Acoustical wave propagation in cylindrical ducts: Transmission line parameter approximations for isothermal and nonisothermal boundary conditions

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Approximate expressions are given for the characteristic impedance and propagation wavenumber for linear acoustic transmission through a gas enclosed in a rigid cylindrical duct. These expressions are most complicated in the transition zone where the thermoviscous boundary layers are on the order of the tube radius. The approximations are accurate to within 1% for all frequencies and tube diameters except within the transition zone where the approximations are accurate to within 10%. A simple modification of the transmission line parameters is presented for the case where the tube walls are nonisothermal.

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INTRODUCTION

The subject of sound propagation in a rigid cylindrical tube has a long history. Following the work of Stokes,[1] Helmholtz,[2] and others on the effect of viscosity on sound propagation in tubes, Kirchhoff[3] included the effects of both viscosity and heat conduction on acoustic propagation through a rigid cylindrical tube. He assumed that the oscillatory flow is laminar, the particle displacement is sufficiently small that all nonlinear terms in the equations of motion can be neglected, and the inner tube wall is isothermal.

Many applications in acoustics require calculation of the oscillatory pressure/flow relationships within a cylindrical tube. Thus it is helpful to find accurate aproximations to Kirchhoff's exact solution which can be computed easily. Rayleigh[4] was one of the first to obtain useful approximations. One case was that of the very narrow tube, or, alternatively, the acoustic boundary layer large relative to the tube radius. Both Rayleigh and Kirchhoff discussed the opposite case where the acoustic boundary layer is much smaller than the tube radius. More recently, Brown[5] extended this approximation to frequencies such that the acoustic boundary layer is on the order of or smaller than the tube radius. Benade[6] has given an approximate form of the solution which spans the entire frequency range. However, Backus[7] has pointed out errors in the approximations used by Benade.

This article sets out an approximate form of the transmission line parameters for thermoviscous propagation in rigid cylindrical ducts. The difficulty in approximating the exact solution arises in the transition region where the viscous and thermal boundary layers are the same order of magnitude as the tube radius. However, the approximations in Sec. I have sufficient accuracy so that the error does not exceed 10% at any frequency. Outside this transition region, the errors are within 1%.

Many experiments have demonstrated the validity of Kirchhoff's theory. The main deviations between theory and experiment are due to acoustic nonlinearities, nonrigidity of the tube walls, roughness on the inner wall surface, and the fact that the tube walls do not remain isothermal.

The influence of nonlinear acoustics on propagation in tubes has been recently discussed by Crighton[8] and Nakamura, Takeuchi, and Oie.[9] Regarding thermal effects, the validity of Kirchhoff's assumption that the inner tube wall remains at a constant ambient temperature has been clarified by later investigators. In a study of nonlinear wave propagation in cylindrical tubes, Mawardi calculated and measured the increase in the mean temperature of the inner wall surface and the enclosed gas.[10] Franken, Clement, Caubergһs, and van de Woestijne have extended Kirchhoff's linearized solution to take accout of the oscillating temperature at the inner wall surface.[11] A simplified form of their solution which is sufficiently accurate for most applications is derived in Sec. II.

I. ISOTHERMAL TUBE WALL

Consider a smooth cylindrical duct of radius a with rigid walls whose axis extends along the z axis. It is assumed that the frequency is sufficiently low that there is only a single mode which propagates energy over axial distances large relative to the tube diameter. The pressure $p(z)$ and volume flow $u(z)$ for thermoviscous wave propagation in the tube assuming that the tube walls are isothermal are

$$\frac{\partial p}{\partial z} = -Zu, \tag{1a}$$

$$\frac{\partial u}{\partial z} = -Yp, \tag{1b}$$

where the series impedance Z and shunt admittance Y per unit length along the z axis of the acoustic tube regarded as a transmission line are

$$Z = j(\omega/c)R_0/(1-F_v), \tag{2a}$$

$$Y = j[(\omega/c)/R_0][1+(\gamma-1)F_t]. \tag{2b}$$

A time dependence $e^{j\omega t}$ is assumed for all oscillatory quantities. The radian frequency is ω, the free-space speed of sound is c, and the quantity R_0 is defined to be

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$$R_0 = \rho c/\pi a^2, \tag{3}$$

where the equilibrium gas density is ρ. In the absence of thermal and viscous dissipation, R_0 is the characteristic impedance of the acoustic transmission line.

