Acoustic performance of a stretched membrane and porous blanket combination

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The sound absorption performance of an acoustic absorber consisting of a stretched circular membrane placed a short distance in front of a fiberglass blanket was both measured and predicted. Both theoretical and experimental analyses were restricted to plane acoustic waves. Theoretical predictions indicated that the membrane–blanket combination would have a sound power absorption coefficient nearly equal to the sound power absorption coefficient of the blanket alone if the incident acoustic plane wave drove the membrane at one of its resonance frequencies. Theoretical analysis also predicted that the sound power absorption coefficient would approach zero when the membrane was driven at an antiresonance frequency by the incident acoustic plane wave. Experimental agreement with theoretical predictions was good for several membrane–blanket combinations. The results show that membrane–blanket combinations can be effective acoustic absorbers in frequency ranges which do not include the antiresonance frequencies of the membrane. The equations developed may be used to predict the acoustic performance of any membrane–blanket combination.

Subject Classification: [43] 20.30, [43] 20.60, [43] 40.20.

INTRODUCTION

It is difficult to find a good acoustic absorbing material which is impervious to moisture and other contaminants. Acoustic absorbing materials by their very nature are porous. However, there are many applications, such as those in food handling areas, where health or safety requirements prohibit the use of porous materials. One way of approaching the problem of designing an acoustic absorbing material is to use a conventional acoustic absorbing material such as fiberglass or foam, and to face it with or enclose it in an impervious plastic film. Thus, for example, fiberglass is often enclosed in Mylar bags. The basic shortcoming of this approach is that the impervious material can act as a reflector of sound and can diminish the effectiveness of the absorbing material.

This paper reports an investigation to determine if a workable acoustic absorbing material could be made by stretching a thin plastic film across supports in front of a conventional absorbing material. By proper design of the film supports and tension it was hoped to make use of the fact that sound transmission through a stretched membrane at resonance is nearly 100%. This idea was investigated both analytically and experimentally, using normal-incidence simple harmonic plane waves as a basis for investigating the behavior of the membrane–absorber combination.

The membrane–absorber combination investigated is shown schematically in Fig. 1. The arrows represent the travelling acoustic plane waves with wave P_i being the incident sound pressure wave on the composite absorber. For simplicity in both analysis and testing, a circular membrane was used. The membrane was placed at a distance l in front of a conventional fiberglass absorbing material. The basic assumptions for the analysis of the membrane were the usual ones of uniform tension, uniform mass per unit area, negligible stiffness in the membrane, and rigid clamping at the outer boundaries of the membrane.

I. ANALYTICAL INVESTIGATION

The circular membrane has been exhaustively investigated. Analysis of the vibration of a free membrane (independent of the effects of interaction with the acoustic pressures) can be found in most theoretical acoustics tests.[1-3] Other authors have included the effects of acoustic loading from the air in various ways.[4-6] However, the present investigation appears to be unique in considering the combination of a circular membrane with a nearby acoustic absorbing medium. In such a case it is necessary to consider the interactions from both incident and reflected waves on both sides of the membrane.

Of considerable assistance in this work was the analysis of a circular membrane published by Ingard,[7] which showed the relationship between the net driving pressure on a circular membrane and the velocity of the membrane at any point. Ingard gives this relationship
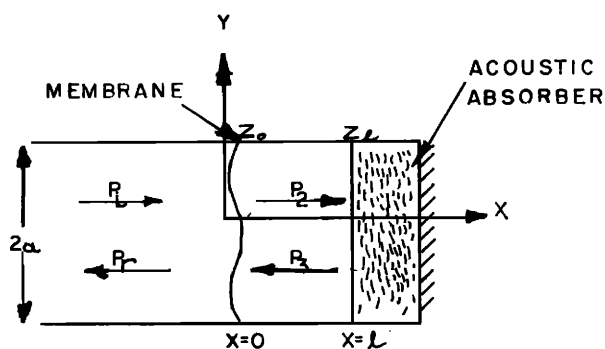
$$P_1 = A \exp(j(\omega T - KX))$$
$$P_2 = C \exp(j(\omega T - KX + \emptyset_2))$$
$$P_r = B \exp(j(\omega T + KX + \emptyset))$$
$$P_3 = D \exp(j(\omega T + KX + \emptyset_3))$$

FIG. 1. Membrane–absorber model.

