

# Active Contour based Document Image Segmentation and Restoration using Split-Bregman and Edge Enhancement Diffusion

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## ABSTRACT

This paper presents PDE based document image segmentation and restoration approach using active contours, non-linear diffusion and split-bregman algorithm. We applied additive (Gaussian, salt & pepper) and multiplicative noise (speckle noise) to clean document images, taken from DIBCO 2009 image dataset. Along with image denoising and segmentation, the edges in the image are enhanced through non-linear diffusion operation. Active contour is also used to get a binarized document image.

### Keywords

Edge enhanced diffusion, split-bregman, level set

## 1. INTRODUCTION

Document images are the images which preserve information digitally. This helps in preserving old literatures, information residing in metal carvings, palm leaf manuscripts, stone carvings etc. In doing this people can know about the past culture. The information in these can be grabbed by scanning or taking a snapshot of it. The images captured like this may have issues of degradations like broken letter, distortion, holes, spots etc. These degradations act as artifacts. In order to understand the images, these degradations have to be suppressed. Then only the portion of the image that holds the relevant information can be extracted correctly for further understanding or analysis. Restoration methods need to apply without losing the information in these images.

From the literature it can be found that the traditional methods like average filter, median filter, Gaussian filter etc are used to reduce the noise. But during this process, the image will get smoothed, which will also smooth away the edge information. In this paper, we have discussed the use of non-linear diffusion, a PDE based method applied to preserve the edge information. This paper discusses 1) how restoration of images is done using non-linear diffusion and split Bregman technique and 2) how relevant portion of the image can be extracted using level set method. The performance of the approach in the paper is analysed by adding additive noise like Gaussian, Salt and Pepper, multiplicative noise like speckle at different noise levels to clean image. Then the same method are applied on to few test images from DIBCO 2009 dataset. In this paper we are investigating the performance of different parameters for non-linear diffusion (edge enhanced diffusion) and split bregman

algorithm to give a denoised image and thereby to segment the different objects in an image through level set method. The paper is organized as follows. In section 2 we give an overview of the theory related to non-linear diffusion, split-

Bregman algorithm, level set method for binarization and segmentation. In section 3 we focus on the results obtained through the experiments. In section 4, we conclude by giving a summary and idea for future work.

## BACKGROUND THEORY:

### 2. PDE BASED EDGE ENHANCED DIFFUSION

Earlier, for denoising application, conventional filter median filter, average filter, Gaussian filter were used. These filters smooth the image which will soften the edges. This created a problem in image restoration process. This effect of smoothing can be reduced with the use of partial differential equation (PDE) based image processing methods [3, 4]. The idea behind PDE based image processing are taken from physical phenomena's like wave propagation, heat and mass transfer [5]. PDE based methods assume that the intensity of illumination on edges varies like geometric heat flow in which heat transforms from a warm environment to a cooler one until the temperature of the two environments reaches a balanced point [4].

PDE's are mainly used for image enhancement and denoising. Initially Perona and Mallik proposed the idea of applying diffusion operation on to image based on heat diffusion equation [4]. Physically, the diffusion process tries to bring an equilibrium when there are concentration differences, without creating or destroying mass [5]. The diffusivity is done by taking derivatives and along the process the image evolves. The equilibrium property of the diffusion can be expressed by Frick's law as

$$j = -D \cdot \nabla u \quad (1)$$

Here 'u' represents the concentration, 'D' is the diffusion tensor matrix which can decide how fast or slow the diffusion can occur, and 'j' is the flux. ' $\nabla u$ ' is the concentration gradient and is the driving force for diffusion. Physically this equation means that when diffusion happens, the flux moves from high concentration area to low concentration one and the magnitude of flux is proportional to concentration gradient. If the diffusion tensor is constant for whole image, then diffusion is called 'isotropic'. Here the flux and gradient will be parallel. If the diffusion is modified depending on the local structure of the image then diffusion is called 'anisotropic' [5] and flux, gradient will not be parallel. This can be expressed with the help of continuity equation as

$$\partial_t u = -\text{div } j \quad (2)$$

where 't' denotes the time. Embedding the Frick's law in this gives

$$\partial_t u = \text{div}(D \cdot \nabla u) \quad (3)$$

If 'D' depends on the image 'u', this becomes a non-linear diffusion filtering [6]. Which sharpens the edge over a range of slope scales with reduction in noise as well as conserving the feature boundaries [8]. Figure 1 is taken from DIBCO 2009 dataset and figure 2 shows the edge enhanced diffusion applied on the grayscale image of figure 1.



