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# Accretion Model for Outbursts of Dwarf Nova

Reiun HŌSHI

*Department of Physics, Rikkyo University, Tokyo 171*

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Structure and stability of an accretion ring around the white dwarf component of a dwarf nova is investigated on the basis of the theory of accretion disks. It is found that the surface density of the accretion ring increases to a certain critical value by continual mass transfer from the red star companion, at which thermal instability develops within the accretion ring and the material of the ring drifts inward to produce a luminous accretion disk around the white dwarf. The theory developed here is in good agreement with observational requirements of dwarf nova.

## § 1. Introduction

Dwarf novae exhibit repeated outbursts of amplitude $2\sim6$ mag with mean interval $10\sim500$ days.[1] Duration time of an outburst is about 1/10 of mean interval. It is estiblished that all dwarf novae are short period close binaries with orbital period ranging from 1 to 20 hours. The close binary systems consist of a white dwarf component (the primary) and a main sequence red star (the secondary) which fills its Roche lobe. Material transferred from the secondary forms an accretion disk around the white dwarf. The masses $0.3\sim1.2M_\odot$[2] were obtained and estimated for the white dwarfs and the main sequence stars from observations of eclipsing dwarf novae.

Rapid coherent oscillations observed in the light curve during outburst suggest that the mass accreting star is a white dwarf. The time scale of oscillations is in the range of $16\sim30$ sec,[2] which is characteristic of the dynamical time of a white dwarf or the orbital period of an accretion disk in the near vicinity of a white dwarf.

The binary light curve of eclipsing dwarf novae exhibits a hump prior to eclipse. The hump is believed to be a hot spot formed at the point where the material transferred from the secondary first collides with the disk. This is a strong evidence of mass transfer from the secondary to the primary.

The most direct evidence that the transferred material forms an accretion disk rather than an amorphous cloud is the behavior of emission lines. According to Smak[3] and Huang[4] the emission line profiles expected from gas disk are double peaked and have extensive wings. In several dwarf novae such profiles have been observed.[2]

It was found from the observation of Z Cha by Warner[5] that at outburst the light curve changed to a deep $V$ shape eclipse, while in the quiescent state this showed a $U$ shape primary eclipse. This important observation, according to Warner, demonstrates that in the quiescent state the disk does not radiate strongly except at inner disk and during outburst the whole disk radiates to a radius of $\sim 10^{10}$ cm.

The presence of an accreting white dwarf in dwarf nova provides a source of energy for outburst. Much work[6] has been done on thermonuclear runaway that could occur eruptively at the base of the accreted material on a white dwarf, when a certain amount of material piles on it. However, it seems that the nuclear burning model is not reconciled with a recently discovered transient $X$-ray source, Aql $X$-1, which closely exhibits $X$-ray outbursts similar to dwarf nova (implication will be discussed in § 7).

As for the mechanism of outburst Bath[8] suggested that the rate of mass transfer by the secondary was modulated by enhanced outflow during outburst due to the instability arising from partially ionized hydrogen zone. Alternatively, Osaki[9] proposed that modulation was caused by some unknown instability within an accretion disk or ring. He demonstrated that amplitude and period-amplitude relation for outburst were naturally explained by this model.

In the present paper we develop a theory of outburst of dwarf novae with the help of the theory of accretion disk which has been extensively investigated to account for observations of $X$-ray star, Cyg $X$-1. The steady state of the accretion ring produced by the material transferred from the secondary is investigated in § 3. It is shown in § 4 that when the surface density of the accretion ring increases to a certain critical value thermal instability develops within the accretion ring and the rate of mass transfer to the primary is greatly increased. Approximate calculation is done for the rate of mass transfer to the primary before and after the instability in § 5. The result of our theory is compared with observations of dwarf novae and the cause of recurrence of outbursts is also briefly discussed in § 6.

## § 2. The model

The structure and stability of the accretion material surrounding the primary are investigated paying attention to the instability which triggers outburst of dwarf nova by means of the following simple model. The matter overflowed from the secondary is considered to form an accretion ring around the primary. Since the ring rotates circularly with Keplerian velocity, velocity gradient is realized between inner and outer part of the ring. Accordingly, angular momentum is transferred from inner matter to outer one and inner part of the ring drifts slowly toward the primary, if turbulent and/or magnetic viscosity operate in the accretion ring. As the result an accretion disk is formed continuously from the ring as illustrated schematically in Fig. 1.