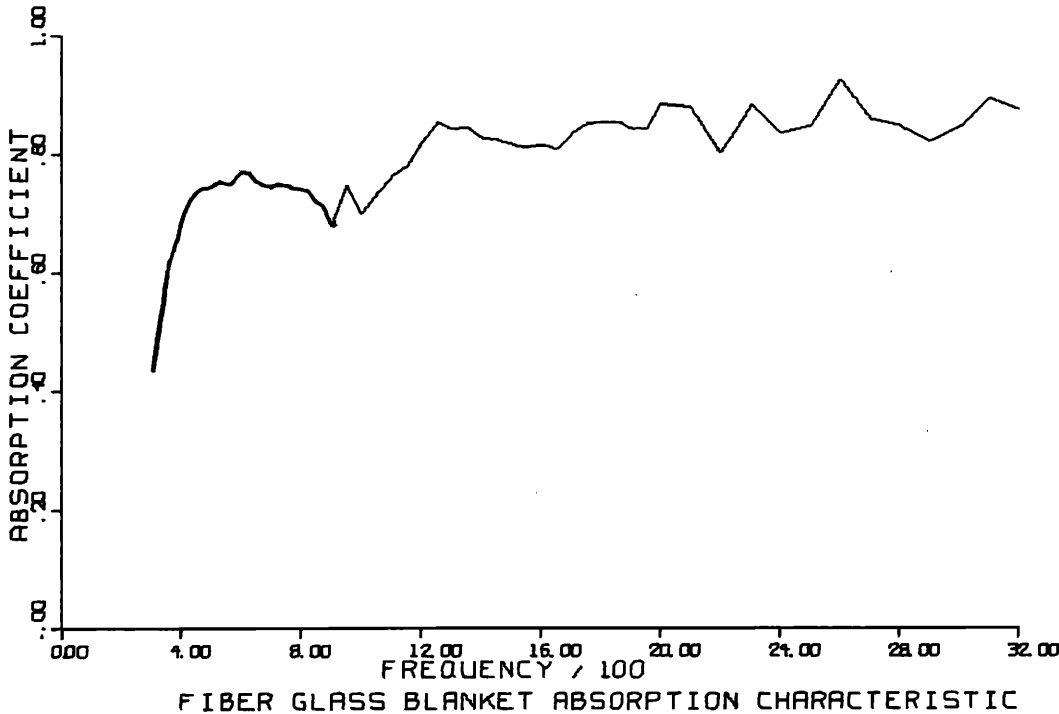FIG. 2. Fiberglass-blanket absorption characteristic.

FIBER GLASS BLANKET ABSORPTION CHARACTERISTIC

as

$$u(r'') = \int G(r'', r^*) F(r^*) dS^* \quad , \tag{1}$$

where $u(r'')$ represents the velocity of the membrane at a point r'' due to a delta function driving force at r^*, $G(r'', r^*)$ is the Green's Function for the membrane, and $F(r^*)$ is the net driving pressure on the membrane at r^*.

The expression for the driving pressure $F(r^*)$ can be deduced for the case under study from the continuity of velocity relationships at the membrane. The particle velocity in a plane wave is related to the acoustic pressure by the well known relationships $u_+ = p/\rho_0 c$ for a right-moving wave and $u_- = -p/\rho_0 c$ for a left-moving wave. Since the net particle motions on both sides of the membrane must be equal, it is possible to show that the amplitude of the plane velocity must satisfy the relation

$$A - B = C - D \quad , \tag{2}$$

which can be rearranged to

$$(A + B) - (C + D) = 2(A - C) \quad . \tag{2'}$$

Examination of the left-hand side of Eq. (2') shows that this is the amplitude of the net driving pressure on the membrane, so that

$$F(r^*) = F = 2(A - C) \quad . \tag{3}$$

Since this driving force is the same for all surfaces on the membrane (because of the plane-wave assumption), Eq. (1) can be rearranged:

$$u(r'') = 2(A - C) \int G(r'', r^*) dS^* \quad . \tag{1'}$$

The velocity of the membrane can thus be seen to be a

function of the amplitudes of the incident and transmitted waves only. With this in mind, it is useful to derive an expression for the mean velocity amplitude of the transmitted wave:

$$u_{\text{mean}} = \frac{C - D}{\rho c} = \frac{1}{S} \int u(r'') dS' \quad . \tag{4}$$

