Active metamaterials: Sign of refractive index and gain-assisted dispersion management

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(Received 27 April 2007; accepted 27 September 2007; published online 5 November 2007)

We derive an approach to determine the causal direction of wavevectors of modes in optical metamaterials, which, in turn, determines signs of refractive index and impedance as a function of *real* and *imaginary* parts of dielectric permittivity and magnetic permeability. We use the developed technique to demonstrate that the interplay between resonant response of constituents of metamaterials can be used to achieve efficient dispersion management. Finally, we demonstrate broadband dispersionless index and impedance matching in active nanowire-based negative index materials. Our work has a potential to open new practical applications of negative index composites for broadband lensing, imaging, and pulse routing. *© 2007 American Institute of Physics.*
[DOI: 10.1063/1.2800309]

Recent research in the area of negative index materials[1] (NIMs) has resulted in a number of exciting applications, including superimaging[2,3] and subwavelength light compression.[4] However, the majority of these applications suffer from substantial material absorption[5] and frequency dispersion of NIM composites. Optical gain has been suggested to minimize and potentially eliminate absorption losses.[6–8] While the effect of gain on propagation length of optical signals is straightforward, its effect on material dispersion has not been completely understood. Furthermore, the very question of sign of the refractive index in active metamaterials is somewhat controversial.[9–11] In this letter, we present a universal approach for imposing causality in active and passive materials and use this technique to analyze the perspectives of gain-assisted dispersion management beyond loss compensation.

Our results can be used to determine the sign of refractive index in active or passive media, as well as in a number of analytical and/or numerical solutions of the Maxwell equations relying on plane-wave representations. Our technique is illustrated using an example of nanowire-based NIM structure, originally proposed in the Ref. 12 and experimentally realized in Ref. 13. It is shown that frequency-independent (negative) index and impedance can be achieved in the same metamaterial with position-dependent gain in weak gain regime. A combination of broadband impedance and refractive index has a potential to open exciting applications of dispersion-managed NIMs in broadband optical processing, packet routing, and nonreflective lensing.

Index of refraction n_{ph} is one of the most fundamental optical properties of the media. Its magnitude relates the magnitude of wavevector $\mathbf{k}$ of plane electromagnetic wave to frequency ω, $|n_{ph}|=|\mathbf{k}|c/\omega$, and thus describes the phase velocities of waves in the material.[14] In particular, n_{ph} enters the equations for reflectivity, Doppler effect, and some nonlinear phenomena. Apart from n_{ph}, reflectivity of a material also depends on its impedance Z. For an isotropic material with (complex) dielectric permittivity ϵ and magnetic permeability μ, n_{ph} and Z are calculated via[14]

$$n_{ph} = \pm\sqrt{\epsilon\mu}, \tag{1}$$

$$Z = \mu/n_{ph} = \pm\sqrt{\mu/\epsilon}. \tag{2}$$

Note that while the magnitude of n_{ph} and Z are completely determined by material paramaters (ϵ and μ),[14] their signs have recently instigated some controversy,[9–11,15,16] which can be traced to different treatments of causality principle. Moreover, Pokrovsky and Efros[15] suggested that the Maxwell equations can be solved correctly regardless the selection of sign of refractive index. Such a freedom of choice, however, is accompanied by a requirement to adjust the signs in equations describing phase velocity-related phenomena, e.g., the Snells law,[15] and still require imposing causality (identical to that of Ref. 9 and 16) when solving the Maxwell equations.

From a mathematical standpoint, imposing causality principle is equivalent to selecting the sign of the wavevector of a plane-wave propagating away from the source. Here, we assume that such a propagation takes place along the positive z direction and therefore focus on the k_z component of the wavevector. Authors of (Refs. 9 and 16) proposed to select the sign of k_z by enforcing positive direction of the Poynting vector (associated with energy flux). Chen *et al.*[10] suggested that causality requires exponential decay [$k_z''>0$ (Ref. 17)] of waves in passive materials and exponential growth ($k_z''<0$) in active media. While all causality requirements discussed above[9–11,15,16] coincide for the case of passive materials, they are not directly applicable for active media, are therefore not universal, and often lead to unphysical results.

