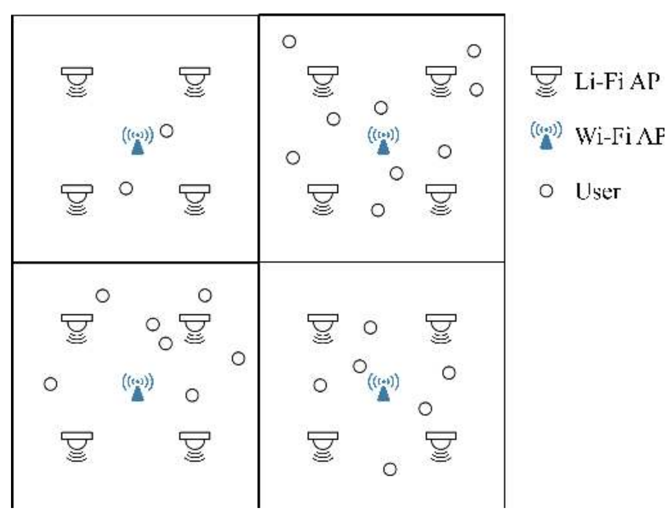



Fig. 1. Schematic diagram of an indoor hybrid Li-Fi and Wi-Fi network.

### B. Li-Fi Channel Model

A VLC channel is comprised of the line of sight (LOS) and non line of sight (NLOS) paths. The geometry of indoor VLC propagation is presented in Fig. 2. It is assumed that each user device is fitted with a PD vertically facing upwards. The LOS path of Li-Fi AP $i$ and user $u$ is the straight line between them, and the corresponding Euclidean distance is denoted by $d_{i,u}$. The angles of irradiance and incidence related to the LOS path are denoted by $\phi_{i,u}$ and $\psi_{i,u}$, respectively. The LOS channel

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2740211, IEEE Transactions on Communications



of Li-Fi is formulated as [16, eq. (10)]:

$$H_{\text{LOS}}^{i,u} = \frac{(m+1)A_{\text{pd}}}{2\pi d_{i,u}^2} \cos^m(\phi_{i,u}) g_f g_c(\psi_{i,u}) \cos(\psi_{i,u}), \quad (1)$$

where $m = -\ln 2/\ln(\cos\Phi_{1/2})$ is the Lambertian emission order, and $\Phi_{1/2}$ is the radiation angle at which the intensity is half of the intensity at the main-beam direction; $A_{\text{pd}}$ denotes the physical area of PD; $g_f$ is the gain of the optical filter; and $g_c(\psi_{i,u})$ is the optical concentrator gain, which is given by:

$$g_c(\psi_{i,u}) = \begin{cases} \frac{n^2}{\sin^2(\Psi_{\max})}, & 0 \le \psi_{i,u} \le \Psi_{\max} \\ 0, & \psi_{i,u} > \Psi_{\max} \end{cases}, \quad (2)$$

where $n$ denotes the refractive index, and $\Psi_{\max}$ is the semi-angle of the field of view (FOV) of the PD.

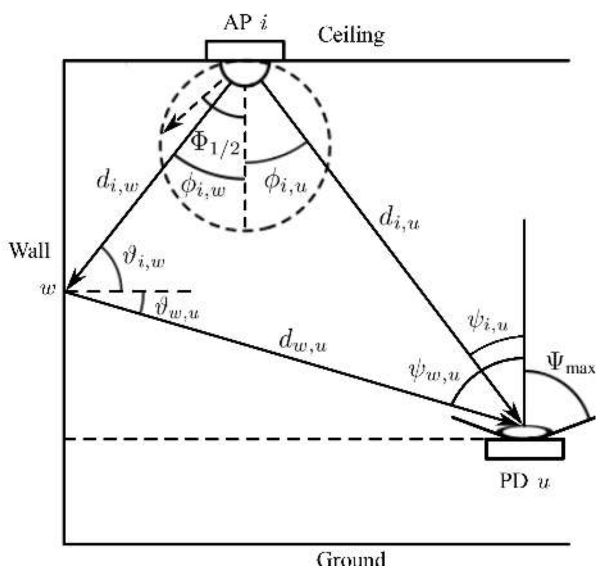



Fig. 2. Geometry of indoor VLC downlink propagation.

For the NLOS path, only first-order reflections are taken into account for the purpose of simplicity. A first-order reflection consists of two segments: i) from the AP to a small area $w$ on the wall; and ii) from $w$ to the user. The Euclidean distances of those segments are denoted by $d_{i,w}$ and $d_{w,u}$, respectively. The angles of radiance and incidence corresponding to the first segment are $\phi_{i,w}$ and $\vartheta_{i,w}$, and for the second segment they are denoted by $\vartheta_{w,u}$ and $\psi_{w,u}$. Since the indoor propagation paths are relatively short, the delay between different paths can be neglected. In other words, the signals of different paths are assumed to arrive at the receiver simultaneously. According to [16, eq. (12)], the NLOS channel of Li-Fi is given in (3), where $A_w$ denotes a small reflective area on the wall, and $\rho_w$ is the wall reflectivity. Combining and (1) and (3), the channel from Li-Fi AP $i$ to user $u$ is given by:

$$H_{\text{Li-Fi}}^{i,u} = H_{\text{LOS}}^{i,u} + H_{\text{NLOS}}^{i,u}. \quad (4)$$

TABLE I
LI-FI CHANNEL PARAMETERS

| Parameter | Value |
|---|---|
| Height between the ceiling and user, $h$ | 2 m |
| The physical area of a PD, $A_{\text{pd}}$ | 1 cm$^2$ |
| The gain of the optical filter, $g_f$ | 1 |
| The refractive index, $n$ | 1.5 |
| Half-intensity radiation angle, $\Phi_{1/2}$ | 60° |
| FOV semi-angle of PD, $\Psi_{\max}$ | 90° |
| Transmit optical power per Li-Fi AP, $P_{\text{opt}}$ | 3 Watt |
| Optical to electric power conversion coefficient, $\kappa$ | 3 |
| Detector responsivity, $R_{\text{pd}}$ | 0.53 A/W |
| Wall reflectivity, $\rho_w$ | 0.8 |
| Bandwidth per Li-Fi AP, $B_{\text{Li-Fi}}$ | 40 MHz |
| PSD of Li-Fi noise, $\mathcal{N}_{\text{Li-Fi}}$ | $10^{-21}$ A$^2$/Hz |

At the receiver, photons are gathered by a PD, and then converted into an electric current. The current value is measured by:

$$I_{\text{elec}} = R_{\text{pd}} H_{\text{Li-Fi}}^{i,u} P_{\text{opt}}/\kappa, \quad (5)$$

where $R_{\text{pd}}$ is the detector responsivity; $P_{\text{opt}}$ is the transmitted optical power; and $\kappa$ denotes the optical to electric power conversion coefficient. The coefficient $P_{\text{opt}}/\kappa$ is equivalent to the optical signal power. The signal-to-interference-plus-noise ratio (SINR) of the desired signal received by user $u$ is written as:

$$\text{SINR}_{\text{Li-Fi}}^{i,u} = \frac{(R_{\text{pd}} H_{\text{Li-Fi}}^{i,u} P_{\text{opt}}/\kappa)^2}{\mathcal{N}_{\text{Li-Fi}} B_{\text{Li-Fi}} + \sum_{j \ne i} (R_{\text{pd}} H_{\text{Li-Fi}}^{j,u} P_{\text{opt}}/\kappa)^2}, \quad (6)$$

where $\mathcal{N}_{\text{Li-Fi}}$ is the power spectral density (PSD) of noise at the PD, and $B_{\text{Li-Fi}}$ is the system bandwidth of a Li-Fi AP. The Li-Fi channel parameters used for simulations are summarised in Table I.

