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Accretion and the stellar mass spectrum in small clusters

I. A. Bonnell,[1] M. R. Bate,[2] C. J. Clarke[1] and J. E. Pringle[1]

[1] *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*
[2] *Max-Planck-Institut für Astronomie, Königstuhl 17, 69117 Heidelberg, Germany*

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ABSTRACT

We investigate the effects of gas accretion on small clusters of young stars. The evolution of clusters containing 3–10 stars and between 0.1 and 90 per cent of their masses in the form of gas is followed using a three-dimensional SPH code with sink-particles to treat the accretion of gas on to the stars. The gas accretion by the stars is highly non-uniform in that a few of the stars accrete significantly more than the rest. The location of the star in the cluster potential and its possible membership in a binary or multiple system are the primary factors in determining the accretion rate of the star. This competitive accretion process results in the formation of a spectrum of stellar masses, even when the initial stellar masses are uniform. Small variations in the initial masses are overwhelmed by the accretion process, whereas larger variations can affect the accretion dynamics as they affect the overall cluster potential. The differential accretion results in the massive stars being formed in the centre of the cluster. Their location is not due to an evolutionary effect of mass segregation. Implications of this competitive accretion process for determining the stellar mass spectrum are discussed.

Key words: accretion, accretion discs – circumstellar matter – stars: formation – stars: luminosity function, mass function – open clusters and associations: general.

1 INTRODUCTION

Since most stars are found to be members of binary and multiple systems (Duquennoy & Mayor 1991), it has been clear for some time that any theory of star formation must of necessity be a theory of how multiple stars form. A review of how single dense cores in molecular clouds collapse to form single stars is given by Shu, Adams & Lizano (1987), and it is evident that many of the processes discussed in that review must be of relevance also to the formation of multiple star systems. There are two main possibilities (Pringle 1993). The first is that as a single core collapses, the dense central regions of the non-homologous collapse become unstable to fragmentation of some kind. This idea has been pursued by a number of authors (e.g. Boss 1993; Bonnell & Bate 1994). We do not consider this mechanism any further here. The second is that the collapse of a dense self-gravitating core is initiated in a quite different manner from that discussed by Shu et al. The idea here is that the collapse is initiated dynamically by some outside agency (e.g. Pringle 1989; Chapman et al. 1992) so that the initial core (which contains by definition one Jeans mass) is transformed on a time-scale which is less than, or of the order of, the dynamical one, into an entity which consists of a number of Jeans masses. This implies that the collapse is highly non-homogeneous, and that collapse occurs simultaneously on to a corresponding number of gravitating centres (e.g. Larson 1978). This form of collapse was called by Pringle 'prompt initial fragmentation', and is a similar physical process to that discussed in the formation of, for example, globular clusters (Fall & Rees 1985; Murray, Clarke & Pringle 1991), in which an initial Jeans mass of $10^6\,M_\odot$ cools rapidly to the extent that the Jeans mass decreases by a factor of 10^6, resulting in a cluster of 10^6 one-solar-mass stars. The process in our case requires that the factor of 10^6 is replaced by a factor of only a few. It is also worth remarking that recent observations (Lada, Strom & Myers 1993; Zinnecker, McCaughrean & Wilking 1993) indicate that star formation frequently occurs in clusters which then disperse in a few dynamical time-scales, and that even in Taurus, where star formation appears to operate in a more distributed (rather than clustered) fashion, there is evidence for clustering into small groups of stars (Herbig 1977; Gomez et al. 1993).

An initial discussion of how the idea of prompt initial fragmentation might give rise to the observed spectrum of binary star parameters, given that each initial gravitating condensation must be initially about a Jeans length (i.e., of order 10^4 au) apart, has been given by Pringle (1989), Clarke & Pringle (1991) and McDonald & Clarke (1995) (see review by Clarke 1996). In these calculations it was assumed that the collapse process occurred in two stages: first, each gravitating condensation collapsed instantly to form a protostar plus surrounding disc, and then the (proto)stars were allowed to interact dynamically, taking into account the fact that the discs evolve in time, and also that disc/disc and star/disc interactions give rise to substantial energy loss, and so modifications of the dynamical orbits. It was found that in this way binaries with much smaller separations than the initial 10^4 au could be formed. However, these authors also pointed out that in fact the time-scale for the separate condensations to collapse individually and become protostars and surrounding discs (using the processes discussed by Shu et al. 1987) is the same as the time-scale on which dynamical interactions occur between neighbouring condensations/protostars. Thus the interaction of the gas with the individual components will be an important process in determining the system's dynamical evolution. This interaction encompasses both gravitational drag from the gas which removes kinetic energy from the cluster (e.g. Larson 1990) and accretion on to the individual stars. Furthermore, if the accretion is non-uniform, with each star competing for the available gas, it can play an important role in determining the final mass of each star. This picture of 'competitive accretion' (Zinnecker 1982) might be responsible for the overall distribution of stellar masses. In this way, the mass of the most massive star will be a direct result of the accretion process (e.g. Larson 1992).