The quantity F_v in Eq. (2a) is given by Kirchhoff's solution and is

$$F_v = \frac{2}{r_v\sqrt{-j}} \frac{J_1(\sqrt{-j}r_v)}{J_0(\sqrt{-j}r_v)}, \tag{4}$$

where J_0 and J_1 are Bessel functions and the dimensionless parameter r_v is

$$r_v = a(\rho\omega/\eta)^{1/2}. \tag{5}$$

Hence r_v may be regarded either as a nondimensional frequency or tube radius. The shear viscosity coefficient is η. The quantity F_t in Eq. (2b) is

$$F_t = \frac{2}{r_t\sqrt{-j}} \frac{J_1(\sqrt{-j}r_t)}{J_0(\sqrt{-j}r_t)}, \tag{6}$$

where

$$r_t = \nu r_v \tag{7a}$$

such that ν is the square root of the Prandtl number as follows:

$$\nu = (\eta C_p/\kappa)^{1/2}. \tag{7b}$$

The coefficient of specific heat of the gas at constant pressure is C_p, and the gas thermal conductivity is κ.

Physically, the dimensionless quantities r_v and r_t are essentially the ratios of the tube radius to the viscous and thermal boundary layers, respectively. The use of r_v as the defining symbol for the ratio in Eq. (5) follows the notation of Ref. 6. There is no clear concensus in the acoustic literature for either a symbol or name to refer to this ratio. This ratio is called the Womersley parameter in the literature of biofluid mechanics.[12] A thermodynamically consistent set of gas parameters for air at standard pressure in the temperature range 290°–310°K has been collected by Benade[6] and relevant values are reproduced in Table I.

It is convenient to express the transmission line properties in terms of a characteristic impedance and propagation wavenumber. The characteristic impedance Z_c of the transmission line is defined to be the input impedance looking into a infinite length of cylindrical tubing and is

$$Z_c = (Z/Y)^{1/2}. \tag{8}$$

The propagation wavenumber Γ of the transmission line is defined to be the phase change per unit length at an arbitrary fixed time along the semi-infinite cylindrical tube and is

$$\Gamma = \alpha + j(\omega/v_p) = (ZY)^{1/2}. \tag{9}$$

The real and imaginary parts of the propagation wavenumber are α and (ω/v_p), respectively, and the phase velocity is v_p. It is convenient to express the series impedance and shunt admittance as

$$Z = j\omega L + R, \tag{10a}$$

$$Y = j\omega C + G. \tag{10b}$$

The series inertance (per unit length of the transmission line) is L, the series resistance is R, the shunt compliance is C, and the shunt conductance is G; all these quantities are real and positive.

The intent of this section is to collect in one place the various approximations to the exact values of these transmission line parameters, and compare the accuracy of the approximations. The limit of small r (i.e., small r_v or r_t) corresponds to the low frequency or small tube limit, and the extreme limit $r_v = 0$ corresponds to steady viscous flow in a cylindrical tube (Poiseuille flow). The small-r approximation is obtained by means of a truncated power series expansion of the Bessel functions in Eq. (4) which are strictly valid for r sufficiently less than unity. However, these approximations are reasonably accurate for $r < 2$ as will be seen.

The limit of large r corresponds to the high frequency or large tube limit. The extreme limit $r_v = r_t = \infty$ corresponds to dissipationless wave propagation in the tube with a characteristic impedance R_0 and a phase velocity c. The large-r approximation[5] is computed using an asymptotic expansion of the Bessel functions which converges asymptotically to the exact result as r assumes large values.