Figure 1. Original image



Figure 2. Edge enhanced diffusion

### 3. BREGMAN APPROACH

In this paper we are utilizing the ability of Bregman iteration which is used to solve constrained optimisation problem equivalent to the original unconstrained optimisation one. Bregman algorithm can solve problems of the form

$$\min_u \{J(u) + H(u, f)\} \quad (4)$$

For a closed and convex set  $X$ , both  $J: X \rightarrow R$  and  $H: X \rightarrow R$  are convex and nonnegative with respect to  $u \in X$  for a 'f'. 'f' can be a vector or matrix based on the problem.  $H(u, f)$  may be differentiable. Initially,

$$u^1 = \min_{u \in X} (J(u) + H(u)) \quad (5)$$

The residual is calculated at each iteration and the converging point is reached when it is minimum [1, 4].

#### Bregman Distance

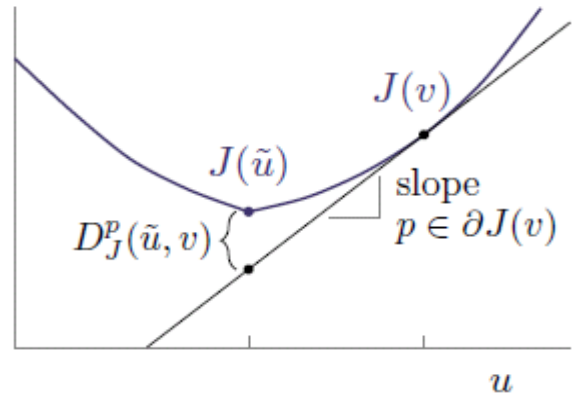


Figure 3. Bregman distance [11]

Bregman distance can be calculated from above figure, it is shown in the below equation.

### 4. SPLIT BREGMAN ALGORITHM

Split Bregman algorithm is a suitable technique in solving convex minimisation problems which are of non-differentiable in nature. [1] Optimisation problems of following format can be solved by using the split Bregman

algorithm  $\phi: X \rightarrow R$   $H: X \rightarrow R$  Constrained function can be defined as, [1]

$$\min_{u \in X} \|\phi(u)\|_1 + H(u) \quad (6)$$

Constrained minimisation problem with a subjected to condition can be expressed as,

$$\min_{u \in X, v \in R} \|v\|_1 + H(u) + (\lambda/2) \|d - \phi(u)\|_2^2 \quad (7)$$

' $\lambda$ ' is lagrangian multiplier, it is used for finding local maxima and minima in the equality constraints. If  $\lambda$  is a positive function, this equation will be,

$$J(u, d) = \|d\|_1 + H(u) \quad (8)$$

So,

$$\min_{u \in X, v \in R} J(u, d) + H(u) + (\lambda/2) \|d - \phi(u)\|_2^2 \quad (9)$$

When we apply Bregman iteration into the minimisation problem, then Bregman distance is calculated recursively to find the solution,

$$(u^{k+1}, d^{k+1}) = \arg \min_{u, d} D_J^p(u, u^k, d, d^k) + (\lambda/2) \|d - \phi(u)\|_2^2 \quad (10)$$

Figure 1, helps to know the calculation of Bregman distance over 'u' and 'v', 'p' is the subgradient [2]

$$D_J^p(u, v) = J(u) - J(v) \langle p, u - v \rangle,$$

$$p \in \partial J(v) \quad (11)$$

Then finding sub-gradient P at points 'u' and 'd' for each iterations [3]

$$\arg \min_{u, d} J(u, d) - \langle P_u^k, u - u^k \rangle + \langle P_d^k, d - d^k \rangle + \frac{\lambda}{2} \|d - \phi(u)\|_2^2 \quad (12)$$

Then the simplified form of Split Bregman can be given as

$$(u^{k+1}, d^{k+1}) = \min_{u \in X, d \in R} J(u, d) - J(u^k, d^k) + \lambda \langle b^k, \phi_u - \phi_u^k \rangle - \lambda \langle b^k, d - d^k \rangle + \lambda / 2 \|d - \phi(u)\|_2^2 \quad (13)$$

$$(u^k, d^k) = \min_{u \in X, d \in R} J(u, d) + (\lambda / 2) \|d - \phi(u) - b^k\|_2^2 + C_2 \quad (14)$$

where  $C_2$  is a constant.

