Active damping of chatter in machine tools - Demonstration with a "Hardware in the Loop" simulator

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Abstract

The motivation of the work is twofold: (i) Understand the physics behind regenerative chatter and the influence of structural damping and (ii) demonstrate an active damping technique based on collocated actuator/sensor pairs. A numerical stability analysis is performed with the Root Locus Method and it is shown that along with the structural poles, eigenvalues due to the delay parameter may contribute to instability. Since experimental demonstration of chatter in real machines is difficult, an alternative way of demonstration via a mechatronic simulator is presented, using the "Hardware in the Loop" concept. The mathematical model of the regenerative cutting process in turning is simulated in a computer and this is interfaced to a beam, representing the structural dynamics of the machine, via a displacement sensor and force actuator. In this way, a hardware and a software loop are combined. In a second step, an additional control loop is added, consisting of an accelerometer sensor and a collocated inertial actuator. Numerical and experimental stability lobes diagrams are compared, with and without active damping.

Keywords: Chatter, Root Locus Method, Hardware in the Loop Demonstrator, Active Damping

1 Notations

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width of cut (m)
a
             damping coefficient (Ns/m)
c
F_f
             cutting force (N)
              feed of the tool (m)
h_0
h(t)
              total chip thickness (m)
k
             stiffness of the system (N/m)
              cutting constant (N/m<sup>2</sup>)
K_f
K_{cut}
              cutting stiffness (N/m)
              mass of the system (kg)
m
N
             spindle speed (RPM)
T = 60/N
             time for one revolution of spindle (s)
y(t-T)
             displacement of the tool during the previous pass (m)
             current displacement of the tool (m)
              acceleration of the tool (m/s<sup>2</sup>)
\ddot{y}
              velocity of the tool (m/s)
\dot{y}
G(s)
              Transfer function between cutting force F_f and tool displacement y
              for a single input single output (SISO) system
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2 Introduction

Chatter is a problem of instability in the metal cutting process, characterized by violent vibrations, loud noise and poor quality of surface finish. Chatter reduces the life of the tool and the productivity of the manufacturing process by interfering with the normal functioning of the machine. The problem is affecting the manufacturing community for quite some time and it is a popular topic for academic and industrial research. Generally two mechanisms are responsible for chatter, a) Mode Coupling b) Regeneration of surface waviness. The latter is by far the most common cause and is considered in the present study. Tobias and Fishwick [1] and almost at the same time Tlusty [2] were among the first to independently propose the phenomenon of regeneration to explain chatter instability. Figure 1 shows the regeneration process in turning where the tool is cutting a cylindrical

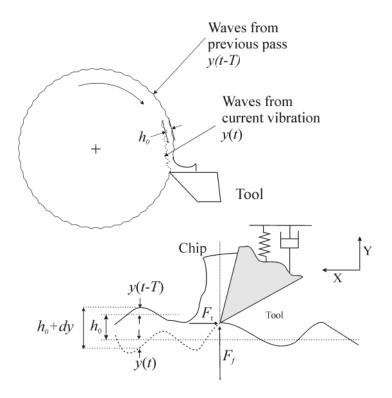


Figure 1: The regeneration process

surface. While machining, the tool may face a hard spot on the surface of the workpiece and some vibrations are triggered. This leaves behind a wavy surface, as shown in the figure and after one full rotation the tool faces the waves left during the previous pass. This causes fluctuation of the cutting forces, further exciting the structure. This in turn leaves more vibration marks on the workpiece surface. This is the process of regeneration.

