

Active Balancing and Vibration Control of Rotating Machinery: A Survey

Shiyu Zhou and Jianjun Shi

ABSTRACT—Vibration suppression of rotating machinery is an important engineering problem. In this paper, a review of the research work performed in real-time active balancing and active vibration control for rotating machinery, as well as the research work on dynamic modeling and analysis techniques of rotor systems, is presented. The basic methodology and a brief assessment of major difficulties and future research needs are also provided.

1. Introduction

Rotating machinery is commonly used in mechanical systems, including machining tools, industrial turbomachinery, and aircraft gas turbine engines. Vibration caused by mass imbalance is a common problem in rotating machinery. Imbalance occurs if the principal axis of inertia of the rotor is not coincident with its geometric axis. Higher speeds cause much greater centrifugal imbalance forces, and the current trend of rotating equipment toward higher power density clearly leads to higher operational speeds. For example, speeds as high as 30,000 rpm are not rare in current high-speed machining applications. Therefore, vibration control is essential in improving machining surface finish; achieving longer bearing, spindle, and tool life in high-speed machining; and reducing the number of unscheduled shutdowns. A great cost savings for high-speed turbines, compressors, and other turbomachinery used in petrochemical and power generation industries can be realized using vibration control technology.

It is well established that the vibration of rotating machinery can be reduced by introducing passive devices into the system (Cunningham, 1978; Nikolajsen and Holmes, 1979). Although an active control system is usually more complicated than a passive vibration control scheme, an active vibration control technique has many advantages over a passive vibration control technique. First, active vibration control is more effective than passive vibration control in general (Fuller et al., 1996). Second, the passive vibration control is of limited use if several vibration modes are excited. Finally, because the active

actuation device can be adjusted according to the vibration characteristic during the operation, the active vibration technique is much more flexible than passive vibration control. The main purpose of this paper is to review and reevaluate the active vibration control techniques for rotating machinery and shed some light on future research directions.

There are two major categories in active vibration control techniques for rotating machinery: direct active vibration control (DAVC) techniques directly apply a lateral control force to the rotor, and active balancing techniques adjust the mass distribution of a mass redistribution actuator. The control variable in DAVC techniques is a lateral force generated by a force actuator such as the magnetic bearing. The advantage of DAVC techniques is that the input control force to the system can be changed quickly. By applying a fast-changing lateral force to the rotating machinery, the total vibration, including the synchronous vibration, the transient-free vibration, and other non-synchronous vibration of the rotating machinery, can be suppressed. The limitation of most force actuators is the maximum force they can provide. In high rotating speed, the imbalance-induced force could reach a very high level. Most force actuators cannot provide sufficient force to compensate for this imbalance-induced force. Under this condition, active balancing methods can be used. In active balancing methods, a mass redistribution actuator (namely, whose mass center can be changed) is mounted on the rotor. After the vibration of the rotating system is measured and the imbalance in the rotating machinery is estimated, the mass center of the actuator is changed to offset the system imbalance. The vibration of the rotating machinery is suppressed by eliminating the root cause of the vibration—system imbalance. Contrary to the force actuator, the mass redistribution actuator can provide large compensating force. However, the speed of the mass redistribution actuator is slow. Although active balancing methods can eliminate imbalance-induced synchronous vibration, they cannot suppress transient vibration and other nonsynchronous vibration.

In this paper, both DAVC and active balancing techniques are reviewed. Because the mathematical model is the foundation of any active vibration control technique, a review of relevant dynamic modeling techniques of rotating machinery is also included in this paper for completeness. The review of dynamic modeling and analysis of rotor systems is presented in Section 2. Section 3 presents a review of DAVC and active balancing techniques. The

Shiyu Zhou, Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109. Jianjun Shi, Industrial and Operations Engineering Department, University of Michigan, Ann Arbor, MI 48109, USA.

limitations of current technologies and future research directions are discussed in Section 4.

2. Dynamic Modeling and Analysis of Rotor Systems

The planar rotor model is the simplest rotor model. Only the motion in the plane, which is perpendicular to the rotating shaft, is considered. The geometric setup of the planar rotor model is shown in Figure 1.

In this model, the imbalance-induced vibration is described by the particle motion of the geometric center of the disk. P is the geometric center of the disk, and G is the mass center of the disk. The motion is represented by the vector \mathbf{r} . It is well known that the governing equation of motion is (Childs, 1993)

$$\begin{aligned} m\ddot{r}_x + c\dot{r}_x + kr_x &= ma_x\dot{\phi}^2 + ma_z\ddot{\phi}, \\ m\ddot{r}_z + c\dot{r}_z + kr_z &= ma_z\dot{\phi}^2 - ma_x\ddot{\phi}, \end{aligned} \quad (1)$$

where m , c , and k are the mass, the viscous damping coefficient, and the shaft-stiffness coefficient, respectively. $[a_x, a_z]$ is the vector from P to G , expressed in the stationary coordinate system. ϕ is the rotating angle of the rotor. For a constant rotating speed, $\dot{\phi}$ is zero. Although the planar rotor is a very simple rotor model, it can be used to study the basic phenomena in rotor dynamics such as critical speed, the effect of damping, and so on.