### C. Wi-Fi Channel Model

The path loss model used for indoor RF propagation consists of the free space loss (slope of 2) up to a breakpoint distance, and a slope of 3.5 after the breakpoint distance [17]. Let $L_{\text{FS}}(\cdot)$, $X_\sigma$ and $d_{\text{BP}}$ denote the free space loss, the shadow fading and the breakpoint distance, respectively. The path loss is written as [15, eq. (3.26)]:

$$L(d) = \begin{cases} L_{\text{FS}}(d) + X_\sigma, & d \le d_{\text{BP}} \\ L_{\text{FS}}(d) + 35\log_{10}\left(\frac{d}{d_{\text{BP}}}\right) + X_\sigma, & d > d_{\text{BP}} \end{cases}, \quad (7)$$

where $d$ is the distance between a user and a Wi-Fi AP; $X_\sigma$ is a Gaussian random variable with zero mean and standard deviation $\sigma$; and $L_{\text{FS}}(\cdot)$ is given by:

$$L_{\text{FS}}(d) = 20\log_{10}(d) + 20\log_{10}(f_c) - 147.5, \quad (8)$$

where $f_c$ is the central carrier frequency.

The multipath propagation of Wi-Fi channel is formulated as [15, eq. (3.13)]:

$$H_{\text{Wi-Fi}} = \sqrt{\frac{K}{K+1}} e^{j\phi} + \sqrt{\frac{1}{K+1}} X_1, \quad (9)$$

$$H_{\text{NLOS}}^{i,u} = \int_{A_w} \frac{(m+1)A_{\text{pd}}}{2(\pi d_{i,w} d_{w,u})^2} \rho_w \cos^m(\phi_{i,w}) g_f g_c(\psi_{w,u}) \cos(\psi_{w,u}) \cos(\vartheta_{i,w}) \cos(\vartheta_{w,u}) dA_w, \tag{3}$$

TABLE II
WI-FI CHANNEL PARAMETERS

| Parameter | Value |
|---|---|
| Breakpoint distance, $d_{\text{BP}}$ | 5 m |
| Shadow fading standard deviation, $\sigma$ (before $d_{\text{BP}}$) | 3 dB |
| Shadow fading standard deviation, $\sigma$ (after $d_{\text{BP}}$) | 5 dB |
| Central carrier frequency, $f_c$ | 2.4 GHz |
| The angle of arrival/departure of LOS, $\phi$ | 45° |
| Transmit power, $P_{\text{Wi-Fi}}$ | 20 dBm |
| Bandwidth per Wi-Fi channel, $B_{\text{Wi-Fi}}$ | 20 MHz |
| PSD of noise, $\mathcal{N}_{\text{Wi-Fi}}$ | -174 dBm/Hz |

TABLE III
MODULATION AND CODING SCHEME [18]

| min. SINR [dB] | Modulation | Code rate | Spectrum efficiency [bit/s/Hz] |
|---|---|---|---|
| - | - | - | 0 |
| 2 | BPSK | 1/2 | 0.5 |
| 4 | BPSK | 3/4 | 0.75 |
| 5 | QPSK | 1/2 | 1 |
| 9 | QPSK | 3/4 | 1.5 |
| 11 | 16QAM | 1/2 | 2 |
| 15 | 16QAM | 3/4 | 3 |
| 18 | 64QAM | 2/3 | 4 |
| 20 | 64QAM | 5/6 | 5 |

where $X_1$ is a complex Gaussian random variable with zero mean and unit variance, $\phi$ is the angle of arrival/departure of the LOS component, and $K$ is the Ricean $K$-factor ($K = 1$ before the breakpoint and $K = 0$ after the breakpoint). The channel gain between Wi-Fi AP $i$ and user $u$ is denoted by $G_{\text{Wi-Fi}}^{i,u}$, and it is calculated by:

$$G_{\text{Wi-Fi}}^{i,u} = \left| H_{\text{Wi-Fi}}^{i,u} \right|^2 10^{-\frac{L(d_{i,u})}{10}}. \tag{10}$$

As no interference is counted in the Wi-Fi system, the SINR of a Wi-Fi user is given by:

$$\text{SINR}_{\text{Wi-Fi}}^{i,u} = \frac{G_{\text{Wi-Fi}}^{i,u} P_{\text{Wi-Fi}}}{\mathcal{N}_{\text{Wi-Fi}} B_{\text{Wi-Fi}}}, \tag{11}$$

where $\mathcal{N}_{\text{Wi-Fi}}$ is the PSD of noise at the receiver; $P_{\text{Wi-Fi}}$ and $B_{\text{Wi-Fi}}$ are the transmit power and bandwidth consumed by a Wi-Fi AP, respectively. The Wi-Fi channel parameters used for simulations are listed in Table II.

### D. User's Throughput, Satisfaction and Fairness

Given the SINR for a user, its spectrum efficiency is determined by the modulation and coding scheme, which is summarised in Table III. Let $\boldsymbol{U}_i$ denote the set of users served by AP $i$. The proportion of the transmission time that AP $i$ spends on user $u$ is denoted by $\tau_{i,u}$, which is constrained to $\sum_{u \in \boldsymbol{U}_i} \tau_{i,u} \leq 1$. The data rate AP $i$ can provide to user $u$ is denoted by $\tilde{R}_{i,u}$, and it is computed by:

$$\tilde{R}_{i,u} = \tau_{i,u} \eta_{i,u} B_i, \tag{12}$$

where $B_i$ is the bandwidth of AP $i$, depending on its type of Li-Fi or Wi-Fi.

The user's required data rate, denoted by $R_u$, is a random variable following a gamma distribution with an average value $\bar{R}$ [19]. The user's satisfaction $S_{i,u}$ is defined as the ratio of the achieved data rate to the required data rate, i.e. $S_{i,u} = \tilde{R}_{i,u}/R_u$. In order to offer a proportional fairness among the users, $\tau_{i,u}$ is calculated by:

$$\tau_{i,u} = \begin{cases} \dfrac{R_u}{\eta_{i,u} B_i}, & \text{if } \sum\limits_{u \in \boldsymbol{U}_i} \dfrac{R_u}{\eta_{i,u} B_i} \leq 1 \\ \dfrac{1}{\sum\limits_{u \in \boldsymbol{U}_i} \dfrac{R_u}{\eta_{i,u}}} \times \dfrac{R_u}{\eta_{i,u}}, & \text{if } \sum\limits_{u \in \boldsymbol{U}_i} \dfrac{R_u}{\eta_{i,u} B_i} > 1 \end{cases}. \tag{13}$$

The fairness among users is measured by Jain's fairness index [20]. Denoting the total number of users by $N_u$, the users' fairness is computed by:

$$\xi = \frac{\left( \sum\limits_{u=1}^{N_u} S_{i,u} \right)^2}{N_u \sum\limits_{u=1}^{N_u} S_{i,u}^2}. \tag{14}$$

## III. CONVENTIONAL ACCESS POINT SELECTION METHODS

For simulation benchmarks we introduce three conventional APS methods: the signal strength strategy (SSS), the load balancing (LB), and the optimisation method. The process of those methods is elaborated in this section, while their respective drawbacks when applied to a hybrid network are discussed in Section IV-A.

