

# Acoustic Sensing and Mitigation of Lean Blow Out in Premixed Flames

Shashvat Prakash<sup>\*</sup>, Suraj Nair<sup>†</sup>, T. M. Muruganandam<sup>‡</sup>, Yedidia Neumeier<sup>‡</sup>,  
Tim Lieuwen<sup>§</sup>, Jerry Seitzman<sup>\*\*</sup> and Ben T. Zinn<sup>††</sup>  
*Georgia Institute of Technology, Atlanta, GA, 30332-0150*

**This paper describes a method for detecting and preventing lean blow out (LBO) in a premixed, swirl stabilized combustor. The acoustic signal is filtered to detect localized extinction ‘events’, which increase in frequency as the flame equivalence ratio,  $\phi$ , approaches the LBO limit,  $\phi_{LBO}$ . As the flame becomes leaner,  $\phi_{LBO}$  can be effectively shifted to lower equivalence ratios by redirecting a fraction of the total fuel into a central, premixed pilot. This actuation can increase the equivalence ratio in the stabilization region at a constant power setting and counter the flame lift associated with LBO. The resulting control system maintains lean flame stability in the presence of flow fluctuations. The combustor can therefore operate with an improved safety margin at lower temperatures and less NOx (nitrous oxides) emissions.**

## Nomenclature

LBO	=	lean blow out
$\phi$	=	flame equivalence ratio, i.e. ratio of flame fuel air ratio to the stoichiometric fuel air ratio
$\phi_{LBO}$	=	equivalence ratio at lean blow out limit
$\tau_{1sec}$	=	number of threshold crossings over the previous 1 second (moving window summation)
$\tau_{1sec, LBO}$	=	number of threshold crossings per second at the LBO limit

## I. Introduction

LEAN, premixed combustion presents a method to burn fuel at lower temperatures and reduce NOx (nitrous oxides) emissions. However, with such approaches lies the inherent risk of lean blow out (LBO) as combustors are made to operate near their LBO limit ( $\phi_{LBO}$ ). When sufficiently lean flames are subject to power setting changes, flow disturbances, or variations in fuel composition, the resulting equivalence ratio perturbations may cause loss of combustion. Such a blowout could cause loss of power and expensive down times in stationary turbines. In aircraft engines, a blowout could result in engine loss during critical military maneuvers or approach and landing phases in flight. To avoid such scenarios, current systems are typically operated at sufficiently rich conditions and over-conservatively schedule changes in load and/or operating conditions. These techniques provide greater safety margins but result in high temperature flames (with higher NOx) and restrict the rate at which settings can be adjusted. Thus, there is a need for more optimum LBO mitigation approaches that would both reduce emissions and increase efficiency without compromising safety over the whole range of the engine’s operating conditions.

Effective LBO mitigation techniques are required to satisfy emissions and efficiency criteria while allowing for safe, lean operation of a flame subject to unanticipated flow disturbances. This requires sensing, in real time, the probability of LBO and actuating to prevent LBO. The sensing and actuation must be part of a capable, coherent control system which can stabilize the flame for a variety of operating conditions and disturbances.

This paper presents an LBO mitigation control scheme replete with actuation and sensing approaches. The objective is to stabilize a lean premixed flame by increasing the safety margin – i.e. the difference between  $\phi$  and

<sup>\*</sup> Graduate Research Assistant, School of Mechanical Engineering, 0405, AIAA Student Member.

<sup>†</sup> Graduate Research Assistant, School of Aerospace Engineering, 0150, AIAA Student Member.

<sup>‡</sup> Adjunct Professor, School of Aerospace Engineering, 0150, AIAA Member.

<sup>§</sup> Assistant Professor, School of Aerospace Engineering, 0150, AIAA Member.

<sup>\*\*</sup> Associate Professor, School of Aerospace Engineering, 0150, AIAA Associate Fellow.

<sup>††</sup> Regents Professor, Schools of Aerospace and Mechanical Engineering, 0150, AIAA Fellow.

