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Accessing nanomechanical resonators via a fast microwave circuit

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We demonstrate how to fully electrically detect the vibrations of conductive nanomechanical resonators up to the microwave regime. We use the electrically actuated vibrations to modulate an *LC* tank circuit, which blocks the stray capacitance and detect the created sideband voltage by a microwave analyzer. We prove the technique up to mechanical frequencies of 200 MHz. Finally, we estimate how one could approach the quantum limit of mechanical systems. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3173826]

Micro and nanomechanical systems[1,2] are increasingly finding use in various sensor applications, where the vibrations of clamped beams or cantilevers are affected by the measured quantities. The detection of mechanical vibrations at the submicron scale in such systems gets notoriously difficult. This essentially stems from the fact that the transduction of mechanical motion into the engineering world based typically on *electrical* measurement techniques is difficult. Also, the detection of higher mechanical resonant frequencies f_0 becomes increasingly difficult. On the other hand, the smallest nanomechanical resonators are the most interesting ones, as sensors due to their small active mass[3] or, in basic research, for observing quantum-mechanical phenomena.[4,5]

The backbone for electronic readout of nanomechanical resonators (NRs) has been the magnetomotive method[6,7] or its variants,[8] which have been proven above 500 MHz.[9] Here, the current-carrying beam is vibrating in a sizable magnetic field, thus inducing an electromotive force. However, the method has a practical constraint limiting its general applicability, since typically a 1–10 T field and hence superconducting magnets and 4 K operation are needed. The high field also suppresses superconductivity, which becomes an issue in delicate measurements close to the quantum limit.

A straightforward electric readout would be valuable, which would desirably work up to $f_0 \sim 1$ GHz, and without the use of cumbersome high Tesla magnets. Truitt *et al.*[10] recently took a step to this direction. They took advantage of the fact that at the drive frequency, a driven NR looks like a series electrical *RLC* resonator, for which they improved the impedance match to $Z_0 = 50\ \Omega$ by coupling the NR to an electrical *LC* matching circuit resonant with the mechanical mode.

In our work, we demonstrate an improved fully electric readout protocol, where the *LC* frequency $f_{LC} \gg f_0$ can be chosen high enough to ensure bandwidths up to the range of 1 GHz, independent of f_0. In our scheme, an electric resonant circuit can be seen as eliminating the external wiring capacitances. Similar techniques where an external resonant ("tank") circuit has been used in order to enhance the detection bandwidth are well known in the mesoscopic electron transport community.[11,12]

Let us consider a NR actuated by the voltage $V(t) = V_{dc} + V_0 \cos(\omega t)$ from a nearby gate (capacitance C_R). About the (fundamental mode) mechanical resonance $\omega_0 = 2\pi f_0$, the NR responds as a Lorentzian $x = F_0/M \cos(\omega t + \Theta)[(\omega\omega_0/Q_M)^2 + (\omega^2 - \omega_0^2)^2]^{-1/2}$. Here, the driving ac force is $F_0 = [(\partial C_R)/(\partial x)]V_{dc}V_0$, $\tan\Theta = \omega_0\omega[Q_M(\omega^2-\omega_0^2)]^{-1}$, M is the effective mass of the fundamental mode (about 0.73 times the total mass of the beam), and Q_M is the quality factor of the nanomechanical mode. In the following, we will denote by x the resonant (maximum) value of the displacement.

The mechanical motion gives rise to a time-varying capacitance $C_R(t) = C_{R0} + [(\partial C_R)/(\partial x)]x$. On the mechanical resonance $\Theta = \pi/2$, current and voltage are in phase and NR looks like a resistance. Toward increasing mechanical frequency, this effective resistance grows rapidly, and when combined with the large wiring stray capacitances, the signal becomes small.

In our approach [Fig. 1(a)], the measurement is performed at a frequency f_{LC} fully different from the actuation frequency $f \sim f_0$. At f_{LC}, the NR looks like a time-varying capacitance [Fig. 1(b)]. A related technique was recently demonstrated by Regal *et al.*[13] However, they used a very high-Q transmission line resonator as the coupling element.