### A. Signal Strength Strategy

The SSS is a straightforward method that always selects the AP offering the highest spectrum efficiency. In a homogeneous network, the receiver experiences the same level of noise power when collecting the signals emitted from different APs. Therefore for the user of interest, the SSS method simply selects the AP that delivers the highest received signal power. In a hybrid network, however, different mechanisms are needed to receive light and radio signals, leading to different noise power per bandwidth between the Li-Fi and Wi-Fi systems. Also, those two systems could use different bandwidths. Thus we adopt signal-to-noise ratio (SNR) instead of signal strength to perform the SSS method in a hybrid network. With AP $i$ sending out the desired signal, the received SNR at user $u$ is denoted by $\gamma_{i,u}$:

$$\gamma_{i,u} = \begin{cases} \dfrac{(R_{\text{pd}} H_{\text{Li-Fi}}^{i,u} P_{\text{opt}}/\kappa)^2}{\mathcal{N}_{\text{Li-Fi}} B_{\text{Li-Fi}}}, & \text{if } i \text{ is a Li-Fi AP} \\ \dfrac{G_{\text{Wi-Fi}}^{i,u} P_{\text{Wi-Fi}}}{\mathcal{N}_{\text{Wi-Fi}} B_{\text{Wi-Fi}}}, & \text{if } i \text{ is a Wi-Fi AP} \end{cases}. \tag{15}$$

The objective function (OF) of the SSS method for a given user is then written as:

$$\begin{aligned} \text{maximise} \quad & \gamma_{i,u} \\ \text{subject to} \quad & i \in \boldsymbol{L}_u \cup \boldsymbol{W}. \end{aligned} \tag{16}$$

where $\boldsymbol{L}_u$ denotes the set of Li-Fi APs in the same room as user $u$, and $\boldsymbol{W}$ is the set of all available Wi-Fi APs.

### *B. Load Balancing*

The LB methods, which consider resource availability as well as channel quality, can be classified into two categories: channel borrowing and traffic transfer. Since Li-Fi and Wi-Fi operate at different spectrum, channel borrowing is infeasible in a hybrid Li-Fi/Wi-Fi network. Here we consider a straight-forward traffic-transfer method, while the optimisation-based LB is deemed as an optimisation method.

Using this LB method, the user is connected to the AP offering the highest SNR if that AP can meet the user's data rate requirement. Otherwise, the user selects the AP that provides the highest user's satisfaction. Note that this AP could still be the one offering the highest SNR. If several APs achieve the highest user's satisfaction, the one having the highest SNR is chosen. In other words, the LB method first maximises the user's satisfaction, and then maximises the channel quality. The corresponding OF is expressed as:

$$\begin{aligned} \text{maximise} \quad & \gamma_{i,u} \\ \text{subject to} \quad & i \in \max\{S_{i,u} | i \in \boldsymbol{L}_u \cup \boldsymbol{W}\}. \end{aligned} \tag{17}$$

### *C. Optimisation Method*

The most commonly used optimisation method is max-sum-log-rate wise [12, eq. (14)]:

$$\text{maximise} \quad \sum_{u \in \boldsymbol{U}} \log(\tilde{R}_u). \tag{18}$$

Substituting (12) into (18), the OF is rewritten as:

$$\Gamma(\boldsymbol{\alpha}) = \sum_{u \in \boldsymbol{U}} \sum_{i \in \boldsymbol{L}_u \cup \boldsymbol{W}} \alpha_{i,u} \log(\tau_{i,u} \eta_{i,u} B_i), \tag{19}$$

where $\alpha_{i,u}$ is a binary value that indicates the connection status: $\alpha_{i,u} = 1$ means user $u$ is connected to AP $i$, and $\alpha_{i,u} = 0$ means otherwise. The elements of $\alpha_{i,u}$ for all pairs of AP and user constitute the matrix $\boldsymbol{\alpha}$, which is a possible solution to the APS for all involved users.

With the constraint that a user can be assigned to only one AP, this optimisation problem is formulated as:

$$\begin{aligned} \text{maximise} \quad & \Gamma(\boldsymbol{\alpha}) \\ \text{subject to} \quad & \sum_{i \in \boldsymbol{L}_u \cup \boldsymbol{W}} \alpha_{i,u} = 1 \quad \forall u \in \boldsymbol{U}; \\ & \alpha_{i,u} \in \{0, 1\} \quad \forall i \in \boldsymbol{L} \cup \boldsymbol{W}, u \in \boldsymbol{U}. \end{aligned} \tag{20}$$

Note that whenever a new user requests access to the network, the optimisation method needs to solve the APS for all users. On the contrast, the LB method selects the AP for the new user based on the current network situation, but does not affect the connection status of existing users. As for the SSS method, the APS is performed for each user individually.

## IV. Proposed Access Point Selection Method

In this section, based on the different characteristics between Li-Fi and Wi-Fi in terms of coverage and capacity, we propose a tailor-made APS method for the hybrid network. The main contribution of this section is three-fold: i) analyse the key issues when conducting the conventional APS methods in a hybrid network; ii) formulate the APS process as a two-stage problem, which firstly determines the users that need service from Wi-Fi and then performs APS for the remaining users as if in a homogeneous Li-Fi network; and iii) apply fuzzy logic to the first stage to rank the user's priority of accessing Wi-Fi. Regarding the second stage, a conventional APS method, such as the SSS and LB, is applicable.

### *A. Discussion about the APS in a Hybrid Network*

With respect to APS, a hybrid network differs from a homogeneous network in two aspects: i) the coverage areas of different systems overlay one another; and ii) the coverage range varies with the AP types. The first point widens the scale of possible options for APS, leading to an exponential increase in the computational complexity required by the optimisation method. See the complexity analysis in Section V-B. Regarding the second point, a Wi-Fi AP has a larger coverage area but less capacity than a Li-Fi AP. In Fig. 3, the Wi-Fi SNR is stronger than the Li-Fi SNR in the green area, which covers 32% of the room, while otherwise in the red area. Considering uniformly distributed users, this means the Wi-Fi AP has to serve 32% users if the SSS is adopted. Meanwhile, in average, each Li-Fi AP serves less than 6% users. Therefore in this situation the Wi-Fi system is prone to be overloaded, i.e. it cannot meet the data rate demands of all served users. Also, it is worth noting the users nearby a Wi-Fi AP are attracted to Wi-Fi, even if they are right beneath a Li-Fi AP (e.g. user 1). As a result, the Li-Fi APs close to a Wi-Fi AP are underused. The LB method can relieve the congestion of Wi-Fi by diverting new users to Li-Fi. However, because of not affecting the AP assignment of existing users, the LB method does not necessarily improve the usage of those underused Li-Fi APs. The lack of efficiency in Li-Fi raises an open question: assigning what kind of users to Wi-Fi (or Li-Fi) is beneficial to the entire hybrid network?