$\phi_{LBO}$  for a given operating condition. In this study, we investigate filtering the acoustic signal to detect LBO precursors and provide a measure of real time LBO probability. A valve actuator is employed to redistribute the injected fuel between the main swirl injector and a premixed, pilot injector that supplies fuel to the flame stabilization region. The objective of the mitigation scheme is to allow for safe, lean operation of a flame in the presence of disturbances that are excited in the system as the LBO limit is approached.

## II. Background

Lean blow out precursors, which occur in sufficiently lean flames and manifest as flow and combustion variations, have been studied by Nicholson and Field<sup>1</sup> and Chao et al.<sup>2</sup>. These studies observed large scale, low frequency flame pulsations and periodic detachment of the flame from its flame holder near the LBO limit. These LBO precursors were determined to consist of brief, localized extinguishment ‘events’ which last from a few short cycles up to several seconds.

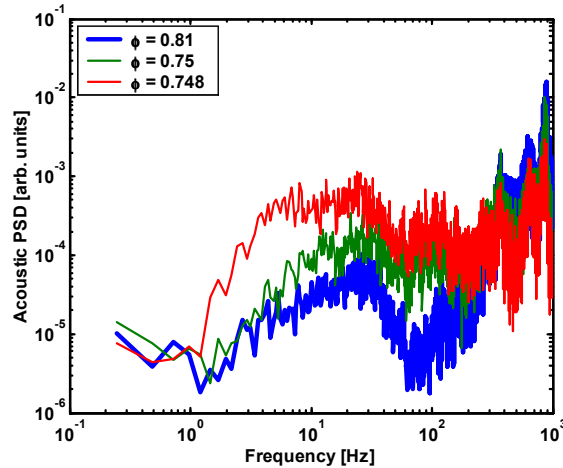
LBO precursors were used by Muruganandam et. al.<sup>3</sup> to determine the proximity of LBO. As the flame became leaner, the frequency of the extinguishment events increased. The study examined both optical OH chemiluminescence and acoustical signals as methods to detect precursors; events were manifest as ‘dips’ in the OH chemiluminescence signal and threshold crossings of a statistically normalized acoustic signal. The investigation noted that the low frequency (10 Hz - 200 Hz) power content of both the acoustic (Figure 1) and optical OH signals increased as the flame became leaner. Muruganandam et. al. postulated that this increase in low frequency power was due to a combination of localized extinction time scales (of the order of 10 ms) and the mean time between such events (of the order of 1s). Both time scales increased to their upper limits as the flame approached its LBO limit. Although the study presented a method for detecting LBO precursors from the acoustic signal, the statistical methods employed involved computing the signal standard deviation over a set time scale. This procedure imposed delays which made the method unsuitable for real time active control implementation, and it was suggested by the authors that optical sensing of ‘dips’ is ‘faster’ than the statistical acoustic based sensing.

Other acoustic LBO precursor sensing methods have been pursued as well. Nair and Lieuwen<sup>4</sup> demonstrated that when LBO precursors are present, the acoustic signal exhibits a repeated pattern, which can be extracted with a wavelet whose shape resembles that of the ‘event’. This approach was also unsuitable, however, for real time LBO detection due to the excessive computation time involved.

Control of lean flames has been previously pursued as a means for NOx reduction. Nakae et. al.<sup>5</sup> demonstrated low NOx operation in a lean, premixed, prevaporized (LPP) combustor where the equivalence ratio was varied based on CO emissions feedback. The controller varied the air distribution in the combustion chamber by modulating the air flow through an inlet downstream of the injection plane. Although emissions feedback provided a failsafe estimate of equivalence ratio, gathering emissions data is a slow process which severely limits control capabilities due to the delay involved.

Higher speed LBO mitigation was demonstrated by Muruganandam et al.<sup>6</sup> using optical, OH-based, sensing and fuel split ratio actuation which distributed the fuel between a swirler and a central, premixed pilot. The optical signal was compared with a threshold to detect and count ‘dips’. The dip frequency over a one second moving window was used to determine whether to increment actuation. Fuel was redistributed from the annular swirler to a central port, maintaining overall combustor equivalence ratio ( $\phi$ ) but increasing the equivalence ratio in the stabilization zones.

This paper describes a study that was undertaken to investigate whether acoustic sensing can be effectively used in LBO control and, thus, provide an alternative to optical sensing. Specifically, this study investigated whether acoustic sensing could sense, in real time, LBO precursors for active control implementation.



