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Accretion disc viscosity: a simple model for a magnetic dynamo

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ABSTRACT

We develop here a simple physical model for the manner in which a magnetic dynamo might operate in an accretion disc and so provide an effective (magnetic) viscosity. In contrast to other dynamo models, the mechanism we discuss does not depend on the existence of some hydrodynamical small-scale turbulent flow hypothesized to be already present in a non-magnetic disc. Rather, the model we present depends on three well-established physical processes: the Parker instability, the Balbus–Hawley instability and magnetic field reconnection. The model gives rise to finite but non-stationary magnetic field configurations. For the set of parameters chosen here we find a time-averaged effective viscosity with Shakura–Sunyaev α-parameter around $\alpha_{SS} \approx 0.4$.

Key words: accretion, accretion discs – instabilities – MHD.

1 INTRODUCTION

One of the main failings of accretion disc theory has been the uncertainty as to the nature and magnitude of the viscosity (Pringle 1981). In early papers (von Weizsäcker 1948) it was argued that hydrodynamic turbulence might provide the appropriate transport mechanism. Our ignorance was neatly encapsulated by Shakura & Sunyaev (1973) into a single parameter which we denote here as α_{SS}. They pointed out that in measuring the viscous force per unit area in the shearing disc, which is $\rho \nu R \mathrm{d}\Omega/\mathrm{d}R$, where $\Omega(R)$ is the angular velocity in the disc and ρ the density, one could write the kinematic viscosity, ν, in the form

$$\nu = \alpha_{SS} C_s^2/\Omega, \tag{1.1}$$

where a_{SS} is a dimensionless measure of the strength of the viscosity and C_s is the sound speed in the disc. For transsonic hydrodynamic turbulence with eddy sizes of order the disc thickness they showed that $\alpha_{SS} \approx 1$, and further argued that since supersonic turbulence dissipates rapidly this should prove an upper limit. However, it has yet to be shown that accretion disc flow is hydrodynamically unstable and thus there is as yet no evidence that such turbulence exists. Even the suggestion that convection in cool accretion discs might prove to be a self-sustaining viscosity mechanism has proved untenable (Ryu & Goodman 1992).

However, Shakura & Sunyaev (1973) also pointed out that magnetic stress could prove a viable transport mechanism and that an accretion disc is fertile ground for the maintenance of a magnetic dynamo. Equating the viscous force (above) to the magnetic stress, $B_R B_\phi/4\pi$, where B_R and B_ϕ are the radial and azimuthal magnetic field components respectively, they found that formula (1.1) still applied but now with

$$\alpha_{SS} = (B_R B_\phi / 4\pi \rho C_s^2). \tag{1.2}$$

Since then there have been a number of papers written about dynamo generation of magnetic fields in disc configurations (Eardley & Lightman 1975; Galeev, Rosner & Vaiana 1979; Parker 1979, ch. 22; Soward 1978; Pudritz 1981a,b; Meyer & Meyer-Hofmeister 1983; Stepinski & Levy 1988, 1990; Vishniac, Jin & Diamond 1990; Campbell 1992). All these authors concur that the azimuthal field is generated from the radial by means of disc shear. The problem arises in determining what closes the cycle and generates radial field from the azimuthal. With the exception of Eardley & Lightman (1975) who postulated closure in an essentially two-dimensional dynamo by means of magnetic reconnection within the disc, the rest of the authors appeal to hydrodynamic flows (usually turbulence or convection) already present in the disc and of unknown origin in order to serve as a mechanism for the usual 'α'-part of the standard '$\alpha\omega$'-dynamo. For this reason this approach does not produce a satisfactory explanation of magnetic viscosity in accretion discs.

We investigate here whether there are plausible physical processes which can give rise to a dynamo mechanism within an accretion disc which is independent of any hypothesized

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internal disc flows which are present in the absence of magnetic fields. In other words, we attempt to construct a self-consistent, but purely magnetic, dynamo process.

In Section 2, we set out our proposed dynamo equations. The physical processes we shall invoke (apart from the usual shearing of radial field) are three processes for which the physical principles are relatively clear, but for which there is no detailed understanding of how they operate on the small scale and of how this relates to the more global scale of interest here. These three processes are: reconnection, the Parker instability (Parker 1979) and the Balbus–Hawley instability (Balbus & Hawley 1991). Our aim in Section 2, therefore, is to write down physically plausible relationships for the interactions between the various components of magnetic field in the light of the underlying mechanism behind the processes concerned. Where possible we have made use of results to be found in the literature as a guide to time-scales and length-scales involved.

In Section 3, we show that the derived equations lead to oscillation around an unstable equilibrium configuration and derive the mean values of the field components, as well as a value for α_{SS}, in that case. We wish to stress from the start, however, that given the nature of the approximations we have made and the analysis we employ, the numerical values we derive from the various quantities should not be taken at face value.

In Section 4, we present discussion and conclusions.

2 THE DYNAMO EQUATIONS

We shall work in terms of the quantities B_R, B_ϕ and B_Z which we regard as the relevant local averages of the radial, azimuthal and vertical fields within the disc. Thus we do not attempt to describe the detailed magnetohydrodynamics which make up the full dynamo process, but, rather, content ourselves with attempting to write down equations or relationships which describe how such averaged quantities interact with one another given the various physical processes taking place in the disc.

In particular we do not solve explicitly for the space dependences of the fields, although it will be necessary to give consideration to what the form of such space dependences might be in certain specific instances. Nor do we take account of any net flow within the disc, but, rather, assume that the time-scales associated with the dynamo process are short compared to flow time-scales. Thus the quantities B_R, B_ϕ and B_Z are local quantities. We shall also ignore for the time being any thermal consequences of the presence of a dynamo. For this reason we treat the disc locally as an isothermal, thin accretion disc with scaleheight, H, determined by balance between gravity and thermal pressure. Thus, to the approximation we are working, we neglect the contribution of magnetic pressure to vertical disc support. As will be seen below, this is an adequate assumption for the equilibrium dynamo.

