Achieving the Capacity of any DMC using only Polar Codes

David Sutter, Joseph M. Renes, Frédéric Dupuis, Renato Renner

suttedav@student.ethz.ch, {renes,dupuis,renner}@phys.ethz.ch

ITW, September 3, 2012, Lausanne

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Background

- Polar codes achieve the *symmetric* capacity of any DMC W having $O(N \log N)$ encoding and decoding complexity, where N denotes the blocklength [Arıkan'09].

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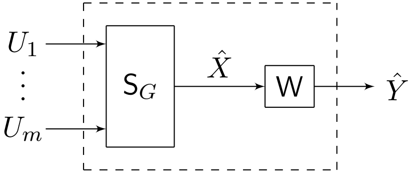
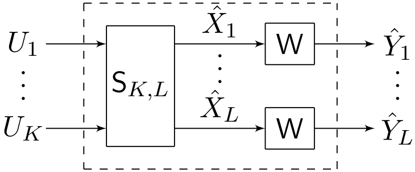
- Using Gallager's shaper idea, polar codes can be used to achieve the capacity of any DMC at the cost of a slightly higher encoding and decoding complexity [Sasoglu,Telatar&Arıkan'09].



- A q-ary input U^m (with $q = 2^m$ for $m \in \mathbb{Z}^+$) whose elements are i.i.d. Bernoulli $\left(\frac{1}{2}\right)$ distributed is shaped into a rational approximation to $X \sim$ Bernoulli (p), i.e. $\hat{X} \sim$ Bernoulli (k/q) where $k \in \mathbb{Z}^+$ and $k/q \approx p$.

Results

- We construct a different shaper $\mathsf{S}_{K,L}$ using polarization-based randomness extraction to simulate the capacity-achieving input *vector* X^L from a uniformly-random input U^K.



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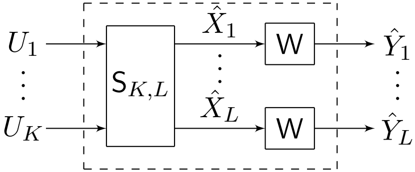

- We use source polarization to obtain an efficient encoder and decoder.

	Gallager's scheme	New scheme
Rate	$C - O\left(\frac{1}{q}\log q\right)$	$C - \frac{o(N)}{N}$
Complexity	$O\left(q \log q \cdot N \log N\right)$	$O\left(N \log N\right)$
Error probability	$O\left(\log q \cdot 2^{-N^{\beta}}\right)$	$O\left(\sqrt{N} 2^{-\frac{1}{2}N^{\frac{\beta}{2}}}\right)$

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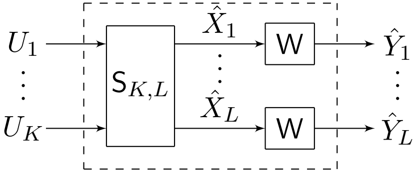

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- An application to the AWGN channel is discussed.