Indeed, enforcing the sign of energy flux is physical only in transparent materials (as described below). Materials with opposite signs of ϵ and μ (known as single-negative materials[4,14]) reflect the majority of incident radiation. Enforcing decay/growth of field based solely on passive/active state of material yields nonphysical results (such as abrupt disappearance of surface plasmon polariton waves) when media undergo smooth transition from low-loss to low-gain state.[18]

To resolve the above controversy,[9,10] we propose to simultaneously consider transparency of the material in the

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TABLE I. The direction of wavevector (and thus the sign of refractive index) in optical material is related to the interplay between transparency and gain/loss state of the media. The table summarizes this dependence. First column represents the transparency state of the material determined by the sign of product $\epsilon'\mu'$; the sign of $|\epsilon|\mu''+|\mu|\epsilon''$ (second column) determines whether the material is passive ($|\epsilon|\mu''+|\mu|\epsilon''>0$) or active ($|\epsilon|\mu''+|\mu|\epsilon''<0$). The sign of the refractive index is selected to satisfy the requirement for wave attenuation ($k_z''>0$) or growth ($k_z''<0$) (third column).

Transparency $\epsilon'\mu'$	Gain(−)/loss(+) $\lvert\epsilon\rvert\mu''+\lvert\mu\rvert\epsilon''$	Wave growth(−)/decay(+) k_z''
+	+	+
+	−	−
−	Any	+

absence of losses and gains, $\epsilon''=\mu''=0$, along with absorption (or gain) state of this material.[19] Clearly, electromagnetic radiation should decay inside all passive media.[14] It should also grow inside *transparent* (double-negative or double-positive) active materials. Nontransparent (single-negative) materials do not support propagating modes. Energy can penetrate these structures only in the form of exponentially decaying waves.[20] Since decay/growth of electromagnetic waves can be related to the sign of imaginary part of the refractive index, our arguments, summarized in Table I, provide complete solution to the problem of selection of direction of the wavevector of plane waves. For isotropic media, the developed technique also provides a solution to selection of the sign of n'_{ph}, which should be identical to that of k'_z, yielding "conventional" Snell's law.[1]

For passive media, our results agree with those of Refs. 15 and 16, and with Refs. 10 and 11, relying on the preselected branch cut when calculating the square root in Eq. (1). Table I, however, cannot be reduced to such a cut. Indeed, an optical material can fall into one of the four cases: it has either negative ($n'_{ph}<0$) or positive ($n'_{ph}>0$) refractive index and it attenuates ($n''_{ph}\geqslant 0$) or amplifies ($n''_{ph}<0$) incoming radiation.[17] Selection of any single complex plane cut in Eq. (1) would immediately limit the number of possible $\{n'_{ph}, n''_{ph}\}$ combinations to two and therefore, in general, is not correct.

Requirements of Table I can be formally satisfied by the following procedure: starting from material parameters ϵ and μ, one first calculates $\sqrt{\epsilon}$ and $\sqrt{\mu}$, cutting the complex plane *along negative part of the imaginary axis*.[18,20] Refractive index and impedance are then calculated as $n_{ph}=\sqrt{\epsilon}\sqrt{\mu}$; $Z=\sqrt{\mu}/\sqrt{\epsilon}$ as was suggested for passive NIMs in Ref. 21.

We note that the above procedure can be generalized to other classes of materials and excitation waves.[7,18,20,22,23]

We now employ the developed technique to analyze the gain-assisted dispersion management in active negative index metamaterials. To illustrate our approach, we select nanowire-based optical NIM system schematically shown in Fig. 1. In the limit of small concentration of nanowires p, effective dielectric permittivity and magnetic permeability of such a mix can be qualitatively described by[12]

$$\epsilon_{\text{eff}} = \epsilon_h + \frac{4pr}{d}\frac{f(\Delta)\epsilon_m}{1+\dfrac{4f(\Delta)\epsilon_m r^2}{l^2}\ln\left(1+\dfrac{\epsilon_h l}{2r}\right)\cos\Omega}, \tag{3}$$