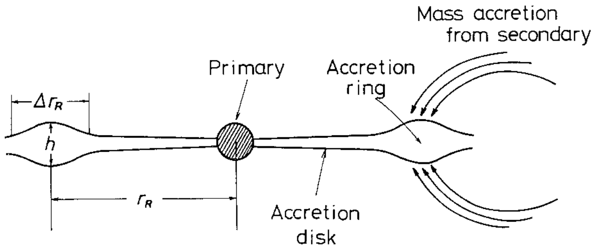


Fig. 1. Schematic sketch of the material around the primary.

As will be discussed in the following sections most of accreted matter transferred from the secondary constitutes the accretion ring. A smaller part of material in the ring drifts to the primary which forms the accretion disk. The radial flux of matter flows in the accretion disk, $\dot{m}_d$, is determined as a function of the temperature and the surface density of the accretion ring (see § 5). If $\dot{m}_d$ is smaller than the rate of supply $\dot{m}_1$ from the secondary to the ring, the surface density of the ring increases with time. When the surface density increases to a certain critical value, instability sets in. Due to the instability the temperature of the ring rises more than ten times that before the instability. At the same time the accretion rate $\dot{m}_d$ also increases abruptly, which is observed as an outburst of a dwarf nova.

From the reason mentioned above we restrict ourselves to the structure and the stability of the accretion ring in the following four sections. In our model the structure of the ring is determined under the following three main assumptions. a) The accretion ring rotates circularly with Keplerian velocity. b) The thickness of the ring is governed by hydrostatic balance between vertical pressure gradient and the vertical component of gravity due to the primary. c) Both viscous heating and the conversion of kinetic energy into heat in collision of gas stream from the secondary with the ring are equated with radiative loss. Assumptions b) and c) hold good approximations, if the growth time of the surface density of the ring is longer than the hydrodynamical and thermal time scales.

## § 3. Steady state structure

We consider a circularly orbiting accretion ring, which is located at a distance $r_R$ from the primary of mass $M$. Let the temperature and the surface density at the center of the ring be $T$ and $\sigma$, and the width and the half thichness of the ring be $\Delta r_R$ and $h$. In what follows we assume $r_R > \Delta r_R \gg h$, and are specially concerned with quantities at the center of the ring.

*a) Hydrostatic equilibrium*

Vertical hydrostatic equilibrium between the pressure gradient and gravity due to the primary is written as

$$\frac{dP}{dh} = -\frac{GM}{r_R^3}\rho h\,, \tag{1}$$

where $P$ is the pressure, $\rho$ the density (the surface density is approximated by $\sigma=2\rho h$), and $G$ the gravitational constant, respectively. If we replace $dP/dh$ by $P/h$ and use the equation of state, $P=(k/\mu m_H)\rho T$, the thickness is given by

$$h \simeq \sqrt{2}\left(\frac{GM}{r_R}\right)^{-1/2}\left(\frac{k}{\mu m_H}\right)^{1/2} T^{1/2}, \tag{2}$$

where $m_H$ is the hydrogen mass, $k$ the Boltzmann constant, and $\mu$ the mean molecular weight, respectively.

*b*) *Energy balance*

In this section we consider the accretion ring to be optically thick. Optically thin state will be discussed separately in § 5. In the optically thick ring the rate of radiative loss from unit area of the surface of the ring is given by

$$F_r = -\frac{ac}{3\kappa\rho}\frac{dT^4}{dh} \simeq \frac{acT^4}{3\kappa\rho h}\,, \tag{3}$$

where $a$ is the radiation density constant, $c$ the light velocity and $\kappa$ the opacity, respectively.

The rate of energy dissipated by viscous stress is given by

$$F_v = \frac{1}{2} r_R W_{r\phi}\left[\frac{d}{dr}\left(\frac{v_\phi}{r}\right)\right]_{r=r_R}\,, \tag{4}$$

where $W_{r\phi}$ is the vertically integrated viscous stress over the thickness of the ring and $v_\phi$ the azimuthal velocity. In the theory of accretion disk,[10] $W_{r\phi}$ is approximated as

$$W_{r\phi} = -2\alpha Ph\,, \tag{5}$$

where $\alpha$ is the ratio of turbulent velocity to sound velocity if the ring is turbulent. If the magnetic viscosity outweighs the turbulent one, $\alpha$ is given by the ratio of magnetic pressure to thermal pressure. At present, however, we have no exact theory of turbulent velocity or magnetic field strength generated by differential rotation in the ring or the accretion disk. In the theory of accretion disk the parameter $\alpha$ is regarded as a constant and is believed to lie between $10^{-3}$ to 1. Recently, Ichimaru[11] developed a theory of magneto-hydrodynamic turbulence appropriate to plasmas in a disk geometry. The formula he derived reduces to Eq. (5) with $\alpha \sim 1/2$,[10),12)] if the radiation pressure is negligible.