2.1 The B_ϕ-equation

We assume that B_ϕ is generated from B_R within the disc by shear (the usual ω-dynamo process). This process could also be regarded as part of the Balbus–Hawley instability which generates B_R and simultaneously B_ϕ from the vertical field B_Z (see below). In an accretion disc the angular velocity profile $\Omega(R)$ is fixed (since viscous and pressure forces within the disc are small). Thus we may write for a Keplerian disc:

$$\left.\frac{\mathrm{d}B_\phi}{\mathrm{d}t}\right|_{\mathrm{gain}} = \frac{3}{2}\,\Omega\, B_R. \tag{2.1.1}$$

We shall assume that the dominant mechanism which leads to loss of azimuthal field from the disc is the Parker instability. We shall find, in general, that in an equilibrium dynamo situation the dominant horizontal field in the disc is the azimuthal one. Parker (1979, p. 330) treats the instability in an isothermal, constant gravity, g, (and so constant scale-height Λ) atmosphere. Since the field is predominantly azimuthal, and since the growth rates and also the wavenumber in the field direction are insensitive to the structure of the unstable mode perpendicular to the field (in the R-direction), we assume here that shear has little effect on the instability, but return to this point below. Thus if τ_P is the growth time-scale for the Parker instability we may write

$$\left.\frac{\mathrm{d}B_\phi}{\mathrm{d}t}\right|_{\mathrm{loss}} = \frac{-B_\phi}{\tau_P}, \tag{2.1.2}$$

and hence the full B_ϕ-equation becomes

$$\frac{\mathrm{d}B_\phi}{\mathrm{d}t} = \frac{3}{2}\,\Omega\, B_R - \frac{B_\phi}{\tau_P}. \tag{2.1.3}$$

We apply Parker's calculations to an accretion disc, and take $g=\Omega^2 H$, where H is the disc semithickness given by $H=C_s\sqrt{2}/\Omega$ and where C_s is the (constant) sound speed. Then we find that

$$\tau_P = \eta H / V_{A\phi}, \tag{2.1.4}$$

where $V_{A\phi}^2 \equiv B_\phi^2/4\pi\rho$, ρ is a representative density in the disc, and η is a constant in the range 2–2.4. We note that Horiuchi et al. (1988) perform similar calculations to Parker, but for an isothermal accretion disc with a z-dependent gravity. They find similar results with η in the range 2–5.

The wavelength of the instability in (for us) the azimuthal direction is found by Parker to be

$$\lambda_{P\phi} = \xi H, \tag{2.1.5}$$

where ξ is in the range 7.3–9.8. We note that we have implicitly assumed that $\lambda_{P\phi} \ll R$, i.e., that $H/R \ll \xi^{-1} \sim 0.1$.

2.2 The B_R-equation

We assume that the major loss of radial magnetic flux is caused, as for the azimuthal flux, by the Parker instability. Indeed, for the regime under discussion with $B_R \ll B_\phi$ we may regard B_R as merely converting B_ϕ into the geometry of a tightly wound spiral. Thus the appropriate time-scale is τ_P (equation 2.1.4), and we may write

$$\left.\frac{\mathrm{d}B_R}{\mathrm{d}t}\right|_{\mathrm{loss}} = -\frac{B_R}{\tau_P}. \tag{2.2.1}$$

We note, however, that since the radial field is subject to shear, the effect of the Parker instability on a general radial field is still subject to debate. There are claims in the literature that the effect of shear is to enhance (Coroniti 1981) and to diminish (Shibata, Tajima & Matsumoto 1990; Vishniac & Diamond 1992) the instability. The most thorough linear stability analysis has been carried out by Shu (1974) who comes to the general conclusion that for the modes of interest to us the effect of shear is small.

In a pair of recent papers (Balbus & Hawley 1991; Hawley & Balbus 1991) attention has been drawn to an instability present in any cylindrical shear flow with a vertical component of the magnetic field and for which $\mathrm{d}\Omega/\mathrm{d}R<0$. This instability has been in the literature for some time – indeed, it is to be found in Chandrasekhar's monograph (Chandrasekhar 1961) – but its significance has been overlooked. The effect of the instability is to tap the energy present in the shear flow and to use this to generate radial (and so, via the shear, azimuthal) field from the initial vertical component. We shall refer to this instability as the Balbus–Hawley instability. The relevant properties to the problem in hand, namely a Keplerian, locally isothermal disc, are as follows (Balbus & Hawley 1991).

The fastest growing mode has a growth rate $\gamma_{\max}\Omega$, and vertical wavelength $\lambda_{\mathrm{BH\,max}}$, where $\gamma_{\max}\approx 0.74$ and

$$\lambda_{\mathrm{BH\,max}}\approx 2\pi V_{\mathrm{AZ}}/\Omega, \tag{2.2.2}$$

where $V_{\mathrm{AZ}}^2\equiv B_Z^2/4\pi\rho$. This mode is the relevant one when $\lambda_{\mathrm{BH\,max}}\lesssim 2H$, that is when $V_{\mathrm{AZ}}/C_{\mathrm{s}}\lesssim\sqrt{2}/\pi$.

There is also a critical wavelength, $\lambda_{\mathrm{BH\,crit}}\simeq 2\pi V_{\mathrm{AZ}}/\Omega\sqrt{3}$, such that for wavelengths $\lambda\lesssim\lambda_{\mathrm{BH\,crit}}$ there are no unstable modes. Thus within the disc, if $\lambda_{\mathrm{BH\,crit}}\gtrsim 2H$ (i.e., if $V_{\mathrm{AZ}}/C_{\mathrm{s}}\gtrsim\sqrt{6}/\pi$), the Balbus–Hawley instability is suppressed. We now need to consider what happens in the range $\sqrt{2}/\pi\lesssim V_{\mathrm{AZ}}/C_{\mathrm{s}}\lesssim\sqrt{6}/\pi$. In this range the mode with the most rapid growth rate has $\lambda\approx 2H$, and from an approximate analytical fit to fig. 1(c) of Balbus & Hawley (1991) we find the growth rate is given by $\gamma_{\mathrm{BH}}\Omega$ where, over the range of interest,

$$\gamma_{\mathrm{BH}}\approx\gamma_{\max}\left[1-\frac{\left(1-\dfrac{\pi V_{\mathrm{AZ}}}{C_{\mathrm{s}}\sqrt{2}}\right)^2}{(1-\sqrt{3})^2}\right]^{1/2}. \tag{2.2.3}$$