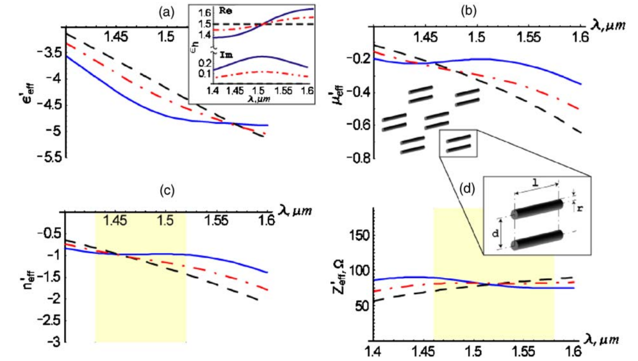

FIG. 1. (Color online) Effective permittivity (a), permeability (b), refractive index (c), and impedance (d) of the passive nanowire NIMs, as schematically shown in the inset in (b); solid lines corresponds to $A=0.04$, dashed to $A=0$, and dash-dotted to $A=0.0175$. The inset in (a) shows the real and imaginary parts of permittivity of host material

$$\mu_{\text{eff}} = 1 + \frac{12pl^2C_2k^2d^2}{rd}\frac{2\tan(gl/2)-gl}{(gl)^3}, \tag{4}$$

where r, l, and d correspond to nanowire radius, length, and separation between two wires, and remaining parameters are given by $\Omega^2=k^2l^2(\ln[l/2r]+i\sqrt{\epsilon_h}kl/2)/(4\ln[1+\epsilon_h l/2r])$, $C_2=\epsilon_h/(4\ln[d/r])$, $g^2=k^2\epsilon_h[1+i/(2\Delta^2 f[\Delta]\ln[d/r])]$, $\Delta=kr\sqrt{-i\epsilon_m}$, and $f(\Delta)=(1-i)J_1[(1+i)\Delta]/(\Delta J_0[(1+i)\Delta])$, with $k=2\pi/\lambda=\omega/c$, λ being wavelength in the vacuum and ϵ_m and ϵ_h being permittivities of nanowires and host materials. Here, we assume that ϵ_m of silver wires is described by the Drude model[24] and host is a polymer ($\epsilon_0\simeq 1.5$) doped with quantum dots,[25] qualitatively described by the Lorentz model,

$$\epsilon_h = 1.5 + \frac{A\omega_0^2}{\omega_0^2-\omega^2-i\omega\gamma}, \tag{5}$$

where ω_0 is the resonant frequency, γ is the damping constant, and A is the macroscopic analog of Lorentz oscillator strength, which formally describes gain in the system and can be related to the concentration of quantum dots and the fraction of quantum dots in excited state. $A>0$ corresponds to lossy materials, $A=0$ represents the case when the number of excited quantum dots is equal to the number of dots in the ground state, and $A<0$ corresponds to the inverted (gain) regime.[26] The permittivity of the host medium and corresponding permittivity of the NIM system for different pump rates are shown in the inset of Fig. 1(a).

Figures 1 illustrates dispersion management in *lossy* ($A\geqslant 0$) nanowire composites with $p=0.1$, $r=25$ nm, $l=700$ nm, $d=120$ nm, $\lambda_0=2\pi c/\omega_0=1.5\ \mu$m, and $\gamma=0.628\ \mu\text{m}^{-1}$. It is clearly seen that dispersion of host media can completely compensate the dispersion of refractive index and impedance of the NIM system. Note broadband refractive index and broadband impedance are realized at different values of oscillator strength A in the single-oscillator model assumed here. We suggest that impedance matching can be combined with index matching in the same system where A is (adiabatically) changed from $A\simeq 0.0175$ corresponding to $\partial Z/\partial\omega=0$ at the interface to $A\simeq 0.04$ corresponding to $\partial n_{ph}/\partial\omega=0$ in the core of the system by changing quantum dot doping or external pumping rate.

