Research Article

Accurate Solution of Navigation Equations in GPS Receivers for Very High Velocities Using Pseudorange Measurements

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GPS is a satellite-based navigation system that is able to determine the exact position of objects on the Earth, sky, or space. By increasing the velocity of a moving object, the accuracy of positioning decreases; meanwhile, the calculation of the exact position in the movement by high velocities like airplane movement or very high velocities like satellite movement is so important. In this paper, seven methods for solving navigation equations in very high velocities using least squares method and its combination with the variance estimation methods for weighting observations based on their qualities are studied. Simulations on different data with different velocities from 100 m/s to 7000 m/s show that proposed method can improve the accuracy of positioning more than 50%.

1. Introduction

Global positioning system (GPS) does the positioning with the help of a group of satellites that each travels in its orbit around the Earth [1]. These satellites by sending L1 and L2 signals with frequency of 1575.42 MHz and 1227.6 MHz, respectively, identify their time and place toward the Earth. A GPS receiver by receiving these signals from at least four satellites organizes the navigation equations and by solving them shows the position of the user [1–4].

At present the only way of positioning a kinematic receiver in real time with high accuracy is by differential mode that is a relative positioning of two receivers [5, 6]. To achieve a higher accuracy, many researches have been done on differential GPS [7–10]. In some other research efforts too, it has been tried to combine GPS with other navigation systems to attain a higher accuracy [11–13]. But none of these methods are suitable for positioning at velocities of up to 7,000 m/s.

Calculation of the exact position in the movement by high velocities like airplane movement or very high velocities like satellite movement using GPS receivers is very important. Jumping at outputs data of GPS receivers' situation and sudden acceleration at very high velocities leads to making too much error in determining the position of receiver. Some

methods like least squares (LS) method that has been presented for solving navigation equations up to now generally have low precision and much error [14]. So we are looking for a method which can solve the navigation equations at very high velocities and significantly decrease positioning errors.

The LS is a standard approach to the approximate solution of overdetermined systems, that is, sets of equations in which there are more equations than unknowns. According to this method, the unknown coefficients are found based on minimizing the sum of squares of the errors made in the results of every single equation.

Suitable weighting GPS data takes place based on qualities of observations. In such a way, we could decrease the effect of noisy data, while using whole data in positioning.

This paper is organized as below. In Section 2, determining the location in GPS system has been studied in brief. In Section 3, we explain how to solve navigation equations by weighted least squares (WLS) method using pseudorange data. Data quality investigating and weighting algorithms are introduced in Sections 4 and 5. In Section 6, the results of applying these algorithms on different data with different velocities from 100 m/s to 7000 m/s are described and compared. Finally, the conclusion is provided.

2. Navigation Equations in GPS

A GPS receiver calculates its position by precisely timing the signals sent by GPS satellites high above the Earth. Each satellite continually transmits messages that include the time the message was transmitted and the satellite position at the time of message transmission.

The receiver uses the messages it receives to determine the transit time of each message and computes the distance to each satellite using the velocity of light. This measured distance is called pseudorange. Because of errors in receiver clock, the pseudorange has many errors. Since this error is equal for all observations, the effect of that can be destroyed. The calculation of distance should be done for at least four satellites. After that with organization of navigation equations as (1), the pseudorange error can be calculated [15]:

$$
\begin{aligned}
\left(X_1 - U_X\right)^2 + \left(Y_1 - U_Y\right)^2 + \left(Z_1 - U_Z\right)^2 &= \left(P_1 - c\Delta T_B\right)^2, \\
\left(X_2 - U_X\right)^2 + \left(Y_2 - U_Y\right)^2 + \left(Z_2 - U_Z\right)^2 &= \left(P_2 - c\Delta T_B\right)^2, \\
\left(X_3 - U_X\right)^2 + \left(Y_3 - U_Y\right)^2 + \left(Z_3 - U_Z\right)^2 &= \left(P_3 - c\Delta T_B\right)^2, \\
\left(X_4 - U_X\right)^2 + \left(Y_4 - U_Y\right)^2 + \left(Z_4 - U_Z\right)^2 &= \left(P_4 - c\Delta T_B\right)^2.
\end{aligned}
\tag{1}
$$

In these equations, (U_X, U_Y, U_Z) are x, y, and z components of receiver (user) position, ΔT_B is the time difference between receiver and satellite, (X_i, Y_i, Z_i) are three components of satellite position, P_i is the exact distance of ith satellite from receiver in ideal conditions, and c shows the light velocity. Calculation of error provides the possibility of position determination by using the pseudorange data.