From Eqs. (4) and (5), and the assumption a) in § 2, the rate of energy supplied by viscous heating is given by

$$F_v = \frac{3}{2}\alpha P\left(\frac{GM}{r_R}\right)^{1/2} h\,. \tag{6}$$

The rate of the conversion of kinetic energy into heat in collision of accreting gas with the ring is given on an average by

$$F_k = \frac{f}{2}\left(\frac{GM}{r_R}\right)\frac{\dot{m}_1}{2\pi r_R \Delta r_R}, \tag{7}$$

where $f$ denotes the efficiency of the conversion of kinetic energy into heat. One of remarkable fetures in dwarf novae is the hot spot. The luminosity of the hot spot is considered to be maintained by the conversion of kinetic energy in the colliding gas stream. It seems that most of kinetic energy is converted directly into radiation to maintain the luminosity of the hot spot. The efficiency parameter $f$ may be as small as $10^{-2}\sim 10^{-1}$.

*c) Physical state of the accreting ring*

In order to see which heating process plays an important role, ranges of the surface density and temperature which satisfies $F_v > F_k$ are examined first. The condition $F_v > F_k$ is written as

$$\sigma > \frac{f}{3\pi\alpha}\left(\frac{k}{\mu m_H}\right)^{-1}\left(\frac{GM}{r_R}\right)^{1/2}\frac{\dot{m}_1}{\Delta r_R T}. \tag{8}$$

The locus which fulfills $F_v = F_k$ is shown in the surface density versus temperature diagram (Fig. 3). In Fig. 3 typical values of parameters appropriate to dwarf nova are adopted. The radius of the accretion ring and the mass of the primary are fixed to $r_R = 10^{10}$cm and $M = 1M_\odot$. The accretion rate is assumed to be $\dot{m}_1 = 2\times 10^{16}$g/s. The width of the ring and efficiency parameters are tentatively selected as $\Delta r_R = 3\times 10^9$cm and $\alpha = f = 1/10$. In the accretion ring which satisfies the condition (8), thermal steady state can be represented by $F_v = F_r$, from which we have

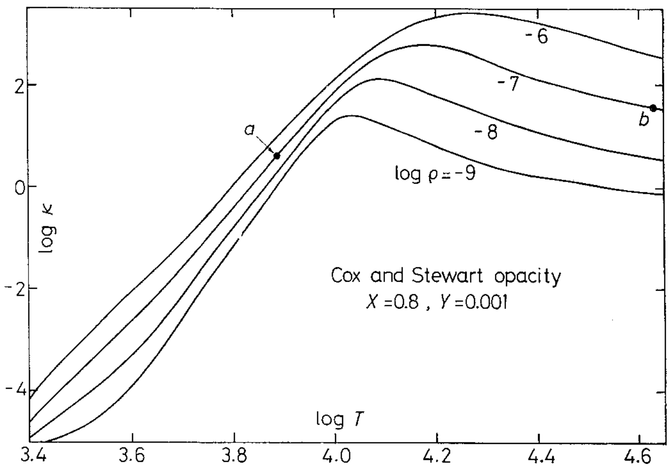


Fig. 2. Cox and Stewart opacity of Pop. II mixture as a function of temperature. Each curve is labeled by the value of density.