The planar rotor model is a special case of the Jeffcott model that was first introduced in 1919. In the Jeffcott

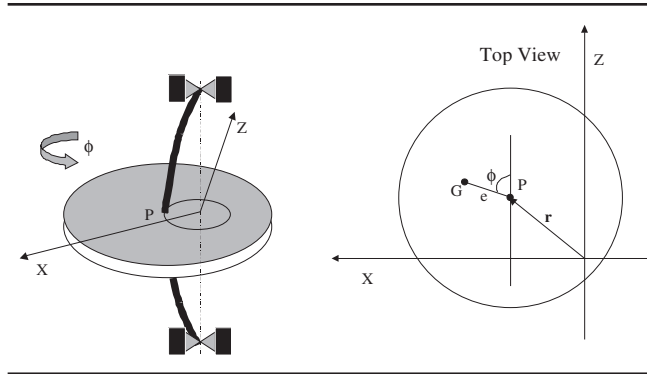


Figure 1. Geometric Setup of the Planar Rotor

In this setup, bearings are modeled as isotropic linear spring and damper. The imbalance is modeled as concentrated mass on the rigid shaft. Two coordinate systems are used: the body-fixed coordinate $oxyz$ and the inertial coordinate $OXYZ$. The body-fixed y -axis is the rotating axis of the shaft, and x and z axes are defined by the other two principal inertia axes of the rotor. The origin of xyz is selected as the geometric center of the shaft. The XYZ coordinate system is the stationary coordinate and coincides with the xyz coordinate system when the body is at rest. The transverse motion of the rotor is described by the position of the geometric center $[R_x R_z]$ and by the orientation of the rigid shaft with respect to the X and Z axes $[\theta \psi]$. A simplified governing equation in state space form is shown as follows (Zhou and Shi, 2000):

$$\frac{d}{dt} \begin{bmatrix} R_x \\ R_z \\ \theta \\ \psi \\ \dot{R}_x \\ \dot{R}_z \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2k}{m} & 0 & 0 & 0 & -\frac{2c}{m} & 0 & 0 & 0 \\ 0 & -\frac{2k}{m} & 0 & 0 & 0 & -\frac{2c}{m} & 0 & 0 \\ 0 & 0 & -\frac{kL^2}{2I_t} & \frac{I_p\ddot{\phi}}{I_t} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{I_p\ddot{\phi}}{I_t} & -\frac{kL^2}{2I_t} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R_x \\ R_z \\ \theta \\ \psi \\ \dot{R}_x \\ \dot{R}_z \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{-m_u u_z}{m} & \frac{m_u u_x}{m} \\ \frac{m_u u_x}{m} & \frac{m_u u_z}{m} \\ \frac{m_u u_x u_y}{m} & \frac{m_u u_y u_z}{m} \\ \frac{I_t}{I_t} & \frac{I_t}{I_t} \\ \frac{m_u u_y u_z}{I_t} & \frac{-m_u u_x u_y}{I_t} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}, \quad (2)$$

model, the rotor was modeled as a rigid disk supported by a massless elastic shaft that was mounted on fixed rigid bearings. This model is also equivalent to a rigid shaft supported by elastic bearings. The major improvement over the simple planar rotor model is that the motion of the rotor is depicted by rigid body motion instead of by particle motion. Although this is a single rigid body model, it can show the basic phenomena in the motion of the rotor, including the forward and backward whirling under imbalance force, critical speeds, the gyroscopic effect, and so on. The fact that the natural frequency is a function of the rotating speed can be predicted by this model. A typical geometric setup of this model is shown in Figure 2.

where L is the length of the shaft; I_p and I_t are the polar and the diametric moments of inertia of the shaft, respectively; m_u, u_x, u_y, u_z are the mass and the position of the imbalance in body-fixed coordinate; and $f_1 = \ddot{\phi} \cos \phi^2 \sin \phi$, $f_2 = \ddot{\phi} \sin \phi + \dot{\phi}^2 \cos \phi$. This model can be used on a real system provided that the rigidity of the shaft is high compared to the supporting bearing. The gyroscopic effect can be studied by this model.

For a more complicated rotor system, a flexible rotor model was developed. This model allows for the elastic deformation of the rotor during rotation. Certainly, it is more accurate than the rigid rotor model. Breaking down a complex system into many simpler components that are easy to

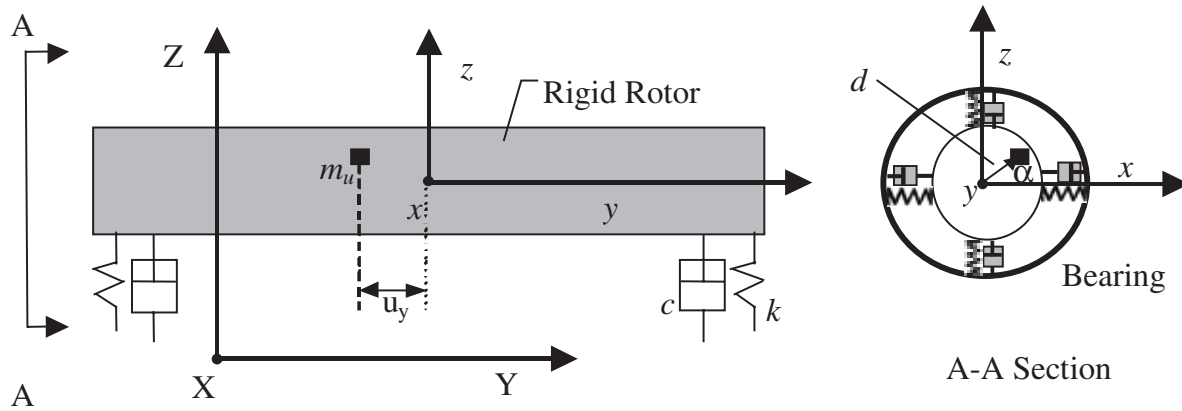


Figure 2. Geometric Setup of the Rigid Rotor Model

analyze is very common in engineering applications. A complicated rotor system is divided into several kinds of basic elements: rigid disk, bearing (usually linear modeled), flexible shaft segments, couplings, squeeze-film dampers, and so on. The equations of motion for each of these components were developed using the appropriate force-displacement and force-velocity relations and the momentum principles or other equivalent dynamic relations. Then, the system equations were assembled using geometric displacement constraints that guaranteed the connectivity of the components.