3. Positioning Using Least Squares Method

The LS is a standard approach to the approximate solution of overdetermined systems. In mathematics, a system of linear equations is considered overdetermined if there are more equations than unknowns [16]. "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation. The intent of error is the difference between the observed data and the amount obtained from the model [17].

Now by using pseudorange data and LS, we determine the position of a moving object. Assuming that the initial coordinates of the receiver are known, (x_0, y_0, z_0), the actual receiver coordinates can be expressed as

$$
x_i = x_0 + \Delta x_i, \qquad y_i = y_0 + \Delta y_i,
$$
$$
z_i = z_0 + \Delta z_i.
\tag{2}
$$

In the above equation, (x_i, y_i, z_i) shows the position of an object at epoch t_i and $(\Delta x_i, \Delta y_i, \Delta z_i)$ explains the unknown amount of movement of the object at epoch t_i. (x_i, y_i, z_i) can be written as in

$$
(x_i, y_i, z_i) = f\left(x_0 + \Delta x_i, y_0 + \Delta y_i, z_0 + \Delta z_i\right).
\tag{3}
$$

Using Taylor's series $f(x_0 + \Delta x_i, y_0 + \Delta y_i, z_0 + \Delta z_i)$ can be expanded as below:

$$
\begin{aligned}
&f\left(x_0 + \Delta x_i, y_0 + \Delta y_i, z_0 + \Delta z_i\right) \\
&= f\left(x_0, y_0, z_0\right) + \frac{\partial f\left(x_0, y_0, z_0\right)}{\partial x_0} \Delta x_i \\
&\quad + \frac{\partial f\left(x_0, y_0, z_0\right)}{\partial y_0} \Delta y_i + \frac{\partial f\left(x_0, y_0, z_0\right)}{\partial z_0} \Delta z_i \\
&\quad + \frac{1}{2!} \frac{\partial^2 f}{\partial x^2} + \cdots .
\end{aligned}
\tag{4}
$$

The other clauses after linear clauses can be ignored. For finding the coefficients of linear clauses of Taylor series, we rewrite each of the equation of (1) as below:

$$
\begin{aligned}
P_i^j &= \rho_i^j + c\Delta T_B \\
&= \sqrt{\left(x^j - x_i\right)^2 + \left(y^j - y_i\right)^2 + \left(z^j - z_i\right)^2} + c\Delta T_B,
\end{aligned}
\tag{5}
$$

where (x^j, y^j, z^j) is the coordination of jth satellite, P_i^j shows the exact distance between jth satellite, and receiver at epoch t_i and ρ_i^j shows the amount of pseudorange between jth satellite and receiver at epoch t_i. Now by using (2) to (5), we can obtain the coefficients of linear clauses in Taylor expansion as follows:

$$
\begin{aligned}
\frac{\partial f\left(x_0, y_0, z_0\right)}{\partial x_0} &= -\frac{x_j - x_0}{\rho_0^j}, \\
\frac{\partial f\left(x_0, y_0, z_0\right)}{\partial y_0} &= -\frac{y_j - y_0}{\rho_0^j}, \\
\frac{\partial f\left(x_0, y_0, z_0\right)}{\partial z_0} &= -\frac{z_j - z_0}{\rho_0^j}.
\end{aligned}
\tag{6}
$$

The equation of pseudorange without considering the unmodelled errors can be rewritten as in

$$
P^j = \rho_0^j - \frac{x_j - x_0}{\rho_0^j} \Delta x_i - \frac{y_j - y_0}{\rho_0^j} \Delta y_i - \frac{z_j - z_0}{\rho_0^j} \Delta z_i.
\tag{7}
$$

Equation (7) can then be simplified as below:

$$
l^j = a_{x_i}^j \Delta x_i + a_{y_i}^j \Delta y_i + a_{z_i}^j \Delta z_i + c\Delta t_i,
\tag{8}
$$

where

$$
\begin{aligned}
l^j &= P^j - \rho_0^j, \qquad a_{x_i}^j = -\frac{x_j - x_0}{\rho_0^j}, \\
a_{y_i}^j &= -\frac{y_j - y_0}{\rho_0^j}, \qquad a_{z_i}^j = -\frac{z_j - z_0}{\rho_0^j}.
\end{aligned}
\tag{9}
$$