**Figure 1. Fourier spectra of the acoustic signal from a premixed flame at varying equivalence ratios.**

### III. Experimental Setup

The experiments were performed on a setup intended to serve as a simple simulator of a lean, premixed gas turbine combustor that includes a swirling premixer section. The intention was to study the nature of lean blow out and LBO precursors.

The atmospheric pressure combustor consisted of a cylindrical quartz tube with a 70 mm inner diameter and 190 mm in length (Figure 2). An annular opening at the base of the tube supplied a swirling, combustible mixture of fuel ( $\text{CH}_4$ ) and air into the combustor and a secondary stream of combustible, richer air-fuel mixture, was supplied through a pilot at the center of the injection plane.

A valve manifold governed the proportions of total fuel that were directed to the main (annular) and pilot ports (Figure 3). The manifold consisted of ten Asco Scientific solenoid valves with a response time of 20 ms. The valves were driven from a control computer running QNX real time operating system with a Power DAQ I/O board running at 2 kHz.

Generally, all the fuel and air were supplied to the combustor through the annular supply ring. When LBO precursors were detected, the valve manifold redirected a portion of the total fuel that depended upon the amount of actuation required into the center pilot port. During testing, the air supply rate was externally perturbed by the operator to simulate disturbances.

Acoustic oscillations were measured with a calibrated, Bruel and Kjaer type 4191 condenser microphone that has a flat frequency response up to 40 kHz. The microphone was located  $\sim 61$  cm from the combustor exit at  $\sim 90^\circ$  from the flow axis. Since the flame noise exhibited little directivity, the results demonstrated weak dependence upon the microphone location. The principal exception to this occurred when the microphone was placed in the combustion chamber exhaust. In this case, hydrodynamic pressure fluctuations substantially increased background noise levels. For analysis, all sensing and control signals were fed into a 12 bit National Instruments A/D board connected to a computer running LabVIEW.

### IV. Flame Structure

The combustor geometry allowed for various flame configurations as the operating conditions varied. Several flame transitions were especially noted as air flow into the combustor was gradually increased, affecting velocity and equivalence ratio throughout the combustion region.

At sufficiently high equivalence ratios, two flames were visible in the combustor; a primary flame with a 'v-shaped' base centered near the swirlers and a secondary torus-shaped flame in the dump region around the primary flame's base. At lower equivalence ratios, such as those immediately above blowout, only the primary flame remained. When the flame transitioned between the two flame

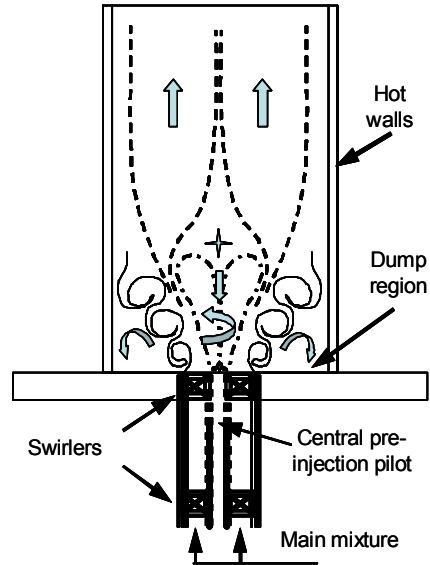


Figure 2. A schematic of the utilized combustor.

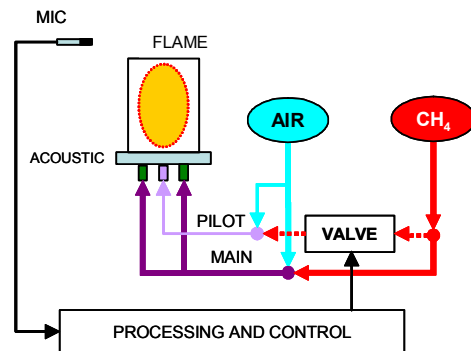


Figure 3. Schematic of fluid and signal flow.