3 Stability against chatter

Referring to the Figure 1, assuming the tool to be flexible only in the Y-direction, the uncut chip thickness h(t) at any instant is given by,

$$h(t) = h_0 + y(t - T) - y \tag{1}$$

where y and y(t-T) are also called the inner modulation and outer modulation respectively. Assuming that the cutting forces are proportional to the frontal area of the chip,

the cutting force in the Y direction is equal to

$$F_f(t) = K_f.a.[h_0 + y(t - T) - y]$$
(2)

Many authors have observed the existence of damping in the cutting process, especially at low spindle speeds. Tobias et al [1], Tlusty [3] and later Minis et al [4] incorporate the displacement variable and its derivative in the cutting force relationship to take into account the damping. Knight [5] presents experimental investigations on the dependance between the cutting force and the cutting velocity, the rake angle of the tool and the feed. Tlusty et al [6] and Sato et al [7] deal with non-linearities such as the tool leaving the workpiece, due to excessive vibrations. Tobias et al [8] and Ulsoy et al [9] relate cutting forces to the power of the chip thickness. However a proportional model has been found to be quite adequate for analysis and is adopted in the present study. The dynamic equation of motion in the Y direction is

$$m\ddot{y} + c\dot{y} + ky = K_f.a.[h_0 + y(t - T) - y]$$
 (3)

Equ. 3 is a Delay Differential equation. In Laplace domain $y(t-T) = y(s).e^{-sT}$. Defining the machine-tool transfer function between the applied force F and displacement y as G(s) and substituting for y_0 , we have in Laplace domain,

$$\frac{h(s)}{h_0(s)} = \frac{1}{1 + K_f \cdot a \cdot G(s)(1 - e^{-sT})} \tag{4}$$

where

$$G(s) = \frac{y(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$
 (5)

Therefore the characteristic equation of the closed loop system is

$$1 + K_{cut} \cdot G(s)(1 - e^{-sT}) = 0 (6)$$

where K_{cut} is the product of K_f and a. This equation is not restricted to a single degree of freedom (SDOF) oscillator but can also be extended to single input single output (SISO) systems with multiple degrees of freedom, provided the appropriate expression for G(s) is used. Merrit [10] introduced a closed loop feedback diagram for regenerative chatter, as shown in Figure 2 and is credited for providing a viewpoint from control engineering

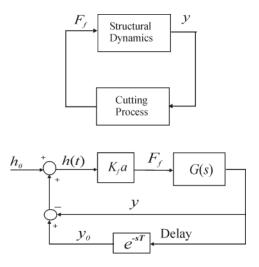


Figure 2: Merrit's closed loop representation of chatter

literature. Under certain combinations of K_{cut} and spindle speed N, the feedback loop becomes unstable, leading to chatter. The traditional method of stability analysis assumes that a root of the characteristic equation is on the imaginary axis, i.e., $s=\pm j\omega_c$, where ω_c is the chatter frequency and then solves for the corresponding limiting value of K_{cut} and spindle speed N. Based on this, Tobias et al [11] introduced the classical stability lobe diagram which is a plot of K_{cut} or a versus N. Tlusty et al in [2] and Merrit [10] and Altintas [12], formulate limiting width of cut a_{lim} as a function of the frequency response function G(s). Minis et al in [13] use the Nyquist Criterion for stability analysis. Figure 3 shows the classical stability lobe diagram. The figure is a plot of the ratio between limiting value of K_{cut} and k and the spindle speed N. The area below the lobes is the stable machining region while the region above is that of instability or chatter. The envelope of the minimum values on the stability lobe diagram is a straight line and this represents the "Asymptotic Stability Margin", which is proportional to the structural damping ratio ξ for a SDOF system. In this paper, we investigate the instability using the Root Locus Method.

4 Role of the delay term in chatter instability

The Root Locus method plots the eigenvalues of the closed loop system for increasing value of K_{cut} . Instability arises when at least a pair of conjugate roots just crosses the imaginary axis. The corresponding value of K_{cut} is the limit of stability for the chosen

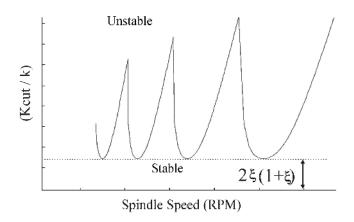


Figure 3: A typical stability lobe diagram

spindle speed and the imaginary part of the root gives the chatter frequency. The whole exercise is repeated for different spindle speeds and the stability lobe diagram and the chatter frequency plot are generated. Equ. 6 is transcendental and has an infinite number of roots, due to the delay term e^{-sT} . Two limit cases arise depending on the value of K_{cut} .