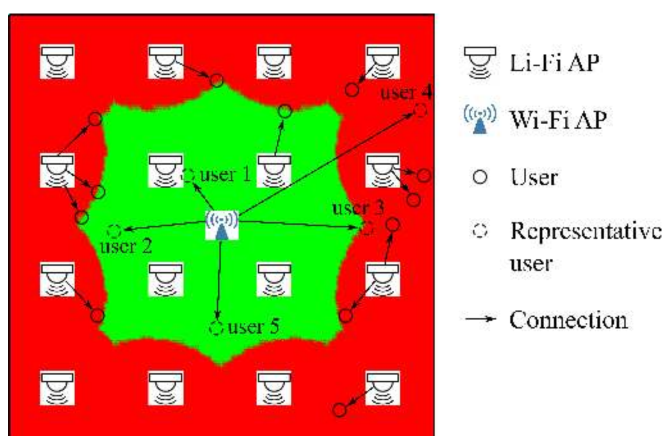



Fig. 3. Representative users for APS in a hybrid Li-Fi and Wi-Fi network.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2740211, IEEE Transactions on Communications



Fig. 3 demonstrates some representative users. Due to the presence of ICI in the Li-Fi network, cell-centre users (e.g. user 1) obtain a much higher SINR and thus a much higher spectrum efficiency than cell-edge users (e.g. user 2). Note that both user 1 and 2 would be connected to Wi-Fi if the SSS is applied. To reach the same data rate, user 2 requires more resource than user 1 if they are both switched to Li-Fi. Hence assigning user 2 to Wi-Fi is better than assigning user 1, though user 1 receives a stronger Wi-Fi signal than user 2 does. User 3 is in a situation similar to user 2, but locates in the field where the Wi-Fi SNR is lower than the Li-Fi SNR. In other words, user 3 is connected to Li-Fi when using the SSS method. Because of receiving a lower Wi-Fi SNR, user 3 has a lower priority than user 2 to use the Wi-Fi resource.

However, not all of the Li-Fi cell-edge users should be switched to Wi-Fi. When a user experiences a very weak Wi-Fi signal (e.g. user 4), it would consume a substantial quantity of Wi-Fi resource. Therefore this user is better to stay in the Li-Fi network in order to avoid draining the Wi-Fi resource. Another case is when the Li-Fi APs adjacent to a user are not in service (e.g. user 5). In this case, the user receives slight interference from distant Li-Fi APs, and thus is better to be served by Li-Fi so as to offload traffic from Wi-Fi.

### *B. Proposed APS Method*

The APS for a hybrid Li-Fi and Wi-Fi network is formulated as a two-stage problem: i) determine the users that need to be served by Wi-Fi; and ii) conduct APS for the remaining users as if they are in a stand-alone Li-Fi network. We apply fuzzy logic (FL) to fulfil the task of the first stage, while the SSS or LB can be used in the second stage. Correspondingly, the formed methods are referred to as the FL-SSS and FL-LB. In the following context, a FL system is developed to measure how well a user should be assigned to Wi-Fi.

Fig. 4 presents the block diagram of the FL system, which is comprised of three steps: *fuzzification*, *rule evaluation* and *defuzzification*. In general, based on some information of a user, the FL system outputs an accessibility score, which indicates the priority of connecting that user to Wi-Fi. Here four parameters are considered as input: the required data rate, the Wi-Fi SNR, the SNR variance of adjacent Li-Fi APs, and the activity of adjacent Li-Fi APs. Taking the network deployment in Fig. 3 as an example, the Li-Fi APs adjacent to a certain user are the four closest ones. The SNR variance of adjacent APs reflects how deeply a user might be affected by interference. A zero variance means the user receives equal signal power from the adjacent APs. Consequently, this user will experience severe interference if those APs are transmitting signals. The activity of an AP is defined as the percentage of time during which the AP is active for data transmission. The average activity of adjacent APs reflects how likely a user is affected by interference.

*1) Fuzzification:* The first step of FL is fuzzification, which converts a single-valued input into the values of a fuzzy set comprised of a number of membership functions (MFs). The only condition an MF must satisfy is that it must vary between 0 and 1, and there is a very wide selection of MFs. Here we adopt a set of MFs commonly used in Matlab: Z-shaped, S-shaped and Π-shaped MFs [21]. Each parameter is classified into three categories: low, medium and high. The category low has a Z-shaped MF:

$$f_{\mathrm{MF}}^{\mathrm{low}}(x;a,b)=\begin{cases}1, & x\le a\\ 1-2\left(\dfrac{x-a}{b-a}\right)^2, & a\le x\le \dfrac{a+b}{2}\\ 2\left(\dfrac{x-b}{b-a}\right)^2, & \dfrac{a+b}{2}\le x\le b\\ 0, & x\ge b\end{cases}. \quad (21)$$

The category high has a S-shaped MF, which is the opposite of a Z-shaped MF:

$$f_{\mathrm{MF}}^{\mathrm{high}}(x;b,c)=1-f_{\mathrm{MF}}^{\mathrm{low}}(x;b,c). \quad (22)$$

The category medium has a simplified Π-shaped MF, which is given by:

$$f_{\mathrm{MF}}^{\mathrm{med}}(x;a,b,c)=\begin{cases}0, & x\le a\\ 2\left(\dfrac{x-a}{b-a}\right)^2, & a\le x\le \dfrac{a+b}{2}\\ 1-2\left(\dfrac{x-b}{b-a}\right)^2, & \dfrac{a+b}{2}\le x\le b\\ 1-2\left(\dfrac{x-b}{c-b}\right)^2, & b\le x\le \dfrac{b+c}{2}\\ 2\left(\dfrac{x-c}{c-b}\right)^2, & \dfrac{b+c}{2}\le x\le c\\ 1, & x\ge c\end{cases}. \quad (23)$$

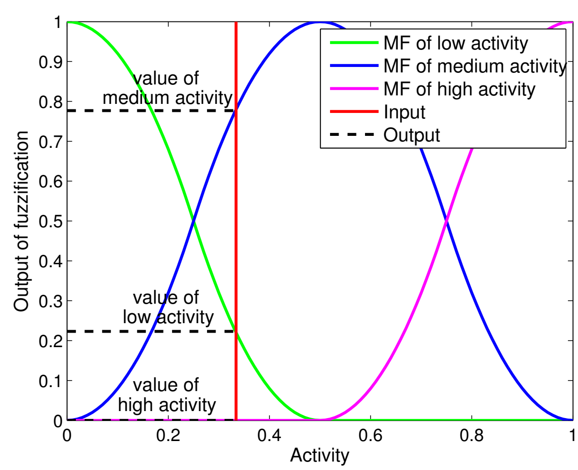



Fig. 5. An example of the fuzzification process.