Equation (1') may be substituted into Eq. (4) and the result rearranged to obtain

$$\frac{C}{A} = \frac{1}{1 + Q} + \frac{D}{A}\left(\frac{1}{1 + Q}\right) \quad , \tag{5}$$

where

$$Q = \frac{2\rho c}{S} \int\int G(r', r^*) dS' dS^* \quad . \tag{6}$$

Definition and integration of the appropriate Green's function for a circular membrane can be found in Ingard.[7] The expression for Q in the plane wave case is

$$Q = -j \frac{2}{k'L} \frac{J_2(ka)}{J_0(ka)} \quad , \tag{7}$$

where

$j = \sqrt{-1}$,

$k' = \omega/c_m = $ wave number in the diaphragm,

$\omega = $ circular frequency,

$c_m = $ speed of wave propagation in the membrane,

$k = $ wave number in the air,

$a = $ radius of the membrane,

$L = \sigma/\rho = $ mass per unit area of the membrane divided by the density of the air.
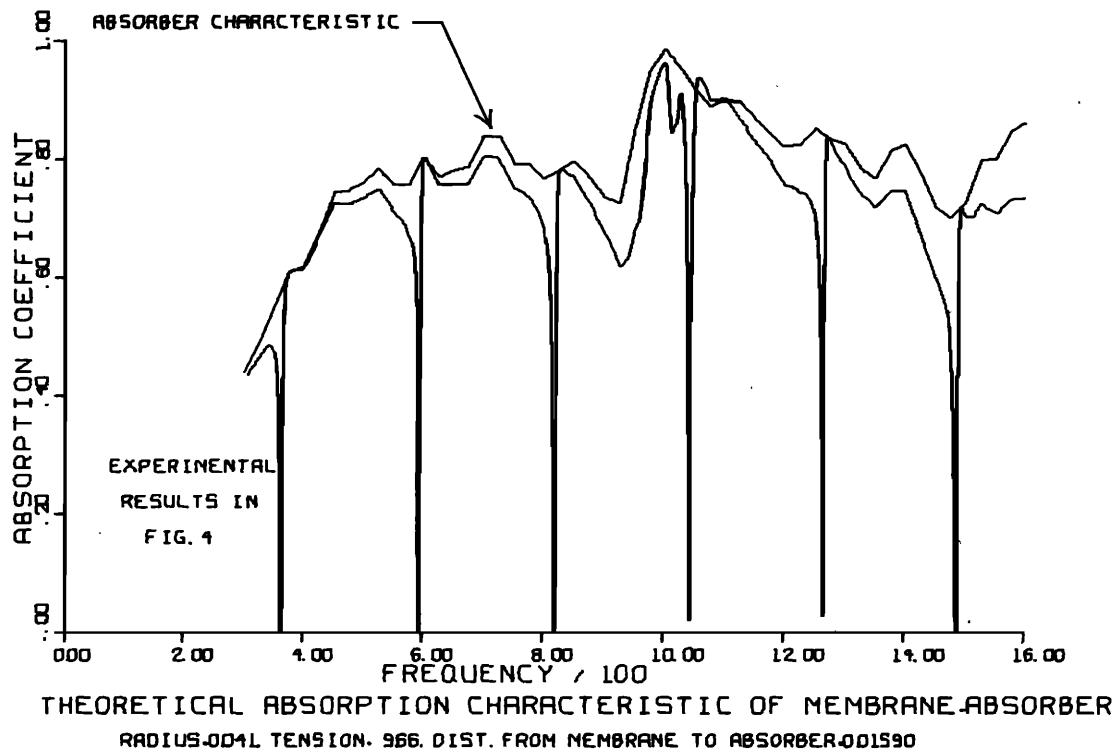
Equation (2) can be rearranged to yield

