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Acoustic daylight imaging via spectral factorization: Helioseismology and reservoir monitoring

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ABSTRACT

The acoustic time history of the sun's surface is a stochastic (t, x, y)-cube of information. Helioseismologists cross-correlate these noise traces to produce impulse response seismograms, providing the proof of concept for a long-standing geophysical conjecture. We pack the (x, y)-mesh of time series into a single super-long one-dimensional time series. We apply Kolmogoroff spectral factorization to the super-trace, unpack, and find the multidimensional acoustic impulse response of the sun. State-of-the-art seismic exploration recording equipment offers tens of thousands of channels, and permanent recording installations are becoming economically realistic. Helioseismology, therefore, provides a conceptual prototype for using natural noises for continuous reservoir monitoring.

INTRODUCTION

The earth and the sun both have a noisy surface. This acoustic noise generates sonic waves that dive into the sphere and emerge at all distances. A process that we call "acoustic daylight imaging[2]" enables us to form various pictures of the interior.

Acoustic daylight imaging: previous work

At the Stanford Exploration Project, we began our interest in acoustic daylight imaging many years ago when theory (Claerbout, 1976) showed that under certain idealized conditions, the autocorrelation of an earthquake seismogram should mimic an echo sounding like those made with explosives for petroleum prospecting.

Early attempts to verify this theory in practice quickly failed and we came to realize that the essential physical feature is real-world three dimensionality while both our data and

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[2] The term "acoustic daylight imaging" is known in oceanography where measurements are not made at the free surface. This difference leads ocean acoustics away from seismology.

our mathematical theory were merely one dimensional. We didn't have a three-dimensional theory, but we did have a conjecture:

> By cross-correlating noise traces recorded at two locations on the surface, we can construct the wavefield that would be recorded at one of the locations if there was a source at the other.

Cole (1995, 1988) initially tried to verify this conjecture on a passive 3-D survey recorded with an array of 4056 geophones covering more than a half kilometer square on the Stanford campus. Unfortunately, however, only twenty minutes of passive seismic data was recorded, and beam steering showed ambient noise was predominately incident from only one direction. His cross-correlation results were not conclusive. The proximity of the San Andreas fault makes the Stanford area difficult to analyze, and we were also troubled by poor coupling between the geophones and the dry summer soil.

Nobody expected the geophones to record plunging waves from great distances but that is exactly what happened. We saw seismic waves apparently coming from the American Midwest. Earthquake seismologists were surprised to learn that we could receive seismic waves from so far at such high frequency (10 Hz) because with their small numbers of seismometers they cannot. Unfortunately, we were not able to observe what we sought, the much smaller scale reflected waves that we would crosscorrelate within our array. Such waves would illuminate the area within drilling distance so proof of concept would interest our sponsors.

Modeling studies (Rickett and Claerbout, 1996) showed that longer time-series, and a white spatial distribution of random noise events would be necessary for the conjecture to work in practice.

Helioseismology

In 1995, solar physicists developed a new instrument for studying the sun. The Michelson Doppler Imager (MDI) instrument measures the Doppler shift of solar absorption lines formed in the lower part of the solar atmosphere. This provides line-of-sight velocity measurements for points on the sun's surface that can be used to study solar oscillations. This amounts to having a million (1024 × 1024) seismometers uniformly distributed on the surface of the sun. Furthermore, the solar seismologists are able to zoom their lens to reposition their million virtual seismometers to give them a magnified view anywhere they choose.

Most helioseismology (e.g. Kosovichev, 1999) has been done in the frequency domain with spherical harmonic functions. Spherical harmonics provide an excellent tool for studying the whole sun at one time. However, small-scale events are only described by harmonic modes of very high-order. Spherical harmonic functions are therefore inefficient for studying small, localized area's of the sun's surface.

Solar seismologists (Duvall et al., 1993) had also come up with the idea of creating 'time-distance' seismograms by crosscorrelating surface noise observations to mimic impulsive sources on the solar surface. They were successful with real data in three dimensions on the sun, before we could do it on earth.

Convective flow in the outer third of the sun leads to a breakdown in reciprocity of time-distance seismograms derived by cross-correlation. Helioseismologists have used this breakdown in reciprocity to estimate the three-dimensional flow velocity structure in the outer third of the sun.