The coefficients $a$, $b$, and $c$ for one parameter are chosen to be its minimum, median and maximum values. Fig. 5 gives an example of the fuzzification process regarding the activity of adjacent Li-Fi APs. As the activity is a value between 0 and 1, here we have $a=0$, $b=0.5$ and $c=1$. For other parameters, these coefficients are determined by statistics. According to the input value and MFs, for each category the fuzzification process outputs one value, which reflects how well the input

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2740211, IEEE Transactions on Communications



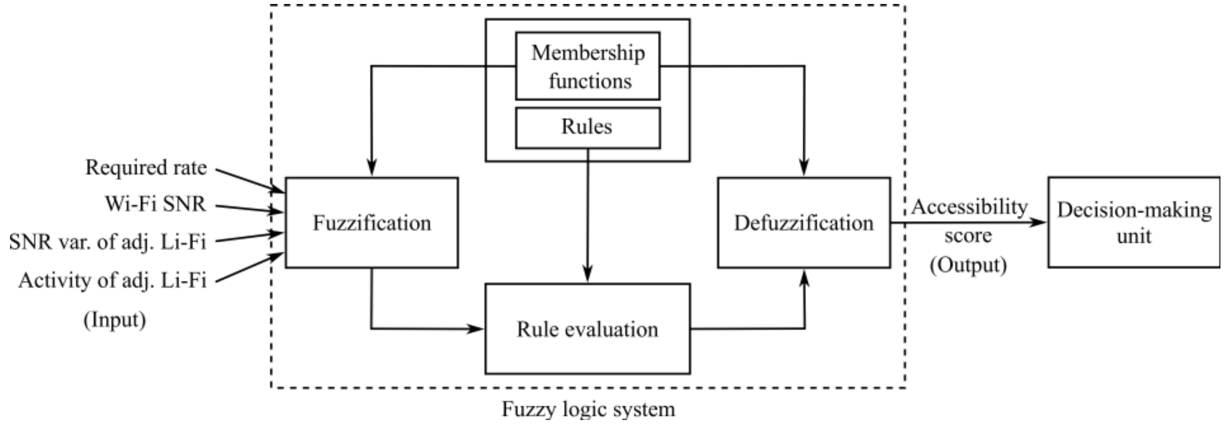



Fig. 4. Block diagram of the fuzzy logic system.

value fits that category. In the example of Fig. 5, the activity has an input of 0.33, indicated by the cross point between the red line and the horizontal axis. The output for each category is the cross point between the red line and the corresponding MF curve. In this case, the outputs for the low, medium and high activity are 0.22, 0.78 and 0, respectively.

*2) Rule Evaluation:* In this step, a number of fuzzy rules are used to measure the benefit or detriment of assigning a certain user to Wi-Fi, as shown in Table IV. These rules are self-explanatory. For example, those users with low SNR variance and high activity (rules 1 and 2) are better to access the Wi-Fi system except under certain circumstances (rule 6). Depending on the operator of AND/OR, the output value of each rule is the minimum or maximum value of all involved components. There are three states of output: 'positive'/'negative' signifies the assignment is beneficial/detrimental to the overall network performance, while 'neutral' stands for a result inbetween. The process of rule evaluation obtains one value for each state, which is the maximum value of the rules of the same state.

*3) Defuzzification:* This step obtains a single-valued score for each user. This score signifies the priority of a user accessing Wi-Fi, and thus is termed accessibility score. Similar to the step of fuzzification, MFs are used to describe the relation between the aforementioned states and the score. Here a Trapezoidal-shaped MF is adopted:

$$f_{\rm MF}(x;a,b,c,d)=\begin{cases}0, & x\le a\\ \dfrac{x-a}{b-a}, & a\le x\le b\\ 1, & b\le x\le c\\ \dfrac{d-x}{d-c}, & c\le x\le d\\ 0, & x\ge d\end{cases}, \tag{24}$$

where $x$ denotes the possible accessibility score.

The settings of coefficients $a$. $b$, $c$ and $d$ for each state are not fixed, but normally need to provide overlap between the MFs of two neighbouring states. In this paper we set $a = 0$, $b = 0$, $c = 0.2$ and $d = 0.4$ for the state 'negative'; $a = 0.2$, $b = 0.5$, $c = 0.5$ and $d = 0.8$ for the state 'neutral'; $a = 0.6$, $b = 0.8$, $c = 1$ and $d = 1$ for the state 'positive'. An example of the defuzzification process is given in Fig. 6. In this example, the values of the 'negative', 'neutral' and 'positive' states yielded by rule evaluation are 0.1, 0.6 and 0.3, respectively. For each state, the area that is below both its MF and the state value is shaded in the figure. This area reflects how significantly that state influences the score. The shaded areas pertaining to the three states merge into a whole shaded area, of which the upper edge is denoted by $f(x)$. Using the centroid method, the accessibility score of user $u$, denoted by $\zeta_u$, is computed by:

$$\zeta_u=\frac{\int_0^1 f(x)x dx}{\int_0^1 f(x)dx}. \tag{25}$$

The obtained score ranges between 0 and 1, and a higher score means assigning the corresponding user to Wi-Fi is more beneficial to the hybrid network. In the example of Fig. 6, the accessibility score output by the defuzzification process is 0.56, indicated by the cross point between the red line and the horizontal axis.

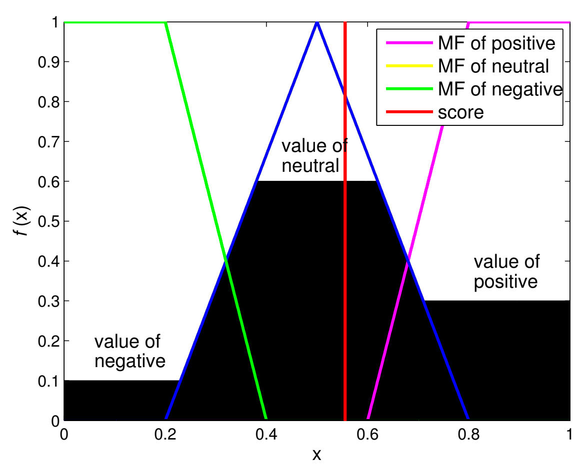



Fig. 6. An example of the defuzzification process.

*4) Decision-making:* A central unit is employed to store and process the accessibility scores of all users. When a new user requests access to the network, its score is compared with

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2740211, IEEE Transactions on Communications



TABLE IV
FUZZY RULES OF ASSIGNING USERS TO WI-FI.

| Rule No. | Operator | Required rate | Wi-Fi SNR | SNR var. of adj. Li-Fi APs | Activity of adj. Li-Fi APs | Assigned to Wi-Fi |
|---|---|---|---|---|---|---|
| 1 | AND | - | High | Low | High | Positive |
| 2 | AND | Low | *not* Low | Low | High | Positive |
| 3 | AND | - | High | Low | Med | Neutral |
| 4 | AND | Med | *not* Low | Med | High | Neutral |
| 5 | OR | - | - | High | Low | Negative |
| 6 | AND | High | Low | - | - | Negative |

the scores of the existing users. The users are assigned to Wi-Fi in a descending order of their accessibility scores, until the Wi-Fi resource is depleted. Those users that fail to access Wi-Fi are then connected to Li-Fi. Note that whenever a Li-Fi AP connects or disconnects a user, the activity status of that AP needs to be updated. Consequently, the accessibility score has to be remeasured for the users adjacent to that AP.