There are two kinds of assembly procedures: the finite element method and the transfer matrix method. Ruhl and Booker (1972) used a finite element model to study the dynamic characteristic of a turbo rotor. In their model, only elastic bending and translational kinetic energy were included, whereas the effects of rotatory inertia, gyroscopic effects, shear deformation, axial torque, axial load, and internal damping were neglected. Dimarogonas (1975) presented a more general model that included rotatory inertia, gyroscopic effects, and internal damping. Gasch (1976) presented a model that was similar to Dimarogonas's but included the effect of distributed eccentricity. At the same time, Nelson and McVaugh (1976) published their model that included rotatory inertia, gyroscopic moments, and axial load. The detailed equations for the elements are expressed in both a fixed and a rotating reference frame. Their work was generalized by Zorzi and Nelson (1977), who included internal damping. Nelson (1980) presented a model that included the shear deformation effects. In general, the governing equation of motion of a flexible rotor can be written as (Lalanne and Ferraris, 1998)

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}(\phi), \quad (3)$$

where \mathbf{q} is the generalized coordinate to describe the motion and \mathbf{M} , \mathbf{C} , and \mathbf{K} are coefficient matrices. The dimensions of these matrices are determined by the number of nodes in the model.

For a complicated rotor-bearing-foundation system, the system matrices in the governing equations are very large. The computer memory storage requirements and computation time will be large. Therefore, most of the recent research on finite element methods for rotor dynamics has

been designed to reduce the order or to reorder the system equations to achieve better computational efficiency while maintaining accuracy. The works of Shiau and Hwang (1989) and of Nelson and Chen (1993) are particularly noteworthy. Their work proposed a modeling procedure using assumed modes to reduce the order of the system matrices. Childs and Graviss (1982) and Chen (1998) used different reordering techniques to increase computational efficiency.

The other important method in rotor dynamics analysis is the transfer matrix method. This method is particularly well suited for "chainlike" structures. It was first used in the area of torsional vibrations. Lund (Lund and Orcutt, 1967; Lund, 1974a, 1974b) presented procedures that use this method for rotor dynamics analysis. The advantage of the transfer matrix method is that it does not require the storage and manipulation of large system arrays. The transfer matrix method uses a marching procedure: it begins with the boundary conditions at one side of the system and successively marches along the structure to the other side. The solution should satisfy all the boundary conditions at all boundary points. The disadvantage of this method is that it is difficult, although not impossible (Kumar and Sankar, 1984), to extend to time domain and nonlinear analysis. Therefore, it is difficult to conduct active balancing controller design using the transfer matrix method.

All methods mentioned above focus on linear systems, which means the system equations are a set of ordinary differential equations that are linearized in the neighborhood of an operating point. Generally, they require that the rotating speed be constant.

Only a few analyses have dealt with speed-varying transient rotor dynamics. The earliest paper on the transient response of rotors may be from Lewis (1932). Using a graphical method, Lewis presented an approximate solution to the problem of running a system that has a single degree of freedom and linear damping through its critical speed from rest at a uniform acceleration. The solution shows that the resonant vibration amplitude is smaller than the corresponding amplitude if the spin speed is held constant at the critical speed. Furthermore, in transient time an apparent shift in the position of the critical speed will occur that is higher than the true critical speed when speed is increasing and lower when speed is decreasing. These

effects are commonly observed in reality. This delay is possibly due to the fact that there is not enough time to accumulate energy at critical speed. Shen (1972) used Newton's laws to derive a mathematical formulation for the analysis of both the transient and the steady-state flexible rotor dynamics. Many effects were included in that formulation, but no further numerical examples were given. Childs (1969, 1972) developed a simulation model for general flexible spinning bodies in his two papers. In this development, Childs attempted to separate the rigid body motion and the flexible motion. Although the modal analysis method was proposed as a possible way for the model order reduction, how to apply it was not stated and no examples were given. So far, no further research work on the transient response has followed these two formulations. Recently, Nelson and Meacham (1981) used the component mode synthesis method to conduct transient analysis of rotor-bearing systems under the finite element framework and found that the number of degrees of freedom in the rotor system model is directly proportional to the number of elements (or modes) implicit in the problem. This requires very large computational effort. Subbiah and Rieger (1988) and Subbiah et al. (1988) proposed a methodology that combines finite element and transfer matrix methods to perform the transient dynamic analysis, thereby overcoming the computational difficulties. This approach uses the finite element method to model symmetric shafts and then transforms the system properties to transfer matrix mode. This is a computational technique rather than an analytical tool.

From the above review on rotor dynamics, it is clear that many powerful tools for the linear system and frequency response are available. However, most of these techniques are targeted at the rotor design analysis. For the active vibration control system synthesis, a suitable analytical model is needed that is small in comparison to the overall system equations while still providing the essential dynamic characteristics.

Maslen and Bielk (1992) presented a stability model for flexible rotors with magnetic bearings. Besides the flexible rotor model itself, their model included the dynamics of the magnetic bearing and the sensor-actuator non-collocation. This model can be used for stability analysis and active vibration synthesis. Most recently, an analytical imbalance response of the Jeffcott rotor with constant acceleration was developed by Zhou and Shi (2001a). The solution quantitatively shows that the motion consists of three parts: the transient vibration at damped natural frequency, the synchronous vibration with the frequency of instantaneous rotating speed, and a suddenly occurring vibration at damped natural frequency. This solution provides physical insight into the imbalance-induced vibration of the rotor during acceleration. It can be used for the synthesis of active vibration control schemes.