The small-r approximation gives the following results for the transmission line parameters:

$$R = 8\eta/\pi a^4, \tag{11a}$$

$$\omega L = \tfrac{4}{3}\rho\omega/\pi a^4, \tag{11b}$$

$$G = (\pi a^2\omega^2/8\rho c^2)(\gamma - 1)\rho C_p/\kappa[1 - (13/384)r_t^4], \tag{11c}$$

$$\omega C = (\pi a^2\omega/\rho c^2)\gamma, \tag{11d}$$

$$\text{Re } Z_c = R_0\left(\frac{2}{\sqrt{\gamma}}\right) r_v^{-1}\left\{1 + \frac{1}{2}\left[\left(\frac{r_v^2}{6}\right) + \frac{(\gamma-1)}{\gamma}\left(\frac{r_t^2}{8}\right)\right]\right\}, \tag{11e}$$

$$-\text{Im } Z_c = R_0\left(\frac{2}{\sqrt{\gamma}}\right) r_v^{-1}\left\{1 - \frac{1}{2}\left[\left(\frac{r_v^2}{6}\right) + \frac{(\gamma-1)}{\gamma}\left(\frac{r_t^2}{8}\right)\right]\right\}, \tag{11f}$$

$$\alpha = \left(\frac{\omega}{c}\right)\left(\frac{2\sqrt{\gamma}}{r_v}\right)\left\{1 - \frac{1}{2}\left[\left(\frac{r_v^2}{6}\right) - \frac{(\gamma-1)}{\gamma}\left(\frac{r_t^2}{8}\right)\right]\right\}, \tag{11g}$$

$$v_p^{-1} = c^{-1}\left(\frac{2\sqrt{\gamma}}{r_v}\right)\left\{1 + \frac{1}{2}\left[\left(\frac{r_v^2}{6}\right) - \frac{(\gamma-1)}{\gamma}\left(\frac{r_t^2}{8}\right)\right]\right\}. \tag{11h}$$

TABLE I. Thermodynamic constants. All of the below are evaluated at $T = 26.85$ °C (300 °K), and are accurate within ± 10 °C of that temperature. The temperature difference relative to 26.85 °C is ΔT.

$\rho = 1.1769 \times 10^{-3}(1 - 0.00335\Delta T)$ g cm^{-3}
$\eta = 1.846 \times 10^{-4}(1 + 0.0025\Delta T)$ g s^{-1} cm^{-1}
$\gamma = 1.4017(1 - 0.00002\Delta T)$
$\nu = 0.8410(1 - 0.0002\Delta T)$
$c = 3.4723 \times 10^4(1 + 0.00166\Delta T)$ cm s^{-1}

The power series expansions in the above equations have been truncated so as to give the best fit in the transition region, where r_v is of order unity. In particular, a term of order r_t^4 is kept in the expansion of G to improve the accuracy.

The large-r approximation may be written in the form,

$$R = R_0\left(\frac{\omega}{c}\right)\sqrt{2}r_v^{-1}\left[1+\left(\frac{3}{\sqrt{2}}\right)r_v^{-1}+\left(\frac{15}{8}\right)r_v^{-2}\right], \quad (12a)$$

$$\omega L = R_0(\omega/c)\left[1+\sqrt{2}r_v^{-1}-(1/\sqrt{2})r_v^{-3}\right], \quad (12b)$$

$$G = [(\omega/c)/R_0]\sqrt{2}(\gamma-1)r_t^{-1}\left[1-(1/\sqrt{2})r_t^{-1}-r_t^{-2}/8\right], \quad (12c)$$

$$\omega C = [(\omega/c)/R_0]\left[1+(\gamma-1)\sqrt{2}r_t^{-1}(1+r_t^{-2}/8)\right], \quad (12d)$$

$$\begin{aligned}\mathrm{Re}\, Z_c = R_0\Bigg\{1 &+ \left(\frac{r_v^{-1}}{\sqrt{2}}\right)\left(1-\frac{(\gamma-1)}{\nu}\right) - \left(\frac{r_v^{-3}}{\sqrt{2}}\right)\\ &\times\left[\frac{7}{8}-\frac{(\gamma-1)}{\nu}+\frac{1}{2}\frac{(\gamma-1)}{\nu^2}+\frac{1}{8}\frac{(\gamma-1)}{\nu^3}\right.\\ &+\frac{3}{2}\left(\frac{(\gamma-1)}{\nu}\right)^2\\ &\left.-\frac{3}{2}\frac{(\gamma-1)^2}{\nu^3}-\frac{5}{2}\left(\frac{(\gamma-1)}{\nu}\right)^3\right]\Bigg\}, \end{aligned} \quad (12e)$$