- For $K_{cut} \to 0$, the roots are the poles of $G(s)(1 e^{-sT})$ which are the poles of G(s) and an infinite number of poles of $(1 e^{-sT})$ at $s = -\infty \pm j(2n\pi/T)$, where $j = \sqrt{-1}$ and n is any integer.
- For cases where $K_{cut} \to \infty$, the roots are the zeros of $G(s)(1 e^{-sT})$, which are the zeros of G(s) and the infinite number zeros of $(1 e^{-sT})$ at $s = \pm j(2n\pi/T)$.

This has been discussed by Olgac et al in [14]. The evolution of roots from very low values of K_{cut} to very high values is investigated with a Root Locus plot in Figure 5. In the present study, Padé Approximation is used, which converts the delay to a rational fraction of two polynomials. The system, with an infinite number of roots is thus transformed to one with finite number of characteristic roots. The quantity e^{-sT} introduces a phase lag proportional to the frequency, which differs from the phase, introduced by the Padé approximation. The difference depends on the order of approximation chosen for the polynomials and value of the quantity sT. In Figure 4, the variation from actual phase, due to various orders of Padé approximation, is shown. The maximum value of sT, which depends on the natural frequency s and the maximum value T (i.e., smallest value of N), decides the order of the approximation required for an accurate solution of the eigenvalue

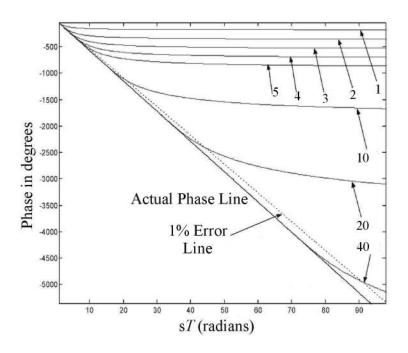


Figure 4: Phase of Padé approximation with increasing order

problem. For multiple degrees of freedom (MDOF) systems, the highest frequency among the modes included in the model of the structure should be considered. For low values of K_{cut} , as shown in Figure 5, the pole (denoted by a cross), closest to the imaginary axis corresponds to the structural mode. The rest of the poles, due to the delay term, ideally should be at infinite distance from the imaginary axis. But due to the approximation of the delay term they can be seen at finite but large distances from the imaginary axis. With increasing value of K_{cut} , all the roots approach the imaginary axis and cross it. They ultimately converge to the $s=\pm j(2n\pi/T)$ points, i.e. $s=\pm j(n/T)$ in Hz units, where n as any integer, for very high values of K_{cut} . Traditional techniques of chatter analysis generally recognize that instability arises from the structural mode of the system. However it will be shown in the following that for certain spindle speeds, there is a possibility that the roots due to the delay may cross over to the right side of the imaginary axis before a structural pole does. The source of instability depends on the relative values of the natural frequency of the structure and the quantity n/T, which is nothing but a harmonic of the spindle speed frequency in Hz. A SDOF system, with a natural frequency of 47 Hz, is chosen and cases are investigated for various spindle speeds via Root Locus Plots. The minimum spindle speed chosen is 1000 RPM and a Padé order of 40 is found to be