If we write (8) for all of the observations (all in-sight satellites), we obtain the developed formula as in

$$
\begin{bmatrix} l^1 \\ l^2 \\ l^3 \\ \vdots \\ l^J \end{bmatrix} = \begin{bmatrix} a_{x_i}^1 & a_{y_i}^1 & a_{z_i}^1 & c \\ a_{x_i}^2 & a_{y_i}^2 & a_{z_i}^2 & c \\ a_{x_i}^3 & a_{y_i}^3 & a_{z_i}^3 & c \\ \vdots & \vdots & \vdots & \vdots \\ a_{x_i}^J & a_{y_i}^J & a_{z_i}^J & c \end{bmatrix} \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ \Delta t_i \end{bmatrix}. \tag{10}
$$

The matrix representation of (10) is stated as in

$$
\vec{L} = A\vec{X}. \tag{11}
$$

Equation (11) is solved using linear LS, as the number of observation equations is greater than number of unknowns. The LS solution for (11) is given by

$$
\Delta x_i = \left(A_i^T A_i \right)^{-1} A_i^T \vec{L}_i, \tag{12}
$$

where $\vec{L}_i$ is a vector of n observations (the number of available satellites should be at least four) at epoch t_i, Δx_i is a vector of four unknowns at epoch t_i, and A_i is the design matrix at epoch t_i with dimension of $n \times 4$. By using this equation the amount of $(\Delta x_i, \Delta y_i, \Delta z_i)$ is resulted. Then from (2) the coordination of the receiver is obtained [14, 18].

4. Observations Weight

Weighting observations is done based on their qualities. The weight of observation is inversely proportional to variance. Because of correlation between data, the covariance matrix is explained as in

$$
\Sigma = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1 x_2} & \cdots & \sigma_{x_1 x_n} \\ \sigma_{x_2 x_1} & \sigma_{x_2}^2 & \cdots & \sigma_{x_2 x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{x_n x_1} & \sigma_{x_n x_2} & \cdots & \sigma_{x_n}^2 \end{bmatrix}, \tag{13}
$$

where $\sigma_{x_i}^2$ is the variance of ith observation and $\sigma_{x_i x_j}$ is the variance between the jth and ith observation [19]. If there is no correlation between data, all of the off-diagonal terms are zero and covariance matrix will be diagonal. In this case, the cofactor matrix of observations can be expressed as follows:

$$
Q_{P_i} = \frac{1}{\sigma_0^2} \Sigma_{P_i} = \begin{bmatrix} \dfrac{\sigma_1^2}{\sigma_0^2} & 0 & \cdots & 0 \\ 0 & \dfrac{\sigma_2^2}{\sigma_0^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \dfrac{\sigma_n^2}{\sigma_0^2} \end{bmatrix}, \tag{14}
$$

where Q_{P_i} is the cofactor matrix of pseudorange observations at epoch t_i and σ_0^2 is the reference variance. So the weight matrix of the observations can be calculated as

$$
W_i = Q_{P_i}^{-1} = \sigma_0^2 \Sigma_{P_i}^{-1}. \tag{15}
$$

In case that the cofactor matrix is diagonal, its reverse would be diagonal. Therefore, the weight matrix is also diagonal. As a result, it could be summarized as below:

$$
w_i(n) = \frac{\sigma_0^2}{\sigma_i^2(n)}. \tag{16}
$$

Now the equation of LS in (12) is changed to

$$
\begin{aligned}
\Delta x_i &= \left(A_i^T W_i A_i \right)^{-1} A_i^T W_i \mathrm{Pr}_i \\
&= \left(A_i^T Q_{P_i}^{-1} A_i \right)^{-1} A_i^T Q_{P_i}^{-1} \mathrm{Pr}_i.
\end{aligned} \tag{17}
$$

5. Variance Estimation Methods: Calculation of Weight Matrix

The functional model for processing GPS phase observations is well understood, but the establishment of an appropriate stochastic model is still an issue. This is mainly due to physical correlations, random noise observation, and unmodelled systematic effects, for example, signal distortion effects [20, 21]. Since the quality of observation has the reverse equation with the variance, so the reverse of variance matrix can be used as weight matrix. Therefore, by assigning less weight to data with more noise, while using noisy data in determining position it can reduce the effect of them and achieve more precision. In this paper, we use two factors for decision about quality (elevation angle and signal-to-noise ratio (C/N_0)) and study different ways for estimating variance of observation based on these quality factors.

5.1. Estimating Variance of Observation with Sigma exp. In this method, variance of each observation is estimated as an exponential function of elevation angle based on

$$
\sigma_i^2(n) = a + \exp\left(\frac{-El_i(n)}{\theta_0} \right), \tag{18}
$$

where $\sigma_i(n)$ is the variance of nth observation at epoch t_i and $El_i(n)$ is the elevation angle of nth satellite at epoch t_i. a and θ_0 are also constant parameters which are obtained experimentally [18].