For the synthesis of DAVC techniques, most researchers used simplified low-order finite element models of the rotor system. Although the techniques developed can be extended to a high-order system theoretically, the computational load will be heavier and the signal-to-noise ratio will have to be higher. The DAVC techniques can be difficult to implement for the high-order system. Therefore, it is necessary to use a low-order system to approximate the high-order system. Model reduction techniques and the

specific impact of the model reduction on the performance of the DAVC schemes require further investigation.

3. Active Balancing and Vibration Control of Rotor Systems

3.1. Active Balancing Techniques

A large body of literature is available on rotor balancing methods. A rough classification of the various balancing methods is shown in Figure 3. The most recent development in active balancing is summarized in the dashed-line box in Figure 3. The rotor balancing techniques can be classified as off-line balancing methods and real-time active balancing methods. Because active balancing methods are extensions of off-line balancing methods, we also provide a review of off-line methods.

3.1.1. OFF-LINE BALANCING METHODS

The off-line rigid rotor balancing method is very common in industrial applications. In this method, the rotor is modeled as a rigid shaft that cannot have elastic deformation during operation. Theoretically, any imbalance distribution in a rigid rotor can be balanced in two different planes (Wowk, 1995). Methods for rigid rotors are easy to implement but can only be applied to low-speed rotors, where the rigid rotor assumption is valid. A simple rule of thumb is that rotors operating under 5000 rpm can be considered rigid rotors. It is well known that rigid rotor balancing methods cannot be applied to flexible rotor balancing. Therefore, researchers developed modal balancing and influence coefficient methods to off-line balance flexible rotors.

Modal balancing procedures are characterized by the use of the modal nature of the rotor response. In this method, each mode is balanced with a set of masses specifically selected so as not to disturb previously balanced, lower modes. There are two important assumptions: (1) the damping of the rotor system is so small that it can be neglected and (2) the mode shapes are planar and orthogonal. The first balancing technique similar to modal balancing was proposed by Grobel (1953). This method was refined in both theoretical and practical aspects by Bishop (1959; Bishop and Gladwell, 1959; Bishop and Parkinson, 1972). Many other researchers also published works on the modal balancing method, including Saito and Azuma (1983) and Meacham et al. (1988). Their work resolved many problems with the modal balancing method such as how to balance the rotor system when the resonant mode is not separated enough, how to balance the rotor system with residual bow, how to deal with the residual vibration of higher modes, and how to deal with the gravity sag. An excellent review of this method can be found in Darlow (1989). Most applications of modal balancing use analytical procedures for selecting correction masses. Therefore, an accurate dynamic model of the rotor system is required. Generally, it is difficult to extend the modal balancing method to automatic balancing algorithms.

Unlike the modal balancing method, the influence coefficient method is an experimental method. It was originally proposed by Goodman (1964), refined by Lund and Tonneson (1972), and verified by Tessarzik and others (1972).