Permanent recording installations

Ebrom et al. (1998) discuss the economic justification for installing permanent recording installations in marine environments over producing hydrocarbon fields. The incremental cost of additional 3-D seismic surveys is much less for permanent monitoring systems than for conventional repeated 3-D. The quality of ocean bottom cable data has also increased dramatically in recent years, and so permanent monitoring systems also provide a higher level of repeatability.

For these reasons, permanent installations are already installed in many basins worldwide. In the future, explorationists may take advantage of continual recordings and acoustic daylight imaging to continually monitor reservoir production.

RAW DATA

The Solar Oscillations Investigation project (http://soi.stanford.edu) provided us with a cube of data recorded by their MDI instrument. They transformed the coordinate system to Cartesian coordinates by projecting high-resolution data from an area approximately 18° square onto a tangent plane.

Figure 1 shows a time slice through a cube of raw velocity data from the MDI. The object in the center of the panel is a sun-spot. Figure 2 shows a time-distance section through the same cube. The sampling spacing is 1 minute on the time-axis and approximately 68 km on the two spatial axes.

Events on Figure 2 fall into two distinct spectral windows. The low temporal frequency events ($<$1.25 mHz) are related to solar convection (super-granulation), while the higher frequency events are related to acoustic wave propagation. We were interested in studying acoustic wave phenomena; so as a preprocessing step, we removed the lower frequency spectral window by applying a $\frac{1-Z}{1-\rho Z}$ low-cut filter to the data.

3-D KOLMOGOROFF SPECTRAL FACTORIZATION

A simple linear model for the observed solar oscillations consists of a convolution of a source function with the impulse response of the sun's surface. The source function is stochastic in nature, and may be characterized as being spectrally white in time and space