We may now write the full B_R-equation in the following form:

$$\frac{\mathrm{d}B_R}{\mathrm{d}t}=\begin{cases}\gamma_{\max}\Omega B_Z-B_R/\tau_{\mathrm{P}} & \dfrac{V_{\mathrm{AZ}}}{C_{\mathrm{s}}}\le\dfrac{\sqrt{2}}{\pi},\\[2ex] \gamma_{\mathrm{BH}}\Omega B_Z-B_R/\tau_{\mathrm{P}} & \dfrac{\sqrt{2}}{\pi}<\dfrac{V_{\mathrm{AZ}}}{C_{\mathrm{s}}}\le\dfrac{\sqrt{6}}{\pi},\\[2ex] -B_R/\tau_{\mathrm{P}} & \dfrac{\sqrt{6}}{\pi}<\dfrac{V_{\mathrm{AZ}}}{C_{\mathrm{s}}}.\end{cases} \tag{2.2.4}$$

We further note that the scale of the instability in the z-direction, λ_{BH}, is given by

$$\frac{\lambda_{\mathrm{BH}}}{2H}=\begin{cases}1 & \dfrac{V_{\mathrm{AZ}}}{C_{\mathrm{s}}}>\dfrac{\sqrt{2}}{\pi},\\[2ex] \dfrac{\pi V_{\mathrm{AZ}}}{\sqrt{2}C_{\mathrm{s}}} & \dfrac{V_{\mathrm{AZ}}}{C_{\mathrm{s}}}<\dfrac{\sqrt{2}}{\pi}.\end{cases} \tag{2.2.5}$$

2.3 The B_Z-equation

Generation of B_Z comes about because the fundamental (and fastest growing) mode for the Parker instability in a thin disc has the effect of directly converting horizontal to vertical field (Horiuchi et al. 1988). Thus we may write

$$\left.\frac{\mathrm{d}B_Z}{\mathrm{d}t}\right|_{\mathrm{gain}}\simeq B_\phi/\tau_{\mathrm{P}}+B_R/\tau_{\mathrm{P}}, \tag{2.3.1}$$

cf. equations (2.1.2) and (2.2.1). However, since we are working in the regime $B_R\ll B_\phi$, we shall neglect the second term on the right-hand side in what follows.

The equations so far have set up an efficient mechanism for taking an initial B_Z and generating further B_Z from it at the expense of the shear energy in the Keplerian disc. We now need to consider what limits the growth of B_Z. In fact it is necessary to do more than just limit B_Z growth (which is done anyway by the Balbus–Hawley mechanism – Section 2.2), since all we have so far is a mechanism for converting shear energy to magnetic energy. The fields so formed will not be able to operate as a 'magnetic viscosity', and so will not be able to drive an accretion disc, unless some mechanism is identified which leads to flux loss from or in the disc, either by dissipation (conversion to thermal energy within the disc) or by bodily removal.

At this point we need to make explicit something which has been partially implicit in some of the above discussion, and that is that we are considering mainly field strengths and configurations which are relevant to the equilibrium magnetic dynamo (Section 3). Thus it needs to be borne in mind that the equations we derive here may need rethinking for magnetic configurations far from equilibrium – for example when the disc is subject to a strong externally imposed vertical field. We return to this point below. For the present, we are able to note that the structure of the vertical field produced by the fundamental mode of the Parker instability is one of alternating sign across the disc. Furthermore we note that horizontal motions are driven within the disc by both the Parker and the Balbus–Hawley instabilities with velocities $\sim V_{\mathrm{AZ}}$. Thus the B_Z field lines are rattled around and pushed into each other with motions which give rise to net local excursions of order $\sim V_{\mathrm{AZ}}/\Omega$.

We shall assume, therefore, that the predominant flux loss mechanism is reconnection. Patches of B_Z of opposite sign come together and reconnect, leading to some dissipation of energy within the disc, but probably mainly loss of flux into the regions above and below the disc. If the mean distance between two neighbouring patches of opposite sign is λ_{rec}, we shall assume that B_Z reconnects and is removed from the disc at a rate

$$\tau_{\mathrm{rec}}=\lambda_{\mathrm{rec}}/\Gamma V_{\mathrm{AZ}}. \tag{2.3.2}$$

Here $\Gamma^{-1}\sim\ln(\mathscr{R}_{\mathrm{m}})$, where $\mathscr{R}_{\mathrm{m}}$ is the magnetic Reynold's number and Γ is expected to be in the range 0.1 to 0.01 (Parker 1979, p. 395). Given these assumptions, the B_Z-equation can now be written in the form

$$\frac{\mathrm{d}B_Z}{\mathrm{d}t}\simeq\frac{B_\phi}{\tau_{\mathrm{P}}}-\frac{B_Z}{\tau_{\mathrm{rec}}}. \tag{2.3.3}$$

The main problem now is to determine τ_{rec} or, equivalently, λ_{rec}. In order to estimate the relevant mean distance between neighbouring patches of B_Z with opposite sign, it is convenient to consider the azimuthal and radial directions separately, and so to obtain $\lambda_{\mathrm{rec}\,\phi}$ and $\lambda_{\mathrm{rec}\,R}$, respectively. We then have that $\lambda_{\mathrm{rec}} = \min(\lambda_{\mathrm{rec}\,\phi}, \lambda_{\mathrm{rec}\,R})$.

2.3.1 Azimuthal reconnection

The fundamental scale for reconnection imposed by the Parker instability in the ϕ-direction is $\frac{1}{2}\lambda_{\mathrm{P}\phi}$, where $\lambda_{\mathrm{P}\phi}$ is given by equation (2.1.5). We must also note, however, that the azimuthal field is generated by the shearing of B_R which is produced from B_Z. For a uniform B_Z, this process produces an azimuthal field with a structure such that B_ϕ alternates sign in the z-direction on a length-scale $\frac{1}{2}\lambda_{\mathrm{BH}}$, where λ_{BH} is defined in equation (2.2.5). We conclude therefore that

$$\lambda_{\mathrm{rec}\,\phi} \sim \frac{1}{2}\lambda_{\mathrm{P}\phi} \times \frac{1}{2}(\lambda_{\mathrm{BH}}/2H). \tag{2.3.4}$$