In this paper we model some aspects of this process. We start our calculations at a stage when a small fraction of the mass of the original core has condensed into gravitating centres, and follow the evolution of the core as these gravitating centres continue to accrete from their surroundings as well as simultaneously to interact dynamically with each other. Since we are modelling a scenario in which this residual gas eventually accretes on to the pre-existing centres (rather than itself fragmenting into new protostars) it is necessary that the Jeans number in the gas is not high. This implies that at the start of the calculation the gas is hotter than it must have been when the seed fragments were formed. We envisage that such initial conditions might be realized in the case that the core was first destabilized by some impulse (e.g., a core–core collision; Chapman et al. 1992), leading to rapid cooling and the onset of fragmentation, and then reheated before it could all fall on to the seed fragments (i.e., fragmentation is inefficient). We do not, however, attempt to model the discs which might form around the condensing protostars nor the interactions of such discs with other material and/or stars, as this requires much more detailed calculations (e.g. Heller 1995; Hall, Clarke & Pringle 1996).

A discussion on simulating accretion is included in Section 2, and the calculations are presented in Section 3. Section 4 discusses the dynamics of the gas in the cluster, while Section 5 investigates the accretion process. Section 6 discusses how accretion can affect the final stellar mass spectrum. Section 7 discusses the dependency on initial mass variations, and Sections 8 and 9 discuss binary and disc formation, respectively. The implications of the results on mass segregation in clusters and on the stellar mass function are discussed in Section 10. Our conclusions are presented in Section 11.

2 SIMULATING ACCRETION

In order to follow the evolution of both stellar and gaseous components, it is necessary to be able to treat the accretion of the gas on to the (proto)stars. There are several different methods that can be envisaged, from the most rudimentary where the accretion rate depends solely on the mass of the star (e.g. Zinnecker 1982), to analytical expressions of the accretion rate from Bondi–Hoyle accretion (e.g. Ruffert 1996), to a full hydrodynamical treatment of the gaseous component and the accretion process (e.g. Bate, Bonnell & Price 1995; Bate & Bonnell 1997; Bate 1997) as is considered here.

The reason why full hydrodynamical simulations are required is that the gas component reacts to the ever changing potential of the system and thus needs to be followed. The alternative, relying on the Bondi–Hoyle formalism (Bondi & Hoyle 1944; Bondi 1952), requires knowledge of the surrounding medium which is not available.

The Bondi–Hoyle accretion rate,

$$\dot{M}_{\rm BH} \approx 4\pi\rho_\infty \frac{(GM)^2}{(v_\infty^2 + c_{\rm s}^2)^{3/2}},$$

uses the stellar velocity v_∞, and assumes a uniform gas density ρ_∞. This assumption does not hold for a small cluster of stars where the gas is continually stirred up by the motions of the stars and the changing potential. In order to use this approach, an approximation of ρ_∞, and hence the gas dynamics, is needed. One can imagine two simple ways of treating the gas component in this fashion. The first assumes that the gas component is static and is accreted only when a star passes nearby. The second would assume that the gas component can move, but that the overall form of the gas distribution is unchanged and that the mass is evenly accreted from the whole distribution. The problem with the former treatment is that, with static gas, the stars have to search out and find each and every element of the gas for it to be accreted, an obviously unsatisfactory solution. The problem with the latter treatment is that it assumes that the gas can adapt at infinite speeds to the accretion process, which is similarly unphysical. Both of these methods give widely different results from the full hydrodynamical simulation. The former requires a much longer time-scale to accrete the gas, while the latter accretes it much too quickly.