$$\begin{aligned}-\mathrm{Im}\, Z_c = R_0\Bigg\{&\left(\frac{r_v^{-1}}{\sqrt{2}}\right)\left(1-\frac{(\gamma-1)}{\nu}\right)+r_v^{-2}\left[1-\frac{(\gamma-1)}{\nu}\right.\\ &\left.+\frac{1}{2}\frac{(\gamma-1)}{\nu^2}+\frac{3}{2}\left(\frac{(\gamma-1)}{\nu}\right)^2\right]\\ &+\left(\frac{r_v^{-3}}{\sqrt{2}}\right)\left[\frac{7}{8}-\frac{(\gamma-1)}{\nu}+\frac{1}{2}\frac{(\gamma-1)}{\nu^2}\right.\\ &+\frac{1}{8}\frac{(\gamma-1)}{\nu^3}+\frac{3}{2}\left(\frac{(\gamma-1)}{\nu}\right)^2\\ &\left.-\frac{3}{2}\frac{(\gamma-1)^2}{\nu^3}-\frac{5}{2}\left(\frac{(\gamma-1)}{\nu}\right)^3\right]\Bigg\}, \end{aligned} \quad (12f)$$

$$\begin{aligned}\alpha = \left(\frac{\omega}{c}\right)\Bigg\{&\left(\frac{r_v^{-1}}{\sqrt{2}}\right)\left(1+\frac{(\gamma-1)}{\nu}\right)+r_v^{-2}\left[1+\frac{(\gamma-1)}{\nu}\right.\\ &\left.-\frac{1}{2}\frac{(\gamma-1)}{\nu^2}-\frac{1}{2}\left(\frac{(\gamma-1)}{\nu}\right)^2\right]\\ &+\left(\frac{r_v^{-3}}{\sqrt{2}}\right)\left[\frac{7}{8}+\frac{(\gamma-1)}{\nu}-\frac{1}{2}\frac{(\gamma-1)}{\nu^2}\right.\\ &-\frac{1}{8}\frac{(\gamma-1)}{\nu^3}-\frac{1}{2}\left(\frac{(\gamma-1)}{\nu}\right)^2\\ &\left.+\frac{1}{2}\frac{(\gamma-1)^2}{\nu^3}+\frac{1}{2}\left(\frac{(\gamma-1)}{\nu}\right)^3\right]\Bigg\}, \end{aligned} \quad (12g)$$

$$\begin{aligned}v_p^{-1} = c^{-1}\Bigg\{1&+\left(\frac{r_v^{-1}}{\sqrt{2}}\right)\left(1+\frac{(\gamma-1)}{\nu}\right)\\ &-\left(\frac{r_v^{-3}}{\sqrt{2}}\right)\left[\frac{7}{8}+\frac{(\gamma-1)}{\nu}-\frac{1}{2}\frac{(\gamma-1)}{\nu^2}\right.\\ &-\frac{1}{8}\frac{(\gamma-1)}{\nu^3}-\frac{1}{2}\left(\frac{(\gamma-1)}{\nu}\right)^2\\ &\left.+\frac{1}{2}\frac{(\gamma-1)^2}{\nu^3}+\frac{1}{2}\left(\frac{(\gamma-1)}{\nu}\right)^3\right]\Bigg\}. \end{aligned} \quad (12h)$$