2.3.2 Radial reconnection

The value of $\lambda_{\mathrm{rec}\,R}$ depends on the coherence of the Parker instability in the radial direction. Were it not for shear, the Parker instability would exist on all radial scales. We wish to discover what is the longest relevant scale (in the presence of shear) as this will give us the reconnection time-scale for the bulk of B_Z. We denote this coherence length of the instability in the radial direction by ΔR and define ΔR by requiring that in one Parker growth time, τ_{P}, the annuli at R and $R+\Delta R$ have sheared relative to one another by a distance of order $\lambda_{\mathrm{rec}\,\phi}$. We find that

$$\Delta R \simeq \frac{2}{3\sqrt{2}}\,\eta^{-1}\,\frac{V_{\mathrm{A}\phi}}{C_{\mathrm{s}}}\,\lambda_{\mathrm{rec}\,\phi} \tag{2.3.5}$$

and conclude that $\Delta R \lesssim \lambda_{\mathrm{rec}\,\phi}$. Thus we expect reconnection in the radial direction to dominate, and we conclude that one estimate of $\lambda_{\mathrm{rec}\,R}$, which we denote as $\lambda^{\mathrm{a}}_{\mathrm{rec}\,R}$, becomes

$$\lambda^{\mathrm{a}}_{\mathrm{rec}\,R} \simeq \Delta R. \tag{2.3.6}$$

This gives rise (via 2.3.4) to a corresponding estimate of τ_{rec}, which we denote as $\tau^{\mathrm{a}}_{\mathrm{rec}\,R}$.

We must also note, however, that the shearing process itself tends to reduce length-scales in the radial direction and so to enhance the reconnection. Consider a patch of B_Z of initial ($t=0$) size ΔR in the radial direction and $\lambda_{\mathrm{rec}\,\phi}$ in the azimuthal direction. At times $t \gtrsim \Omega^{-1}$, the radial length-scale of such a patch is decreased to a value of l_R, where

$$l_R \simeq \frac{2}{3}\lambda_{\mathrm{rec}\,\phi}(\Omega t)^{-1}. \tag{2.3.7}$$

Thus l_R decreases with time until the radial length-scale is sufficiently small that reconnection takes over. In this way we have a second estimate of the radial reconnection time-scale, $\tau^{\mathrm{b}}_{\mathrm{rec}\,R}$, which we define as being the time by which the time-scale for decrease of l_R is equal to the reconnection time-scale across the distance l_R, that is

$$l_R/\dot{l}_R \sim l_R/\Gamma V_{\mathrm{AZ}}. \tag{2.3.8}$$

This gives

$$\tau^{\mathrm{b}}_{\mathrm{rec}\,R} = \left(\frac{2\lambda_{\mathrm{rec}\,\phi}}{3\Gamma\Omega\, V_{\mathrm{AZ}}}\right)^{1/2}. \tag{2.3.9}$$

We conclude that the relevant value of τ_{rec} in equation (2.3.3) is given by

$$\tau_{\mathrm{rec}} = \min(\tau^{\mathrm{a}}_{\mathrm{rec}\,R}, \tau^{\mathrm{b}}_{\mathrm{rec}\,R}), \tag{2.3.10}$$

and we find that

$$\frac{\tau^{\mathrm{a}}_{\mathrm{rec}\,R}}{\tau^{\mathrm{b}}_{\mathrm{rec}\,R}} = \begin{cases} \dfrac{1}{2^{3/4}\sqrt{3}}\,\dfrac{\xi^{1/2}}{\eta\Gamma^{1/2}}\,\dfrac{V_{\mathrm{A}\phi}}{C_{\mathrm{s}}}\left(\dfrac{C_{\mathrm{s}}}{V_{\mathrm{AZ}}}\right)^{1/2} & \dfrac{V_{\mathrm{AZ}}}{C_{\mathrm{s}}} > \dfrac{\sqrt{2}}{\pi}, \\[2ex] \dfrac{\sqrt{\pi}}{2\sqrt{3}}\,\dfrac{\xi^{1/2}}{\eta\Gamma^{1/2}}\,\dfrac{V_{\mathrm{A}\phi}}{C_{\mathrm{s}}} & \dfrac{V_{\mathrm{AZ}}}{C_{\mathrm{s}}} < \dfrac{\sqrt{2}}{\pi}. \end{cases} \tag{2.3.11}$$

3 SOLUTION OF THE EQUATIONS AND THE MAGNETIC VISCOSITY

In this section we consider the solutions of the disc dynamo equations we derived in the previous section, paying particular attention to the equilibrium solutions and their stability.

3.1 The trivial ($\boldsymbol{B}=0$) solution

It is evident, and physically necessary, that $\boldsymbol{B}=0$ is a solution of the dynamo equations. Here we investigate the stability of that solution. For small $\boldsymbol{B}$, that is for $B^2/4\pi\rho \ll C_{\mathrm{s}}^2$, we note that $\tau^{\mathrm{a}}_{\mathrm{rec}\,R} \ll \tau^{\mathrm{b}}_{\mathrm{rec}\,R}$. In this approximation the equations become

$$\frac{\mathrm{d}B_R}{\mathrm{d}t} = k_1 B_Z, \tag{3.1.1}$$

$$\frac{\mathrm{d}B_\phi}{\mathrm{d}t} = k_2 B_R \tag{3.1.2}$$

and

$$\frac{\mathrm{d}B_Z}{\mathrm{d}t} = -k_3 B_Z/B_\phi, \tag{3.1.3}$$

where k_1, k_2 and k_3 are constants. Note that for self-consistency we need to consider solutions for small $\boldsymbol{B}$ for which $B_Z \ll B_\phi$. We note that within the context of the model, these equations display an algebraically growing solution of the form

$$\boldsymbol{B} = (c, ck_2 t, 0), \tag{3.1.4}$$

where c is a constant. We conclude that the equilibrium solution at $\boldsymbol{B}=0$ is unstable, and has a growth time-scale of order $k_2^{-1} \sim \frac{2}{3}\Omega^{-1}$.

3.2 The equilibrium dynamo

We now look for a non-trivial equilibrium solution to the dynamo equations, that is, a solution with $\mathrm{d}B_\phi/\mathrm{d}t = \mathrm{d}B_R/\mathrm{d}t = \mathrm{d}B_Z/\mathrm{d}t = 0$. Because the growth terms for B_ϕ and B_R involve the most rapid time-scale in the disc, Ω^{-1}, whereas the loss terms involve somewhat larger time-scales, e.g., τ_{P}, we find that an equilibrium occurs only when the growth of B_R via the Balbus–Hawley instability is inhibited by the presence of a strong B_Z. Thus (equation 2.2.4) we may take $V_{AZ} \simeq \zeta C_s$, where we expect $\sqrt{2}/\pi < \zeta < \sqrt{6}/\pi$. In this regime we also note from equation (2.2.5) that $\lambda_{BH} = 2H$.