Active quantum dots $A<0$ can simultaneously reduce absorption in the system and provide versatile gain-assisted

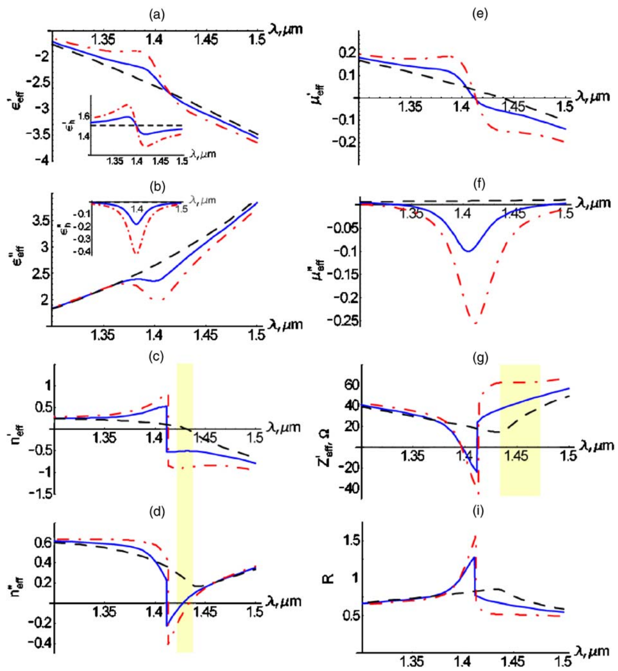

FIG. 2. (Color online) Real and imaginary parts of the effective permittivity [(a) and (b)], permeability [(e) and (f)], refractive index [(c) and (d)], real part of impedance (g) of active nanowire NIMs, and reflection from the semi-infinite slab of this system (i). Solid, dashed, and dash-dotted curves correspond to $A=-5.05\times10^{-3}$, $A=0$ and $A=-0.012$, respectively. The insets in (a) and (b) show the real and imaginary parts of permittivity of host material, respectively.

dispersion management. Note that such a modulation of $n_{\rm ph}$ or Z does not require full compensation of propagation losses.

Gain-assisted dispersion management in active nanowire composites with $A<0$, $\lambda_0=1.4\ \mu$m, $\gamma=0.129\ \mu\text{m}^{-1}$, $p=0.09$, $r=25$ nm, $l=720$ nm, and $d=120$ nm is shown in Fig. 2. Note that refractive index, impedance, as well as reflectivity between vacuum and nanowire metamaterial are continuous when material switches between active and passive states. In contrast to this behavior, transition between transparent and "metallic" regimes yields a discontinuity in reflectivity (the discontinuity disappears when thickness of gain region is finite). This discontinuity[23] is accompanied by enhanced reflection ($R>1$) and has a physical origin similar to the one of enhanced reflectivity reported in Refs. 18 and 23 for gain media excited by evanescent waves in total internal reflection geometry.

To conclude, we developed a universal approach to determine the sign of refractive index and impedance in active and passive media. We have further utilized this approach to demonstrate versatile dispersion management, achieving $\partial n_{\rm ph}/\partial\omega=0$ and $\partial Z/\partial\omega=0$ regimes in nanowire-based NIM system with bandwidth equivalent to picosecond optical pulses. The developed technique can be readily utilized to determine sign of the refractive index in different classes of materials and can be used to optimize the dispersion of these structures for applications at micro- and nano-scales.[27,28]

The authors acknowledge fruitful discussions with A. L. Efros. This research has been partially supported by PRF (ACS) Grant No. 43924-G10, ARO, ONR, NSF PREM (No. DMR 0611430), NSF NCN (No. EEC-0228390), NSF CREST No. HRD 0317722, and NASA URC (No. NCC3-1035).

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[20] Excitation of waves in active singe-negative materials is equivalent to excitation of waves in conventional active media (dye solution) by evanescent waves, commonly realized in SPP experiments, in which case the eigentransparency criterion becomes $\epsilon'\mu'\omega^2/c^2-k_x^2$, with k_x being wavevector component directed along the interface. As shown in Ref. 18, selection of exponentially growing solution in active singe-negative materials will lead to unphysical results such as disappearance of SPP mode.
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