#### SPLIT-BREGMAN ALGORITHM [10]

Initialize  $k = 0$   $u^0 = 0$   $b^0 = 0$

while  $\|u^k - u^{k-1}\|_2^2 > tol$  do

$$u^{k+1} = \min_u H(u) + \frac{\lambda}{2} \|d^k - \phi(u) - b^k\|_2^2$$

$$d^{k+1} = \min_d |d| + \frac{\lambda}{2} \|d - \phi(u^{k+1}) - b^k\|_2^2$$

$$b^{k+1} = b^k + \left\| \phi(u^{k+1}) - b^k \right\|_2^2$$

$k = k + 1$

end while

## 5. SEGMENTATION

Segmentation is mainly used in image processing and computer vision applications.[17]. Segmentation is the process of dividing the digital image into multiple set of pixels. Image segmentation is mainly used for identifying the object and boundaries in an image. In this paper we are using Level set based image segmentation in which the objects in an image are segmented using contours. The image after segmentation may consist of set of contours. Level set method is used for identifying the shape. Level set methods have advantages like performing computation for curves and surfaces. Level set is mainly used for satellite imagery, biomedical imaging system video image analysis etc. In medical science by using Segmentation we can find the size and shape of the tumours in our body. Two types of segmentation are commonly used

- Active Contour Model(Snakes)
- Geodesic Active Contour Model

Active contour model is one of the variational method in image segmentation[19]. Active contour is the representation of an object outlines from a noisy image. It generate a curve under some constraint to extract the desired objects. Its mainly used in computer application. For segmenting an image the active contour use parameterized representation of the evolving curves. The active contours can be represented as [16]

$$c(s) = (x(s), y(s)) \quad (15)$$

Where  $x(s)$  and  $y(s)$  are the co-ordinates of the contour and  $s \in [0, 1]$

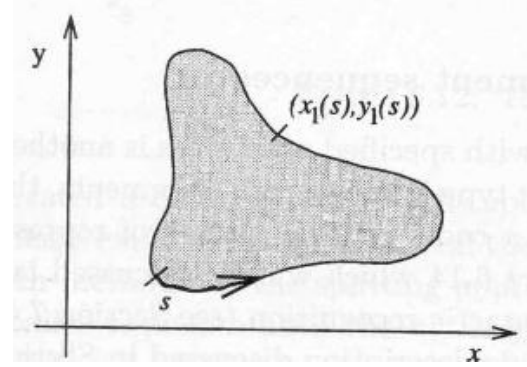


Figure 4.

The level set. Image is taken from [16]. Geodesic Active Contour is used for identifying the boundaries of an object [20]. For applying level set into an image we require a mesh or a grid over an image. Depending upon the finness of the mesh the level set would be more accurate. Once the grid is formed the function  $\phi(x, y, t)$  is applied where  $(x, y)$  is a point at time 't'. These functions can be initialized at time  $t=0$ . This is called 'zero level set'. This can be represented by  $C = \{(x, y) \phi(x, y) = 0\}$ ,  $\phi$  can be initialized at each point of the mesh or grid.

$$\phi(x, y, t) = \begin{cases} \text{dist}((x, y), C) & \text{if } (x, y) \in \text{inside } C \\ 0 & \text{if } (x, y) \in C \\ -\text{dist}((x, y), C) & \text{if } (x, y) \in \text{outside } C \end{cases}$$

The positive sign denotes that the point is inside the curve and the negative sign denotes that the point is outside the curve. There are two types of approaches.

1. Edge Based Model
2. Region Based Model

### 5.1 EDGE BASED MODEL

This model is mainly used for detecting the edges of an image. Here we introduce an edge stopping function  $g(x, y)$ . This can be represented as

$$g(x, y) = \frac{1}{1 + |\nabla G_\sigma * f(x, y)|^2} \quad (16)$$

Where  $G_\sigma$  is the Gaussian filter which is obtained by normal distribution with standard deviation  $\sigma$ . When these edges are multiplied with Gaussian filter the diffusion occurs and the edges become blurred.