Then from the ϕ-equation (2.1.3) we find

$$\frac{3}{2}\Omega B_R = B_\phi \tau_P^{-1}, \tag{3.2.1}$$

where τ_P is given by equation (2.1.4). From the z-equation (2.3.3) we have

$$B_\phi \tau_P^{-1} = B_Z \tau_{rec}^{-1}, \tag{3.2.2}$$

where τ_{rec} is to be determined (equation 2.3.10).

If we assume that $\tau^a_{rec\,R} < \tau^b_{rec\,R}$ then, using (2.3.2), (2.3.5) and (2.3.6), we obtain from (3.2.2):

$$V^3_{A\phi}/C^3_s = 6\sqrt{2}\Gamma \eta^2 \zeta^2/\xi, \tag{3.2.3}$$

and from (3.2.1):

$$V_{AR}/C_s = (\sqrt{2}/3\eta)(V_{A\phi}/C_s)^2, \tag{3.2.4}$$

where we define $V^2_{AR} \equiv B^2_R/4\pi\rho$.

To solve the B_R-equation (2.2.3 and 2.2.4) we write $\zeta = \sqrt{6}(1-\varepsilon)/\pi$, and require $\varepsilon \ll 1$. Then we find to first order in ε:

$$\varepsilon = \frac{24}{\pi^2}\left(1 - \frac{1}{\sqrt{3}}\right)\gamma_{max}^{-2}\xi^{-2}\Gamma^2$$
$$= 2.9 \times 10^{-4}\left(\frac{\Gamma}{0.1}\right)^2\left(\frac{\gamma_{max}}{0.74}\right)^{-2}\left(\frac{\xi}{8}\right)^{-2}, \tag{3.2.5}$$

and conclude that for the appropriate parameters the approximation is valid. To this order we find from (3.2.3) that

$$V_{A\phi}/C_s = 0.8\left(\frac{\Gamma}{0.1}\right)^{1/3}\left(\frac{\eta}{3}\right)^{2/3}\left(\frac{\xi}{8}\right)^{-1/3}, \tag{3.2.6}$$

and from (3.2.4) that

$$\frac{V_{AR}}{C_s} = 0.1\left(\frac{\Gamma}{0.1}\right)^{2/3}\left(\frac{\eta}{3}\right)^{1/3}\left(\frac{\xi}{8}\right)^{-2/3}. \tag{3.2.7}$$

For completeness we add

$$\frac{V_{AZ}}{C_s} = 0.8. \tag{3.2.8}$$

Thus in this regime we estimate using equation (1.2) that

$$\alpha_{SS} = 0.09\left(\frac{\Gamma}{0.1}\right)\left(\frac{\eta}{3}\right)\left(\frac{\xi}{8}\right)^{-1}. \tag{3.2.9}$$

For the values obtained here we find from (2.3.11) that

$$\frac{\tau^a_{rec\,R}}{\tau^b_{rec\,R}} = 0.9\left(\frac{\xi}{8}\right)^{1/6}\left(\frac{\eta}{3}\right)^{-1/3}\left(\frac{\Gamma}{0.1}\right)^{-1/6}, \tag{3.2.10}$$

and thus $\tau^a_{rec\,R} \lesssim \tau^b_{rec\,R}$, justifying the assumption made above.

3.3 Stability of the equilibrium dynamo

We investigate stability for the case in which $\tau^a_{rec\,R}$ is the relevant reconnection time-scale for B_Z. A similar analysis holds when $\tau^b_{rec\,R}$ is relevant. Writing $v_R = V_R/C_s$, $v_\phi = V_\phi/C_s$ and $v_Z = V_Z/C_s$, and defining a dimensionless time $\tau = \Omega t/\eta\sqrt{2}$, the equations, close to the equilibrium solution found in Section 3.2, take the form

$$\frac{dv_R}{d\tau} = \gamma_{max}\,\eta\sqrt{2}v_Z\left[1 - \frac{\left(1 - \frac{\pi v_Z}{\sqrt{2}}\right)^2}{(1-\sqrt{3})^2}\right]^{1/2} - v_R v_\phi, \tag{3.3.1}$$

$$\frac{dv_\phi}{d\tau} = \frac{3\eta}{\sqrt{2}}v_R - v_\phi^2, \tag{3.3.2}$$

and

$$\frac{dv_Z}{d\tau} = v_\phi^2 - \frac{v_Z^2}{v_\phi}\frac{6\sqrt{2}\eta^2\Gamma}{\xi}. \tag{3.3.3}$$

We now let the equilibrium solution of these equations, defined by equations (3.2.6), (3.2.7) and (3.2.8), be $(v_R^{eq}, v_\phi^{eq}, v_Z^{eq})$, and define $w_R = v_R/v_R^{eq}$, etc. The equations can then be written approximately for $w_Z \approx 1$ in the form

$$\frac{dw_R}{d\tau} = \lambda_R[(1-w_Z)^{1/2}w_Z - \varepsilon_0^{1/2}w_R w_\phi], \tag{3.3.4}$$

$$\frac{dw_\phi}{d\tau} = \lambda_\phi[w_r - w_\phi^2], \tag{3.3.5}$$

and

$$\frac{dw_Z}{d\tau} = \lambda_Z[w_\phi^2 - w_Z^2/w_\phi]. \tag{3.3.6}$$

Here,

$$\lambda_R = \left(\frac{2\sqrt{3}}{\sqrt{3}-1}\right)^{1/2}\frac{\gamma_{max}\,\eta\sqrt{2}v_Z^{eq}}{v_R^{eq}}, \tag{3.3.7}$$

$$\lambda_\phi = v_\phi^{eq}, \tag{3.3.8}$$

$$\lambda_Z = (v_\phi^{eq})^2/v_Z^{eq}, \tag{3.3.9}$$

and

$$\varepsilon_0/(v_\phi^{eq}/\lambda_R)^2. \tag{3.3.10}$$

We note that $\varepsilon_0 \ll 1$, and that, to first order in ε_0, the equilibrium solution is

$$w = \left(1 - \frac{4}{3}\varepsilon_0, 1 - \frac{2}{3}\varepsilon_0, 1 - \varepsilon_0\right). \tag{3.3.11}$$