5.2. Estimating Variance of Observation with Sinusoidal Function. The most widely used model is sinusoidal function, which can be expressed by (19) or (20):

$$
\sigma_i^2(n) = \frac{1}{\sin^2\left(El_i(n) \right)}, \tag{19}
$$

$$
\sigma_i^2(n) = a^2 + \frac{b^2}{\sin^2\left(El_i(n) \right)}, \tag{20}
$$

where a and b are the experimental values [22].

5.3. Estimating Variance of Observation with Tangential Function. In this method, variance is obtained by using tan of elevation angle as bellow:

$$
\sigma_i^2(n) = \frac{1}{\tan^2\left(El_i(n) - \theta_0 \right)}, \tag{21}
$$

where θ_0 is a slight adjustment to avoid the singularity $\tan(90) = 0$ [23].

5.4. Estimating Variance of Observation with Sigma ε.

This model uses the measured signal-to-noise ratios for determination of the variance of observations:

$$\sigma_i^2(n) = a \exp\left(\frac{C/N_0}{10}\right), \tag{22}$$

where C/N_0 is a measured carrier-to-noise power-density ratio expressed as a ratio (equal to $10^{(C/N_0)/10}$ for C/N_0 expressed in dB-Hz) and a is a model parameter that is equal to $0.244 \text{ m}^2 \text{ Hz}$ [24].

5.5. Estimating Variance of Observation with Sigma Δ.

Let us introduce Δ as the difference between the measured C/N_0 value and the template value at the appropriate satellite elevation. To investigate the distinction between a clear signal and a signal affected by diffraction effects, Brunner et al. found out that such signals generally have lower C/N_0 values than clear signals at the same elevation [25]. Thus, they constructed C/N_0 versus elevation plots, termed template values. Now Δ can be used as an indicator for diffraction noise, which yields a larger variance than $\sigma_i(n)$ from (22). Therefore, we can estimate the variance using

$$\sigma_i^2(n) = a \exp\left(\frac{C/N_0 - \alpha \cdot \Delta}{10}\right), \tag{23}$$

where Δ is a deviation from C/N_0 template (dB-Hz), α is empirical constant factor (equal to 2.0), and a is a model parameter that is equal to $0.244 \text{ m}^2 \text{ Hz}$ [25]. This model uses the C/N_0 value and the carrier tracking loop bandwidth to estimate variance by

$$\sigma_i^2(n) = \sqrt{\frac{B}{C/N_0}} \frac{\lambda}{2\pi}, \tag{24}$$

where B is the carrier tracking loop bandwidth (Hz) and λ is the wavelength of the carrier (m) [26].

6. Simulations Results

Rohde & Schwarz GNSS simulator is used to generate raw GPS data such as pseudorange, integrated carrier phase, Doppler shift, and satellite ephemeris. The GNSS simulator in the R&S SMBV100A includes the ability to simulate realistic transmission conditions through the use of multipath signal generation and modeling of various atmospheric effects. In the motion of an aircraft containing GNSS receiver, simulator is used to model effects that impact GNSS receiver performance, such as atmospheric conditions, multipath reflections, antenna characteristics, and interference signals.

The simulations are done for six different scenarios in Matlab. The first scenario corresponds to the movement of a mobile object in the air with velocity of 90 m/s. The second to fourth scenarios correspond to the moving object with circular motions along the z and y-axis with different
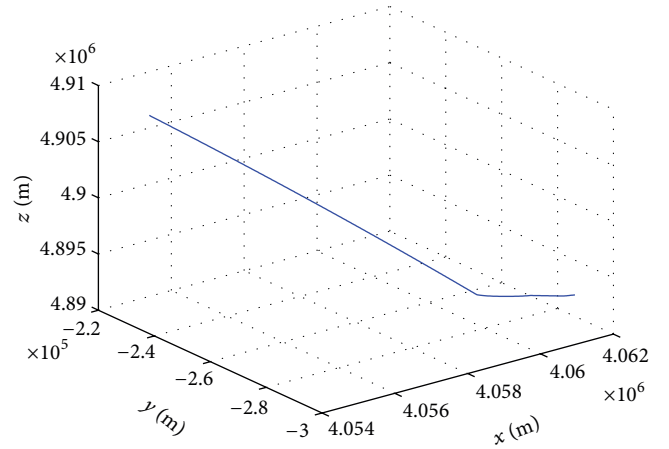

