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Active Vibration Absorber Design via Sliding Mode Control

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Abstract: A new, simple control method for vibration absorber design is presented in this paper. A nonlinear robust control scheme based on a variable structure is designed and simulated. Robust synthesis of the discontinuity surface based on classical frequency loop-shaping and the Edge Theorem is discussed. The proposed control scheme has two advantages over the current existing vibration absorber design methodologies: (i) it is completely insensitive to changes in the stiffness and damping of the absorber, and it is strongly robust against parametric uncertainties of the primary vibrating structure; (ii) it is capable of suppressing both cyclic and random vibrations over a very wide range of frequencies.

Key words: Sliding mode control, variable structure, discontinuity surface, vibration absorber.

I. Introduction

The vibration absorber technique refers to the use of a mass-spring-damper system that is attached to a primary vibrating structure in order to suppress its vibration. Vibration absorbers can be passive, active, or passive-active in nature. A variety of active vibration absorbers, such as the dual-frequency fixed-delay resonator [1], have been designed recently. Many adaptive control algorithms have also been proposed, e.g., [2]. In these works, the convergence of the parameter estimation for such adaptive control schemes is an important issue, and the modelling error is not fully addressed. Recently, a band-pass vibration absorber design has been proposed in [3], however, as stated by its authors, the proposed absorber itself is not stable even though the whole system is stable. A demonstration of the feasibility of a nonlinear robust control scheme, based on the principle of sliding mode control, for the design of a stable controller that is capable of suppressing random and cyclic vibrations of unknown frequencies in the presence of significant modelling error, is presented in this paper.

II. Vibration Absorber Model

The construction of a vibration absorber device consists of one mass-spring-damper trio $[m_a, c_a, k_a]$ attached to another mass-spring-damper trio $[m, c, k]$. In state-space representation, the equation of motion for the combined system is:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}_1 u + \mathbf{B}_2 f. \tag{1}$$

where: $\mathbf{X} = \begin{bmatrix} x_a & x & \dot{x}_a & \dot{x} \end{bmatrix}^T$,

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_a/m_a & k_a/m_a & -c_a/m_a & c_a/m_a \\ k_a/m & -(k_a+k)/m & c_a/m & -(c_a+c)/m \end{bmatrix},$$

$\mathbf{B}_1 = [0 \;\; 0 \;\; -1/m_a \;\; 1/m]^T$, $\mathbf{B}_2 = [0 \;\; 0 \;\; 0 \;\; 1/m]^T$, u is the control input, and f is the unknown disturbant force. For vibration absorber design, the control purpose is to minimise the displacement x of mass m while keeping the displacement x_a of mass m_a bounded.

III. Sliding Mode Controller Design

The design of the sliding mode controller consists of two phases. A stable discontinuity surface is first designed to describe the dynamics of the system in sliding mode. A switching control law is then formed to guarantee that all states can converge to this surface. The discontinuity surface, S, is a linear combination of the state variables:

$$S = \mathbf{C}\mathbf{X}. \tag{2}$$

System stability during sliding mode, i.e., when S = 0, can be achieved by proper selection of **C**. Given that the matrix **CB** is nonsingular, the equivalent control during sliding mode is then described by:

$$u_{eq} = -(\mathbf{CB})^{-1}\mathbf{CAX}. \tag{3}$$

Finally, the control law is expressed as:

$$u = u_{eq} + K\,sign(S) \quad , \tag{4}$$

where K is chosen to be large enough to account for uncertainty in the magnitude of the disturbance f. Asymptotic stability of the controlled system is guaranteed once the following reaching condition is satisfied:

$$S\dot{S} \le -\eta|S|, \quad \eta > 0. \tag{5}$$

IV. Discontinuity Surface Synthesis

Equation (1) is equivalent to:

$$\begin{cases} m\ddot{x} + c\dot{x} + kx - c_a(\dot{x}_a - \dot{x}) - k_a(x_a - x) = u + f, \\ m_a\ddot{x}_a + c_a(\dot{x}_a - \dot{x}) + k_a(x_a - x) = -u. \end{cases} \tag{6a,b}$$