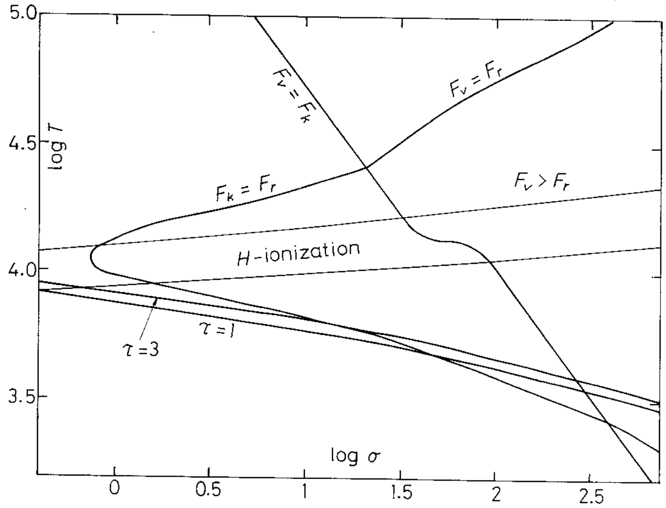


Fig. 3. Steady state temperature of the accretion ring as a function of surface density.

$$T=3\alpha^{1/2}\left(\frac{1}{ac}\right)^{1/2}\left(\frac{k}{\mu m_H}\right)\left(\frac{GM}{r_R}\right)^{-1/4}\kappa^{1/2}\rho\,. \tag{9}$$

The temperature and half thickness are determined from Eqs. (9) and (2) as a function of $\rho$ (or $\sigma$), if the opacity is given as a function of $\rho$ and $T$. We have adopted Cox and Stewart opacity[13] of Population II mixture ($X=0.800$, $Z=1\times10^{-3}$). Figure 2 shows the value of the opacity as a function of temperature for various values of the density.

Temperature determined from Eq. (9) is shown in Fig. 3 by a thick solid curve as a function of surface density. It is noted that Eq. (9) determines two different states, that is, two different steady state temperatures for a given surface density. This property is deeply related to the opacity that has large peak between $10^4$ and $2\times10^{4}$°K due to the ionization of hydrogen. For a given $\rho$ Eq. (9) gives $T\kappa^{-1/2}=\text{const.}$ As is seen from Fig. 2, this relation is satisfied at two different points on the locus of opacity of constant $\rho$ (for example, these points are labeled by $a$ and $b$ on the opacity curve of $\rho=10^{-7}\text{g/cm}^3$).

If the accretion ring satisfies $F_v<F_k$, the temperature is given by

$$T=\left\{\frac{3}{\sqrt{2}}\frac{f}{2\pi}\left(\frac{k}{\mu m_H}\right)^{1/2}\left(\frac{1}{ac}\right)\left(\frac{GM}{r_R}\right)^{1/2}\frac{\dot{m}_1\kappa\rho}{\Delta r_R}\right\}^{2/7}. \tag{10}$$

The locus of $T$ determined from Eq. (10) is also shown in Fig. 3 in the left-hand side of the line $F_v=F_k$. In Fig. 3 we have also shown the loci of the optical depth $\tau(=\kappa\rho h)=1$ and 3, and the hydrogen ionization zone. The upper and lower curves correspond to the ionization degree $x=0.9$ and 0.1.

## § 4. Instability in the accretion ring

*a) Stability of the accretion ring*

The surface density vs temperature diagram (Fig. 3) splits into two domains according to whether $F_v$ (or $F_k$) is larger than $F_r$ or not. In the middle domain enclosed by $F_v$ (and $F_k$) $=F_r$, energy supply overweighs radiative loss so that if the ring falls to this domain the temperature rises until it reaches a steady state on the upper half of the curve $F_v$ (or $F_k$) $=F_r$. In the other domain the reverse is true. It is to be noticed that the lower half of the curve $F_v$ (or $F_k$) $=F_r$ is unstable, since a small perturbation of temperature causes the ring more and more deviate from the curve. For example, if the temperature increases slightly from a steady value, the ring falls in the domain $F_v$ (or $F_k$) $>F_r$. On the other hand, the upper half of the curve is stable against the perturbation of temperature. One can also see from Fig. 3 that no steady state exists if the surface density is smaller than $\sigma_0=0.7\text{g/cm}^2$.

*b) Steady state for the optically thin accretion ring*

To ask for the steady state of the ring with thin surface density we examine the optically thin state of the ring. For the optically thin ring the rate of radiative loss is given by

$$F_r\,(\text{thin}) = \Lambda h\,, \tag{11}$$

where $\Lambda$ is the emission rate per unit volume. As for the most important cooling processes we have considered the excitation of low lying levels of the positive ions, $Si^+$ and $Fe^+$ by electron impact. In the relevant range of temperature of the ring, by assuming the relative abundance

$$n\,(\text{H}) : n\,(\text{Si}) : n\,(\text{Fe}) = 10^6 : 1.2 : 0.7\,,$$

the cooling rate is approximated by[14]