3 CALCULATIONS

The calculations reported in this paper were performed with a three-dimensional smoothed particle hydrodynamics (SPH) code that includes sink-particles to model the stars. This code uses a common adaptation of the SPH equations (Benz 1990) with tree-code gravity (Benz et al. 1990). The inclusions of sink-particles allows for the modelling of nongaseous bodies such as stars and the accretion of the gas on to these bodies (Bate et al. 1995). The sink-particles interact only through gravity. The sink-particles use a constant sink radius ($R_{\rm sink}$) and any gas matter which falls within this radius and is bound to the sink-particle is accreted by it. The boundary conditions used in other studies of accretion (Bate et al. 1995; Bate & Bonnell 1997) are not included, as it is assumed that any accretion flow is supersonic by the time it reaches the sink radius, and that no discs are present. The gravitational forces from the sink-particles are calculated directly (not in the tree), using a Plummer potential with ε constant. The sink radius was chosen to be 2×10^{-3} of the initial radius of the cluster, $R_{\rm cl}$, and $\varepsilon = 5 \times 10^{-4} R_{\rm cl}$. All plots included are in units of the dynamical time of the system, $t_{\rm dyn} = (GM_{\rm tot}/{\rm R}_{\rm cl})^{1/2}$.

The clusters are modelled as containing between three and 10 stars of equal masses (except for Section 6), and a gas component which comprises between 0.1 and 90 per cent of the cluster mass. The simulations are assumed to start within approximately one crossing time of the initial fragmentation, in which time the stars and gas have virialized due to their initial interaction. The stars are then in virial equilibrium with the total mass inside the cluster. The gas component is modelled by 10^4 SPH particles with equal masses. The gas is isothermal and initially in equilibrium with the gravitational potential of the cluster, bounded by a small external pressure. It is fully contained within $R_{\rm gas} < 2R_{\rm cl}$. Additional calculations were also performed with the gas and stars out of virial equilibrium ($E_{\rm kin} + E_{\rm therm} = 0.05 E_{\rm grav}$), with no significant differences in the results except for the shorter accretion time-scale, corresponding to the shorter crossing time after the cluster has virialized.

To test the dependency on the numerical resolution, calculations were also performed with 2×10^3 and 5×10^4 particles. These calculations showed no qualitative differences from the 10^4 particle run. Furthermore, the form of the gas distribution is not crucial, as initially uniform distributions gave qualitatively similar results, the only difference being the time-scale for accretion.

Test cases of the Bondi–Hoyle accretion process have shown that in order to resolve the accretion process adequately with the method described here, it is necessary (Bate 1995) that the sink radius, $R_{\rm sink}$, be smaller than the Bondi–Hoyle radius $R_{\rm BH}$,

$$R_{\rm BH} = \frac{2GM}{v_\infty^2 + c_{\rm s}^2}.$$

This is always the case throughout the simulations. Unfortunately, it is not possible to obtain accurate Bondi–Hoyle accretion rates for the simulations, as the gas is not uniform (there is no ρ_∞) and the stars are not isolated. Best guesses for ρ_∞ give Bondi–Hoyle accretion rates of the same order of magnitude as the simulated rates.

The calculations here include no feedback from the stars on the gas. This should be adequate for low-mass star formation as the stars will not have strong stellar winds and the intrinsic jets are well collimated, and hence will not affect the overall gas distribution significantly. Although these jets and associated outflows may stir up the surrounding medium with the injection of kinetic energy, they evidently do not stop the accretion process; accretion on to the circumstellar discs can still proceed off the jet axis. Larger systems will result in the formation of massive stars which can remove the surrounding gas, effectively halting the accretion process.

4 STAR–GAS DYNAMICS

The presence of gas in a stellar cluster can have several effects on the evolution of the system. The most obvious is that the gas contributes to the gravitational potential such that its removal can unbind the system (e.g. von Hoerner 1968; Lada, Margulis & Dearborn 1984; Verschueren & David 1989). In addition to this superficial effect on the evolution of the system, there are several more fundamental ways in which the gas can contribute to the evolution. First, there is mass loading, where the gas is accreted on to the individual stars, increasing their masses without changing their momenta and hence decreasing their kinetic energies. This increases the binding energy of the cluster. Secondly, the presence of the gas can act as a drag term on the stars as they excite wakes in their passage. This gravitational drag, which accelerates the gas and allows it to be accreted in the Bondi–Hoyle accretion process, decelerates the stars, removing kinetic energy from the cluster (e.g. Larson 1990). The kinetic energy deposited in the gas is then mostly removed through shocks and radiated away. These two processes both contribute to making the cluster more bound, increasing the chances of forming a bound system when the remaining gas is removed.