Simple manipulation of (6a,b) gives:

$$\begin{cases} m\ddot{x} + c\dot{x} + kx + m_a\ddot{x}_a = f, \\ m_a\ddot{x}_a + c_a(\dot{x}_a - \dot{x}) + k_a(x_a - x) = -u. \end{cases} \tag{7a,b}$$

Assume a linear discontinuity surface of the form:

$$S = a_1 x_a + a_2 x + a_3\dot{x}_a + a_4\dot{x}. \tag{8}$$

The system dynamics during sliding mode is described by:

$$\begin{cases} m\ddot{x} + c\dot{x} + kx + m_a\ddot{x}_a = f, \\ a_1 x_a + a_2 x + a_3\dot{x}_a + a_4\dot{x} = 0. \end{cases} \tag{9a,b}$$

Taking the Laplace transform of (9a,b) and substituting (9b) into (9a) gives:

$$X_F(s) = \frac{a_3 s + a_1}{D(s)}, \quad X_{aF}(s) = -\frac{a_4 s + a_2}{D(s)} \quad , \tag{10a,b}$$

where: $D(s) = M(s)(a_3 s + a_1) - m_a s^2(a_4 s + a_2)$. (10c)

X_F and X_{aF} are the transfer functions from f to x and from f to x_a, respectively, and $M(s) = ms^2 + cs + k$. The problem then reduces to designing a discontinuity surface S such that the closed-loop transfer functions X_F and X_{aF} have desired shapes. The 3rd-order closed-loop characteristic equation D(s) has 4 coefficients, a_1, a_2, a_3, a_4, which can be designed by specifying the desired closed-loop poles. Thus, any desired closed-loop dynamics can be achieved by proper selection of the discontinuity surface. For a given primary system $[m, c, k]$, there is a trade-off between the vibration attenuation level, the settling time of the absorber, and its maximum allowable displacement.

V. Stability & Robustness Analyses

Stability of this control law is guaranteed if (i) the discontinuity surface is attractive, and (ii) the roots of D(s) have negative real parts. The first condition can be met with the choice of the equivalent control law as shown in (3), while the second condition can be satisfied by a judicious choice of the discontinuity surface gain. The dynamics of the coupled system during sliding mode is completely insensitive to changes in the stiffness and damping of the absorber, as evident from (10a,b). Robustness against parametric uncertainties in the primary system $[m, c, k]$ and the absorber mass m_a can be analysed using the Edge Theorem [4].

VI. Simulation & Discussions

The performances of the proposed control scheme are assessed via simulation. The parameters of the two trios are: $[m, c, k]$ = [5kg, 100Ns/m, 16000N/m], and $[m_a, c_a, k_a]$ = [1kg, 1Ns/m, 3200N/m]. Both the absorber and the primary system have a resonant frequency at ω_0=56.57rad/s. Assume that the active absorber is expected to operate over the frequency band [40rad/s, ∞]. Note that the suppression frequency band has no upper limit. The Bode plot of X_F is shaped such that it has a cut-off frequency at ω_c=10rad/s, and its response in the frequency range [0, 40rad/s] is not amplified compared with that of the passive system. This loop-shaping gives an attenuation level of approximately 30dB in the suppression band. Figure 1 shows the Bode plots of X_F and X_{aF}. The required closed-loop poles are then placed at -7± 7.1414j and -1, which corresponds to the discontinuity surface C=[0.0063 -14.2602 0.0071 -0.9646]. In order to avoid control chattering (due to the fact that the switch can not be infinitely fast), a boundary layer switching scheme is employed [5]. The switching gain is set to K=100, with a boundary layer of thickness 0.001. Figure 2(a) shows the closed-loop time responses when the system is excited by a sine wave of amplitude 1N, with frequencies swept from 25rad/s to 105rad/s and back to 25rad/s. As seen from this figure, vibrations of frequencies greater than 40rad/s are suppressed efficiently. Figure 2(b) shows the closed-loop time response when the system is excited by a band limited white noise generated by Matlab Simulink. The noise power is set to 0.01 and the sampling time is 0.01. Active control gives significant improvements: the peak amplitude of the primary displacement is 23.3% and its r.m.s. value is 25.5% that of the passive control. The simulations have been carried out without the equivalent control component u_{eq}. This demonstrates the simplicity of the proposed control scheme implementation.