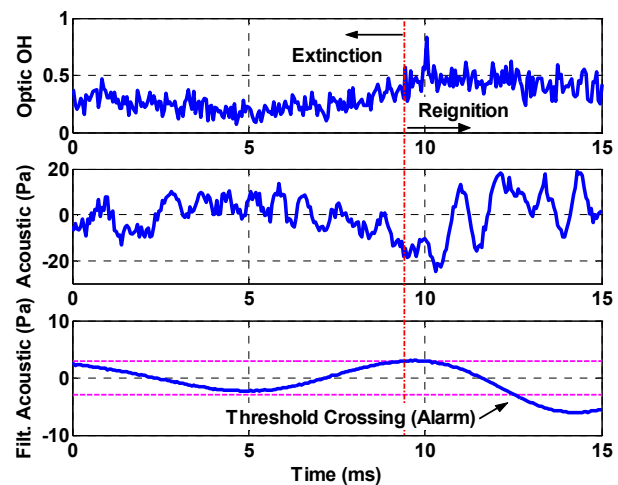


Figure 4. Anatomy of an extinction event, as captured by OH chemiluminescence (top), acoustic (middle), and bandpass filtered acoustic, 10.6 Hz to 95.5 Hz (bottom).

and single flame modes, the secondary flame pulsed, vanishing and reappearing at random intervals. When the secondary flame existed the primary flame was situated lower in the tube, most likely due to the extra heat and radicals generated by the secondary flame. Likewise, when there was no secondary flame, the primary flame shifted higher. Near LBO, the single primary flame would lift even higher and also exhibit large scale pulsations, of the same type observed in earlier studies<sup>1,2</sup>.

## V. Sensing

Although previous LBO mitigation schemes have employed optical chemiluminescence to detect precursors, acoustic methods offer several advantages. Many land based gas turbines are already instrumented with dynamic pressure transducers, which can be easily installed and shielded in a harsh engine environment. Furthermore, unlike optical methods, acoustic sensors are not subject to field of view limitations, i.e. they can sense attributes of the entire combustion region.

### A. Acoustic Signal

Analysis of acoustic emissions provides means for detecting transient flame holding events because they are proportional to the temporal rate of change of heat release. Since combustion noise is generated by the unsteady expansion of reacting gases, the acoustic emissions of turbulent flames are dominated by unsteady heat release<sup>7</sup> processes (as opposed to flow noise) that excite acoustic waves over a broad range of frequencies, typically between  $\sim 10$  Hz – 25 kHz<sup>8</sup>. Thus, acoustic measurements can be used to detect either global changes in heat release rate or fluctuations in heat release at certain time scales by measuring their acoustic emissions in the corresponding frequency bands.

Earlier studies<sup>3</sup> have determined that as a lean flame approaches its LBO limit, the frequency of localized extinction – reignition “events” increases and that such events typically last from 10-15 milliseconds. An optical sensor sensing such an event will detect an abrupt drop in OH chemiluminescence during the extinction phase followed by a sharp increase during reignition (Figure 4, top plot). On the other hand, the acoustic signal’s amplitude slightly increases during extinction, but with reduced amplitude fluctuation and the reignition phase is distinguished by a sudden “appearance” of a large amplitude oscillation as the flame deficient region is reignited with a noticeable ‘pop’ (Figure 4, middle plot). As the flame becomes leaner, the extinction regions may grow larger, and event durations may become longer.

Prior studies of acoustic sensing of extinction events have investigated the statistical kurtosis<sup>3</sup> and wavelet analysis<sup>4</sup> approaches. It was noted that while these methods can capably detect extinction events, they require excessive computation cycles and would not be suitable for use in real time control.

### B. Filtering Scheme

It has been determined<sup>3</sup> that localized extinction events boost the low frequency (10 Hz – 200 Hz) content of the acoustic signal. By selectively filtering the excited frequencies, precursor ‘signatures’ could be extracted from the

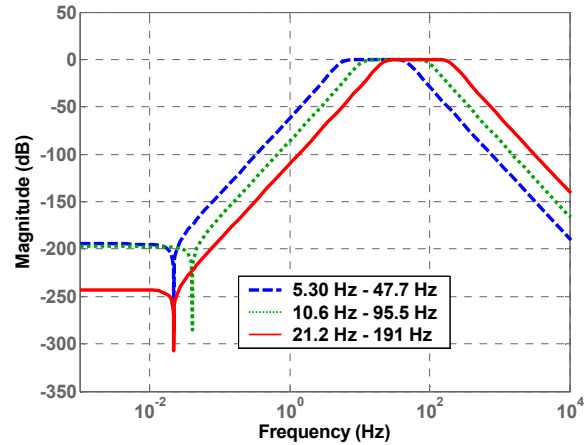


Figure 6. Magnitude plot of the bandpass filters used to detect precursor events.