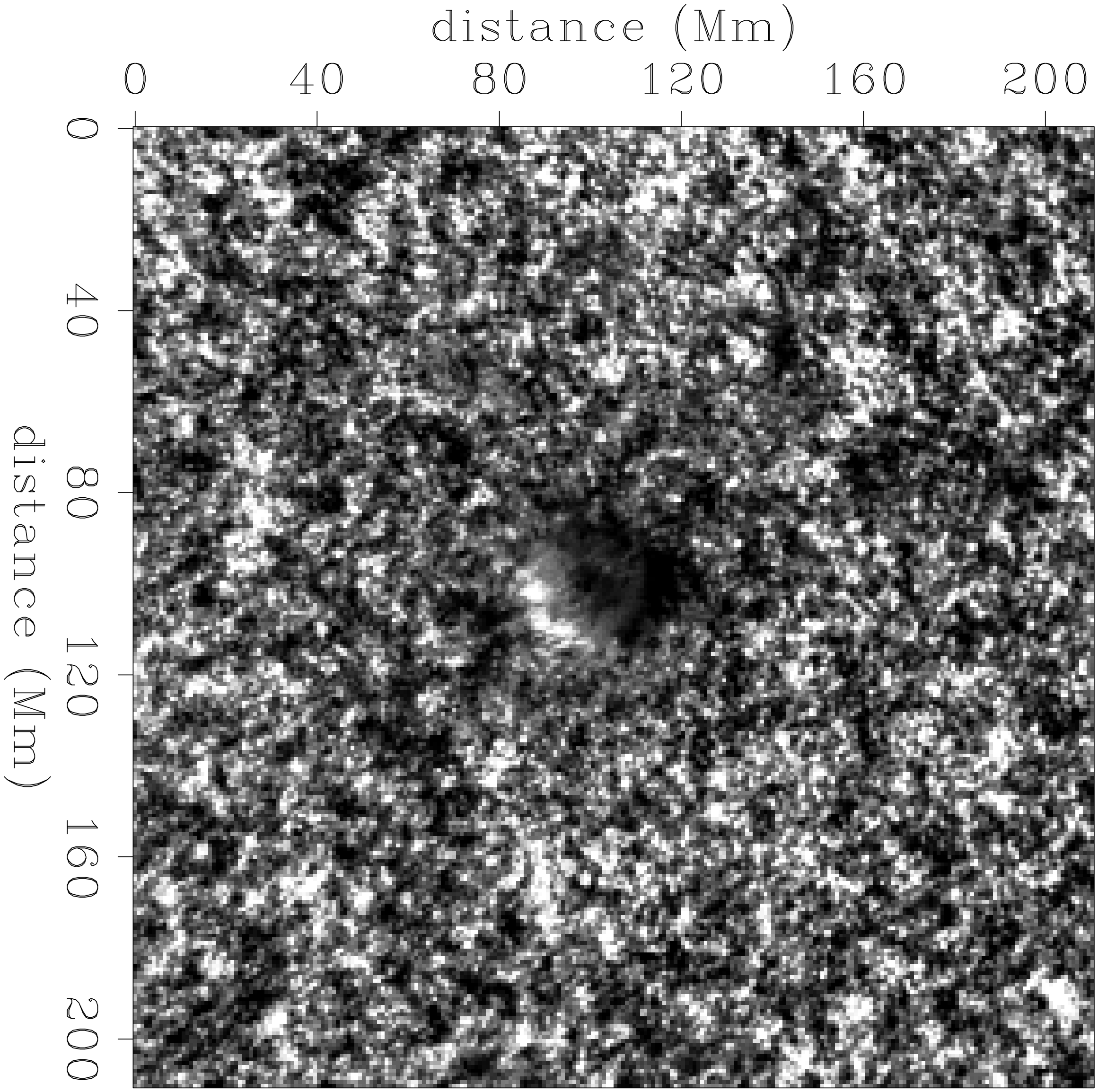

Figure 1: Time-slice through the raw data. The object in the center is a sun-spot. Distances are in Megameters (thousands of kilometers), and time units are kiloseconds. james1-timedata [CR]