Note in Eqs. (12a)–(12d) that R and L depend only on the viscous coefficient (r_v), and G and C depend only on the thermal coefficient (r_t). In Eqs. (12e)–(12h) the characteristic impedance and propagation wavenumber include both viscous and thermal effects. The thermal coefficient r_t has been replaced by νr_v in this latter set of equations. It is convenient to rewrite these equations for Z_c and Γ for the special case of air at 300 °K and atmospheric pressure using the values in Table I as follows ($r_v > 2$):

$$\mathrm{Re}\, Z_c = R_0(1+0.369r_v^{-1}), \quad (13a)$$

$$-\mathrm{Im}\, Z_c = R_0(0.369r_v^{-1}+1.149r_v^{-2}+0.303r_v^{-3}), \quad (13b)$$

$$\alpha = (\omega/c)(1.045r_v^{-1}+1.080r_v^{-2}+0.750r_v^{-3}), \quad (13c)$$

$$v_p^{-1} = c^{-1}(1+1.045r_v^{-1}). \quad (13d)$$

The perturbation series approximation is mathematically valid in the region $r \ll 1$ and the asymptotic expansion is valid in the region $r^{-1} \ll 1$. One difference between Eqs. (12) and (13) is that the order r_v^{-3} terms in the real part of the characteristic impedance and the phase velocity have been

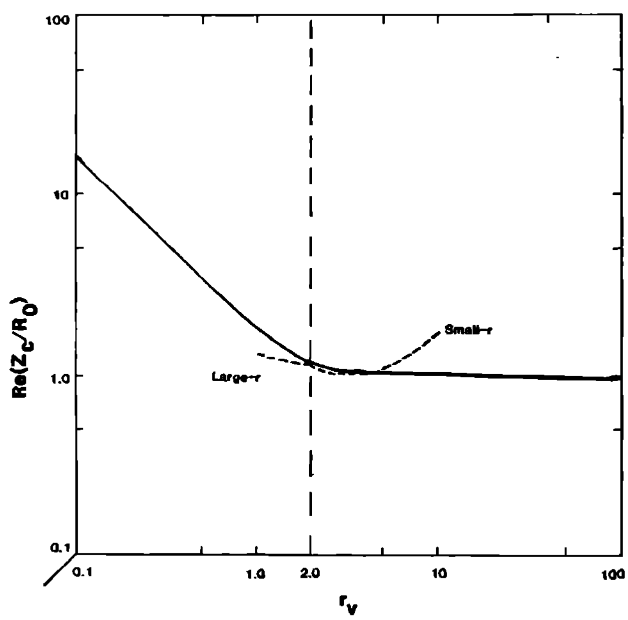

FIG. 1. The real part of the characteristic impedance normalized to the duct area as a function of the dimensionless parameter r_v.

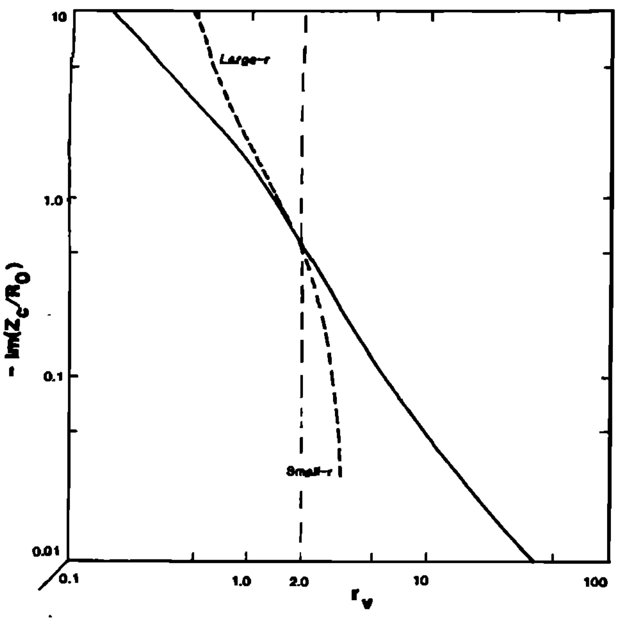

FIG. 2. The imaginary part of the characteristic impedance normalized to the duct area as a function of the dimensionless parameter r_υ.