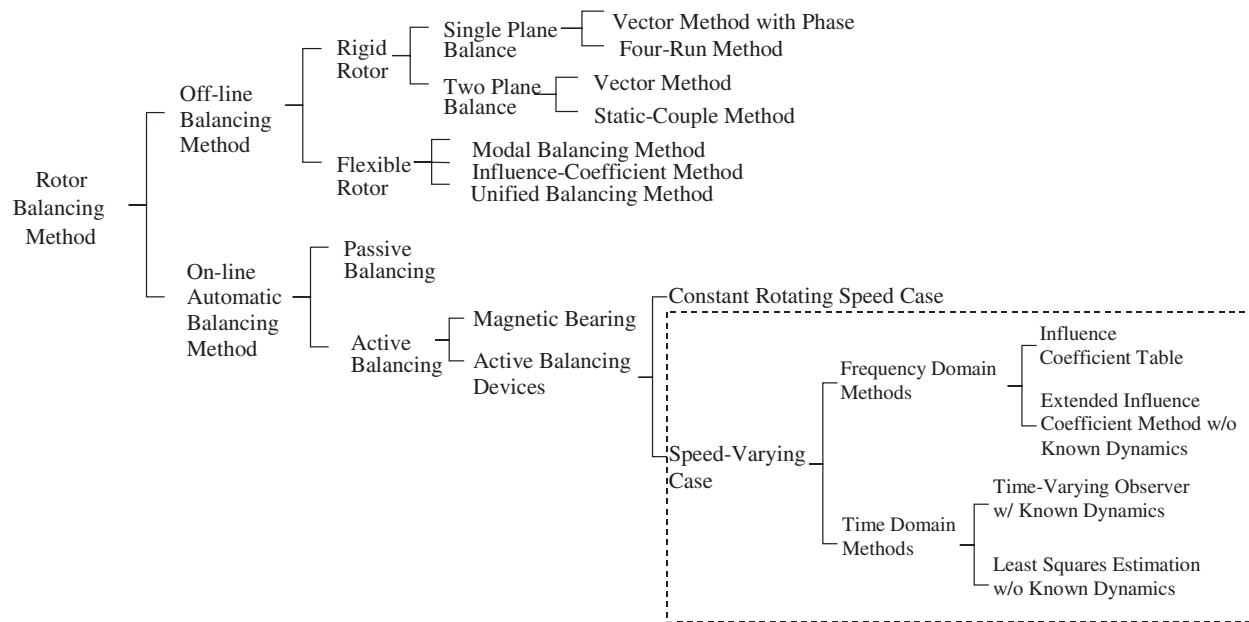


Figure 3. Classification of Balancing Methods

The basic principle used in the influence coefficient method is

$$\mathbf{v}_w = \mathbf{C}\mathbf{w}, \quad (4)$$

where \mathbf{v}_w is a complex number representing the magnitude and phase of the rotor imbalance response, \mathbf{w} is a column vector representing the imbalances in these planes, and \mathbf{C} is a matrix whose elements are the influence coefficients relating the imbalance and the rotor response. The influence coefficient is a function of the sensor/actuator position and the rotating speed. The assumptions behind equation (4) are that (1) the rotor response is proportional to the imbalance and (2) the effect of a set of imbalances can be obtained by superimposing each individual unbalance. These two assumptions have been generally accepted if the imbalances, and hence the imbalance-induced vibration, are not very large. Considering both the system imbalance and the controlled imbalance, the overall vibration is

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{C}\mathbf{w}, \quad (5)$$

where \mathbf{v} is the vibration at the sensor position, \mathbf{v}_0 is the vibration caused by the unknown imbalance at the sensor position, \mathbf{C} is the influence coefficient matrix, and \mathbf{w} is the correction imbalance mass vector. If \mathbf{C} is a square matrix, meaning that the correction mass planes are equal to the sensor planes, then \mathbf{w} is simply $\mathbf{w} = -\mathbf{C}^{-1}\mathbf{v}_0$. If \mathbf{C} is not a square matrix, the least squared solution can be used to find \mathbf{w} , where

$$\mathbf{w} = -(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T\mathbf{v}_0. \quad (6)$$

The superscript T is the conjugate transpose operator. Moreover, many least squares estimation techniques (e.g., weighted least squares estimation) can be used in this case.

Some researchers (Pilkey and Bailey, 1979; Pilkey et al., 1983) have extended the least squares estimation method

to the constraint optimization method to control the possible range of the correction weight. A good review of this method can be found in Darlow (1989). The influence coefficient method is an entirely experimental procedure and is easily automated. Therefore, this method has been expanded for application to the automatic balancing scheme. A disadvantage of this method is that more trial runs are needed, and if the operation speed changes, all the experiments have to be done over again because influence coefficients are functions of rotating speeds. If an active automatic balancer is used, these disadvantages will disappear.

The unified balancing method attempts to combine the modal and the influence coefficient balancing methods to achieve a better result with fewer trial runs. The theoretical basis, practical procedures, and experimental verifications of this method are described in detail in Darlow (1987).

3.1.2. REAL-TIME ACTIVE BALANCING METHODS

The real-time balancing methods can be classified into passive balancing methods and active balancing methods according to which kinds of balancing devices are used.

Automatic balancing using passive devices. Very little research has been done on passive auto-balancing devices. The first passive balancer was proposed by Thearle (1950). Thearle described a device in which one or two particles that are subject to viscous damping are free to move in a groove on the rotor. Thearle showed that plane rotors with this device exhibit auto-balancing, a property due to the dynamic characteristic of the plane rotor. Bovik and Hogfors (1986) used perturbation theory to show that some fairly general rotor systems exhibit auto-balancing. Their analysis was based on a simple rotor model. For more complicated models (i.e., the nonplanar rotor), the system requires axial motion of the particles. However, this is not usually feasible in industry. Thus,

the passive balancing method is not widely implemented in industry.

Active real-time balancing using active mass redistribution balancers. The first research on the active mass redistribution balancer was presented by Van De Vegte (1964). The operation of the mass redistribution balancer was based on the motion of correction masses along two perpendicular axes fixed to the rotating system. The masses were driven by two small servomotors. The power for these two servomotors was supplied through slip rings. Therefore, the balancing action could be performed during the operation of the rotor. The only input to the active balancing system was the measurement of the imbalance-induced vibration at the supporting bearings. Van De Vegte found that if the rotating speed is far from the critical speed, the inherited system imbalance (not the imbalance provided by the mass redistribution balancer) can be calculated from the vibration measurement and the predetermined influence coefficient. However, if the rotating speed is close to critical speed, the errors in the measurement and influence coefficient estimation will lead to serious errors in the balancer correction. In this case, a modal balancing method is needed instead. In Van De Vegte's follow-up research (Van De Vegte and Lake, 1978; Van De Vegte, 1981), two operator-controlled balancing heads provided shifting of the correction masses in two planes. Because there was no specialized motor controls, it was impossible to coordinate the weight adjustments in both planes simultaneously. The adjustment had to be conducted one plane at a time, or "sequentially." Three different kinds of control schemes were used: (i) sequentially minimize the imbalances, (ii) sequentially minimize the sum of the squares of the bearing vibration amplitudes, and (iii) sequentially minimize the sum of the bearing vibration amplitudes. The procedure of sequential correction mass shifting proposed by Van De Vegte (1981) was criticized by other researchers (e.g., Bishop, 1982). Bishop (1982) pointed out that if the critical speeds account for all significant vibrations and those critical speeds are spaced well apart, good balancing result is possible using only one balancing plane. The procedure proposed by Van De Vegte is unnecessarily complicated. Gosiewski (1985, 1987) presented his research on the automatic balancing of flexible rotors. A digital computer was used as the controller in his control scheme. Gosiewski's method is an extension of the influence coefficient method using the particular mass redistribution actuator proposed by Van De Vegte and Lake (1978). In this method, it is assumed that the influence coefficients for several spin-speed ranges are known beforehand and have been embedded into the computer's memory. Then, the position and magnitude of the correction mass are calculated based on the vibration measurement and the predetermined influence coefficient. Because the mass redistribution balancer can be adjusted during the rotating of the rotor, Gosiewski's method can handle the situation in which the system imbalance is time varying. Gosiewski also pointed out that operating the movable masses enables the influence coefficient matrix to be determined without stopping the rotor, but