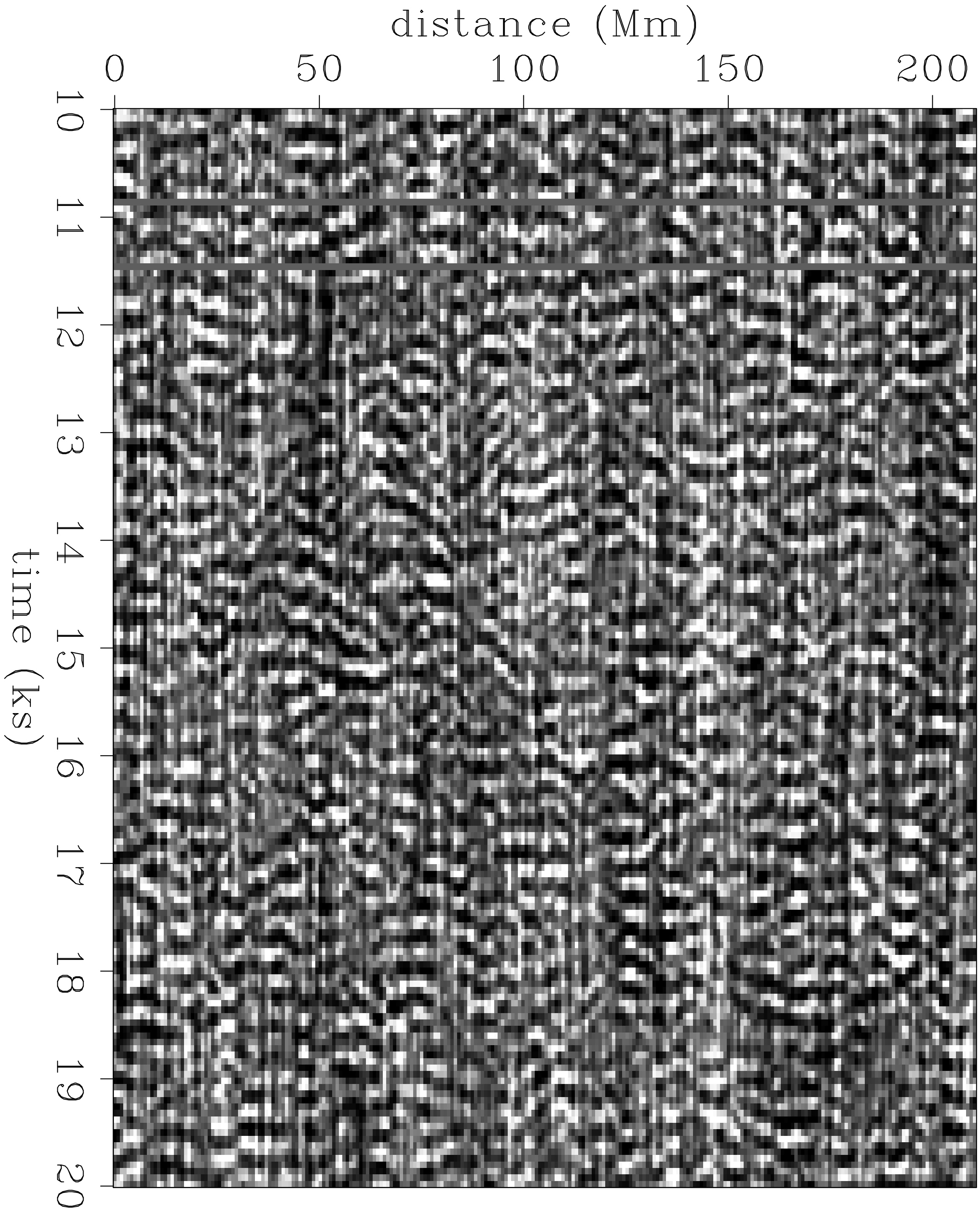

Figure 2: Time-space section through a subset of the raw data. james1-txdata [CR]

with random phase. The impulse response contains the spectral color, and is commonly a minimum-phase function.

If this model holds true, then estimating the source function reduces to estimating a minimum-phase function with the same (w, k_x, k_y) spectrum as the original data: multi-dimensional spectral factorization.

Helical boundary conditions (Claerbout, 1998) provide a framework for converting a multi-dimensional problem into an equivalent problem in only one dimension, and allow us to solve the three-dimensional spectral factorization problem efficiently.

We perform the spectral factorization rapidly in the frequency domain in three steps. Firstly, we transform the multi-dimensional signal to an equivalent one-dimensional signal using helical boundary conditions. Secondly, we perform one-dimensional spectral factorization with Kolmogoroff's (1939) algorithm. Finally, we remap the impulse response back to three-dimensional space. We reduce wrap-around effects by padding the spatial axes.

Figure 3 shows the impulse response derived from Kolmogoroff spectral factorization as a function of radial distance from the impulse. It looks very similar to the cross-correlation time-distance seismogram shown in Figure 4, and those displayed by (Kosovichev, 1999). However, for the dataset described above, this operation was approximately twenty times faster than cross-correlating every trace in either $(w, \mathbf{x})$ or $(t, \mathbf{x})$. The speed-up becomes apparent when you consider that cross-correlating every trace with every other trace requires $O(N_x^2\ N_y^2\ N_t)$ operations, whereas one-dimensional spectral factorization requires only $O(N \log N)$ operations where $N = N_x N_y N_t$.

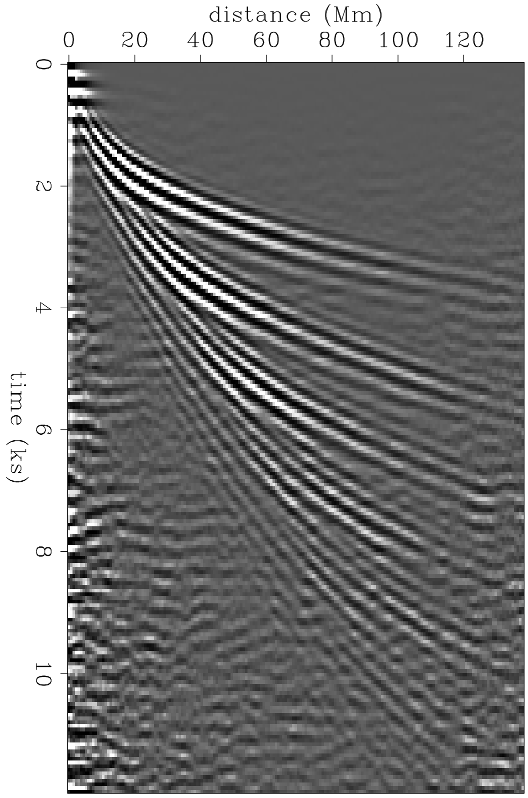

Figure 3: Impulse response derived by Kolmogoroff spectral factorization binned as a function of radial distance from the impulse. james1-kolstack [CR]