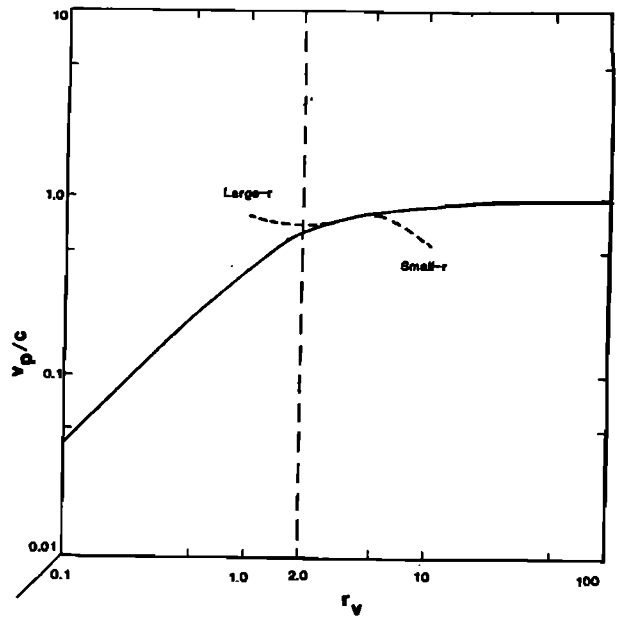

FIG. 4. The phase velocity normalized by the free space speed of sound as a function of the dimensionless parameter r_υ.

omitted in Eq. (13) to improve the accuracy. This is an example of a general property of asymptotic series expansions[13]; namely, that an asymptotic expansion with a fewer number of terms may lead to more accurate results.

It is fortuitous that the small-r approximation works well for r larger than unity, and the large-r approximation is reasonably accurate near unity as Figs. 1–4 bear out. Figures 1 and 2 show the real and imaginary parts of (Z_c/R_0) as a function of r_υ. The exact result as computed from tables of Bessel functions is shown in solid lines, and the small and large-r approximations are shown in dotted lines. Similarly, Figs. 3 and 4 show the normalized attenuation coefficient $\alpha/(\omega/c)$ and phase velocity (υ_p/c). These figures are plotted assuming that the gas parameters $\gamma = 1.402$ and $\nu = 0.841$, as is the case for air.

These figures show that the approximations agree to within 1% with the exact results except in the region $1 < r_\upsilon < 5$ where some discrepancies exist—the maximum error in $\mathrm{Re}(Z_c)$ is 3%, in $\mathrm{Im}(Z_c)$ is 10%, in α is 6%, and in υ_p is 2%. Figures 1–4 imply that the value $r_\upsilon = 2$ is a good value to use for the transition betweeen the small and large-r approximations.

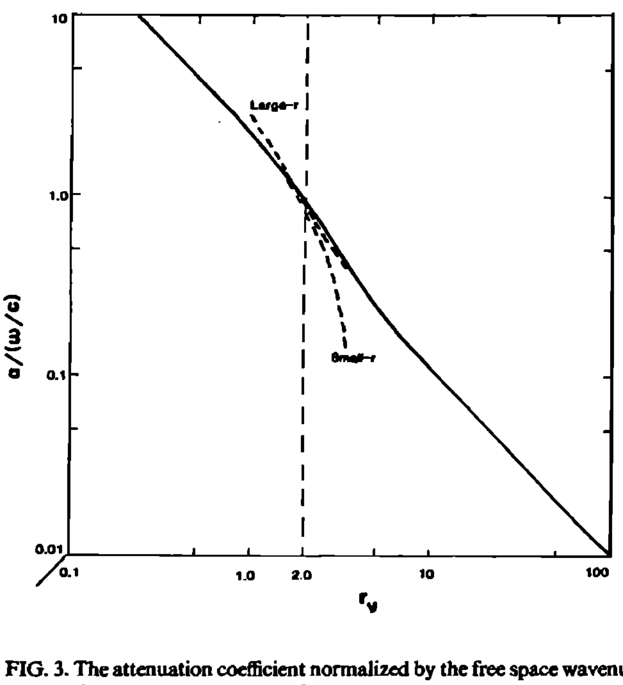

FIG. 3. The attenuation coefficient normalized by the free space wavenumber as a function of the dimensionless parameter r_υ.