## V. OPTIMALITY AND COMPLEXITY OF THE PROPOSED METHOD

Because of the heuristic nature and non-linearity of fuzzy logic, it is very difficult to derive an analytical expression for the system performance of the proposed method. As an alternative, an experimental comparison between the proposed method and the benchmarks methods is undertaken. See the system setup in Section II and VI. We demonstrate that the proposed method provides a near-optimal throughput at significantly reduced complexity.

### A. Optimality

Fixing the average required data rate at 10 Mbps, Fig. 7 shows users' average throughput as a function of the number of users. Due to the excessive complexity required, up to 50 users are considered for the optimisation method. As shown, the SSS achieves a much lower throughput than other methods. By balancing the traffic loads between the Li-Fi and Wi-Fi networks, the LB method can effectively improve the system throughput over the SSS, as expected. However, there is still a pronounced gap between the LB and the optimisation method. In contrast, the proposed method obtains a throughput very close to the optimal result. In the case of 50 users, the throughput achieved by the FL-SSS is only 2.2% less than the optimal result, and this value reduces to 1.4% when the FL-LB is applied.

### B. Complexity

The SSS selects the AP with the highest SNR from the sets of $\boldsymbol{L}_u$ and $\boldsymbol{W}$. Thus the computational complexity of the SSS can be written as:

$$O\left(N_{\boldsymbol{L}_u} + N_{\boldsymbol{W}}\right), \tag{26}$$

where $N_{\boldsymbol{L}_u}$ and $N_{\boldsymbol{W}}$ are the numbers of APs in $\boldsymbol{L}_u$ and $\boldsymbol{W}$, respectively. The LB method also selects one from those APs,

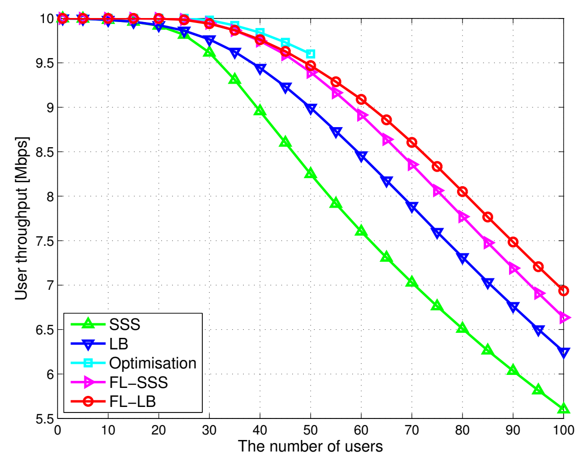



Fig. 7. User throughput versus the number of users ($\bar{R} = 10$ Mbps).

but with an additional calculation of the user' satisfaction, which requires complexity of $O(N_u)$. Also, maximising the user's satisfaction costs complexity of $O\left(N_{\boldsymbol{L}_u} + N_{\boldsymbol{W}}\right)$. The complexity of the LB is then computed by:

$$O\left(2\left(N_{\boldsymbol{L}_u} + N_{\boldsymbol{W}}\right) + N_u\right). \tag{27}$$

The optimisation method is an $\mathcal{NP}$-hard problem, which is only solvable (if possible) in exponential time. In this paper, the optimal solution to (20) is obtained by exhaustively searching all of the possible connections between users and APs. There are $N_{\boldsymbol{L}_u} + N_{\boldsymbol{W}}$ options for each user in (20), and the corresponding complexity is estimated as:

$$O\left(\prod_{u=1}^{N_u}\left(N_{\boldsymbol{L}_u} + N_{\boldsymbol{W}}\right)\right). \tag{28}$$

In the proposed method, $K_1 = 4$ inputs are combined for each user to obtain the accessibility score based on $K_2 = 6$ rules, causing complexity of $O(K_1K_2)$. Then the users are sorted according to their scores. In general, sorting algorithms demonstrate complexity of $O\left(N^2\right)$. Therefore the complexity of FL is approximately:

$$O\left(N_u^2 K_1 K_2\right). \tag{29}$$

Also, the proposed method needs to perform the SSS or LB in its second stage, and the corresponding complexity is added to the total complexity.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2740211, IEEE Transactions on Communications



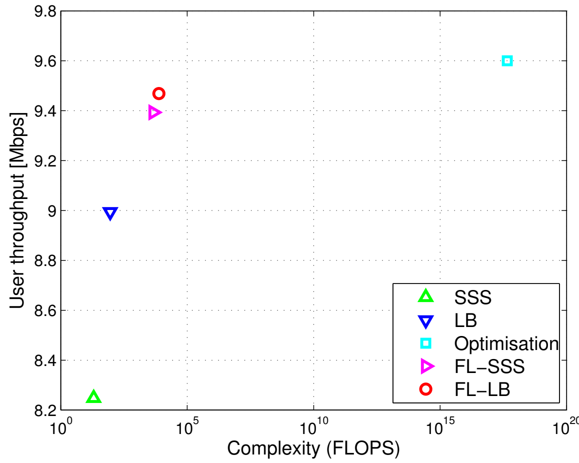



Fig. 8. User throughput versus computational complexity ($\bar{R} = 10$ Mbps and $N_u = 50$).

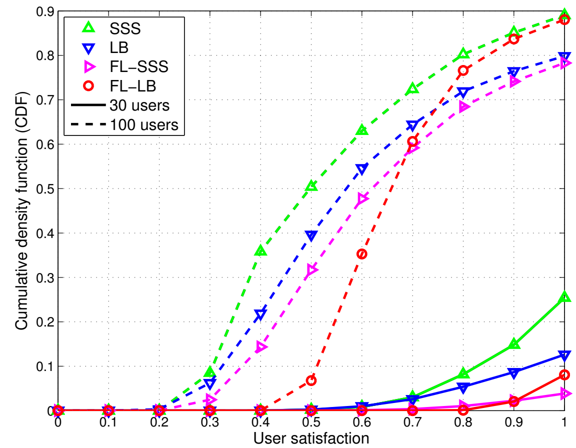
It is evident that the complexity required by the SSS is linear to the size of network, and is independent of the number of users. The complexity of the LB is linearly related to both the size of network and the number of users. The proposed method requires additional complexity proportional to $N_u^2$. As for the optimisation method, its complexity is exponentially increased with both the size of network and the number of users. In Fig. 8, the computational complexity required by various methods is measured by floating-point operations per second (FLOPS). As shown, the optimisation method needs a tremendous number of calculations to reach the maximum throughput. In contrast, the proposed method can significantly reduce the complexity with a slight decrease in throughput.