FIG. 3. Theoretical absorption characteristic of membrane-absorber.

$$\frac{B}{A}=1-\frac{C}{A}+\frac{D}{A}=1-\frac{Q}{1+Q}-\frac{D}{A}\left(\frac{1}{1+Q}\right)+\frac{D}{A} \ . \tag{2''}$$

Since D/A can be written as $(D/C)(C/A)$, the D/A ratio may be expressed as a function of Q and D/C, as is shown below:

$$\frac{D}{A}=\frac{C}{A}\frac{D}{C} \ , \tag{8a}$$

$$\frac{D}{A}=\frac{Q}{1+Q}+\frac{D}{A}\frac{1}{1+Q}\frac{D}{C} \ , \tag{8b}$$

or

$$\frac{D}{A}=\frac{Q}{1+Q-D/C}\frac{D}{C} \ . \tag{8c}$$

Equation (8c) may be substituted into Eq. (2'') to yield the final form for the B/A ratio for the membrane–absorber combination:

$$\frac{B}{A}=1-\frac{Q}{1+Q}+\frac{D}{C}\frac{Q}{1+Q-D/C}\frac{Q}{1+Q} \ . \tag{9}$$

This result is, of course, dependent upon knowledge of the D/C ratio. The D/C ratio can be calculated if the impedance of the absorbent material is known. If Z_t is the impendance of the absorbent material, then

$$\frac{D}{C}=e^{-j2kl}\frac{Z_1-\rho c}{Z_1+\rho c} \ . \tag{10}$$

Substitution of these relationships (or values) into Eq. (9) yields the ratio of reflected to incident pressure amplitudes, B/A. The sound power absorption coefficient can then be calculated by the familiar relationship

$$\alpha=1-|B/A|^2 \ . \tag{11}$$

II. EXPERIMENTAL VERIFICATION

Experimental verification of Eq. (11) was obtained by testing a membrane–absorber combination in a standing wave tube. A polyethylene membrane with a thickness of 0.03175 mm (0.0125 in.) and a mass per unit area of 0.0294 kg/m^2 was stretched 0.0159 m in front of a 0.191-m-thick fiberglass blanket. The fiberglass blanket was placed against a rigid steel end cap on the standing wave tube. The membrane had an outer radius of 0.041 m and was stretched to a tension of 9.66 N/m.

The acoustic properties of the fiberglass blanket were required as input information for the analysis. Consequently, the properties of the blanket to be used in the experimental program were determined separately in the standing wave tube. The results are shown in Fig. 2. These results were used as input data in the calculations of the sound power reflection coefficient for the membrane–absorber combination.