We will write down the equations governing the flow of information between the different frequencies present in the

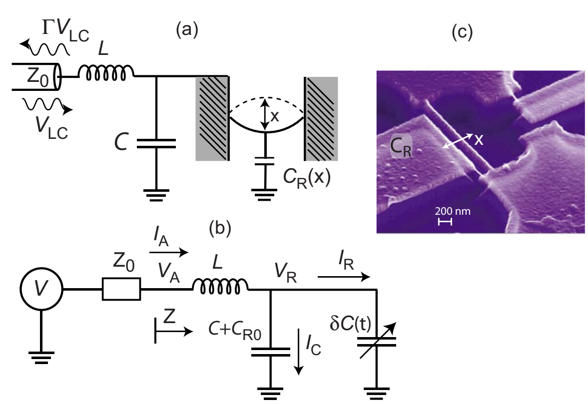

FIG. 1. (Color online) The readout of nanomechanical resonators by the use of a low-Q_{LC} *LC* tank circuit, which blocks the stray capacitance in the external cabling. The vibrations of the beam at the mechanical frequency $\omega_0 \ll \omega_{LC} = (LC)^{-1/2}$ modulate the total capacitance of the tank circuit. (b) The circuit appears as one with a time-dependent capacitance when sensed at a frequency different from the actuation frequency ω_0. The input voltage $V(t) = V_{dc} + V_0\cos(\omega_0 t) + V_{LC}\cos(\omega_{LC} t)$ drives the beam at ω_0 and probes the tank circuit at its frequency ω_{LC}. (c) SEM image of a $l = 1.8$-μm-long and $W = 160$-nm-wide aluminum beam.

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problem, namely, f_0, $f_{LC}=(2\pi\sqrt{LC})^{-1}$, and the mixing products (sidebands) $f_\pm \equiv f_{LC}\pm f_0$ but neglecting higher-order mixing terms [a related analysis for the radio-frequency single-electron transistor (rf-SET) is in Ref. 14].

The voltages at the various frequencies in the middle of the tank circuit, at the point R in Fig. 1(b) are then

$$V^R = V^R_{dc} + V^R_0 \exp(i\omega_0 t) + V^R_{LC} \exp(i\omega_{LC} t) + V^R_+ \exp[i(\omega_{LC}+\omega_0)t] + V^R_- \exp[i(\omega_{LC}-\omega_0)t]. \quad (1)$$

Using Eq. (1) and the expression for the time-dependent capacitance, we get a relation for the currents I^R flowing through the NR

$$I^R(t) = \frac{d}{dt}[C_R(t)V^R(t)] \simeq \frac{x\left(\frac{\partial C_R}{\partial x}\right)}{2}\{\omega_0 V^R_{dc}\exp(i\omega_0 t) + \omega_{LC}(V^R_+ - V^R_-)\exp(i\omega_{LC}t) + \omega_+ V^R_{LC}\exp(i\omega_+ t) - \omega_- V^R_{LC}\exp(i\omega_- t)\} + I_{R0}, \quad (2)$$

where I_{R0} is the current through the constant part of the capacitance C_{R0}. We use the Kirchoff's voltage and current laws, which allow us to solve the circuit at the three frequencies, when substituted by Eqs. (1) and (2)

$$V - I^A(Z_0 + i\omega L) - V^R = 0,$$

$$V^R = \frac{I^A - I^R}{i\omega(C + C_{R0})}. \quad (3)$$

If $\omega_0 \ll \omega_{LC}/Q_{LC}$ and in the limit of small capacitance modulation $x[(\partial C_R)/(\partial x)] \ll C_{R0}$, we obtain from Eq. (3) the measured quantity, namely, the voltage amplitudes $V_\pm = Z_0 I^A_\pm$ of either sideband

$$V_+ = V_- = \frac{x\left(\frac{\partial C_R}{\partial x}\right)V_{LC}}{2\omega_{LC}(C+C_{R0})^2 Z_0} \simeq \left(\frac{C_{R0}}{C+C_{R0}}\right)^2 \frac{Q_M V_{dc} V_0 V_{LC}}{2(x_0+W)^2 M \omega_{LC} Z_0 \omega_0^2}, \quad (4)$$

where, for the last form, we approximated $[(\partial C_R)/(\partial x)] \sim C_{R0}/(x_0+W)$ where x_0 is the vacuum gap between the beam (width W) and the gate. The capacitance C_R is modeled as that for two parallel beams of length l, $C_R = \pi\varepsilon_0 l\{\ln[x_0/W + 1 + \sqrt{(x_0/W)^2 + 2x_0/W}]\}^{-1}$.