To ascertain the relative importance of the two processes (mass loading and gas drag), we have rerun some of the evolutions without the full gas-dynamics but with the masses evolved due to the accretion as in the full gas-dynamical evolution. Thus the effects of mass loading without any gravitational drag from the gas can be determined. Fig. 1 plots the evolution of the total energy of the stellar component for the two cases. The energy contribution of the gas component (potential, thermal and kinetic) is not included. The gas comprises half of the mass of the system, so neglect-

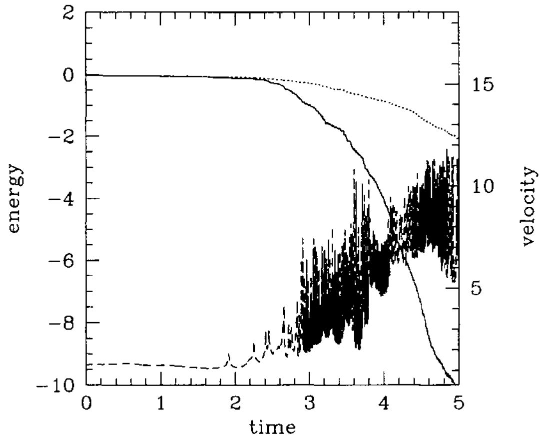

Figure 1. The evolution of the total energy of the stellar component due to the full SPH run with mass loading and gas drag (solid line), and due to mass loading alone (dotted line). The mean stellar velocity, in units of the gas sound speed, is also plotted (dashed line).

ing its contribution means that the initial energy is close to zero. The solid line shows the total energy for the full SPH evolution, while the dotted line includes only the effects of mass loading. The dashed line is the mean stellar velocity in units of the gas sound speed. The rapid oscillation of the mean stellar velocity indicates the formation of a central binary system. The total energy decreases in both cases, as potential energy of the accreted matter is added without any kinetic energy. In the full SPH run, additional energy is lost due to the gas drag. The two energy curves are similar during the early stages of the run, but diverge later on. The point at which they diverge is where the mean stellar velocity becomes significantly supersonic. By this point, 40 per cent of the gas has been accreted. Put simply, mass loading dominates the energetics as long as the stellar velocities are not significantly supersonic, whereas gas drag is dominant when the stellar velocities are supersonic.

The large decrease in total energy of the system (note that the majority of the gas has been accreted by the end of the evolution) results in the cluster being much tighter than it was initially. It also helps in forming closer binaries than are otherwise formed by dynamical capture. Of course, in clusters that contain most of their mass in gas, but which do not accrete all of it before it is removed by stellar winds, this decrease in total energy need not result in a bound cluster.

5 DYNAMICS OF ACCRETION

The accretion of the gas component on to the individual stars occurs in a highly non-uniform manner. Each star accretes the gas that is in its vicinity, creating underdense regions. This occurs relatively quickly as the gas close to each star is accelerated to supersonic velocities during infall. These underdense regions are in turn replenished by the infall of gas from larger distances. This replenishment is then dependent not only on the gravitational attraction of the individual star, but also (and mostly) on the overall gravitational potential of the system. This results in unequal accretion rates for each star, depending on their position in, and the dynamics of, the cluster. Fig. 2 demonstrates this phenomenon. It shows the radius of each star in the cluster and the corresponding percentage of gas mass accreted versus time during the evolution. Each star accretes significantly different amounts of gas, such that only a few stars accrete the majority of the gas, while others accrete negligible amounts of gas. The mass accretion depends predominantly on the star's position in the cluster. The stars closest to the centre of the cluster (heavy-solid line in Fig. 2) have the benefit of the cluster potential in attracting gas close enough to be accreted. Stars that spend most of their time in the outer regions of the cluster do not have this benefit, so they do not accrete nearly as much gas. This is shown by the heavy-dashed line in Fig. 2, where the star is ejected early on from the central regions of the cluster and does not accrete much gas thereafter. The heavy-dotted curves in Fig. 2 show how a star that initially is in the outer regions of the cluster does not accrete much gas until it comes near the centre of the potential. The stars that stay on the outside of the cluster (top curves of the left-hand panel of Fig. 2) never accrete appreciable amounts of gas (lowest curves on right-hand panel of Fig. 2).