FIGURE 1: Move in the air with velocity of 90 m/s.
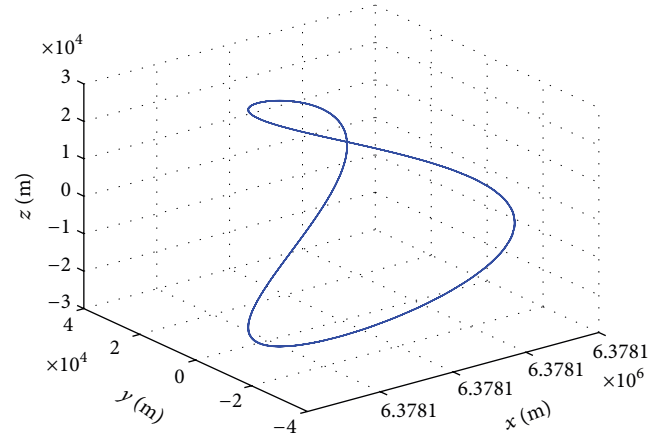

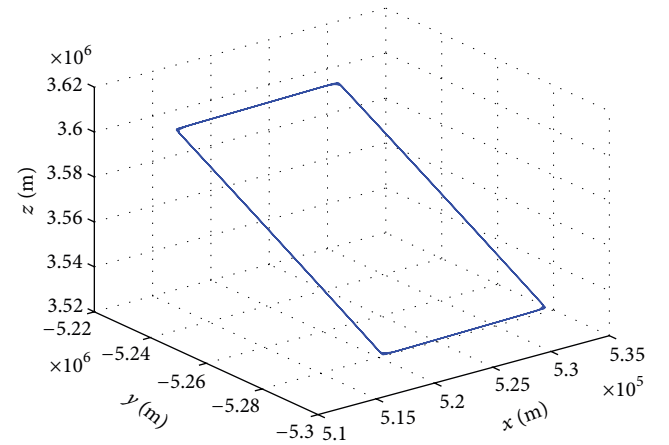FIGURE 2: Circular motion with velocity of 3500 m/s.

FIGURE 3: Rectangular motion with velocity of 3200 m/s.

velocities (100 m/s, 500 m/s, and 3500 m/s). The fifth scenario corresponds to a rectangular motion with maximum velocity of 3200 m/s and the last scenario corresponds to a movement in space with velocity of 7300 m/s.

The movement paths of these objects are shown in Figures 1, 2, 3, and 4. Meanwhile, the results of positioning based on WLS method that is combined with seven methods of variance estimation are presented in Table 1. Figures 5, 6, 7,

TABLE 1: The RMS error [m] of positioning for eight methods.