Perturbing about this solution in the form

$w_R = [1 - (4/3)\varepsilon_0](1 + \delta_R)$

etc., we find that δ_R, δ_ϕ and δ_Z satisfy the linear equations

$$\frac{d\delta_R}{d\tau} = -\frac{\lambda_R}{2\sqrt{\varepsilon_0}}\delta_Z, \tag{3.3.12}$$

$$\frac{d\delta_\phi}{d\tau}=\lambda_\phi(\delta_R-2\delta_\phi), \tag{3.3.13}$$

and

$$\frac{d\delta_Z}{d\tau}=\lambda_Z(3\delta_\phi-2\delta_Z). \tag{3.3.14}$$

We look for solutions of these equations of the form $\delta\propto\exp(\sigma\tau)$ and find that σ satisfies the cubic equation

$$\sigma(\sigma+2\lambda_Z)(\sigma+2\lambda_\phi)+\frac{3}{2}\lambda_R\lambda_\phi\lambda_Z/\varepsilon_0^{1/2}=0. \tag{3.3.15}$$

Since λ_ϕ and λ_Z are of order unity and $\lambda_R/\varepsilon_0^{1/2}\sim 10^3\gg 1$, we see that one root is given approximately by $\sigma_1\approx -[(3/2)\lambda_R\lambda_\phi\lambda_Z/\varepsilon_0^{1/2}]^{1/3}$, and note that $|\sigma_1|\gg\lambda_\phi$, λ_Z. We then note that we may write the sum of the other roots as $\sigma_2+\sigma_3=2(\lambda_\phi+\lambda_Z)-\sigma_1>0$. Thus, in general, at least one root is an exponentially growing one. For the particular values derived in Section 3.2 we find the roots to be $\sigma_1=-16.4$, $\sigma_{2,3}=6.6\pm 13.2i$, and conclude that the equilibrium is overstable.

3.4 Solution of the full dynamo equations

We have shown above that the two equilibrium solutions are unstable. So we undertake a numerical integration of the equations in order to determine the general form of the solutions to be expected. We now write

$$\tau_{\mathrm{rec}\,R}^{-1}=(\tau_{\mathrm{rec}\,R}^{\mathrm{a}})^{-1}+(\tau_{\mathrm{rec}\,R}^{\mathrm{b}})^{-1} \tag{3.4.1}$$

in order to obtain a smooth approximation to equation (2.3.10). Using the same notation as in Section 3.3 the full dynamo equations become

$$\frac{dw_R}{d\tau}=\begin{cases}\lambda_R' w_Z-\lambda_\phi w_R w_\phi & 0<w_Z<1/\sqrt{3},\\[4pt] \lambda_R'\left[1-\dfrac{(1-w_Z\sqrt{3})^2}{(1-\sqrt{3})^2}\right]^{1/2} w_Z-\lambda_\phi w_R w_\phi & \dfrac{1}{\sqrt{3}}<w_Z<1,\\[4pt] -\lambda_\phi w_R w_\phi & 1\le w_Z,\end{cases} \tag{3.4.2}$$

$$\frac{dw_\phi}{d\tau}=\lambda_\phi(w_R-w_\phi^2), \tag{3.4.3}$$

and

$$\frac{dw_Z}{d\tau}=\begin{cases}\lambda_Z(w_\phi^2-w_Z/w_\phi\sqrt{3})-\mu_Z w_Z & 0\le w_Z<\dfrac{1}{\sqrt{3}},\\[4pt] \lambda_Z(w_Z^2-w_Z^2/w_\phi)-\mu_Z' w_Z^{3/2} & \dfrac{1}{\sqrt{3}}\le w_Z.\end{cases} \tag{3.4.4}$$

Here we have

$$\mu_Z=\eta(12\Gamma/\pi\xi)^{1/2}, \tag{3.4.5}$$

$$\mu_Z'=\eta(24\sqrt{3}\Gamma/\pi\xi)^{1/2}, \tag{3.4.6}$$

and

$$\lambda_R'=\lambda_R[(\sqrt{3}-1)/2\sqrt{3}]^{1/2}. \tag{3.4.7}$$

We first tested the solutions near $\boldsymbol{w}=0$. This required some care as the equations are stiff in this regime and we were forced to use a backward differentiation scheme (DO2EBF from Numerical Algorithms Limited, Oxford, NAG). We tried initial conditions with $|\boldsymbol{w}|$ as small as 10^{-4} and confirmed that the fields always grow from that value.

For general numerical integration we used a Runge–Kutta Merson method (DO2BHF from NAG). In order to be specific we took the relevant parameters to be as in Section 3.2, that is $\Gamma=0.1$, $\gamma_{\max}=0.74$, $\xi=8$ and $\eta=3$. Whatever the initial conditions we found that the magnetic fields stay finite, and eventually oscillate about the unstable equilibrium position found in Section 3.2. In no cases did the fields grow indefinitely.

In Fig. 1 we show the computed time variations of field strength for the case where we took as initial condition $w_R=w_\phi=w_Z=0.01$. The fields converge to an oscillating state with a period of $\tau=1.69$, or $P=2.4(\eta/3)\,\Omega^{-1}$. It is clear that the cycle is controlled predominantly by the value of w_Z. When $w_Z<1$, the Balbus–Hawley instability leads to a rapid growth in w_R, and slower growth in w_ϕ. This then leads to growth in w_Z. When w_Z then exceeds unity, w_R and w_ϕ decay, followed by w_Z. Thus the instability in the cycle appears driven by the time delay in the feed-back loop. We note that, although for Fig. 1 it is evident that at peak w_R exceeds w_ϕ and w_Z by a factor of 4 or so, it is also true (see Sections 3.2 and 3.3) that B_R remains smaller than B_ϕ at all times. However, both $v_{A\phi}$ and v_{AZ} exceed C_s at parts of the cycle, although not by a great amount. This implies that strictly the contribution of time-dependent magnetic pressure to the disc structure needs to be taken into account. In Fig. 2 we show the corresponding time-dependent behaviour of the viscosity parameter α_{SS}. We find a time-averaged value to be $\langle\alpha_{SS}\rangle\simeq 0.4$, and that the parameter varies in the range 0.1 to 0.7.

We have also followed the time behaviour of the equations with $\tau_{\mathrm{rec}\,R}=\tau_{\mathrm{rec}\,R}^{\mathrm{a}}$ and with $\tau_{\mathrm{rec}\,R}=\tau_{\mathrm{rec}\,R}^{\mathrm{b}}$. The time dependence of the magnetic fields is not strongly altered in either of these cases, and we show the time dependence of α_{SS} for these cases in Fig. 2 also.