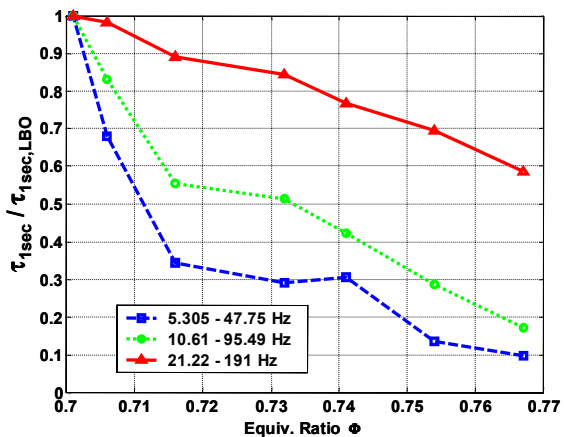


Figure 5. Dependence of the number of threshold crossings (alarms) per second ( $\tau_{1sec}$ ) normalized with the number of alarms at LBO ( $\tau_{1sec,LBO}$ ) upon the filter frequency range and equivalence ratio ( $\phi$ ).

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acoustic signal, thereby attenuating noise from combustion and flow processes. To study this method, the measured acoustic data were analyzed by three different bandpass filters. The filters were Butterworth 8<sup>th</sup> order (4<sup>th</sup> order high pass and 4<sup>th</sup> order low pass) with center frequencies of 15.9, 31.8 and 63.7 Hz and a bandpass ratio of 3 (Figure 6). These filters spanned 5.30-47.7, 10.6-95.5 and 21.2-191 Hz. frequency ranges, respectively.

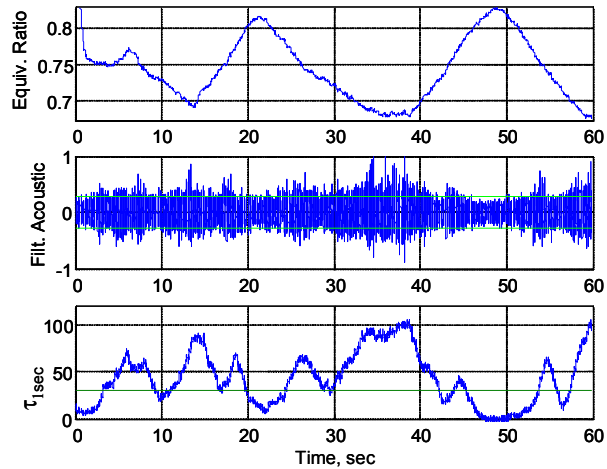
The filtered signal was compared with a threshold set at  $4\sigma$ , where  $\sigma$  is the standard deviation of the absolute value of the acoustic signal from a flame with equivalence ratio 0.75. When the signal exceeded the threshold, an alarm was counted. With this criterion, the dependence of the average number of threshold crossings (or “alarms”) per second ( $\tau_{1sec}$ ), normalized by the corresponding value of crossings at LBO, upon the equivalence ratio was determined (Figure 5). Figure 5 shows that while the low frequency filter exhibited the greatest overall (relative) increase in alarm ratio, the rate of increase did not become significant until the equivalence ratio was below 0.72. The middle frequency filter (10.6 – 95.5 Hz) was comparatively more linear and also exhibited a large increase in alarm ratio over the equivalence ratio range. The bottom plot of Figure 4 shows how an acoustic signal passed through this filter (10.6-95.5 Hz) behaves during an event cycle. The extinction and reignition phases combined last approximately 15 ms, corresponding to a 66.67 Hz event ‘cycle’. Since this frequency lies within the range allowed by the employed bandpass filter, a threshold crossing occurs in the filtered signal.