Scenarios		Methods						
	WLS	WLS + sigma Δ	WLS + sigma ε	WLS + sigma exp	WLS + sigma sin 1	WLS + sigma sin 2	WLS + sigma tan	WLS + sigma SNR
Air	14.777	8.453	14.259	8.420	11.039	10.688	10.438	12.867
Circular 1	12.293	8.399	11.788	6.720	9.965	8.928	7.616	9.974
Circular 2	12.288	8.399	11.786	6.720	9.964	8.927	7.616	9.973
Circular 3	12.080	8.397	11.792	6.720	9.966	8.928	7.616	9.976
Rectangular	25.267	10.053	24.734	9.273	16.564	14.976	13.134	21.834
Space	8.798	3.528	9.144	**2.868**	4.701	4.231	3.633	7.260
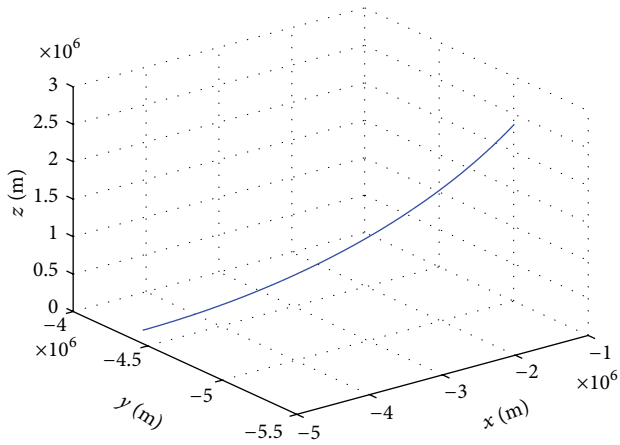
FIGURE 4: Move in space with velocity of 7200 m/s.
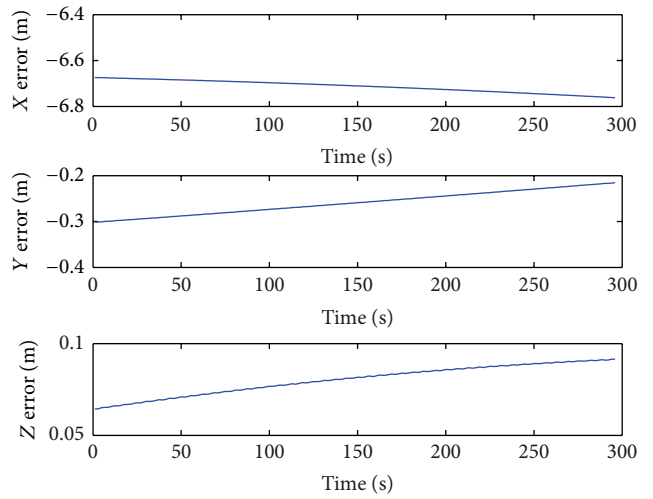
FIGURE 6: Positioning error for circular motion with velocity of 100 m/s.
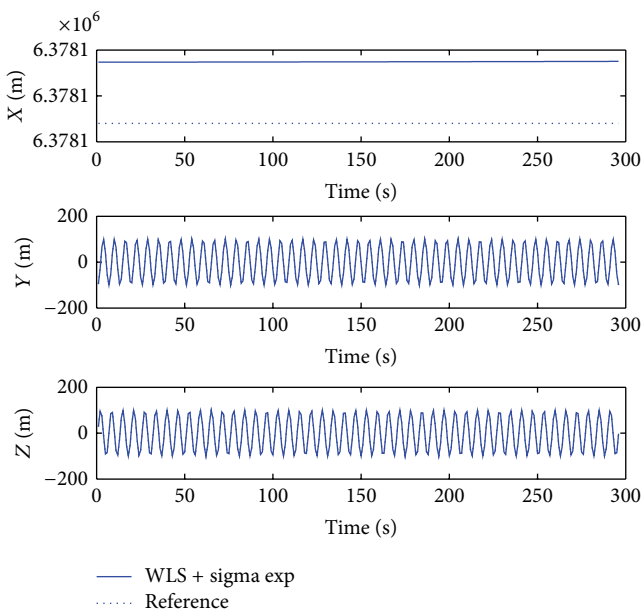
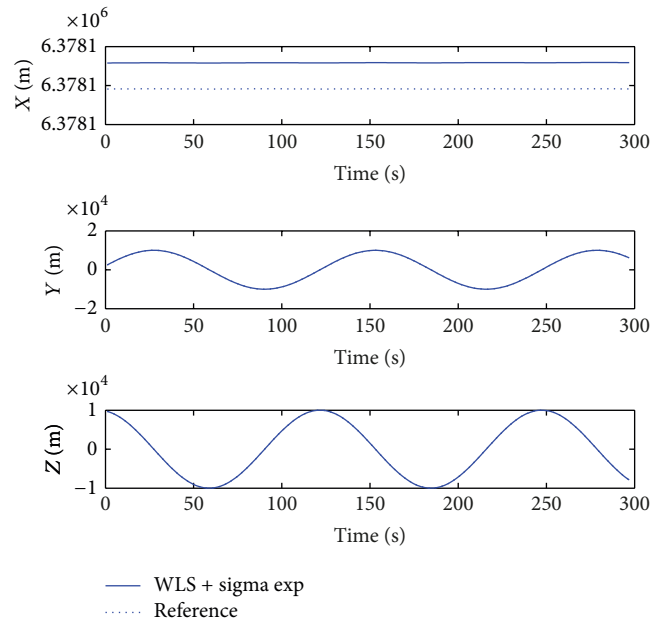FIGURE 5: Positioning result for circular motion with velocity of 100 m/s.