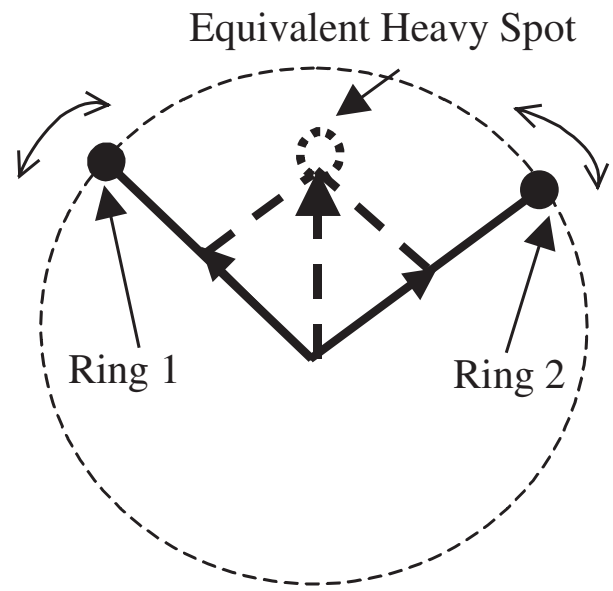


Figure 4. Diagram of the Balancer

how this is done and how it affects the control scheme were not clearly stated.

A successful extension of the influence coefficient method to the on-line estimation and active control was achieved by Dyer and Ni (1999). In their work, an adaptive control scheme that combines the on-line estimation of the influence coefficient and flexible rotor balancing method was successfully implemented using an active mass redistribution actuator. A diagram of the balancer is shown in Figure 4.

The balancer is mounted on the spindle. It consists of two rings. These two rings are not balanced. They can be viewed as two heavy spots. When the balancer is not activated, these two rings are held in place by a permanent magnetic force. The two heavy spots can rotate with the spindle and also can be controlled to rotate with respect to the spindle. The combination of these two heavy spots is equivalent to a single heavy spot whose magnitude and position can change. The details of this actuator can be found in Dyer et al. (1998).

The control algorithm used in Dyer et al. (1998) is an extension of the off-line influence coefficient method. The input to the algorithm is the vibration measurement. The imbalance provided by the balancer is assumed known. In equation (6), the influence coefficient matrix is obtained through experimental trial runs. Ideally, this control law needs only one movement if the estimated influence coefficients are perfect and the vibration measurement is accurate. In practice, however, several control iterations are needed to minimize the imbalance-induced vibration. For the k th iteration,

$$\mathbf{v}_k = \mathbf{v}_0 + \mathbf{C}\mathbf{w}_k \quad (7)$$

The objective is to reduce the overall vibration \mathbf{v}_k . Therefore, the control action for the $(k + 1)$ th iteration is

$$\mathbf{w}_{k+1} = -(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T\mathbf{v}_0 \quad (8)$$

However, the vibration induced by system imbalance is unknown. Equation (7) has to be used to estimate \mathbf{v}_0 as $\mathbf{v}_k - \mathbf{C}\mathbf{w}_k$. Substituting this estimation into equation (8) yields the control law

$$\mathbf{w}_{k+1} = \mathbf{w}_k - (\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T\mathbf{v}_k. \quad (9)$$

The influence coefficient matrix \mathbf{C} is estimated on-line in each control iteration by equation (10):

$$c_{ij,k+1} = \frac{v_{i,k+1} - v_{i,k}}{w_{j,k+1} - w_{j,k}}, \quad (10)$$

where $v_{i,k}$ is the vibration measurement of the k th iteration at the i th plane, $w_{j,k}$ is the balancer position at the j th plane of the k th iteration, and $c_{ij,k+1}$ is the estimation of the influence coefficient from the j th plane to the i th plane at the $(k+1)$ th iteration. This control law only works at constant rotating speed because the influence coefficients change with the rotating speed.

Although balancing at a single working speed is common in practice, balancing during speed-varying periods is also needed. For example, in high-speed machining, the machining tool will engage in cutting as soon as the spindle speed reaches its steady state. If an active balancing scheme is used on such a machine, the balancing has to be completed during the acceleration period to avoid increasing the cutting cycle time. Furthermore, the maximum vibration of a rotor usually occurs when it passes through its critical speeds. To avoid this hostile vibration, balancing during acceleration is needed.

Compared to the constant rotating speed case, there are many technical challenges to active balancing during the speed-varying period. In the constant rotating speed case, only the response at a single excitation frequency (the rotating speed) is important. Hence, a simple rotor model (e.g., the influence coefficient model) can be used to develop an active balancing algorithm. However, the overall dynamics of the rotor are excited in the acceleration case. A more comprehensive rotor model needs to be studied, and a more complicated active balancing scheme needs to be developed.