### C. Sensing Performance

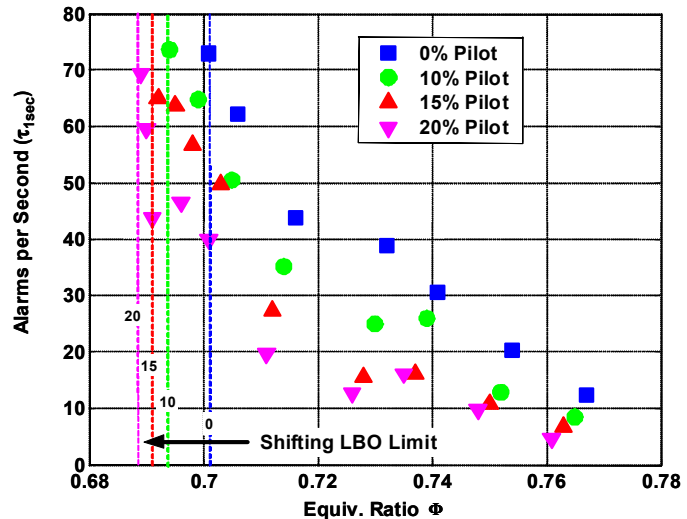
A bandpass filtered acoustic signal, by allowing only frequencies associated with LBO precursors, should, in amplitude, reflect a flame’s proximity to its LBO limit. The resulting signal should provide a reliable measurement on which to base LBO mitigation control response.

Results of the effect of varying the air flow rate to the combustor are shown in Figure 8. The top plot shows the equivalence ratio variation over time, and the middle plot displays the bandpass filtered acoustic signal. The threshold crossing frequency was computed in a moving window summation. The ‘alarms per second’ value ( $\tau_{1sec}$ ) was computed at a given time instant by summing the total threshold crossings over the previous 1 second. This parameter is displayed in bottom plot of Figure 8.

Generally, the frequency of threshold crossings (events) increases as the flame approaches its LBO limit and vice versa. However, there is an intermediary increase in the events per second as the secondary flame in the recirculation zone also exhibits LBO precursors. After the secondary flame extinguishes (below an equivalence ratio of approx. 0.75) the relationship between events per second and equivalence ratio becomes linear again.



**Figure 8. Characteristics of an unpiloted flame undergoing equivalence ratio variations over time. The equivalence ratio is shown in the top plot and the bandpass filtered acoustic signal (10.61 to 95.49 Hz) with thresholds is shown in the middle plot. The bottom plot displays the total events over the past 1 second.**



**Figure 7. Effect of pilot fraction on the LBO limit, as determined by acoustic based sensing. Increasing the fraction of fuel directed to the pilot effectively shifts the LBO**

The results indicate that, below a certain equivalence ratio, the threshold crossings increase linearly with proximity to the LBO limit, causing the events per second signal to mirror the time dependence of the equivalence ratio. The events per second signal can now be used as a basis of mitigation techniques that sense the onset of LBO and improve safety margin.

## VI. Control

A viable LBO mitigation control approach should incorporate sensing and actuation in a coherent system that capably stabilizes a lean flame subject to equivalence ratio variations. The control scheme must provide actuation when the flame approaches its LBO limit and reduce actuation when the number of precursors subsides. The scheme must ensure that the actuation can capably improve safety margin for a given equivalence ratio, and that the commanded actuation is just large enough to ensure proper safety margin.

### A. Actuation

The objective of actuation is to counter the lift and detachment mechanism associated with LBO without increasing the input power. Muruganandam et al.<sup>6</sup> have shown that redistributing part of the fuel in a premixed swirl stabilized combustor to a central premixed pilot can effectively shift the LBO limit to lower equivalence ratios. As the lean flame lifts from the flame holding device, an increase of the equivalence ratio at the central flame stabilization region reattaches the flame. When the pilot is activated, the total fuel flow supplied to the combustor remains constant.

The LBO control experiments of this study were conducted on the same apparatus as in the referenced studies<sup>3,4,6</sup>. The objective of this study was to determine whether acoustic sensing methods could also be used to detect LBO precursors and stabilize a combustor operating near the LBO limit. Figure 7 presents some of the results of this study. It shows that increasing pilot fraction shifts the LBO limit to a lower equivalence ratio and also decreases the number of alarms detected, indicating that the flame has become more stable.