$$\Lambda = 2.28\times10^{-24} g T^{-1/2} n_e n\,(\text{H})\ \text{erg/cm}^3\cdot\text{s}\,, \tag{12}$$

$$g=\frac{n\,(\text{Si}^+)}{n\,(\text{Si})}+0.75\,\frac{n\,(\text{Fe}^+)}{n\,(\text{Fe})}\,,$$

where $n_e$, $n\,(\text{Si})$, $n\,(\text{Fe})$ and $n\,(\text{H})$ are the number density of electron, silicon, iron and hydrogen and $n\,(\text{Si}^+)$ and $n\,(\text{Fe}^+)$ the number density of singly ionized silicon and iron, respectively. Since the density of the ring is so dense even in the optically thin state that the collisional process plays a dominant role to determine the ionization rate. $n_e$, $n\,(\text{Si}^+)$ and $n\,(\text{Fe}^+)$ are calculated by solving Saha equations for Population II mixture.

Steady state is solved by equating $F_v$ (or $F_k$) with $F_r$ (thin). It is to be noticed that other processes we have discarded may contribute with equal importance.

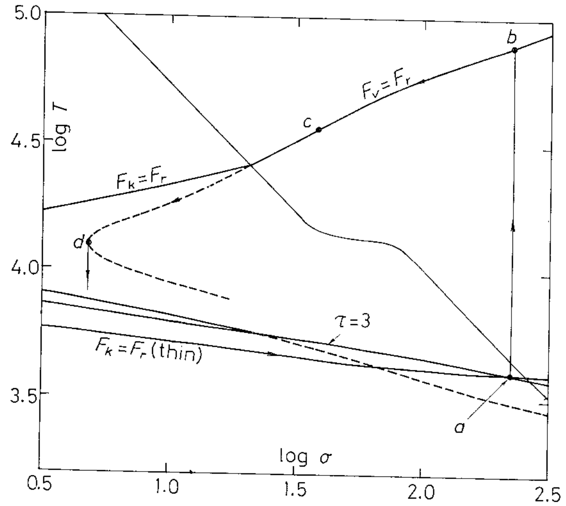


Fig. 4. Time development of the accretion ring in the temperature-surface density diagram. As for details see the text.

However, inclusion of those processes changes the temperature only slightly, since the electron density depends critically on temperature through Saha equations. If we increase the numerical constant in $\Lambda$ by a factor 10, the difference in temperature is only 10%. The temperature of the optically thin ring is shown in Fig. 4 by a solid curve labeled by $F_k = F_r$ (thin). It is also confirmed that the steady state thus determined is stable against the perturbation of temperature.

### c) *Evolution of the accretion ring*

A parameter which characterizes the evolution of the accretion ring is the surface density. If the accretion rate supplied from the secondary surpasses the inward flow $\dot{m}_d$ to the accretion disk, the surface density increases with time. In the relatively early stage the surface density is thinner than $\sigma_0$. Only possible steady state is the optically thin one. The ring evolves in Fig. 4 along the curve $F_k = F_r$ (thin) toward the right as indicated by the arrow.

When the surface density increases, the accretion ring becomes optically thick. We have assumed in Fig. 4 that the ring becomes optically thick when $\tau = 3$ (at the point $a$ in Fig. 4). However, this determination is not accurate. It is appropriate to state that the ring becomes optically thick when it evolves in the vicinity of the point $a$ in Fig. 4. In the optically thick ring steady state must be expressed by $F_k = F_r$. In the vicinity of the point $a$, however, no steady solution exists. The ring becomes unstable since $F_k > F_r$ at the point $a$. The temperature increases until it reaches $b$ on the upper half of the curve $F_v = F_r$, since excess energy raises the temperature of the accretion ring. At the same time, as will be discussed in the next section, the rate of accretion $\dot{m}_d$ to the primary increases suddenly. Thus, sudden accretion of matter to the primary is triggered by this instability, which is observed as the outburst of a dwarf nova.

## § 5. Accretion rate

Since the accretion ring rotates with Keplerian velocity, its inner part rotates faster than outer part. Shear of viscous flow transfers angular momentum outward.