Another factor which can significantly increase the accretion rate is whether the star is a member of a binary or multiple system. In this case, the combined gravitational potential aids in attracting gas and can outweigh the effect of the competitive accretion between the two components. Close encounters that result in ejections of stars from the cluster or the gas-rich centre of the cluster halt any further accretion on to the star involved.

6 ACCRETION AND STELLAR MASSES

The competitive accretion process outlined above can result in the formation of a significant range of stellar masses. Fig. 3 shows the evolution for a cluster of five stars in a cluster that initially contains 75 per cent of its mass in the form of gas. In this calculation, each of the five stars initially comprises 5 per cent of the total system mass. By the end of the evolution, the five stars have significantly different masses due to the accretion. The most massive star now comprises 40 per cent of the total system mass, whereas the least massive star has only increased its mass to ≈ 6.5 per cent of the system mass. Even the second most massive star has less than one-half the mass of the most massive. The accretion rate, and thus the final masses, can be understood as being

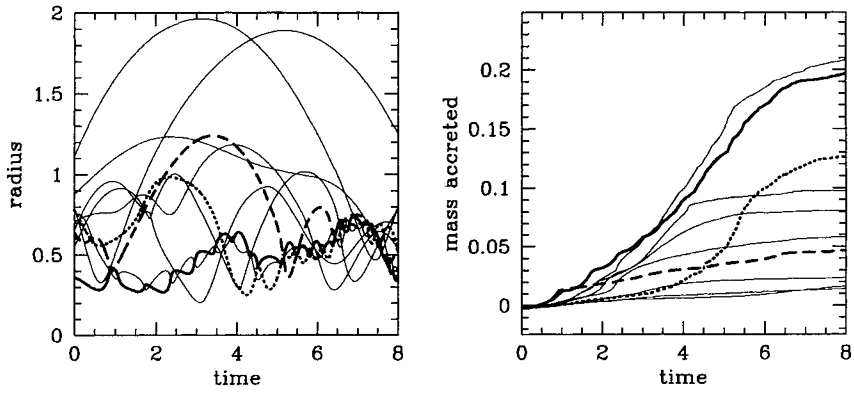

Figure 2. The evolution of a cluster of 10 stars undergoing gas accretion is shown. The radius of each star from the centre of the cluster (left) and the mass accreted by each star (right) are given as functions of time. The gas initially comprises 10 per cent of the total mass of the system. Three individual stars are highlighted (see text). Time is given in units of the system dynamical time, mass in units of the total gas mass available, and radius in units of the initial cluster radius.

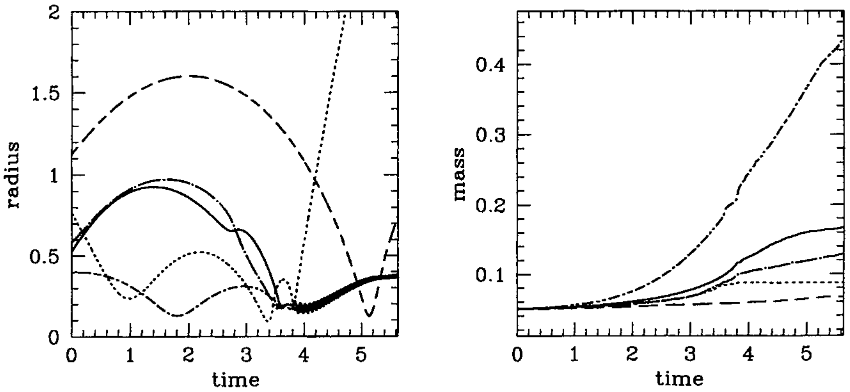

Figure 3. The evolution of the radius (left) and stellar mass (right) of each star in a cluster of five stars. The gas initially comprises 75 per cent of the total system mass.

primarily due to the position of each star in the cluster. The stars closest to the centre accrete the majority of the gas. The ejection of a star (dotted line in Fig. 3) effectively halts the accretion process and thus finalizes the mass of the star.

This large range in final masses suggests that the determining factor in setting the stellar mass can be the accretion process. In all cases, only a few stars accrete significantly more than the rest, and thus have much larger masses. This process can then account for the formation of a large range of stellar masses where the majority of stars have relatively small masses and only a small subset have large masses.