Most recently, Zhou and Shi (2000, 2001b, 2001c) and Shin (2001) developed several active balancing methods for the speed-varying condition. Shin's work was based on the concept of positive real. He found that if the sensor and the actuator are located at the same position on a rotor, the rotor system is positive real. A direct adaptive control scheme can be constructed based on this positive realness property. The resulting active balancing law is simple: instead of using the estimated influence coefficient as in equation (10), a constant influence coefficient can be used in equation (9). Shin discussed the stability and guidelines for the selection of this constant influence coefficient. Zhou and Shi (2000) adopted the recursive least squares method to estimate the unknown dynamics and the imbalance of the rotor system during the speed-varying period. The estimation was based on the equation of motion of the rotor system and the time domain vibration signal. After the system imbalance was estimated, the mass distribution of the actuator was adjusted to offset the system imbalance. If the system dynamic parameters are known, the time-varying observer technique can be used to obtain the position of the system imbalance (Zhou and Shi, 2001c). A

rigid rotor model was used to present this technique. First, a speed-varying transient rigid rotor model was developed in the state space form as shown in equation (2). The states of this model were augmented to include imbalance forces and moments. A time-varying observer could then be designed for the augmented system using canonical transformation. After obtaining an estimation of the imbalance forces and moments as the states of the augmented system, the estimated imbalance could be directly calculated. This estimation method can be used in the active vibration control or active balancing schemes for a rigid rotor. Zhou and Shi (2001b) presented a one-plane active balancing scheme to eliminate the imbalance-induced vibration for a rigid rotor system during acceleration. There are two vibration modes in the vibration of a rigid rotor system. In general, the optimal positions of the balancer to suppress different vibration modes are different. To balance these two modes, a switching function for the balancer from balancing the first mode to balancing the second mode is needed. In Zhou and Shi (2001b), an optimal one-plane active balancing problem was formulated to minimize the imbalance-induced vibration during acceleration. The optimal switching time and switching function could be obtained by solving this optimization problem based on the analytical solution of the imbalance-induced rotor vibration during acceleration. The switching function was found to be a simple step function.

The active balancing methods eliminate the imbalance-induced vibration by eliminating the root cause of the vibration—system imbalance. However, a mass redistribution actuator needs to be mounted on the spindle, which could not be allowed. In this situation, the DAVC technique can be used.

3.2. DAVC for Rotating Machinery

Active vibration control for rotating machinery is a special case of active vibration control for a flexible structure. The general topic of active vibration control was discussed by Meirovitch (1990) and Inman and Simonis (1987). The difference between rotating machinery and other flexible structures is that the dynamics of the rotor changes with the rotating speed of the rotor system. Best control performance will be obtained if control gains vary with rotating speed. Also, because the shaft is a moving part, a non-contact actuator is used to apply the control force to the rotating shaft. There are many types of actuators for direct active vibration control, including electromagnetic, hydraulic, piezoelectric, and so on. The active magnetic bearing is an established industrial technology with a rapidly growing number of applications. A good example of the application of magnetic bearings in the machine tool industry can be found in Bleuler et al. (1994).

Magnetic bearings can be used to apply a synchronous force to the shaft to control the imbalance response, either to cancel the force transmitted to the base or to compensate for the vibration displacement of the shaft. Knospe and colleagues (Knospe et al., 1996; Knospe et al., 1995; Knospe, Tamer, and Fittro, 1997; Knospe, Tamer, and Fedigan, 1997) presented an adaptive open-loop control method for the imbalance displacement vibration control using magnetic bearings. A synchronous force that consists of sinusoids that are tied to the shaft angular position via a

key phasor signal was generated and applied to the rotor through the magnetic supporting bearings. The magnitude and phase of these sinusoids were periodically adjusted so as to minimize the rotor unbalance response. The model relating the imbalance vibration and the applied open-loop force is

$$X = TU + X_0, \quad (11)$$

where X is an $n \times 1$ vector of the synchronous Fourier coefficient (the corresponding frequency is the rotating speed of the rotor) of n vibration measurements, U is an $m \times 1$ vector of the synchronous Fourier coefficients of the m applied synchronous forces, X_0 is an $n \times 1$ vector of the synchronous Fourier coefficient of uncontrolled vibration, and T is an $n \times m$ matrix of influence coefficients relating the applied force to the vibration measurements. This model is identical to the model in equation (5). Knospe and colleagues' control method is also an extension of the influence coefficient method in off-line balancing and is similar to equation (9). The off-line and on-line estimation of the influence coefficient matrix were presented. Moreover, the control problems under slowly varied spin speed were addressed. This shed light on the imbalance control during transient time. Knospe and colleagues' work also presented the robustness and stability results of the adaptive open-loop control. The underlying theoretical foundation of this work was the influence coefficient balancing method. The magnetic bearings were used to emulate the imbalance-induced force to offset the force induced by the system imbalance. Therefore, Knospe and colleagues' methods are called "active balancing" methods rather than "DAVC" methods. Other researchers such as Herzog et al. (1996) and Lum et al. (1996) published their work on the imbalance transmitted force controlled by magnetic bearings. The basic idea is to use a notch filter to blind the control system of the supporting magnetic bearing to the imbalance induced response. Therefore, no synchronous forces can be generated by the magnetic bearings. The rotor will then rotate about its own principal inertia axis provided that the gap between the shaft and the bearing is large enough. Fan et al. (1992) presented a vibration control scheme for an asymmetrical rigid rotor using magnetic bearings.

Other researchers working in DAVC for rotating machinery adopted a state space representation of a rotor system. The control inputs are lateral forces. Balas (1978) pointed out that for a feedback control system for flexible systems, the control and observation spillover due to the residual (uncontrolled) modes could lead to potential instabilities. In Stanway and Burrows (1981), the dynamic model of the flexible rotor was written in the state space format and the controllability and observability of the model were studied. Stanway and Burrows concluded that the lateral motion of the rotor can, under certain conditions, be stabilized by the application of a single control input to a stationary component. Ulsoy (1984) studied the characteristics of rotating or translating elastic system vibration problems that are significant for the design of active controllers. The basic conclusions of his research were that a controller gain matrix that is a function of the rotating speed is required to maintain a desired closed-loop eigenstructure and that a residue model spillover should be

handled carefully by the active controller to avoid instability. Firoozian and Stanway (1988) adopted a full-state observer technique to design a feedback active control system. The stability of the closed-loop system was also studied.