Therefore, matter in inner part of the ring, as loosing angular momentum, drifts inward to form an accretion disk around the primary. In this chapter estimation is made for the accretion rate $\dot{m}_d$ from the ring to the accretion disk. To do this we consider a Keplerian ring with no accretion and an approximate calculation is made for the drift time, that is, the time for the surface density to decrease half of its original value. In terms of the drift time the drift velocity $v_r$ and corresponding accretion rate are calculated. Our approach gives a reasonable approximation, if $\dot{m}_1$ is small as compared with $\dot{m}_d$, but in the contrary case certain errors may be involved in the result. So we should keep in mind that our result is applicable so far as an order of magnitude estimation is concerned.

The time dependent equation of continuity is written as

$$2\pi r\frac{\partial\sigma}{\partial t}=\frac{\partial}{\partial r}\dot{m}_d\,, \tag{13}$$

and the equation of conservation of angular momentum is written as

$$\begin{aligned}&\dot{m}_d\frac{\partial r v_\phi}{\partial r}=\frac{\partial}{\partial r}(2\pi r^2 W),\\ &W=|W_{r\phi}|\,.\end{aligned} \tag{14}$$

From the above equations by eliminating $\dot{m}_d$ we have

$$r\frac{\partial\sigma}{\partial t}=\frac{\partial}{\partial r}\left\{\left[\frac{\partial}{\partial r}(rv_\phi)\right]^{-1}\frac{\partial}{\partial r}(r^2W)\right\}. \tag{15}$$

Here, we define the drift time by $t_d=\sigma(\partial\sigma/\partial t)^{-1}$ which is approximated by

$$\begin{aligned}t_d&=2\sigma r\left\{\frac{\partial}{\partial r}\left[\left(\frac{r}{GM}\right)^{1/2}\frac{\partial}{\partial r}(r^2W)\right]\right\}^{-1}\\ &\simeq\frac{\sigma r}{2}v_\phi W^{-1},\end{aligned} \tag{16}$$

where we have replaced $\partial/\partial r$ by $1/r$. In terms of the sound velocity $c_s(=kT/\mu m_H)^{1/2}$ and Eq. (5), Eq. (16) is written as

$$t_d=\frac{r_R}{2\alpha}\left(\frac{v_\phi}{c_s}\right)\frac{1}{c_s}=0.22\mu\left(\frac{\alpha}{0.1}\right)^{-1}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{r_R}{10^{10}\text{cm}}\right)^{1/2}\left(\frac{T}{10^4{}^\circ\text{K}}\right)^{-1}\text{yr}\,. \tag{17}$$

By defining the inward directed drift velocity,

$$v_r=\frac{r_R}{t_d}=2\alpha\left(\frac{c_s}{v_\phi}\right)c_s=1.4\times10^3\mu^{-1}\left(\frac{\alpha}{0.1}\right)\left(\frac{M}{M_\odot}\right)^{-1/2}\left(\frac{r_R}{10^{10}\text{cm}}\right)^{1/2}\left(\frac{T}{10^4{}^\circ\text{K}}\right)\text{cm/s}\,, \tag{18}$$

the corresponding accretion rate is given by

$$\dot{m}_d = 2\pi r_R \sigma v_r = 4\pi\alpha r_R \sigma c_s \left(\frac{c_s}{v_\phi}\right)$$

$$= 9.0\times 10^{15}\mu^{-1}\left(\frac{\alpha}{0.1}\right)\left(\frac{\sigma}{10^2\mathrm{g/cm^3}}\right)\left(\frac{r_R}{10^{10}\mathrm{cm}}\right)^{3/2}\left(\frac{M}{M_\odot}\right)^{-1/2}\left(\frac{T}{10^{4\circ}\mathrm{K}}\right)\mathrm{g/s}\,. \tag{19}$$

The identical formula is obtained from Eq. (13) by replacing $\partial\sigma/\partial t$ and $\partial\dot{m}_d/\partial r$ by $\sigma/t_d$ and $\dot{m}_d/r$.

In terms of Eq. (19) one can calculate $\dot{m}_d$ at each point on the evolutionary path of the optical thin case in Fig. 4. If $\dot{m}_d$ becomes equal to $\dot{m}_1$ before reaching the point $a$ on the path, a steady state in mass flow is attained, that is, the same amount of mass supplied from the secondary is lost from the ring, and the surface density keeps a constant value which is given by equating Eq. (19) with $\dot{m}_1$. In such case no outburst occurs since the surface density holds a smaller value than $\sigma_a$ (surface density at the point $a$). If $\dot{m}_d$ calculated at the point $a$ in Fig. 4 is smaller than $\dot{m}_1$, the accretion ring can evolve to the unstable region. One of necessary conditions for outburst is that the accretion rate from the secondary to the ring must be greater than $\dot{m}_d$ calculated at the point $a$. By inserting the temperature and the surface density at the point $a$ into Eq. (19), the critical value of $\dot{m}_1$ is given by $(\dot{m}_1)_{cr} = 7\times 10^{15}$g/s for parameters we have adopted.