The value of  $g(x, y)$  become smaller when it is closer to the edge and the value become larger when it is away from the edge. This is the reason why we chose this  $g(x, y)$  as the edge stopping factor.

## 5.2 REGION BASED MODEL

Depending upon the similarity in pixel intensities we can partition the image into object and boundaries. This method is more robust than any other method. We have to minimize the energy by the function .[14]

$$\min_{C, c_1, c_2} E(C, c_1, c_2) = \int_{\text{inside}(C)} (f - c_1)^2 dx dy + \int_{\text{outside}(C)} (f - c_2)^2 dx dy + \lambda \text{length}(C) \quad (17)$$

$c_1$  represents the average pixel value inside  $C$  and  $c_2$  represent the average the pixel outside  $C$ .  $\lambda$  is the regularizing parameter which gives weightage in the minimization process .We can calculate  $c_1$  and  $c_2$  by the following equations.

$$c_1 = \frac{\int_{\Omega} f(x, y) H(\phi) dx dy}{\int_{\Omega} H(\phi) dx dy} \quad (18)$$

$$c_2 = \frac{\int_{\Omega} f(x, y) (1 - H(\phi)) dx dy}{\int_{\Omega} (1 - H(\phi)) dx dy} \quad (19)$$

We use calculus of variation for calculating  $\phi$  using the relation[14]

$$\frac{\partial \phi}{\partial t} = - \frac{\partial E}{\partial \phi} \quad (20)$$

## 6. EXPERIMENTS AND RESULTS

The experimental part of this paper is done by using MATLAB. The document images for the experiments are taken from DIBCO 2009 dataset [21]. This method is applied for both noisy and clean images which is mixed with different types of noises. As a first step of this experiment we are applying edge enhanced diffusion process so that it highlights the edges. After that apply Split Bregman technique to denoise the image without losing the edges. The final binarized output is obtained after applying Level set based image segmentation into it.

By setting proper  $\mu$  and lambda, the actual denoised, edge enhanced, binarized output is obtained.



Figure 4. a) Original image taken from DIBCO 2009 dataset b) image obtained after edge enhanced diffusion operation c) image obtained after applying split bregman algorithm d) contour formed using level set method e) segmented and binarized output

ORIGINAL+NOISE ADDED



Figure 4. a) Original image taken from DIBCO 2009 dataset b) Gaussian noise added to the image c) image obtained after edge enhanced diffusion operation d) denoised image obtained after applying Split Bregman algorithm e) contour formed using level set method f) segmented and binarized output.

INPUT IMAGE	TYPES OF NOISE	NOISE RATE	PSNR	TIME
Image	Gaussian	0.01	68.68	5.52 sec
		0.03	65.18	5.36 sec
		0.05	63.97	6.00 sec
	Speckle	0.01	69.81	7.14 sec
		0.03	66.10	7.77 sec
		0.05	64.79	7.36 sec
	Salt and pepper	0.01	72.89	7.54 sec
		0.03	69.40	6.99 sec
		0.05	67.60	6.82 sec

Table 1:Experiments And Results



Figure (5) Image (a) Original (b) Gaussian 0.01 Noisy (c) Split Bregman



Figure (6) Image(a) Original (b) Gaussian 0.03 Noisy (c) SplitBregman



Figure (7) Image (a) Original (b) Gaussian 0.05 Noisy (c) Split Bregman



Figure (8) Image(a) Original (b) Speckle 0.01 Noisy (c) SplitBregman



Figure (9) Image (a) Original (b) Speckle 0.03 Noisy (c) SplitBregman



Figure (10) Image(a) Original (b) Speckle 0.05 Noisy (c) SplitBregman



Figure (11) Image(a) Original (b) Salt & pepper 0.01 Noisy (c) SplitBregman



Figure (12) Image (a) Original (b) Salt & pepper 0.03 Noisy (c) Split Bregman



Figure (13) Image (a) Original (b) Salt & pepper 0.05 Noisy (c) Split Bregman

## 7. CONCLUSION

In this paper we are making a study of how denoising using Split Bregman technique can be made more effective by using edge enhanced diffusion and level set based segmentation in a noisy image as well as a clean image mixed up with noise.

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