The acoustic absorption coefficient predicted for the membrane–absorber combination by Eq. (11) is shown in Fig. 3. This prediction is superimposed on a plot of the acoustic absorption characteristics of the fiberglass blanket alone to facilitate determination of the effect of the membrane. It can be seen from Fig. 3 that the absorption coefficient of the membrane–absorber assumes values as high as or higher than the absorption coefficient of the fiberglass blanket and as low as zero. The frequencies at which the absorption coefficient goes to zero are those frequencies at which the reflection coefficient of the membrane–absorber combination goes to unity. These frequencies can be determined by setting Eq. (7) equal to zero and solving; they are those frequencies where
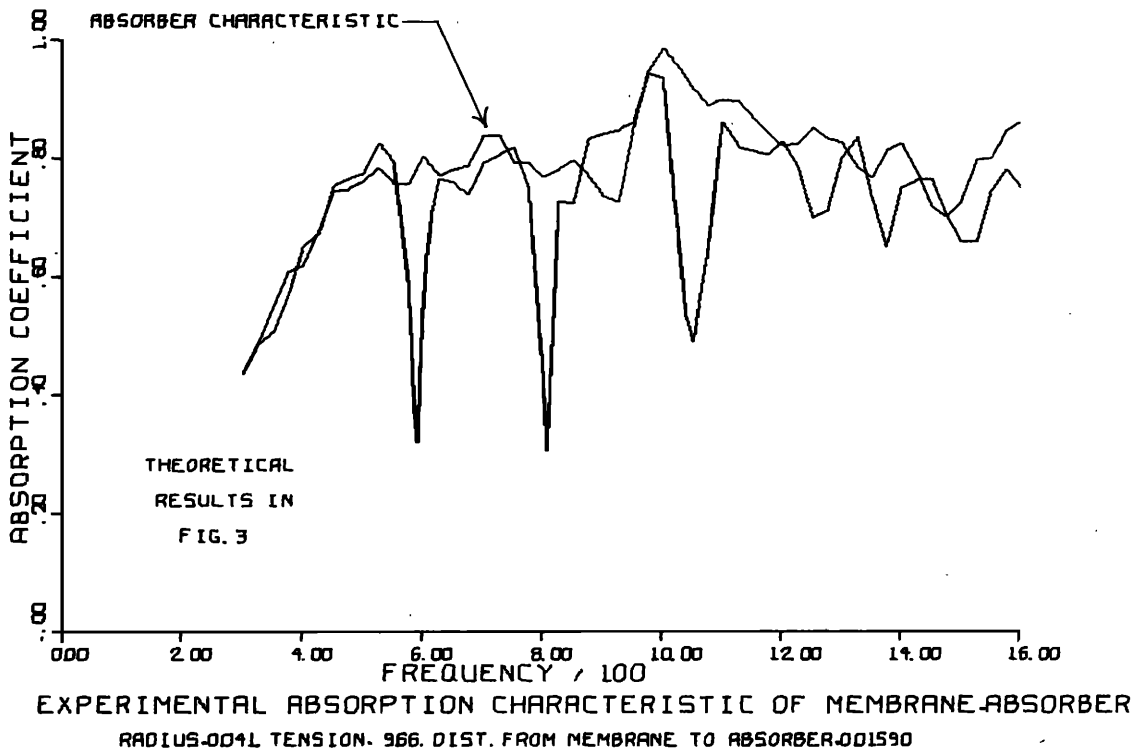
ABSORBER CHARACTERISTIC

ABSORPTION COEFFICIENT

THEORETICAL
RESULTS IN
FIG. 3

FIG. 4. Experimental absorption characteristic of membrane-absorber.

FREQUENCY / 100

EXPERIMENTAL ABSORPTION CHARACTERISTIC OF MEMBRANE-ABSORBER
RADIUS.004L TENSION. 966. DIST. FROM MEMBRANE TO ABSORBER.001590

$ka = 5.14, \ 8.41, \ 11.61, \ \dots \ ,$

or

$f = 281.4/a, \ 460.4/a, \ 635/a, \dots \text{Hz} \ ,$

where a is the membrane radius in meters.

The frequencies where $J_2(ka)$ goes to zero are the membrane antiresonance frequencies. At membrane antiresonance the membrane vibrates in such a way that its average displacement is zero. When the average displacement of the membrane is zero, no plane wave is transmitted, and all of the acoustic energy is reflected.

The resonant frequencies of the membrane occur at frequencies which make $J_0(ka)$ equal to zero. This occurs for those frequencies where

$ka = 2.405, \ 5.520, \ 8.654, \ 11.792, \ \dots \ ,$

or

$f = 131.3/a, \ 301.3/a, \ 472.4/a, \ 643.7/a, \ \dots \text{Hz}.$

These resonant frequencies permit virtually complete transmission of the sound through the membrane. At those frequencies, Eq. (9) can be simplified to

$$\frac{B}{A} = \frac{D}{C} \frac{Q}{1 + Q - D/C} \ . \tag{9a}$$

Of course, the membrane resonances and antiresonances represent discrete frequencies which are primarily useful in designing the membrane and determining its performance in frequency regions nearby. It is interesting and important to note, however, that the absorption coefficient predicted for the membrane–absorber combination was actually above that for the absorber alone in some frequency ranges.

The results from the standing wave tube experiments on the membrane–absorber combination under study are shown in Fig. 4. These results are superimposed on a plot of the absorbing blanket characteristics so that the effect of the membrane may once again by easily determined. A direct comparison between theoretical predictions and experimental results for this case is presented in Fig. 5. As may be seen from this figure, agreement between theoretical predictions and experimental results was good over most of the frequency range of interest. The acoustic properties of several other experimental membrane–absorber combinations were predicted and measured, and the agreement between theory and experiment was about the same as for the example shown in this paper.