## § 6. Outburst and recurrence

If $\dot{m}_1 > (\dot{m}_1)_{cr}$, the instability develops in the accretion ring when the surface density becomes as dense as $\sigma_a$. During the instability the temperature rises continuously and reaches $7.8\times 10^{4\circ}$K at the stage when the instability ceases. Sudden enhancement in the accretion rate $\dot{m}_d$ is realized, since Eq. (19) is proportional to temperature. In terms of the temperature and the surface density at the point $b$ in Fig. 4, the enhanced accretion rate is $\dot{m}_d = 3\times 10^{17}$g/s, which is greater by a factor of 40 than that before instability and can account for the burst luminosity $L_{\mathrm{burst}} = (GM/R_p)\dot{m}_d = 3\times 10^{34}$erg/s, on time scale of 6 days from Eq. (17). In deriving the burst luminosity the radius $R_p$ of the primary is taken to be $10^9$cm by assuming the primary as a white dwarf. These numerical values involve some uncertainty, since we have determined $\sigma_a$ supposing the instability begins at $\tau = 3$. If we adopt a different value of $\tau$, somewhat different values of $\dot{m}_d$ and $L_{\mathrm{burst}}$ are obtained.

We have so far adopted $r_R = 10^{10}$cm for the radius of the accretion ring supposing that the separation of two binary components is the order of $10^{10}$cm, which provides the orbital period of $\sim$hours if the mass of each component is $\sim 1M_\odot$. The mean interval of outbursts is estimated as $P_r = 2\pi r\Delta r\sigma_a/\dot{m}_1 \sim$ a few months, if $\dot{m}_1$ is in the range of $10^{16}$g/s. Though our theory involves an unknown parameter $\alpha$ which at present cannot be determined accurately, the above numerical estimates are in good agreement with the basic eruptive behavior of dwarf novae (see §1).

Due to intense mass loss at the stage just after the instability the surface density of the accretion ring decreases with time. This behavior is shown in Fig. 4 where the evolutionary curve in this stage is indicated by a solid curve from the point $b$ toward the left along the curve $F_v = F_r$. The mass loss rate from the ring $\dot{m}_d$ decreases during this stage and eventually becomes comparable with $\dot{m}_1$ (at the point $c$), since it is proportional to both the temperature and the surface density. A steary state is established at the point $c$ and, if this is the case, no recurrence of outburst occurs.

The above scenario is, however, correct so far as the accretion rate $\dot{m}_1$ keeps constant during the decaying phase of outburst. The constancy of $\dot{m}_1$ must be interpreted carefully. Since outburst occurs at a distance as close as $\sim 10^{10}$cm, the secondary is exposed to intense radiative flux which may be larger by several order of magnitude than its own intrinsic luminosity. The rate of mass loss would be increased since the surface layer of the secondary expands to overflow Roche lobe due to absorption of incident radiative flux. At the stage when outburst ceases the surface layer is far from thermal equilibrium. The surface layer contracts away from Roche lobe and then readjust to thermal equilibrium. Overflow eventually ceases until the stellar surface comes to Roche surface.

During the period of no accretion the surface density of the ring decreases along the curve $F_v = F_r$. If no accretion period continues long enough to reduce the surface density below $\sigma_d$, corresponding to the point $d$ in Fig. 4, the accretion ring jumps from the point $d$ to an optically thin state given by $F_v = F_r$ (thin), which is not indicated in Fig. 4, since no possible steady state exists for the optically thick ring. When accretion recurs, the ring evolves along the curve $F_k = F_r$ (thin) toward the point $a$. Thus, the next outburst occurs, when the surface density grows to $\sigma_a$.

In order to confirm the above scenario, however, detailed investigation concerning dynamical behavior of the surface layer of the secondary exposed to intence radiative flux must be performed.