Assume now, that the values m, c, k, and m_a are perturbed within ± 20% of their nominal values. From equation (9b):

$$\frac{X(s)}{X_a(s)} = -\frac{a_3 s + a_1}{a_4 s + a_2}. \tag{11}$$

The requirement that x is attenuated in the range [40rad/s, ∞] can only be achieved if the magnitudes of a_1 and a_3 are much smaller than those of a_2 and a_4. Combined with (10c), this automatically enables the controller to have a strongly robust performance, which can be tested via the Edge Theorem as follows. From (10c), the perturbed closed-loop characteristic polynomial is:

$$\delta(s) = p_1 E_1(s) + p_2 E_2(s) + p_3 E_3(s) + p_4 E_4(s) \tag{12}$$

where: $E_1(s) = a_3 s^3 + a_1 s^2$, $E_2(s) = a_3 s^2 + a_1 s$, $E_3(s) = a_3 s + a_1$, $E_4(s) = -(a_4 s^3 + a_2 s^2)$, and p_1, p_2, p_3, p_4 vary within the uncertainty ranges of m, c, k, and m_a respectively. All p_i except one, e.g. p_n, are fixed at the upper or lower bounds of their uncertainty ranges, while p_n is allowed to sweep over its own uncertainty range. The process is then repeated for n = 1, 2, 3, 4. This procedure guarantees that all the exposed edges are captured [4]. The root loci of the perturbed system are plotted in Figure 3 which shows that the real parts at -7 are only perturbed within [-7.2, -6.83], and the real part at -1 is only perturbed within [-1.1, -0.95].

VII. Conclusions

Preliminary simulation results show that the control approach via sliding mode is feasible for designing an active vibration absorber in order to suppress both cyclic vibrations of unknown frequencies and white noise disturbances. The proposed control scheme is simple for implementation and has fast transient response since it does not employ any adaptive algorithms. Robust performance with respect to modelling uncertainty is also demonstrated. Current studies are focused on examining the effects of measurement noise and time delay.

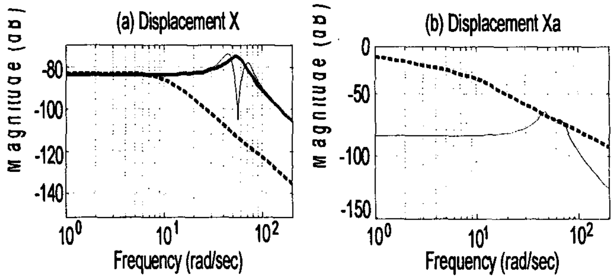

Figure 1. Frequency responses of (a) primary mass displacement, thick line: uncontrolled, thin line: passive control, dotted line: active control; (b) absorber mass displacement, continuos line: passive control, dotted line: active control.

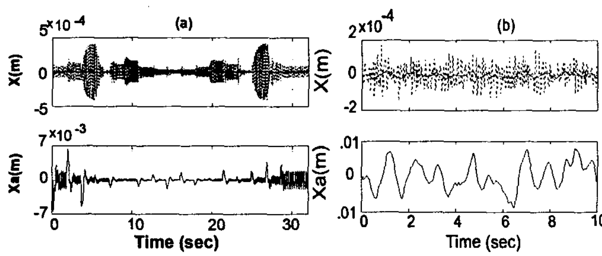

Figure 2. Time responses of x and x_a. Dashed lines: passive control, continuous lines: active control. (a) with sinusoidal disturbances of amplitude = 1N, swept frequencies from 25rad/s to 105rad/s and back to 25rad/s , with step of 10rad/s every 2sec. (b) with band-limited white noise, noise power 0.01, sample time 0.01.

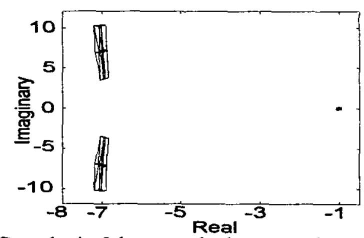

Figure 3. Root loci of the perturbed system along the 32 edges.

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