As one would intuitively think, the signal depends strongly on how much stray capacitance C one has *within* the resonant circuit, the part which is not cancelled by the inductor. The signal is proportional to both actuation parameters (V_{dc} and V_0) and to how much the system responds, given by Q_M, as well as to the measurement strength V_{LC}. Also, the signal can be interpreted as being proportional to the quality factor $Q_{LC} \sim (\omega_{LC} C Z_0)^{-1}$ of the tank circuit. The bandwidth, typical of a modulation scheme, is given by the response time of the electrical tank circuit as f_{LC}/Q_{LC}.

For the fabrication of micron-scale suspended, metallized of fully metallic nanomechanical beams, a multitude of methods have been developed.[7,9,15–17] Our process represents the simplest end of the spectrum in terms of the complexity of the process, and the number of steps needed. The beam itself is metallic, and hence there is no separate metallization needed. The fabrication involves e-beam lithography and a single metallization layer on top of a high-resistivity (~ 3 kΩ m) silicon. In the end, a properly timed SF_6 dry etch suspends the beam [Fig. 1(c)], while leaving the clamps in both ends well hooked to the substrate.

For the measurements, the chip was wire-bonded to a surface-mount inductor $L = 10-30$ nH. The tank circuit capacitance, $C \sim 0.3$ pF, comes from the stray capacitances of the bonding pad and of the inductor. The measurements were made at a temperature of 4 K, in a vacuum of $\sim 10^{-2}$ mBar. The voltages at dc, at the NR drive frequency, and at the measurement frequency were combined at room temperature using bias tees. The signal reflected from the sample was fed to a spectrum analyzer via a circulator and room temperature microwave amplifiers having a noise temperature $T_N \sim 100$ K.

We studied a total of four samples having slightly different parameters. The beams had a length of $l \sim 2$ μm, and a width of $W \sim 150$ nm. The measured mechanical frequencies agreed within 25% of the stress-free prediction given as $f_0 \simeq (W/l^2)\sqrt{E/\rho}$, where E is the Young modulus and ρ is density. The highest measured mechanical frequency was $f_0 = 202$ MHz. All the samples displayed good mechanical Q values $Q_M = (1.7-4.4)\times 10^3$, in agreement with previous 4 K experiments on aluminum beams,[9] and thus do not indicate damage to the beam material due to the processing. In Fig. 2(a), we show data for a representative sample. Toward increasing vibration amplitude, the resonance bends to the right, an indication that the resonator is entering the regime of nonlinear oscillations.

As the basic test of the scheme, we studied the expected linear drive voltage dependence of the peak sideband voltage, as shown in Fig. 2(c). In order to compare with Eq. (4), we used the independently estimated values for all the other quantities except for the rf attenuation and for $x_0 + W$, which come from the simple capacitance model. The linear fit is plotted with $x_0 = 125$ nm and $W = 140$ nm, which come close to measured values.

As a further test of the integrity of our model, we studied the shift of the linear-regime resonant frequency as a function of the applied dc voltage V_{dc}. Due to electrostatic softening of the effective spring constant $k^*(V_{dc}) = k - (1/2)V_{dc}^2[(\partial^2 C_R)/(\partial x^2)]|_{V_{dc}}$, the resonance is expected to shift left with increasing dc voltage

$$\omega_0(V_{dc}) = \omega_0(0)\left[1 - \frac{C_{R0}V_{dc}^2}{2M\omega_0^2(0)(x_0+W)^2}\right]. \quad (5)$$

In Fig. 2(d), we plot the extracted resonance frequencies, which display the expected quadratic dependence on the dc voltage. We also show a fit to Eq. (5) which employed the same values of x_0 and W as Fig. 2(c).