## §7. Discussion

So far we have discussed the physical property of the accretion ring in particular on the basis of the widely accepted formula for viscous stress, so called $\alpha$-model, in which the parameter $\alpha$ is assumed to be constant. Although this formula has been used extensively in the theory of accretion disks, its legitimation is not yet fully confirmed theoretically or experimentally. The nature of the accretion ring discussed in the preceding sections is, however, preserved even if the parameter $\alpha$ is a slowly varying function of $\rho$ and $T$. In order to investigate the structure and stability of the accretion ring more in details and in distinct manner it is necessary to develop theory of viscous stress appropriate to plasma in an accretion disk or ring geometry.

The most ambiguous part of our theory lies in the decaying phase of outburst. Our theory requires that accretion from the secondary is to be stopped or reduced greatly from its steady value. Though a possibility has been discussed in § 6, more effort must be made for the dynamical study of the secondary subject to the critical Roche surface when it is exposed to intense radiative flux.

It is to be noticed that our theory is reconciled with Bath's explanation[8] that enhanced material flows through the accretion disk following an unstable Roche lobe overflow of the secondary. Bath considered that the enhanced flow from the secondary produces directly an accretion disk and the time variation of $\dot{m}_1$ at the unstable surface of the secondary determines the rise and decay profile of outburst. However, our theory indicates that the mass supplied from the secondary is stored in the accretion ring. When the amount of the mass in the ring is increased to $\sim 2\pi r_R \Delta r_R \sigma_a$, the instability begins to develop within it and at the same time the mass in the ring drifts through the accretion disk. Then, the drift time given by Eq. (17) determines the duration time of an outburst.

We conclude this section with a comment on the thermonuclear runaway model. The transient $X$-ray source, Aql $X$-1, exhibits repeated flares (outbursts) in $X$-rays of amplitude $L_{X,\max}/L_{X,\min} > 500$ with mean interval 435 days.[7] A flare shows a relatively rapid flux increase with rise time $\sim 5$ days followed by a slow decay (decay time $\sim 30$ days). A 1.3-day modulation has been reported from 1975 flare data, which is ascribed to the orbital period of an underlying binary system. The close similarity of the flare light curve for Aql $X$-1 with that for optical dwarf novae suggests[15] that Aql $X$-1 is a $X$-ray dwarf nova and flares (outbursts) in $X$-ray and optical dwarf novae are driven by the same underlying mechanism. The presence of a neutron star in place of a white dwarf component will bear the difference of emergent radiation, $X$-rays and optical, and probably of energy fluxes at the peak of a flare, $10^{36} \sim 10^{38}$ erg/s for Aql $X$-1 (distance to it is not known) and $10^{32} \sim 10^{34}$ erg/s for dwarf novae.

We can easily show that flares in Aql $X$-1 is difficult to drive by the thermonuclear runaway model. Let the duration time of a flare and the mean interval be $\tau_d$ and $\tau_i$, and the material accumulated on the accreting neutron star in a time interval $\tau_i$ be $\Delta m$. Let the nuclear energy released in the conversion of 1g of hydrogen into helium and the gravitational energy of accreting material be $E_N (= 6 \times 10^{18}$ erg/g) and $E_g (= GM_n/R_n \simeq 2 \times 10^{20}$ erg/g) where $M_n$ and $R_n$ are the mass and the radius of the neutron star. If eruptive nuclear burning is responsible for flares (here, we suppose that thermonuclear runaway develops at the base of accreted material on the neutron star), the mean luminosity during flare is $L_{X,f} = E_N \Delta m/\tau_d$. On the other hand, in quiescent state $X$-rays are emitted by the conversion of gravitational energy into radiations. Since the accretion rate is given by $\Delta m/\tau_i$, the luminosity is $L_{X,q} = E_g \Delta m/\tau_i$, and the ratio is

$$\frac{L_{X,f}}{L_{X,q}}=\frac{E_N}{E_g}\frac{\tau_i}{\tau_d}\sim 0.3 \quad \text{for} \quad \text{Aql } X-1\,,$$

where we have used $\tau_i/\tau_d \sim 10$. This is to be compared with $L_{X,\max}/L_{X,\min}$ ($\gtrsim 500$ for Aql $X$-1), and indicates that flares are not driven by nuclear burning process but by gravitational energy of accreting material such as the one discussed in the present paper.

If the accretion process discussed in this paper or alternatively quasi-periodic unstable Roche lobe overflow of the secondary really works in $X$-ray dwarf novae, it is natural to invoke the same mechanism for optical dwarf novae.

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