The threshold crossing parameter ( $\tau_{1sec}$ ) may be further considered as an overall gauge of flame stability, as LBO occurs, regardless of pilot, at approximately the same alarm frequency (65-75 alarms per second).

Although piloting at a fixed equivalence ratio increases NOx (nitrous oxides) emissions by creating a localized region of high temperature, it enables operation at leaner equivalence ratio where overall NOx emissions are at a minimum. Furthermore, for a given safety margin (i.e. the difference between operating and LBO equivalence ratios), Muruganandam et al.<sup>6</sup> have shown that the pilot-stabilized combustor produces less NOx than the unpiloted combustor. Hence, we require a control algorithm which maintains safety margin (during lean operation) across a range of power settings.

### B. Control Algorithm

The control algorithm is tuned to move the system gradually towards a no pilot (minimum NOx) operation while allowing for rapid corrections when excessive precursors are detected. This requires two logic modes (Figure 9): one for steadily decreasing the valve parameter (i.e. pilot fraction) and one for rapid increase in case excessive precursors are detected. The number of events per second was described above as a method for gauging a flame's proximity to its LBO limit, and it was also described above how a filtered acoustic signal will exhibit a 'spike' during the reignition phase of an event. The control algorithm counts the number of signal spikes over a fixed time window and, if the sum is greater than a prescribed limit, corrective measures are taken (piloting is increased). If no threshold crossings occur over a set time period, the pilot fraction is incrementally decreased.

To minimize the effect of noise, two thresholds are employed to detect acoustic signal spikes. The 'start' of an event is declared if the signal crosses above the higher threshold and the 'end' is declared once it returns below the

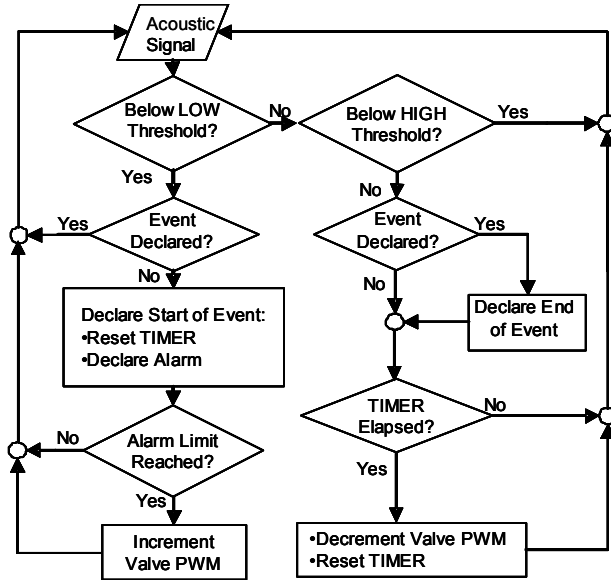


Figure 9. Flow chart of control algorithm

lower threshold. This dual threshold technique minimizes false alarms, especially for the case where signal noise near a single threshold can result in erroneously detected crossings.

Actuation is commanded in increments of 1/10 of a valve, over a total of 10 valves. Fractionally open valves are operated in pulse width modulation (PWM) mode at 20Hz with a duty cycle corresponding to the commanded fraction plus the valve response time of 20ms, with a maximum duty cycle of 50%. Commanded fractions greater than 50% were distributed among two PWM valves to prevent response delays from interfering with performance.