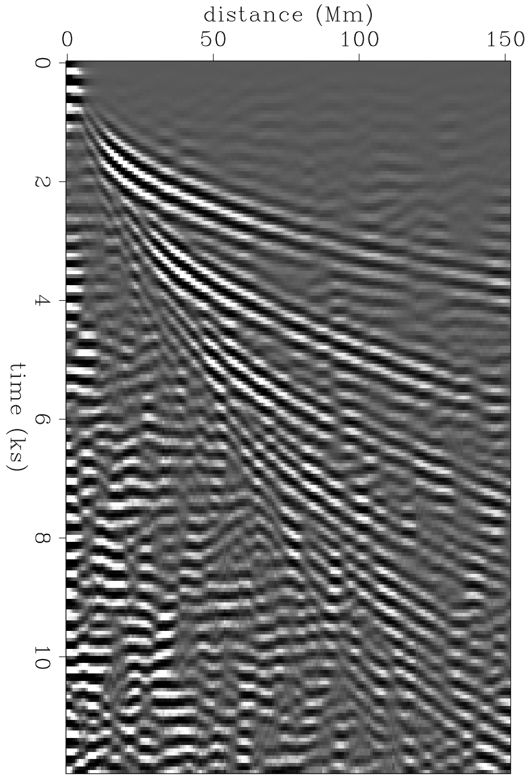

Figure 4: Impulse response derived by cross-correlation binned as a function of radial distance from the impulse. More noise is present in this Figure compared to Figure 3 because less data was used in the calculation. james1-xcorr [ER]

INTERPRETATION

The lack of sharp velocity contrasts mean there are no clearly observable reflections in the impulse response. The visible events are diving waves (refractions) of increasing order. The first arrival is the the direct wave that would be characterized as the 'P' arrival in terrestrial geophysics. The second arrival is equivalent to a 'PP' event (or first order multiple) having bounced once on the solar surface. About six distinct arrivals are visible in Figure 3, corresponding to multiples up to fifth-order.

Since there are no distinct velocity interfaces, only a smooth velocity gradient, the head waves continually curve upward. The contrast between this behavior and head waves on terrestrial exploration seismograms is illustrated in Figure 5.

Radial trace analysis

The radial trace domain provides a natural way to view the seismograms. Since multiples arrive at integer multiples of primary traveltime and distance. The radial trace domain becomes periodic. Rather than preserving traveltimes in the radial trace transform, and stacking all azimuths together, we preserve x and y coordinates.

Figure 6 shows a constant velocity slice through the Kolmogoroff impulse response. Multiple bounces produce the series of concentric circles. Breakdown in reciprocity due to convective flow in the sun, will manifest itself as azimuthal anisotropy in this domain.