The range of stellar masses will obviously be a function of the amount of gas available in the system. Fig. 4 plots the ratio of the maximum to minimum stellar mass as a function of the amount of gas accreted by the system (in terms of the total initial stellar mass). The plot contains the results for each simulation at different times. Clusters with 10 stars have a larger range of stellar masses than do similar clusters with five stars. This is basically due to the reduced mass of each star (relative to the total) in the larger clusters. Hence the few that accrete most can gain more mass relative to those that do not accrete significantly. In clusters where the gas content is very high initially, the accretion will give a large range of stellar masses. In larger clusters, this range will be even greater.

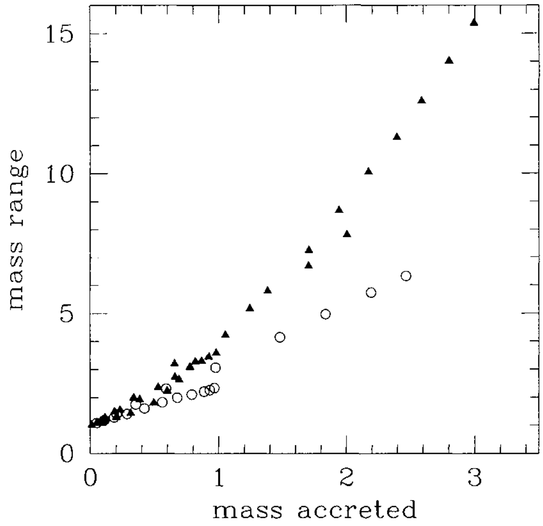

Figure 4. The mass range (defined as the ratio of the maximum to minimum stellar mass) as a function of the mass accreted by the system (in terms of the initial total stellar mass) are plotted. The filled triangles are for clusters containing 10 stars, while the open circles are for clusters containing five stars.

7 DEPENDENCE ON THE INITIAL MASS DISTRIBUTION

The above section assumes a uniform initial stellar mass. We know that the stellar mass will affect the accretion rate and can modify the above results. In order to test how sensitive the results are to the initial stellar mass spectrum, we repeated one of the calculations with different initial stellar masses. This was done by choosing the three stars that accreted most of the gas and reducing their initial masses, increasing that of the other stars accordingly to maintain the same total initial stellar mass. The specific kinetic energy of each star remains unchanged as much as possible, with small changes necessary to maintain the centre of mass and centre of mass velocity of the system.

Fig. 5 plots the evolution for the three cases where the initial masses were modified for the evolution with 10 stars and an initial gas mass of 50 per cent of the total cluster mass. The top panel shows the evolution where all the masses are initially identical. The three stars which then accrete the most are highlighted (heavy solid line). They accrete the most gas because they spend more time near the centre of the cluster. The middle panel shows the evolution of the same cluster with variations in the initial stellar masses of 20 per cent of the mean stellar mass. The three stars that accreted the most in the uniform case are again highlighted (heavy solid line). Even though they start with less mass initially, two of them are still able to accrete enough gas to be amongst the most massive stars in the system. In the bottom panel, these three stars are given a mass only 40 per cent that of the mean stellar mass. In this case, they do not accrete significant amounts of gas, even though they are close to the centre of the cluster, as they do not contribute as significantly to the cluster potential. The three heavy-dotted lines in each panel indicate three stars that, in the uniform stellar mass case, do not accrete signifi-

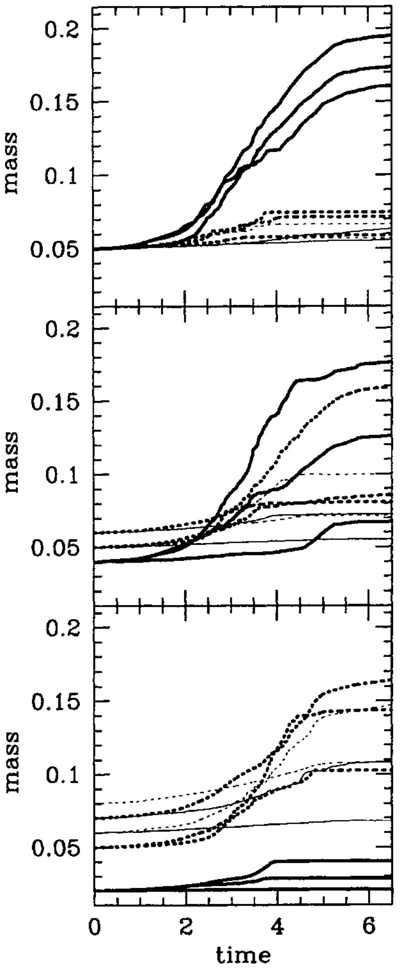

Figure 5. The evolution of the stellar mass of each star in a cluster of 10 stars. The gas initially comprises 50 per cent of the total mass. The top panel has equal initial stellar masses, whereas the bottom two panels contain increasingly larger initial mass spectra.

cant amounts of gas but accrete more when the initial masses are varied. They are able to accrete more gas due to the dynamics of the cluster, such that with the modified masses, they move closer to the centre of the potential. It should be noted that in most cases (except when the initial mass is significantly different from the mean) there is not a direct correspondence between the initial and final stellar mass. The dynamics of the cluster play a crucial role in determining the accretion rates, and thus the mass, of each star.