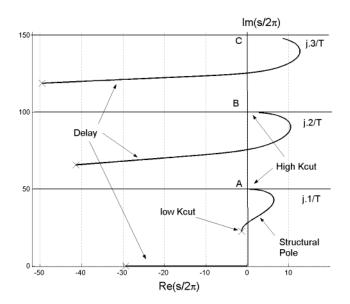


Figure 5: The locus of closed loop poles with increasing K_{cut}

sufficient for the analysis. For 3000 RPM the spindle speed frequency is 1/T = 50 Hz, which is higher than the natural frequency of 47 Hz. It is seen from Figure 6 a), that the instability is arising from the structural mode as it migrates towards the j.1/T point with increasing values of K_{cut} . The poles due to the delay term migrate towards points, which are higher multiples of j.1/T. Only the locus of the delay pole, closest to the structural mode, is shown for reasons of clarity of the figure. For 2820 RPM the spindle speed frequency is equal to the natural frequency of the structure. The locus, corresponding to the delay now becomes unstable, as shown in Figure 6 b). This is reflected in a sudden change in the chatter frequency diagram in Figure 7 at that spindle speed. The pole due to the delay continues to be unstable with further reduction of spindle speed and the distance of the locus from the real axis decreases with reduction of the spindle speed. At 2340 RPM, the root from the delay has migrated to a lower position, compared to the structural poles. The pole for the delay migrates towards j.1/T now, whereas the structural poles go towards the j.2/T point and become unstable, as shown in Figure 6 c). This change in behaviour explains the sudden changes in chatter frequencies at certain spindle speeds from very high chatter frequencies to values close to the natural frequency of the structure. It is observed that for a further decrease of the spindle speed, the instability arises either from the structural pole or the higher placed delay poles, as shown in the final Figure 6 d). Interaction between structures with multiple modes and the delay will be dealt with in a later part of the work.

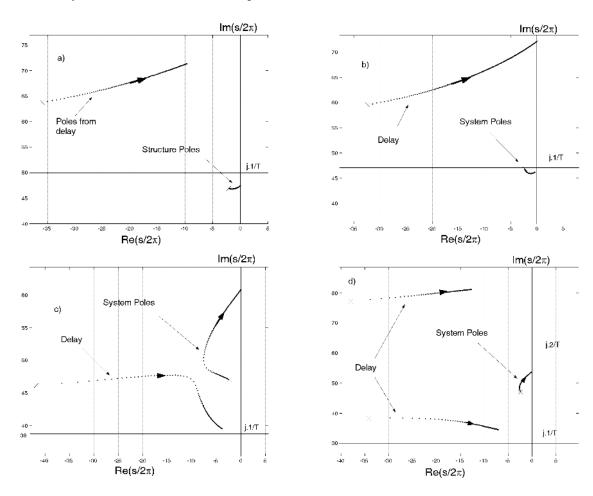


Figure 6: Loci of eigenvalues for a)3000 RPM b)2820 RPM c)2340 RPM d)2000 RPM

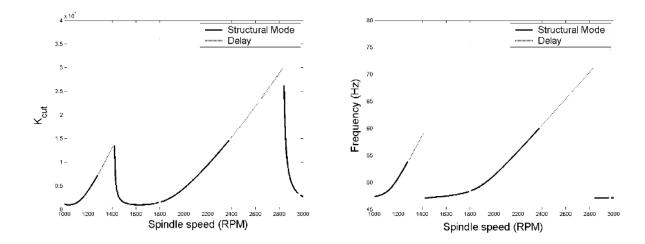


Figure 7: Stability lobe diagrams and chatter frequencies showing regions of instability arising from the structural mode and the delay

The investigations show that an unstable condition may arise from the delay for certain spindle speeds. The traditional stability analysis technique does not distinguish between parts of the stability lobe diagram on the basis of the source of instability, as shown in Figure 7. Higher stability is found at speeds, where the roots from the delay are becoming unstable. This gives an insight that high stability machining is achievable by operating at selected spindle speeds. The eigenvalues are always found to cross the imaginary axis at frequencies, higher than the natural frequency of the structure confirming that chatter frequencies in turning are higher in comparison to the natural frequency, as observed in reference [15].