## VI. Simulation Results

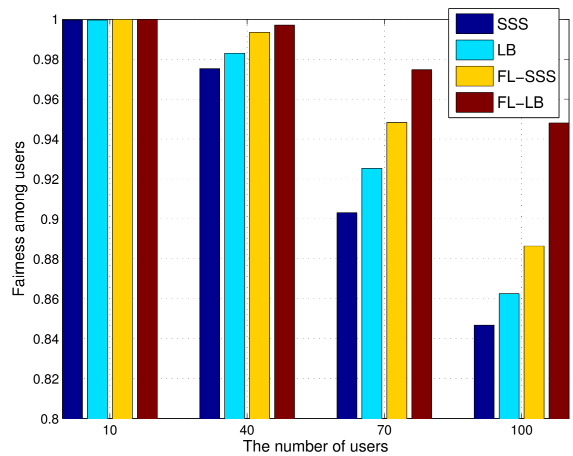
In this section, Monte Carlo simulations are conducted to validate the performance of the proposed method in comparison with the conventional methods. Consider an indoor scenario with 4 rooms as shown in Fig. 1, and each room is square with a side length of 10 m. On the ceiling of each room, 16 Li-Fi APs are placed in a layout of a square matrix, with a separation of 2.5 m between the closest two. The users are randomly distributed with a uniform probability distribution. In addition, the number of available Wi-Fi channels is assumed equal to the number of Wi-Fi APs, except when analysing its effects on the network performance.

### *A. Performance Comparison*

Fig. 9 presents the users' satisfaction and fairness of various methods when the average required data rate is 10 Mbps. As shown in Fig. 9(a), the proposed method can significantly increase the users' satisfaction over the SSS and LB, especially for a large number of users. When 30 users are present, using the SSS can meet the data requirements for only 74.6% of the users. This value is increased to 87.4% by employing the LB instead of the SSS. When using the FL-SSS and FL-LB, the proportion of satisfied users is 96.1% and 91.9%, respectively.




(a) Satisfaction




(b) Fairness

Fig. 9. System performance versus the number of users ($\bar{R} = 10$ Mbps).

Note that there is a cross point between the curves of the FL-SSS and FL-LB. This is because using the LB in the proposed method can improve the performance of deeply-unsatisfied users, by decreasing the number of satisfied users. In Fig. 9(b), the fairness among users is shown for different numbers of users. Two outcomes are observed: i) the fairness of all methods equals 1 given a small number of users, e.g. $N_u = 10$; ii) as the number of users increases, the fairness decreases for all methods, but the fairness of the FL-LB decreases much slower than that of the other methods. At $N_u = 100$, the fairness of the FL-LB achieves 0.95, while the remaining methods have a fairness below 0.9.

In Fig. 10, the users' throughput is shown as a function of the average required data rate. When $\bar{R}$ increases from 1 Mbps, all methods are at first able to meet the data requirement. As $\bar{R}$ further increases, the proposed method gradually outperforms the benchmarks. In addition, the gain achieved by the proposed method becomes more significant when the number of users increases. At $N_u = 100$, the FL-LB improves the users' throughput against the SSS and LB by up to 24% and 11%, respectively.

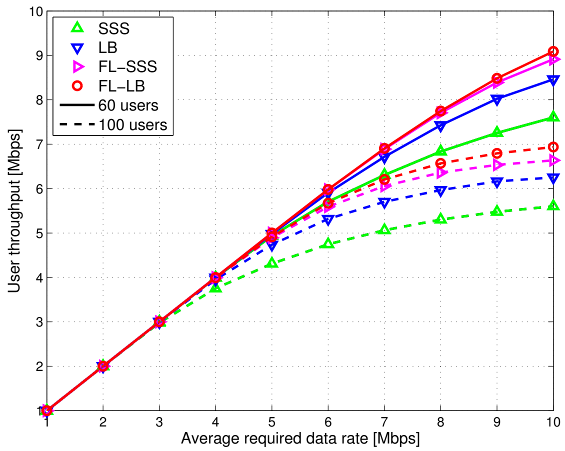



Fig. 10. User's throughput versus average required data rate.

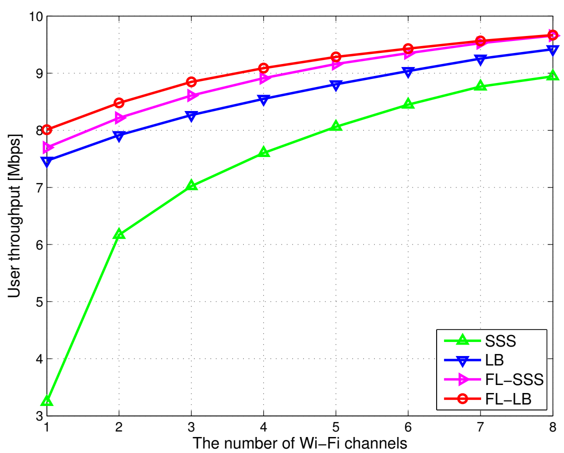



Fig. 11. User's throughput versus the number of Wi-Fi channels ($N_u = 60$).

### *B. Effects of the Number of Wi-Fi channels*

As shown in Fig. 11, in general, the users' throughput decreases when the number of Wi-Fi channels is reduced. Among all the methods, the performance of the SSS decreases the most significantly. This is because the SSS assigns users to Wi-Fi regardless of its capacity and availability. Thus when the Wi-Fi capacity is reduced, the performance of those Wi-Fi users is severely compromised. Unlike the SSS, the other methods have the ability to balance the loads between Li-Fi and Wi-Fi, and thus are less affected by a reduced number of Wi-Fi channels. In addition, as the number of Wi-Fi channels increases, the performance of the FL-SSS gradually approaches that of the FL-LB. The reason for this trend is that when more users are migrated to Wi-Fi, the gap between using the LB and SSS in Li-Fi becomes smaller.

## VII. Conclusion

In this paper, a two-stage APS method was proposed for hybrid Li-Fi and Wi-Fi networks, by exploiting the distinguishing characteristics between those two networks. The proposed method at first determines the users that need service from Wi-Fi, and then assigns the remaining users as if in a homogeneous Li-Fi network. The concept of fuzzy logic is applied in the first stage to rank the user's priority of accessing Wi-Fi. In the second stage the SSS or LB can be employed, and the proposed method is named the FL-SSS or FL-LB correspondingly. Based on experimental results and complexity analysis, it is shown that compared to the optimisation method, the proposed method achieves a near-optimal throughput at significantly reduced complexity. In addition, the FL-LB marginally outperforms the FL-SSS with a slight increase in complexity. Compared with the SSS and LB, results show that FL-LB can improve the network throughput by 24% and 11%, respectively. Future research will involve cellular network in the context of a hybrid network.

## Acknowledgement

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) Grant EP/L020009/1: Towards Ultimate Convergence of All Networks (TOUCAN). Professor Harald Haas greatly acknowledges support from the EPSRC under Established Career Fellowship Grant EP/K008757/1.

## References

[1] X. Wu, D. Basnayaka, M. Safari, and H. Haas, "Two-stage access point selection for hybrid VLC and RF networks," in *IEEE 27th Annu. Int. Symp. on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, Valencia, Spain, Sept. 2016, pp. 1–6.

[2] "Cisco visual networking index: Global mobile data traffic forecast update, 2015-2020," Cisco, Tech. Rep., Feb. 2016.