FIGURE 7: Positioning result for circular motion with velocity of 500 m/s.
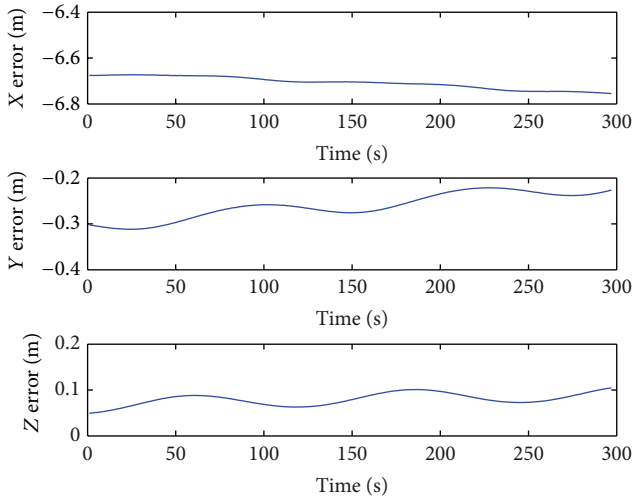
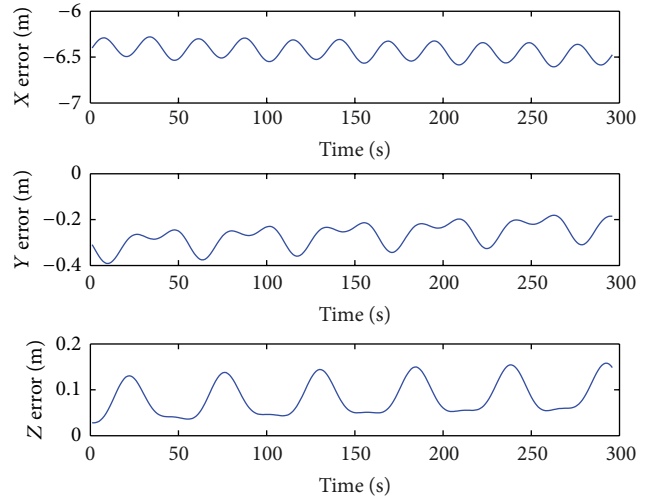FIGURE 8: Positioning error for circular motion with velocity of 500 m/s.

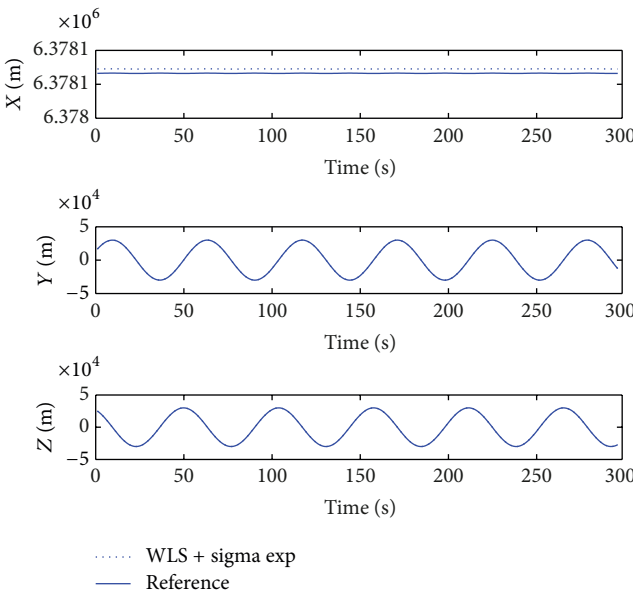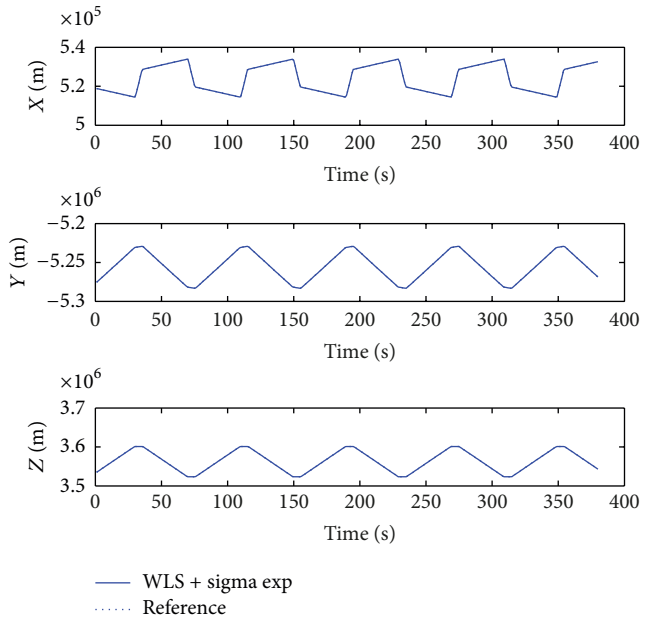FIGURE 10: Positioning error for circular motion with velocity of 3500 m/s.

.... WLS + sigma exp
——— Reference

FIGURE 9: Positioning result for circular motion with velocity of 3500 m/s.

——— WLS + sigma exp
.... Reference

FIGURE 11: Positioning result for rectangular motion with velocity of 3200 m/s.