III. OBSERVATIONS AND CONCLUSIONS

The results of the experimental and theoretical investigations show that it is possible to use a stretched membrane in front of an acoustic absorbing material without destroying its acoustic performance at the majority of frequencies. The performance of the film and absorber combination at most frequencies can be as good as, or slightly better than, the performance of the absorber by itself. However, in order to have an effective acoustic absorber it is necessary to avoid poor acoustic performance at the frequencies near membrane antiresonance. Furthermore, it has been found that the membrane is an efficient transmitter of sound only for those frequencies below the fourth or fifth membrane antiresonance. Above these frequencies the membrane begins to reflect the sound, and very little acoustic energy is transmitted to the absorbing blanket.

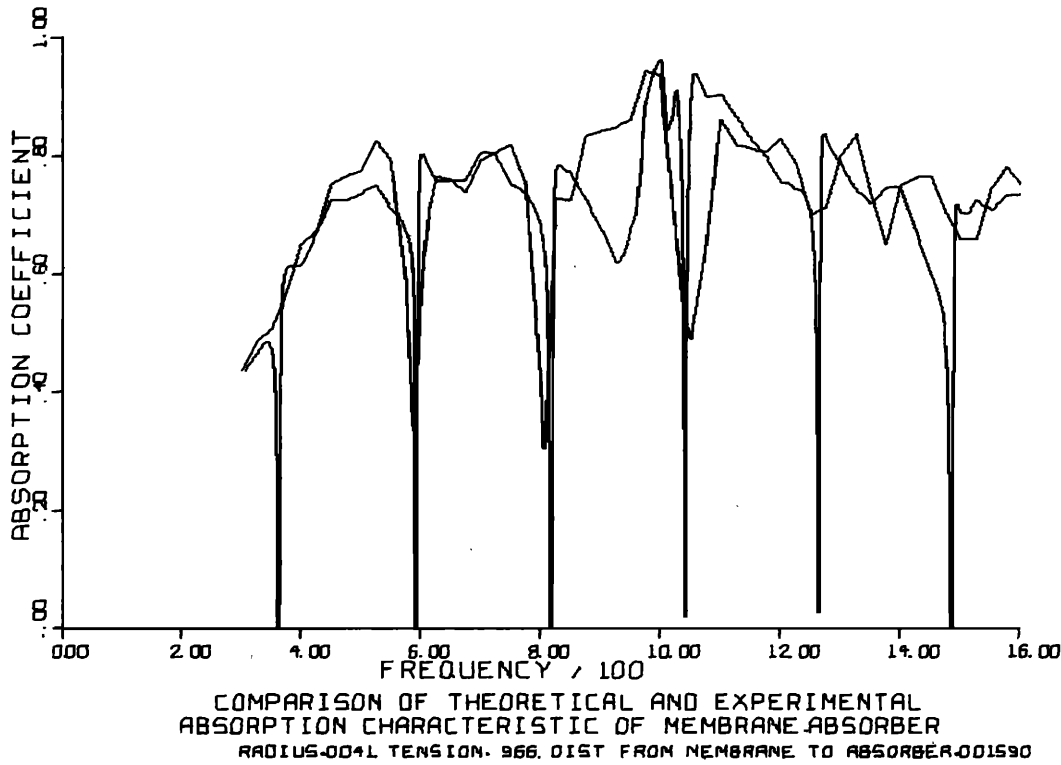Reflection from the membrane was even more pro-

FIG. 5. Comparison of theoretical and experimental absorption characteristic of membrane-absorber.

nounced in the experimental observations than in the theoretical analysis. This was expected, however, since the theoretical analysis neglected the possibility of damping in the membrane. This damping became significant at the higher frequencies, making it more difficult to excite the higher mode shapes.