5 A "Hardware in the Loop" chatter demonstrator

In the previous section a mathematical perspective is presented to understand regenerative chatter. However in real machining it is difficult to understand chatter due to the involvement of a huge number of parameters. Based on the well-established mathematical model of regenerative chatter by [1, 2, 10] and recent advances in signal processing technologies, an alternative way to study chatter via a mechatronic simulator, without conducting actual cutting tests, is presented. An aluminium cantilever beam is used to represent the MDOF dynamics of a turning machine and a voice coil actuator at its end generates the cutting force signal. A corner cube reflector is mounted on the other side

of the tip, which is a part of a HP laser interferometer setup, acting as the displacement sensor. The 16 bit position information from the interferometer is passed on to a DSP board where the regenerative cutting process is simulated in real time. The cutting force values, thus calculated, are fed back through the digital to analog converter of the DSP board and a current amplifier into the voice coil. The closed loop system thus consists of a hardware component, the beam, which represents the machine tool structure, the sensor and actuator and a software layer simulating the cutting forces and the delay part of the system. The setup is shown in Figure 8. The "Hardware in the Loop" concept is illustrated in Figure 9.

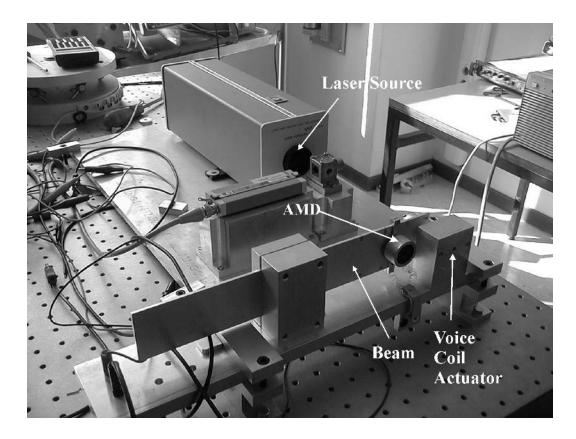


Figure 8: The demonstrator setup

The dSpace Control Desk software provides a graphic user interface, enabling the change of K_{cut} and T in real time on the DSP board. In other words the spindle speed and the cutting condition can be changed just as in real machining. Certain combinations of the two parameters, lead to an unstable feedback loop, resulting in a growth in the oscillations of the beam, thus representing a chatter situation. The feasibility of chatter

control by active damping is also investigated in the latter portion of the study. The beam has an inertial actuator, also called an Active Mass Damper (AMD)(manufactured by Micromega Dynamics) mounted on its side (details in Figure 12) which acts as an active damper to stabilize the system. This aspect of the demonstrator will be dealt with in the section on control of chatter.

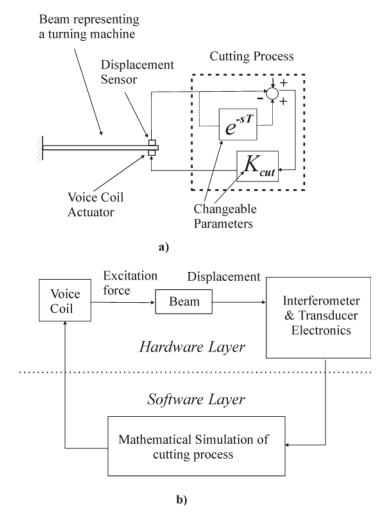


Figure 9: a)The "Hardware in the Loop" setup b) The software and hardware layers

5.1 Numerical simulation of chatter

This portion deals with investigations on stability with a numerical model of the beam structure. The frequency response data for the beam is obtained experimentally. The state space model of the beam is generated, using the Matlab based Structural Dynamics Toolbox, developed by SDTools [16], by curve fitting on the experimental frequency

response plot, using the Pole-Residue method in frequency domain. The identification is done without dismounting the AMD, in order to include its dynamics and generate a realistic model for the system. The Root Locus Method is used to generate the stability lobe diagram and to check for the nature of instability for various spindle speeds. The system is more complicated than the previous example, because of the involvement of multiple modes in the chatter process. The stability lobe diagrams and the chatter frequency for the demonstrator are plotted in Figure 10. As in the case of the SDOF system, there are different sources of instability, as shown in the figure. The Root Locus plots, showing the migration of the eigenvalues of the system are not included in the present work due to space constraints. The interaction between the structural poles and the delay is similar to that of the SDOF system, the only difference arising from the involvement of more than one mode in the process of instability.