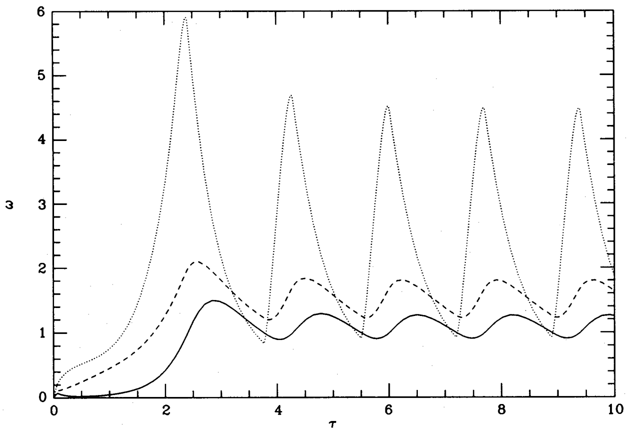

Figure 1. Time dependence of components of the magnetic field through the dynamo cycle. The magnetic field components are normalized to their equilibrium values (Section 3.2) and the dimensionless time, τ, is defined in Section 3.3. The solid line corresponds to the z-component, the dashed line to the ϕ-component and the dotted line to the R-component. The case shown had spatial values of $w_R=w_\phi=w_Z=0.01$, parameters $\Gamma=0.1$, $\xi=8$ and $\eta=3$, and $\tau_{\mathrm{rec}\,R}$ as defined in equation (3.4.1).

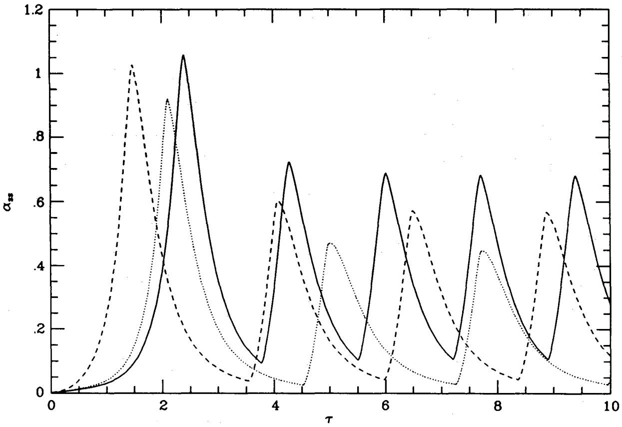

Figure 2. The solid line shows the time-dependent behaviour of the viscosity parameter α_{SS} defined in equation (1.2) corresponding to the magnetic field behaviour shown in Fig. 1. The other curves show the same except that for the dotted line we took $\tau_{\mathrm{rec}\,R}=\tau^{\mathrm{a}}_{\mathrm{rec}\,R}$, and for the dashed line we took $\tau_{\mathrm{rec}\,R}=\tau^{\mathrm{b}}_{\mathrm{rec}\,R}$ (Section 2.3).

4 DISCUSSION

We have proposed a simplified model of a self-sustaining magnetic dynamo which operates in an accretion disc. We have shown that the dynamo is unstable to the presence of a small seed field which it amplifies on the shear time-scale (Section 3.1). The dynamo is also unstable about the only other equilibrium point (Sections 3.2 and 3.3). Because of the cut-off of the Balbus–Hawley instability at large B_Z, it is evident (equation 2.2.4) that the magnetic field cannot grow indefinitely. In two dimensions (that is, if only two components of the magnetic field were involved) these conditions would imply the existence of at least one limit cycle. In this case no such theorem can be simply invoked and so we have integrated the equations numerically for a variety of initial conditions (Section 3.4). We find that the magnetic field (and therefore the value of the viscosity) is never steady but hovers round about the equilibrium values found in Section 3.2.

The dynamo is driven ultimately by shear in the disc and in return provides a self-consistent mechanism for disc viscosity. The approximate strength of the viscosity we obtain (for example the equilibrium value of α_{SS} – equation 3.2.9) agrees to within an order of magnitude with what is known about viscosity strength from observations. We stress again, however, that the precise value we obtain should not be taken too seriously. Of greater relevance are the physical reasons behind the value we obtain. The value of B_Z (equation 3.2.8) comes about because the growth of the dynamo cycle is limited by the cut-off in the Balbus–Hawley instability for large B_Z, which renders the shear less able to deform the vertical field in the horizontal directions. Although B_R and B_ϕ both escape from the disc on the same time-scale $\tau_{\mathrm{P}}\sim 4\,\Omega^{-1}$ (equations 2.1.4 and 3.2.6), B_R is converted into B_ϕ on the shear time-scale $\sim 0.7\Omega^{-1}$. Thus it is not surprising that in the steady-state solution we find $B_\phi \sim 6B_R$. The vertical disc field has a loop structure with typical loop size of order $\Delta R \lesssim H$ and is able to reconnect, and so release itself from the disc, on a time-scale of $\tau_{\mathrm{rec}}\sim 10\Omega^{-1}\gtrsim \tau_{\mathrm{P}}$. Thus in equilibrium we expect $B_\phi \lesssim B_Z$. Given these considerations, the model gives rise to a quasi-steady-state dynamo with corresponding viscosity given by

$$\alpha_{\mathrm{SS}}\sim\left(\frac{V_{\mathrm{AZ}}}{C_{\mathrm{s}}}\right)^2\left(\frac{\tau_{\mathrm{P}}}{\tau_{\mathrm{rec}}}\right)^2\left(\frac{1}{\Omega\tau_{\mathrm{P}}}\right)$$
$$\sim\frac{6}{\pi^2}(\tau_{\mathrm{P}}/\Omega\tau^2_{\mathrm{rec}}). \tag{4.1}$$

For plausible parameters this gives α_{SS} less than, but not greatly less than, unity.