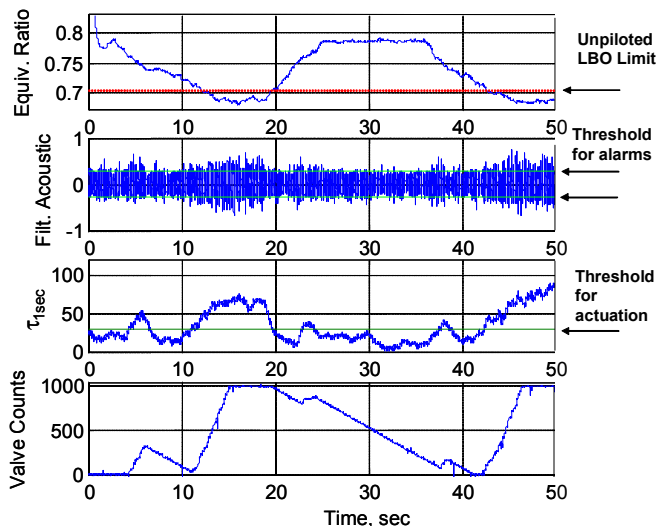
### C. Control Results

The control system should adequately sense precursors, actuate to improve safety margin, and reduce the probability of LBO for a desired equivalence ratio setting. To investigate the performance of the control system, the air flow to the combustor was varied over time manually to test the control system response to external disturbances.

Figure 10 shows the time dependence of various parameters associated with the control system's response to varying air flow rate. The top plot shows how the equivalence ratio varies as air flow changes with time. The next two plots are the filtered acoustic signal and the alarm count signal. As the filtered acoustic signal crosses thresholds, alarms are counted. The alarm sum over a previous 1 second window is the overall alarm level ( $\tau_{1sec}$ ), which determines the actuation (valve counts, bottom plot of Figure 10). Valve counts are given in whole numbers from 0 to 1000, with 1000 corresponding to 10 valves open and 20% pilot fuel fraction. Each 100 counts correspond to a fully open valve, and the remainder fractions are sent as pulse width modulation (PWM) commands to the next valve or valves.

The results show that as valves are opened to increase the pilot fuel split, the alarms subside. This indicates the flame was made more stable (i.e., safety margin increased) as a result of the actuation. Also evident are the two logic modes. When excessive alarms are detected, the actuation rises rapidly. In contrast, as the flame becomes richer, actuation declines at a steady rate.

The control system has thus been shown to capably detect the onset of LBO and command the appropriate actuation only when necessary. Piloting can now be utilized to operate a flame closer to its LBO limit and reduce emissions.



**Figure 10. Results from varying the air flow rate to the combustor. The top plot shows the equivalence ratio variation, the second plot shows the filtered acoustic signal with thresholds, the third plot shows the number of threshold crossings over the past 1 second, and the bottom plot displays the valve actuation, with 1000 corresponding to 20% pilot.**

## VII. Conclusion

A viable control system was developed for LBO mitigation, thus allowing for lean operation at lower temperatures and lower NOx emissions. The acoustic signal was used to detect LBO precursor 'events' by taking into consideration that leaner flames emit more acoustic power at low frequencies (<200 Hz). This low frequency rise is mainly the result of an increasing number of localized extinction-reignition 'events'.

The study showed that these events are amplified in an acoustic signal bandpassed from 10.6-95.5 Hz. The filtered signal can be compared with a double threshold (for noise rejection) to determine the frequency of threshold crossings. The alarms per second parameter ( $\tau_{1sec}$ ) summed the total threshold crossings over the previous 1 second in a moving window. It was further determined that  $\tau_{1sec}$  is a reliable gauge of flame stability, regardless of piloting conditions.

This study also verified that splitting the total fuel between a swirling, premixed annular port and a central, premixed pilot in increasing pilot fractions can improve safety margin by shifting  $\phi_{LBO}$  to leaner values. Although NOx at a given equivalence ratio increases with pilot, piloting enables safe operation at lower equivalence ratios,

thereby minimizing overall NO<sub>x</sub><sup>6</sup>. Hence, the need for comprehensive control strategies which provide actuation only to improve safety margin when needed to allow for overall lean, low NO<sub>x</sub> combustion.

An LBO mitigation control strategy was developed with two logic branches. One branch allowed for rapid pilot response to decreasing equivalent ratio, while the other branch, in the absence of precursors, gradually reduced pilot fraction to minimize NO<sub>x</sub>. The resulting system was shown to stabilize a flame driven to a level leaner than its unpiloted LBO limit.

The described technique can allow for lean, low NO<sub>x</sub> combustor operation with a nominally reduced safety margin. The control system can boost safety margin (by increasing the fraction of total fuel sent to the pilot) as needed when flame disturbances are present, and reduce this actuation when it is no longer needed.

### Acknowledgments

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