To build an active vibration control system for flexible structures, the sensor/actuator deployment needs to be studied. A review is given in the following section.

3.3. Design for Active Balancing and Vibration Control for Rotating Machinery

The issue of actuator/sensor placement for control of flexible structures is an active research area. This problem is often formulated as a constraint optimization problem. The constraints of this optimization problem are the limited available locations for the actuators and sensors. The objective function of this optimization problem is closely related to the control algorithm used for the flexible structure.

The main possible optimal cost functions for sensor and actuator placement are for system identification, state estimation (which is represented by the observability) and indirect control performance (which is represented by the controllability), and direct control performance (e.g., the transient response, stability). A review of the optimal actuator/sensor placement follows.

System identification. The objective function to be maximized by Qureshi et al. (1980) is the determinant of the Fisher information matrix associated with the parameters to be identified. This cost function depends on the spatial locations of the observation points. An early survey on the sensor location problem for system identification can be found in Kubrusly and Malebranche (1985).

State estimation. Gawronski (Gawronski and Lim, 1996; Gawronski, 1997) used the balanced representation of the system to do the actuator and sensor placement. The system is balanced if its controllability and observability gramian are equal and diagonal. Lim (1993) considered the observability and controllability separately. The so-called effective independence (EI) contributions of the actuator and sensor were considered. Liu et al. (1994) used the singular value decomposition of the input matrix \mathbf{B} and observation matrix \mathbf{C} directly to determine the degree of controllability and observability.

Direct control performance. The transmission zeroes were taken as the target function by Maghami and Joshi (1993a, 1993b). Sepulveda and Schmit (1991) considered the optimization problem of structure design and actuator/sensor placement in one framework. Several different control objectives, including the structural mass, control effort, number of actuators, stability margins, controllability and observability, and so on, were taken into account simultaneously. Dhingra and Lee (1994) attempted to optimally select the actuator/sensor positioning and feedback gain simultaneously.

Other optimization criteria include the spillover effect (Barker and Jacquet, 1986), system performance under possible component failure conditions (Vander Velde and

Carignan, 1984), and hyperstability of the system (Stieber, 1988).

As for the rotor active vibration control issues, very little literature deals with the actuator/sensor placement problem. Bishop (1982) pointed out that it is possible to balance a flexible shaft as it is rotating provided that the critical speeds are spaced well apart. He also pointed out that the balancing plane should not be located near a node in any of the lowest n principal modes. However, Bishop did not conduct quantitative analysis of where to put the balancer. Pilkey et al. (1983) proposed a technique for optimizing the axial location of balance planes. By knowing the vibration induced by rotor imbalance and the influence coefficients at certain locations, and through the use of the linear programming method, one can find the optimal correction weights and axial locations of the balancing planes that minimize the residue vibration. This method was based on an off-line balancing scheme. Kim and Lee (1985) used a structural dynamics modification algorithm to determine the optimal active balancing head location on flexible rotors. They attempted to minimize the amount of correction imbalance required to control the imbalance forces at the critical speeds of interest. Although their theoretical derivation is based on the modal balancing technique, it is clear that their result leads to the maximum of influence coefficients. Their work requires the constant rotating speed condition. Most recently, Zhou et al. (2001) proposed a new optimal balancing plane determination procedure for the active balancing scheme. This optimization procedure is based on an analytical expression of the influence coefficients. Besides the balancing capacity, the influence of the measurement uncertainty on control performance is considered in the cost function. The formulated problem is solved by a multicriteria optimization technique.

4. Conclusion

Rotating machinery is widely used in industry. The dynamic analysis and active vibration control of the rotating machinery are important engineering problems for both industry and academia. In this paper, a review of the active balancing and direct vibration control for rotating machinery was conducted.

The major problem faced by the active vibration control scheme is the use of a limited number of actuators to control an infinite number of vibration modes. To design an active control scheme, a reduced-order model should be used and the effect of the spillover of higher vibration modes assessed. Although the available techniques developed for dynamic analysis, rotor imbalance estimation, and active real-time balancing and vibration control can be extended to high-order systems theoretically, the computational load will be heavier and the signal-to-noise ratio of the vibration measurement will have to be higher. Hence, the available techniques could be difficult to implement in high-order systems. Therefore, it is necessary to use a low-order system to approximate the high-order system. The gyroscopic effect caused by the rotating motion and the moment of inertia of the rotating body is a unique dynamic effect in a rotor system and should be considered in model reduction. The specific impact of this model

reduction on the performance of the active balancing should also be investigated in the future.

In many active balancing and vibration control methods, the imbalance estimation is coupled with the control strategy. So far, there are no systematic methods available to show the relationship between the estimation and the control strategy. A control action is preferable if it can obtain small imbalance-induced vibration and excite the system to obtain the good imbalance estimation at the same time. Thus, the coupling effect should be investigated by considering the estimation algorithm, the system dynamics, and the control performance. This research can also lay a scientific foundation for the design of an efficient and reliable generic adaptive control system.

It is clear that active balancing can suppress the imbalance-induced vibration. It is also clear that the active balancing can improve product quality and improve the fatigue life of the machine and cutting tools and, hence, reduce the system cost. However, the installation and maintenance of an active vibration system for rotating machinery will increase the system cost. How to assess the active vibration control system from a cost-effective point of view and on a higher process level is not well studied in the literature. We believe this is an interesting and important problem in the active balancing and vibration control of a rotating system.

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