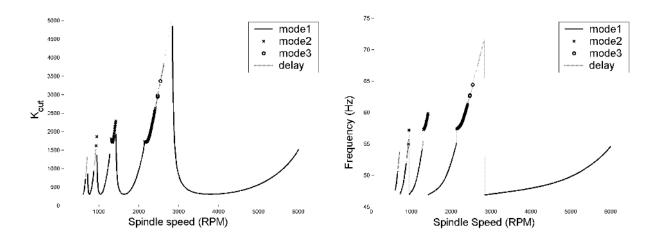


Figure 10: Stability lobes and chatter frequency diagrams for the demonstrator

5.2 Experimental simulation of chatter

Stability lobe diagrams are also generated experimentally. A value of the delay T is chosen. K_{cut} is increased step by step and for each step the displacement response of the beam, due to a computer generated impulse excitation is checked on the computer display. A stable system is characterized by a decaying response in contrary to an unstable response, which grows with time. In case the system is critically stable, the response to impulse does not grow or decay with time and the oscillations are sustained. The corresponding value of K_{cut} is stored as the limiting value for the chosen spindle speed. The frequency of the

displacement signal is approximately measured from an oscilloscope. This experiment is repeated for many spindle speeds. The experimental stability lobes and chatter frequencies are compared with those obtained by the Root Locus Method in Figure 11. Three sets of experiments are performed in order to check for the repeatability of the results. The match between experimental and the numerical analysis is good.

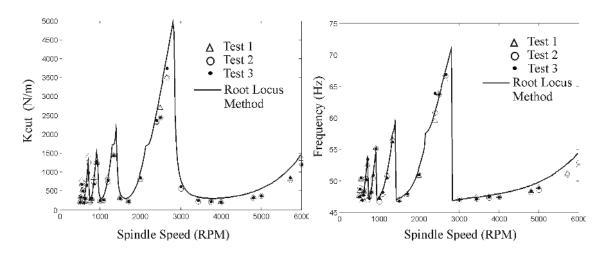


Figure 11: Comparison between experimental and numerical analyses on chatter instability

6 Application of active damping

Control of chatter is the most important aspect of research on this subject. The stability lobe diagram, as proposed in [1, 11], is an attempt to avoid chatter by prescribing a limit to the cutting condition for a chosen spindle speed. Efforts have been made by Slavicek [17] to avoid chatter by changing the geometry of the tool. However this approach is not practical since it makes the design of cutters specific to cutting conditions. Many authors have proposed on-line techniques for chatter avoidance either by spindle speed modulation or spindle speed selection. The spindle speed modulation technique involves periodic variation of the spindle speed with very low frequency, as proposed by Hoshi et al [18] experimentally and also by Sexton et al [19] and Lin et al [20]. The speed modulation technique is costly and limited by the inertia of the rotating parts of the machine. Tlusty et al [21], present an online speed selection control system, which iteratively finds out the spindle speed, corresponding to the maximum stability region in the stability lobe diagram. Soliman et al [22] present a control system that ramps up the spindle speed

until a stable machining situation is reached.

Vibration control techniques have been used by various researchers. Nachtigal et al in [23, 24] propose a feedforward strategy of using the cutting force signal for chatter control in turning. In many studies on chatter, it has been observed that machining stability can be enhanced by increased damping of the whole system. Tlusty [3] shows that damping in the cutting process stabilizes chatter. Merrit [10] shows that increase in the structural damping would raise the asymptotic threshold of stability and the region of stable machining would increase. Passive damping techniques such as the "Lanchester" damper have been proposed in [25], impact dampers in [26] and tuned mass dampers in [27]. But the amount of damping achievable is limited and the performance of tuned mass dampers depends on the accurate tuning between the damper frequency and the structural modal frequency. In the present work, active damping is applied using a collocated system with velocity feedback. Such a collocated configuration ensures unconditional stability in the closed loop [28].