We now consider the energy flow within the dynamo cycle, paying particular attention to where and how the accretion disc energy is deposited and dissipated. In the model presented here, the generation of radial and azimuthal field occurs through a combination of Balbus–Hawley instability and shear, and does so in an efficient manner. Thus at this stage, shear energy has been converted directly into primarily azimuthal field. The azimuthal field is then Parker-unstable to what is essentially an interchange instability. In the equilibrium dynamo we are in the regime $\beta=8\pi\rho C_{\mathrm{s}}^2/(B_R^2+B_\phi^2)\sim 3$, and thus the development of the Parker instability is likely to be fairly gentle with heating of disc material through adiabatic compression as it settles into the disc plane through loss of magnetic support, but no shockheating (Matsumoto et al. 1988). Thus the amount of heating occurring in the disc per unit area due to the loss of azimuthal flux is

$$Q^+_{\mathrm{Parker}}\sim 2H\left(\frac{B_\phi^2}{8\pi}\right)\tau_{\mathrm{P}}^{-1}. \tag{4.2}$$

The rate at which energy is removed from shear (per unit area) is roughly

$$Q^+_{\mathrm{total}}\sim \nu\Sigma(R\Omega')^2$$
$$\sim 6H\left(\frac{B_RB_\phi}{4\pi}\right)\Omega. \tag{4.3}$$

Since, as we remarked above, $B_R\sim(\Omega\tau_{\mathrm{P}})^{-1}B_\phi$, we see that Q^+_{Parker} is comparable to, but a fraction of, Q^+_{total}. A comparable amount of energy to Q^+_{Parker} is converted to vertical magnetic energy. Since the vertical field strength in the disc decays because of reconnection, and since reconnection occurs fastest in regions of low density (high Alfvén speed), we expect most of it to occur predominantly at the upper and lower boundaries of the disc. We conclude that in this model about a half of the energy released from the disc is either dissipated in a small fraction of the disc gas, or lost from the disc entirely in some sort of magnetic wind (cf. Kato & Horiuchi 1986). In either case this model predicts an active chromosphere and/or corona for an accretion disc. We also note that any spectrum calculated for an accretion disc will of necessity have to take into account that a sizeable fraction of the energy released is deposited at low optical depth (cf. Shaviv & Wehrse 1991).

The most detailed estimation of accretion disc viscosities has taken place in the comparison of theory with observations of dwarf nova outbursts (see, for example, Bath & Pringle 1985; Osaki 1990; and references therein). Mantle & Bath (1983) showed that the relationships between decline rate from outburst and orbital period could be explained if the decline rate measured the viscous time-scale and if α_{SS}

was a universal constant of order unity. Since then, however, the limit cycle model of dwarf nova outbursts has become widely accepted and this modifies, but does not substantially change, the Mantle & Bath result. A detailed comparison between theory and observations for the outbursts of two dwarf novae for which ultraviolet spectrophotometry was available through outburst (VW Hyi and CN Ori) was carried out by Pringle, Verbunt & Wade (1987). Although they had problems in fitting some of the details of the light curve, the general outburst properties – e.g., length and size of outburst, length of outburst cycle – could be fitted in the following manner. The VW Hyi outbursts could be fitted as Smak type A events (Smak 1984) in which the outburst starts at the outer disc edge. In agreement with previous work (Meyer & Meyer-Hofmeister 1984; Smak 1984) these were found to require that α_{SS} decrease by about a factor of 10 (here from 0.3 to 0.03) in going from outburst to quiescence. The CN Ori outbursts could be fitted as type B events (Smak 1984) in which the outburst starts at the inner disc edge. These outbursts could be fitted with $\alpha_{SS} \simeq 0.15$ both in outburst and in quiescence. We conclude from the above that the main contradiction between measures of α_{SS} and the value predicted by the dynamo model presented here is that the model predicts a universal value of α_{SS}, independent of other disc properties, whereas to fit some dwarf nova outbursts using the standard model some variation of α_{SS} with, for example, H/R is required (cf. Meyer & Meyer-Hofmeister 1983, 1984).

We note, however, that on decline from outburst, the dwarf nova accretion disc undergoes a thermal jump as a cooling front passes through the disc, running from outside to inside. The time-scale at each point in the disc on which the jump occurs can be as short as the thermal time-scale $t_{th} \sim \alpha^{-1} \Omega^{-1}$ (Pringle 1981) which is less than the time-scale on which the dynamo can be set up. We therefore speculate that in some cases, when the thermal jump occurs particularly rapidly, the magnetic field configuration in the newly cooled disc can end up far from the steady-state dynamo configuration. In particular, if the disc cools rapidly and on a shorter time-scale than the decay of magnetic field, then the disc emerges in a state in which $v_{AZ} \gg C_s$, so that the dynamo cycle ceases, and in which $\beta \ll 1$. There are now two possibilities. The more conservative one is that B_ϕ and B_R now escape from the disc leaving a strong B_Z in place. Decay of B_Z now takes place but at a much slower rate than before because (a) the disc is now denser so that V_{AZ} is decreased and (b) the absence of both Parker and Balbus–Hawley instabilities severely reduces the random motions within the disc. The more radical alternative is that a low-β disc can set up an equilibrium of its own which either maintains itself at low β or at least takes some time to decay from it (cf. Pustilnik & Shvartsman 1974; Shibata, Tajima & Matsumoto 1990; see also Pringle 1989). For example, one can envisage a situation in which all the disc material is in the form of separated, highly magnetized blobs, which therefore collide with each other in an essentially elastic and so dissipationless manner. Such a configuration would give the appearance of a very low-α disc. In this picture the equilibrium dynamo is set up again (so permitting a further outburst) only when sufficient high-β material has been accreted into the disc.

It is evident, therefore, that the behaviour of such a dynamo far from equilibrium is worth investigating more thoroughly than we have space for here. There are two other areas where the dynamo equations as given above may need modification. The first is for accretion discs in which radiation pressure plays a dominant role. The behaviour of the Parker and of the Balbus–Hawley instabilities in such discs has yet to be fully investigated, although various authors have commented upon this regime (Coroniti 1981; Sakimoto & Coroniti 1981; Meyer & Meyer-Hofmeister 1982). To first order, however, it seems to us that radiation pressure changes the above picture very little, except that ρC_s^2 must now be taken to represent the total (and not just gas) pressure. The second is for accretion discs which are not fully ionized, such as might be found in the regions of the discs around pre-main-sequence stars. Here the situation is rather complicated, as one must take into account that the magnetic field is tied strongly only to the ionized component, and that the neutral component acts as a drag on the motion of the ionized component, and thus on the field itself (cf. Königl 1989).

Finally, we note that in a full magnetic dynamo model for an accretion disc, it will be necessary to take into account the spatial behaviour of the magnetic fields, and in particular whether such a dynamo is stationary or propagating in the radial, or azimuthal, direction. What we have presented here is, of necessity, a very simple first step, but we hope that it is at least a step in the right direction.

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