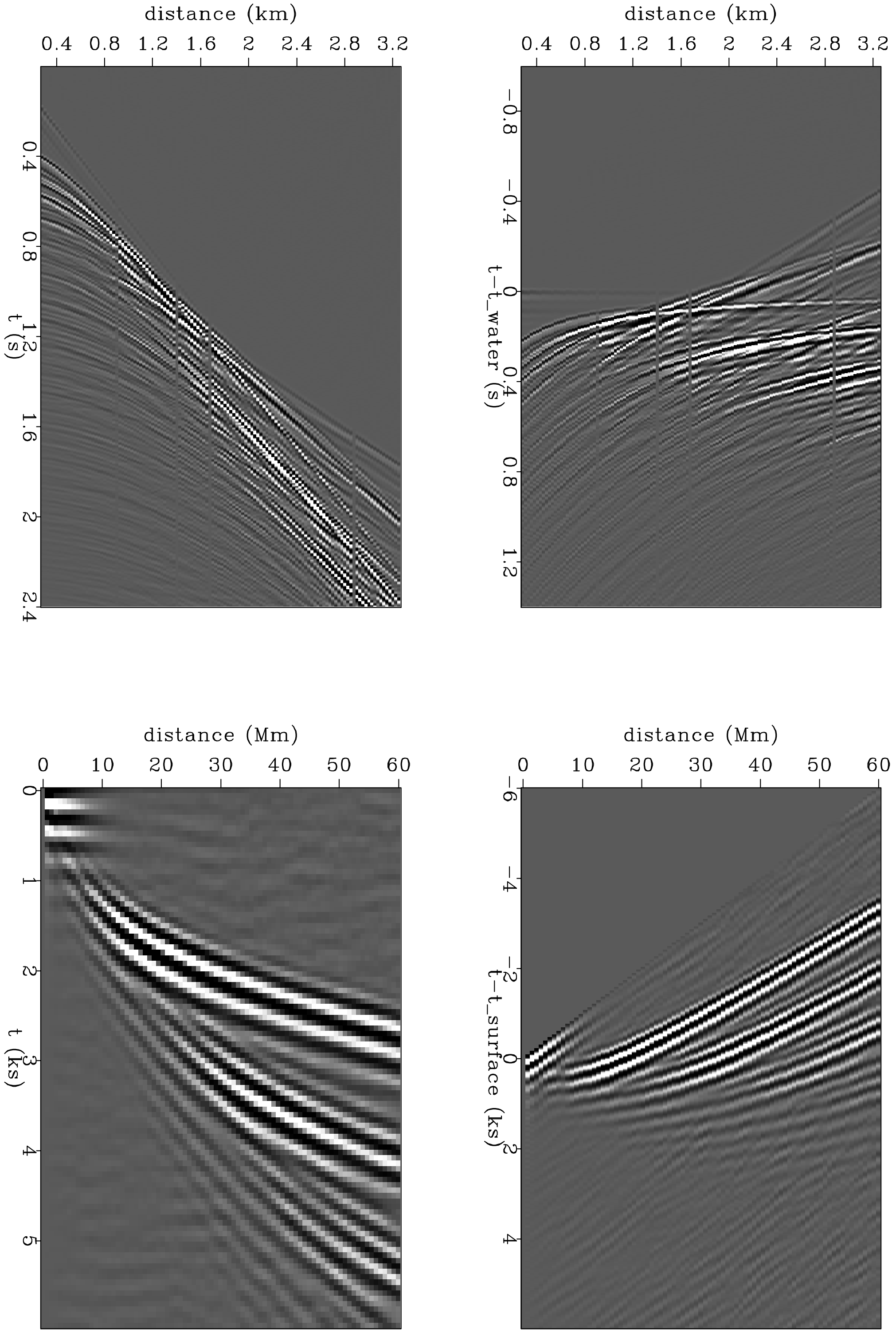

Figure 5: The top two panels show a shot gather from the Gulf of Mexico before (left) and after (right) linear moveout with the water velocity. The lower two panels show the solar impulse response before (left) and after (right) linear moveout at 10 km/s. james1-lmo [CR]

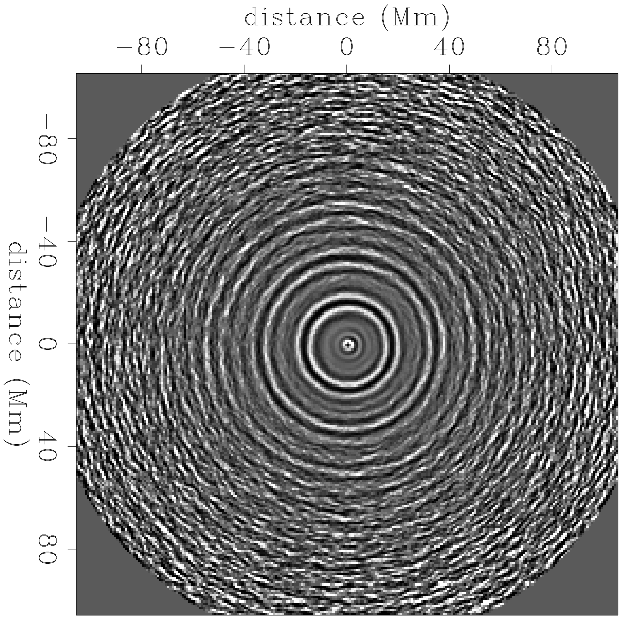

Figure 6: Constant velocity slice through radial transform. james1-radialcube [CR]

CONCLUSIONS

Helioseismology validates a long-standing geophysical conjecture that the cross-correlation of noise traces may provide impulse response seismograms.

Additionally, we have shown that combining one-dimensional Kolmogoroff spectral factorization algorithms with helical boundary conditions allows us to calculate the acoustic impulse response an order of magnitude faster than cross-correlation in either (t, x, y) or (ω, x, y).

This is all very interesting, but our sponsors trust that we are looking towards matters that can have significant practical and financial implications. Not long ago that would have seemed far fetched. But electronics and communications revolutions have been revolutionizing seismology for decades. Many oil fields are already instrumented with multiple hundreds of geophones. We are now prepared to evaluate the prospects for the multiple tens of thousands of geophones that are already practical.

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