In order to go beyond the regime of linear oscillations, we numerically simulated the full equation of motion of the lowest mechanical mode[18]

$$\ddot{x}(t) + \frac{M\omega_0}{Q_M}\dot{x}(t) + \omega_0^2 x(t) + \frac{E}{18\rho}\left(\frac{2\pi}{L}\right)^4 x^3(t) = F(t), \quad (6)$$

which takes into account the nonlinearity due to beam stretching here, $F(t) = F_0\cos(\omega t)$. We substitute the resulting motion into Eq. (4) and sum the result with a background sideband voltage of 2 nV, thereby obtaining Fig. 2(b). This is seen to present a good agreement to the measured resonance

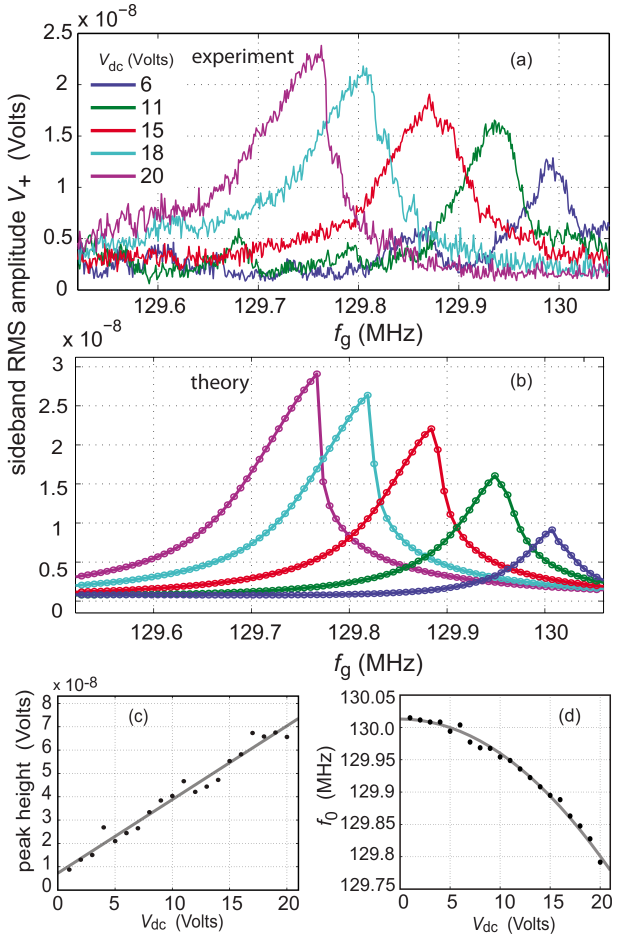

FIG. 2. (Color online) The response of the nanomechanical resonator (2.5-μm-long beam, estimated frequency f_0=120 MHz) using the electric sideband detection, with input values V_0=100 mV and V_{LC}=30 mV, at T=4.2 K. (a) Resonance curves of the fundamental mode for increasing dc voltage (from right to left). The quality factor in the linear regime $Q_M = 4.4\times 10^3$. (b) Simulation of the response [Eqs. (4) and (6)]. (c) Amplitude of the peak extracted from (a), and a fit to Eq. (4). (d) Center frequency of the mechanical peak extracted from (a), fitted to theory [Eq. (5)].

peaks, including the fact that the system approaches the hysteretic regime at V_{dc}=20 V.

Our method offers a sensitive and in-principle tabletop characterization method for the nanomechanical resonators. A major figure of merit is the displacement sensitivity s_x. It is obtained from Eq. (4) as the value of x, which would correspond to a voltage spectral density equal to the noise voltage $\sqrt{2k_B T_N Z_0}$ set by the amplifiers

$$s_x \simeq 2\sqrt{2k_B T_N Z_0^3}\,x_0 \frac{\omega_{LC}(C + C_{R0})^2}{C_{R0}V_{LC}}, \qquad (7)$$

where T_N is the noise temperature of the system. Through minimization of the tank circuit capacitance C by fabricating the NR coupled to an on-chip spiral coil, for example, L =17 nH and $C\sim 25$ fF, the method might also offer a path toward investigating the quantum limit of mechanical motion. Let us consider an aluminum beam having a length l=1.5 μm and frequency f_0=240 MHz. Let us also take f_{LC}=5 GHz and T_N=4 K. If probed even with a decent V_{LC}=3 mV, Eq. (7) yields a displacement sensitivity $s_x \sim 45$ fm/$\sqrt{\text{Hz}}$, which comes interestingly close to the zero-point amplitude $x_{zp}\sim 25$ fm.

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