It is instructive to study Eq. (7) to see if there are

any combination of design parameters that could be utilized to allow construction of a membrane–absorber combination which is effective across all of the frequencies of interest in noise control work. The first such approach might be to decrease the membrane radius and increase the membrane tension to place the first membrane antiresonance above the range of human hearing. However, this approach yields very poor per-
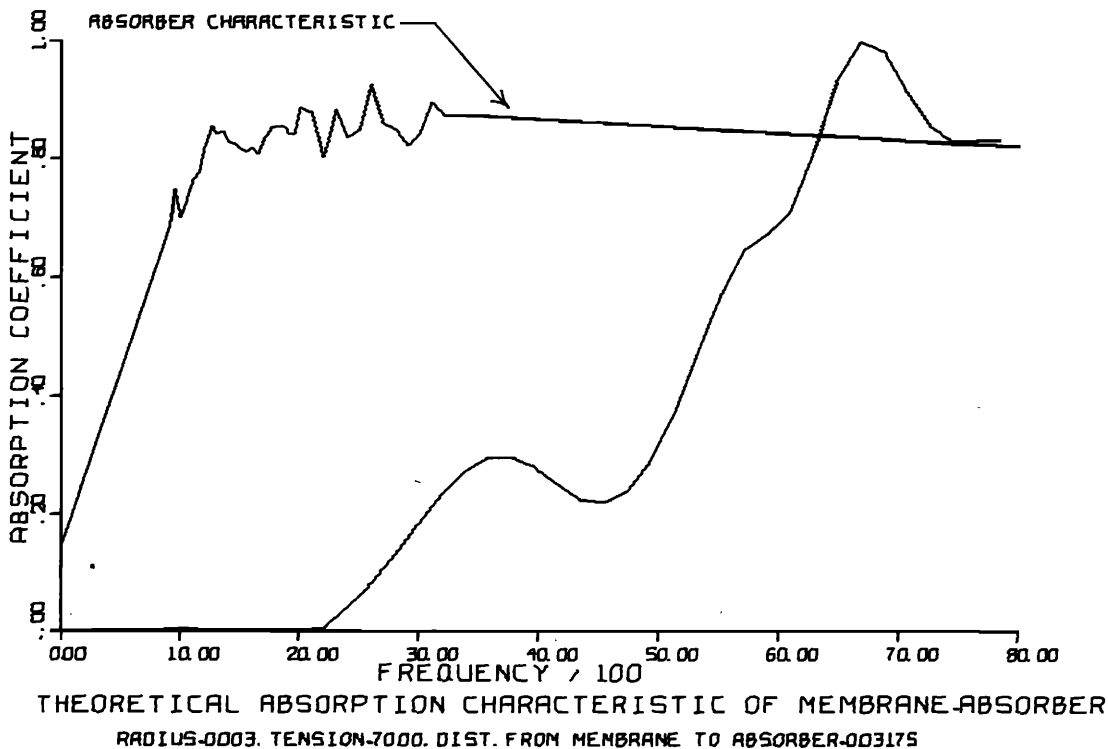

FIG. 6. Theoretical absorption characteristic of membrane-absorber.

formance at low frequencies. A theoretical absorption curve for a combination of this type is shown in Fig. 6 for an assumed absorber characteristic. Clearly, a membrane–absorber with these characteristics would be useful only in the region of 6000 Hz.

Another approach to the design of a useful membrane–absorber would be to attempt to construct a membrane with a low mass per unit area. This would allow better transmission of sound at the higher frequencies. Lower mass per unit area could be obtained by using a thinner membrane or by changing the membrane material. For a given membrane material the limiting factor is its ultimate tensile strength. Thus, as the membrane is made thinner the tension to which it is stretched may have to be reduced. The reduction in tension will lead to lower frequencies for the membrane antiresonances, which in turn limit the frequency range over which the membrane–absorber combination may be useful. A stronger, less dense membrane material would be the best alternative.

The membrane–absorber combination under investigation could provide excellent absorption characteristics over specified frequency ranges. These frequency ranges were centered about the membrane resonant frequencies and between membrane antiresonant frequencies. This suggests that a properly designed array of different membranes could, when arranged on a panel, provide reasonable acoustic absorption characteristics across a broad enough frequency range to be useful. The plane-wave equations for the design of such a composite absorber have now been developed.

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