[3] H. Burchardt, N. Serafimovski, D. Tsonev, S. Videv, and H. Haas, "VLC: Beyond point-to-point communication," *IEEE Commun. Mag.*, vol. 52, no. 7, pp. 98–105, July 2014.

[4] H. Haas, L. Yin, Y. Wang, and C. Chen, "What is LiFi?" *J. Lightw. Technol.*, vol. 34, no. 6, pp. 1533–1544, Mar. 2016.

[5] D. Tsonev, H. Chun, S. Rajbhandari, J. McKendry, S. Videv, E. Gu, M. Haji, S. Watson, A. Kelly, G. Faulkner, M. Dawson, H. Haas, and D. O'Brien, "A 3-Gb/s single-LED OFDM-based wireless VLC link using a gallium nitride LED," *IEEE Photonics Technol. Lett.*, vol. 26, no. 7, pp. 637–640, Apr. 2014.

[6] H. Haas, "Visible light communication," in *2015 Optical Fiber Communications Conf. and Exhibition (OFC)*, Los Angeles, CA, Mar. 2015, pp. 1–72.

[7] M. Rahaim, A. Vegni, and T. Little, "A hybrid radio frequency and broadcast visible light communication system," in *2011 IEEE GLOBECOM Workshops*, Houston, TX, Dec. 2011, pp. 792–796.

[8] D. Basnayaka and H. Haas, "Hybrid RF and VLC systems: Improving user data rate performance of VLC Systems," in *IEEE 81st Vehicular Technology Conf. (VTC Spring)*, Glasgow, UK, May 2015, pp. 1–5.

[9] K. Son, S. Chong, and G. D. Veciana, "Dynamic association for load balancing and interference avoidance in multi-cell networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3566–3576, July 2009.

[10] Y. Bejerano, S. J. Han, and L. Li, "Fairness and load balancing in wireless lans using association control," *IEEE/ACM Trans. Networking*, vol. 15, no. 3, pp. 560–573, June 2007.

[11] Y. Wang, X. Wu, and H. Haas, "Distributed load balancing for internet of things by using Li-Fi and RF hybrid network," in *IEEE 26th Annu. Int. Symp. on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, Hong Kong, Aug. 2015, pp. 1289–1294.

[12] X. Li, R. Zhang, and L. Hanzo, "Cooperative load balancing in hybrid visible light communications and WiFi," *IEEE Trans. Commun.*, vol. 63, no. 4, pp. 1319–1329, Apr. 2015.

[13] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, June 1965.

[14] H. Burchardt, S. Sinanovic, Z. Bharucha, and H. Haas, "Distributed and autonomous resource and power allocation for wireless networks," *IEEE Trans. Commun.*, vol. 61, no. 7, pp. 2758–2771, July 2013.

[15] E. Perahia and R. Stacey, *Next Generation Wireless LAN: 802.11n and 802.11ac*. Cambridge University Press, 2013.

[16] J. Kahn and J. Barry, "Wireless infrared communications," *IEEE Proceedings*, vol. 85, no. 2, pp. 265–298, Feb. 1997.

[17] V. Erceg, L. Schumacher, P. Kyritsi, and et al., *TGn Channel Models*. IEEE 802.1103/940r4, 2004.

[18] *IEEE standard for information technology- Telecommunications and information exchange between systems- Local and metropolitan area networks- Specific requirements Part II: Wireless LAN medium access control (MAC) and physical layer (PHY) specifications*. IEEE Standard 802.11g, 2003.

[19] P. J. Smith, P. A. Dmochowski, M. Shafi, and C. Neil, "On the computation of per-user rate distributions from sum-rate distributions," *IEEE Wireless Commun. Lett.*, vol. 3, no. 4, pp. 381–384, Aug. 2014.

[20] R. Jain, D. Chiu, and W. Hawe, "A quantitative measure of fairness and discrimination for resource allocation in shared systems," Digital Equipment Corporation, DEC-TR-301, Tech. Rep., Sept. 1984.

[21] Matlab, "Foundations of fuzzy logic," Available: https://uk.mathworks.com/help/fuzzy/foundations-of-fuzzy-logic.html.


**Xiping Wu** (S'11-M'14) received the B.Sc. degree from Southeast University, Nanjing, China, in 2008, the M.Sc. degree with distinction and the Ph.D. degree from the University of Edinburgh, Scotland, United Kingdom, in 2011 and 2015, respectively. From September 2011 to August 2014, he was a Marie-Curie Early-Stage Researcher (ESR), funded by the European Unions Seventh Framework Programme (FP7) project GREENET. From December 2013 to April 2014, he was on secondment to the Department of Electrical and Information Engineering, University of LAquila, LAquila, Italy. Dr. Wu is currently a research associate with the Institute for Digital Communications (IDCOM) at the University of Edinburgh, funded by the British Engineering and Physical Sciences Research Council (EPSRC) project TOUCAN. His main research interests are in the areas of wireless communication theory, visible light communications, and wireless network management. Dr. Wu was the recipient of the Scotland Saltire Scholarship by the Scottish Government in 2010.

**Majid Safari** (S'08-M'11) received the B.Sc. degree in Electrical and Computer Engineering from the University of Tehran, Iran, in 2003, the M.Sc. degree in Electrical Engineering from Sharif University of Technology, Iran, in 2005 and the Ph.D. degree in Electrical and Computer Engineering from the University of Waterloo, Canada, in 2011. He is currently an assistant professor in the Institute for Digital Communications at the University of Edinburgh. Before joining Edinburgh in 2013, he held postdoctoral fellowship at McMaster University, Canada. Dr. Safari is currently an associate editor of IEEE Communication letters and was the TPC co-chair of the 4th International Workshop on Optical Wireless Communication in 2015. His main research interests are in the application of information theory and signal processing in optical communications including fiber-optic communication, free-space optical communication, visible light communication, and quantum communication.

**Harald Haas** (S'98-A'00-M'03-SM'16) received the Ph.D. degree from The University of Edinburgh in 2001. He currently holds the Chair of Mobile Communications with The University of Edinburgh. He first introduced and coined spatial modulation and Li-Fi. Li-Fi was listed among the 50 best inventions in TIME Magazine 2011. His main research interests are in optical wireless communications, hybrid optical wireless and RF communications, spatial modulation, and interference coordination in wireless networks. He is a Co-Founder and the Chief Scientific Officer of pureLiFi Ltd. He holds 31 patents and over 30 pending patent applications. He has authored 300 conference and journal papers including a paper in science. He was a co-recipient of the Best Paper Award at the IEEE Vehicular Technology Conference in 2013 and 2015. In 2012, he was the only recipient of the prestigious Established Career Fellowship from the EPSRC (Engineering and Physical Sciences Research Council) within Information and Communications Technology, U.K. He was also a recipient of the Tam Dalyell Prize 2013 from The University of Edinburgh for Excellence in Engaging the Public with Science. In 2014, he was selected by EPSRC as one of ten RISE (Recognising Inspirational Scientists and Engineers) Leaders. He was an Invited Speaker at the TED Global 2011, and his talk has been watched online over 1.5 million times.