In the present study on the demonstrator, the setup consists of an electromagnetic active mass damper (AMD), which is basically a spring-mass-dashpot system coupled to a voice coil actuator. This generates inertial forces which act upon the beam structure. This adds an extra layer to the "Hardware in the Loop" system. Two loops exist in the setup, i.e., the regeneration-cutting process loop and the active damping loop. This is illustrated in Figure 12. An accelerometer collocated with the AMD senses the acceleration signal. This is integrated and multiplied by a gain g through a controller block and fed into the coil of the AMD. The AMD thus introduces damping into the structure. Such a technique has been adopted in [29, 30]. The effect of active damping on the system under chattering conditions is now investigated.

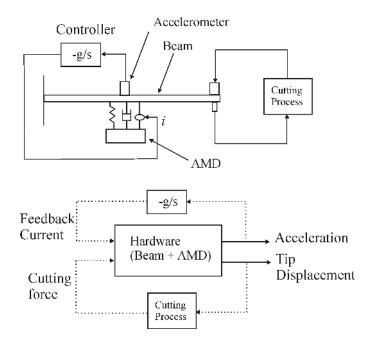


Figure 12: The chatter demonstrator with active damping layer

A Root Locus plot is generated for increasing K_{cut} at a spindle speed of 2000 RPM, without any active damping and for arbitrary feedback gains of 5 and 10 units. The identified model of the beam is used in this numerical exercise. In all the three cases, the instability arises from the first mode as shown in Figure 13. The system at 2000 RPM, without active damping, has an initial damping of 1.9% for the first mode, 1% for the second mode and 0.8% for the third mode. With active damping the first mode is heavily damped to 5% and 9% for 5 and 10 units of feedback gain respectively but the effect on the other two modes is not much. The loci of the eigenvalues for the first mode are much longer than that without any active damping. This explains the rise in the stability of the system against chatter.

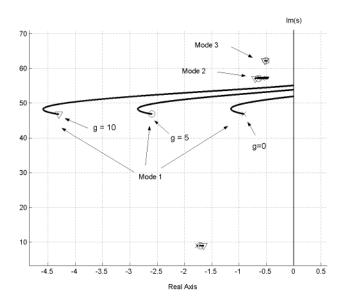


Figure 13: Root locus of eigenvalues for increasing K_{cut} without active damping and with feedback gains g = 5 and 10 units

Finally the stability lobes are determined experimentally to demonstrate the effect of active damping. As described earlier on experimental simulation of chatter, the demonstrator is set to chatter under several spindle speeds with the active damping feedback loop working. The results are plotted in Figure 14. Feedback gains of 5 and 10 units are chosen for the experiment. A rise in the level of stability limits is observed, confirming the role of active damping in increasing the stability against chatter.

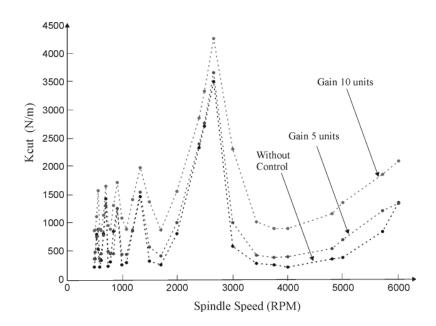


Figure 14: Effect of active damping on stability lobe diagram (Experimental)

7 Conclusions

The first part of this paper has analyzed the effect of damping on the stability lobe diagram for regenerative chatter. A root locus method was used, together with a Pade approximation of the delay, to investigate the stability numerically. It was found that instability may arise not only from the structural modes, but also from the poles due to the delay parameter. In the second part of the study a "Hardware in the Loop" demonstrator is developed to provide an insight into the physics of regenerative chatter in turning. The match between the experimental and numerical stability lobes is good. The demonstrator is also used for application of active damping for chatter suppression. The active control loop consists of a Active Mass Damper collocated with an accelerometer and the configuration has guaranteed stability. It is found that active damping is capable of raising the stability lobes and has a stabilizing effect on chatter. Thus this technique appears to be a good candidate for damping real machines actively and increase their productivity. The active damping scheme is currently being implemented on a large scale milling machine by Micromega Dynamics (http://www.micromega-dynamics.com/amd.htm).

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