7. Conclusion

Vehicle positioning in high velocities is very important. In this paper, seven methods are discussed for determining position using the WLS method combined with methods of variance estimating for weighting the observation based on their qualities in conditions that the velocity of movement is very high. The results of simulation show that the WLS method combined with the sigma exp variance estimation method has the best performance and causes a great decrease of errors than the other methods. If the weight matrix for data that there are correlations between them is computed, it is possible to achieve high precision. Simulations on

8, 9, 10, 11, 12, 13, and 14 show the result of positioning and positioning errors for different scenarios.

It is observed that, among all methods, the WLS method combined with the method of sigma exp has the best performance and cause a great decrease of error than the LS method. The considerable matter about all these methods is that for starting algorithm there is no need to know that the primary point and error of methods are independent of primary point.
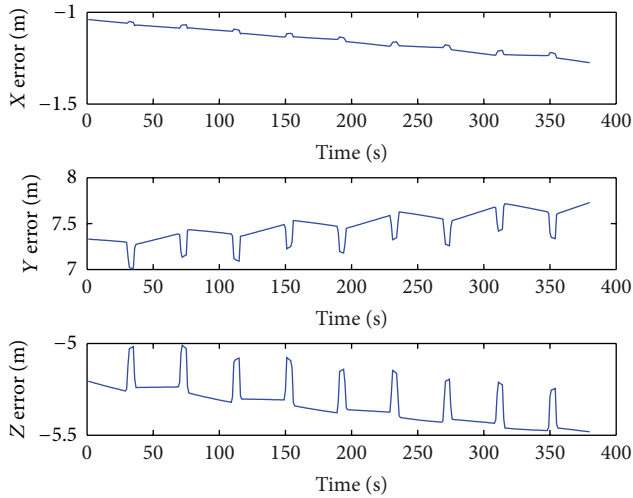
FIGURE 12: Positioning error for rectangular motion with velocity of 3200 m/s.
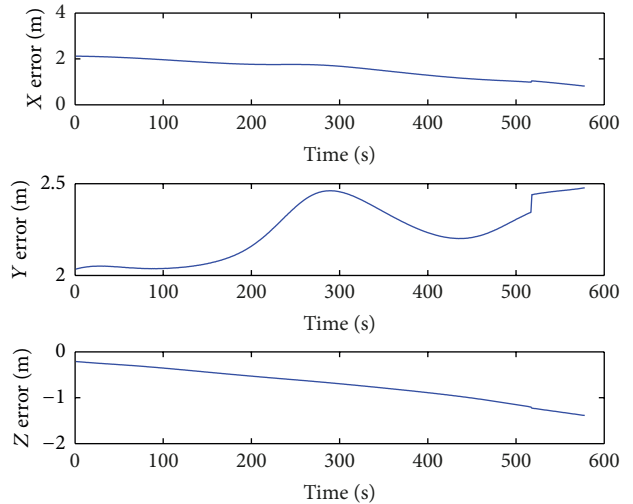

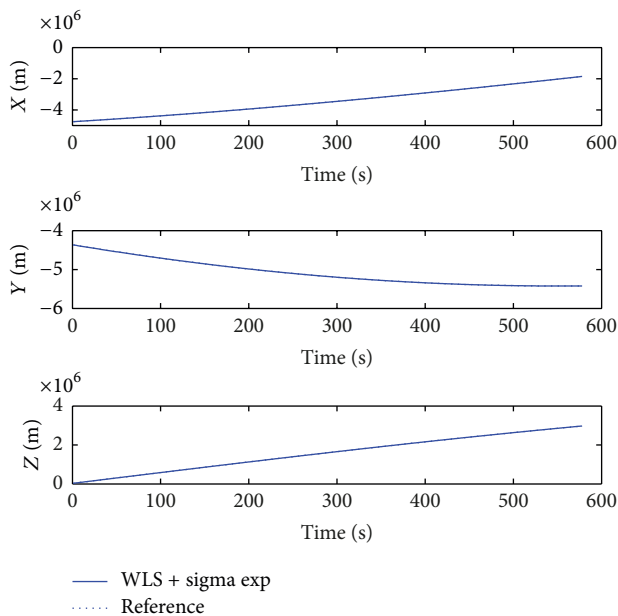FIGURE 14: Positioning error for move in space with velocity of 7300 m/s.

FIGURE 13: Positioning result for move in space with velocity of 7300 m/s.

different scenarios data with different velocities from 100 m/s to 7000 m/s emphasized that proposed method significantly improved the accuracy of the resulting positions more than 50%.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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