8 BINARY FORMATION

The dynamics of small clusters naturally result in the formation of a central binary and the ejection of the rest of the cluster members. If a spectrum of stellar masses is included, the binary forms from the two most massive stars, as the less massive stars are more likely to be ejected. When accretion is included, the cluster dynamics and eventual dissolution are similar, except that the resultant binary is much closer due to the energy lost during the accretion process. The accreted material has, on average, near-zero angular momenta, so that its accretion decreases the specific angular momentum of the binary and hence decreases the separation (Bate 1997; Bate & Bonnell 1997). For example, an equal-mass binary that increases its mass by a factor ≈ 3 will decrease its separation by a factor ≈ 50.

Although the calculations have not been followed long enough to determine the final outcomes, and an insufficient number of simulations have been conducted to determine statistical properties, it is apparent that the accretion will help in forming more stable multiple systems. The removal of kinetic energy through the gravitational drag on the stars and the mass loading tends to help settle the system closer to a stable equilibrium (Smith, Bonnell & Bate 1997).

The mass ratios of the binaries are also determined by the accretion process but depend crucially on the masses of the components, and hence their accretion histories, at the time of the dynamical capture. If a binary is formed with near-equal-mass stars, then the subsequent accretion has little to choose from between the two components; their mass ratio continues to be near unity throughout the accretion. If, on the other hand, the two stars have significantly different masses, then subsequent accretion of basically zero angular momentum matter preferentially increases the mass of the primary (Bate 1997; Bate & Bonnell 1997), leading to very small mass ratio binaries.

9 DISC FORMATION

Protostellar discs are an intrinsic byproduct of the star formation process. Disc formation occurs whenever any non-zero angular momentum is present in the initial gas from which the stars form. These discs can be quite large and can thus affect the dynamics of the stellar interactions (McDonald & Clarke 1995; Hall et al. 1996). Although the simulations here do not include any discs in the initial conditions, they would help form additional binary and multiple systems through star–disc interactions.

Discs can also be formed through the infall and accretion of the intracluster gas. In general, this gas will have some angular momentum which will determine the disc properties. Although the surrounding gas (in the simulations reported here) has zero angular momentum, the mass accreted by each star has some angular momentum due to the dynamics of the cluster. The disc sizes that result can be estimated from the accreted angular momentum, $R_{\rm disc} \approx J^2/GM$, where J is the specific angular momentum of the accreted matter, and M is the mass of the star. All the disc sizes derived in this way are quite small (typically $R_{\rm disc} < 5 \times 10^{-4} R_{\rm cl}$). These values should be considered as lower limits due to the lack of angular momentum in the gas.

There is no definite correlation between accreted mass and disc size. As the lighter stars are more frequently ejected, there is no evidence that the ejected stars would have intrinsically smaller discs prior to ejection. The weak-lined status of these stars would have to be explained as due to the ejection process (Armitage & Clarke 1997).

Finally, we note that the formation of circumbinary discs is *not* a likely outcome for binaries that are formed through dynamical interactions in gas-rich clusters. Circumbinary disc formation requires the infall of material with specific angular momentum greater than that of the binary (Bate

1995; Bate & Bonnell 1997), a situation that is naturally realized if binaries form as single systems fragmenting out of a rotating core – essentially because both binary and infalling gas inherit their angular momentum from the same source. In the present scenario, however, the angular momentum of the binaries is derived from the chaotic dynamical interactions between fragments, and is independent of the angular momentum present in the initial conditions. Thus, as noted above, the binaries accrete relatively low angular momentum material, inconducive to the formation of circumbinary discs. The presence or absence of circumbinary discs may thus be a diagnostic of the mode of binary formation.