II. NONISOTHERMAL INNER TUBE WALL

The pressure/flow characteristics of a thermoviscous gas in a rigid tube has been discussed under the condition that the inner tube wall remains isothermal during the oscillatory cycle. This is never the case, since a local heating of the gas transfers heat to the tube wall. Since any tube has a finite capacity to carry away the heat, there is a local rise in temperature at the tube wall. Intuitively, this decreases the thermal gradients in the boundary layer of the gas, and thus the heat losses are somewhat reduced compared to the idealized model wherein the inner tube walls are assumed to be isothermal. Franken *et al.* have considered the case where the cylindrical tube has a wall thickness such that the temperature on the outside of the tube wall is isothermal. An accurate asymptotic expansion of their solution is described in this section.

The tube wall material has a mass density ρ_w, a specific heat C_w, and a thermal conductivity κ_w. Recall from the previous section that the series impedance per unit length of the acoustic transmission line is affected only by the viscous losses and the shunt admittance is affected only by thermal losses. Thus a change in the thermal boundary conditions modifies only Y and leaves Z unchanged. The modified shunt admittance is,[11]

$$Y = j(\omega/c)/R_0 \times \left\{1 + (\gamma - 1)F_t \Big/ \left[1 - j\left(\frac{\rho C_p \kappa}{\rho_w C_W \kappa_w}\right)^{1/2} \frac{J_1(\sqrt{-j}r_t)}{J_0(\sqrt{-j}r_t)}\right]\right\}. \quad (14)$$

The magnitude of the modification depends on the dimensionless number ϵ_w defined as

$$\epsilon_w = (\rho C_p \kappa / \rho_w C_w \kappa_w)^{1/2}. \quad (15)$$

It is generally the case that the parameter ϵ_w governs the oscillatory heat transfer characteristics of a solid–gas interface.[14,15] If the gas is air and the tube wall is polyvinylchloride tubing, then ϵ_w is on the order of 0.01. In nearly all combinations of gas and tube wall materials, this ratio is always very small compared to unity. Therefore, a power series expansion in ϵ_w is made along with an asymptotic expansion of the Bessel functions which is valid for large r_v with the result that

$$Y \sim j(\omega/c)/R_0 \times \left[1 + \frac{(\gamma - 1)F_t}{(1 + \epsilon_w)}\left(1 - \frac{1}{2}\sqrt{-j}\,r_v^{-1}(1 - \epsilon_w)\right)\right]. \quad (16)$$

The shift in the coefficient of the term of order r_v^{-1} as well as higher order powers of $(1/r_v)$ due to the presence of ϵ_w is negligibly small, so that Eq. (16) may be rewritten in terms of an effective specific heat γ_e as follows:

$$Y = j(\omega/c)/R_0[1 + (\gamma_e - 1)F_t], \quad (17a)$$

where,

$$(\gamma_e - 1) = [(\gamma - 1)/(1 + \epsilon_w)]. \quad (17b)$$

The large-r approximation to the isothermal wall transmission line parameters corresponding to the case of high frequency or large tubing diameter can easily be modified to include the nonisothermal wall behavior. One need only substitute γ_e for γ in all the expressions in Eq. (12). Franken *et al.* found that only the real part of the shunt admittance is modified in any significant manner due to the presence of temperature fluctuations in the tube wall; the imaginary part of the shunt admittance is not significantly shifted. This is consistent with the large-r_v behavior of Eq. (17), or equivalently Eqs. (12c)–(12d) with γ replaced by γ_e.

An approximation has been obtained in the limit of high frequencies to the solution for thermoviscous wave propagation in a rigid cylindrical tube which is constrained to be isothermal on its outside wall. The criterion of sufficiently high frequency is that the dimensionless parameter r_v defined in Eq. (5) be larger than two. The calculational advantage of this approximation is that the high-frequency (or large tube) form of the transmission line parameters may be simply computed by means of a single modification of the isothermal wall transmission line parameters.

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