10 IMPLICATIONS

10.1 Mass segregation

Observations of young clusters usually indicate that the most massive stars are located near the centre of the cluster. This is true for both small (Hillenbrand et al. 1995; Testi et al. 1996) and large (Zinnecker et al. 1993) clusters. Older open clusters also show evidence for this mass segregation (Pandey, Mahra & Sagar 1992). This is often attributed to the dynamical evolution of the cluster, where the massive stars are more likely to lose energy through interactions and sink to the bottom of the cluster potential. The problem with this option is that it occurs on the relaxation time-scale and these clusters are generally not thought to be old enough for this evolution.

Alternatively, the location of the high-mass stars near the centre of the cluster could be an indication of where they formed. In this case, the high stellar densities present in the centre of clusters like the Trapezium means that the pre-collapse gas densities would have to be very high in order for the Jeans length to be less than the stellar separation (Zinnecker et al. 1993). The Jeans mass (assuming a temperature of ≈ 10–20 K) would then be small ($M_{\rm J} \leq 0.5\ {\rm M}_{\odot}$). If the fragmentation resulted in approximately Jeans-mass fragments (e.g. Bonnell et al. 1991, 1992; Burkert & Bodenheimer 1993), then this implies the formation of relatively low-mass protostellar cores. The formation of the high-mass stars observed would then require significant accretion. This is easily understandable in the context of the model presented here where the eventual mass of the star is a product of its accretion history, which depends primarily on its position in the cluster. The stars near the centre of the cluster are the only ones that can accrete sufficient material to attain a large mass.

10.2 The stellar mass function

The build-up of a star's mass through the competitive accretion process described here is an attractive way of determining the stellar mass function. The dynamics of the cluster and the time-dependent position of each star in the cluster potential are the driving forces behind determining the final mass spectrum. The calculations reported here are not sufficient to derive statistical properties such as the emergent mass function. As most stars probably form in larger clusters of several hundred stars (Lada et al. 1993; Zinnecker et al. 1993), it is necessary to consider the competitive accretion process in larger systems.

There are certain differences that can be expected in larger clusters. First of all, each star contributes less to the overall potential of the system. We thus expect that the accretion rates will be less dependent on the mass of the star and more dependent on its position. This is in contrast to Zinnecker's (1982) prescription of accretion, where the mass of the star is the only factor in determining the accretion rate. This also means that the possible membership in a binary or multiple system will be less important in determining the accretion rate. Secondly, from Fig. 4, we expect that the emergent mass spectrum in larger systems will itself be larger. This is mainly due to the smaller masses that each star will have, in order to comprise the same total stellar mass.

11 CONCLUSIONS

Accretion in small stellar clusters is very dependent on the cluster dynamics. Individual stars accrete at highly different rates, dependent on their position in the cluster potential. Stars that spend more time near the centre of the cluster accrete more than those that are in the outer regions. This happens as the gas falls preferentially into the deepest part of the potential, to be accreted by whichever star is nearest. Stars that are in binary systems also accrete at higher rates due to their added gravitational attraction. Furthermore, they are predominantly located near the cluster centre, as they are more likely to form there through dynamical capture, or afterwards fall towards the centre through interactions with other cluster members.

This differential accretion results in a large variation of final stellar masses. A few stars accrete significantly more mass than the rest. For clusters that contain significant amounts of gas, the final stellar masses and the overall mass spectrum are a direct result of the accretion process. Initial variations in the stellar masses, if they are small, can be overwhelmed by the accretion process so that there is no direct correspondence between the initial and final stellar masses. Larger variations can affect the accretion process as they significantly modify the cluster potential. The emergent mass spectrum is highly dependent on the amount of gas accreted, and the number of stars in the cluster. A larger range of stellar masses results if the cluster is comprised mostly of gas and if there are more stars in the cluster. The latter is due to the lower initial mass of the stars.

The overall effect of the gas accretion is to form a tighter cluster and a closer binary than would form otherwise. The gas accretion removes energy from the system, the thermal energy of the gas and some kinetic energy of the stars, while increasing the depth of the cluster potential by adding to the masses of the stars.

The mass ratios of binaries formed during the evolution of the cluster are determined by their initial mass ratios. If the two components have equal masses when the binary is formed, then the infalling matter has little to choose from between them, and they will accrete equal amounts. If, on the other hand, the binary is formed from unequal-mass components, then the infalling gas, which has basically zero angular momentum, will fall preferentially towards the primary and be accreted by it. This will lead to very low-mass-ratio binaries.

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