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Abstract Isolating the sensitive equipment from vibrating base or the foundation from machinery vibration is of practical importance in a number of engineering fields. With the development of the vibration control techniques and increasing requirements for the higher-performance vibration isolation in industry and everyday life, active vibration isolation exhibits the best performances. In this paper, active vibration isolation reducing vibration transmitted from vibrating base to sensitive equipment and from machinery to foundation was investigated. Controller as static output feedback was considered to design components of active isolation system. An active control is provided by using H_∞ control criteria to design this controller. This criterion is presented as a cost function and then optimized by Particle Swarm Optimization (PSO) algorithm. The approach is validated using numerical simulation. Results show that this static output feedback H_∞ controller using PSO algorithm can get good performance to reduce the effect of unwanted vibration and disturbances.

Keywords Active isolation · Vibration control · Particle Swarm Optimization (PSO) · H_∞ control

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1 Introduction

Vibration that is transmitted to the foundation (supporting structure) from machinery or to sensitive equipment from vibrating base can be problems in large varieties of engineering applications. Vibrating, rotating and impacting systems create machine-induced vibration or shock, which is transmitted into their support systems. For instance, industry rotating machines and equipments that are not properly balanced produce centrifugal forces creating steady state vibration. Machines generating pulses or impacts, such as forging presses, hammers and compressors are the most sources of vibration and shock. The shock is transmitted through the mounting system to foundation, soil and surrounding environment. The transmitted vibration generates disturbance to neighboring equipment and the residential area. On the other hand, vibration affects the reliability and performance of systems, such as high precision equipment, machine-tools, and measuring instruments. The accuracy of measurement and validity of tests are influenced by vibrations and disturbances.

An effective method for reducing vibration is by using a vibration isolation system. The main objective of the vibration isolation system is to reduce transmission of vibratory forces to the machine or the foundation. Additionally, it is also important to ensure that relative displacement, acceleration of machine and structure are minimized as far as possible. These limitations are observed when a passive isolation mount is used [1]. With such passive mounts there is a trade-off between low and high-frequency isolation performances. Statically, the solution of such major problem is to make the mount as stiff as possible to better support the equipment and dynamically, as soft as possible to better isolate the equipment. It is difficult to carry out with passive elastomeric mounts, as described by Crede [2] and Ungar [3]. To provide a more approving static and dynamic stiffness compromise, active isolation solutions such as skyhook damping [4] must be used, which are usually based on mounts and actuators.

Generally the best isolation performance is achieved by using an active system in combination with a passive mount. There are mainly two approaches towards developing an active vibration control mount: the feedforward approach and the feedback approach [1]. Feedforward control involves feeding a signal related to the disturbance input into the controller which then generates a signal to drive a control actuator in such a way as to decrease the disturbance [1, 5]. On the other hand, feedback control uses signals measured from the system response to drive a control actuator so as to assure the response. The use of feedback control for vibration isolation has been studied by several researchers [6–8]. The static output feedback is one of the most important issues in control theory and applications. The reason for this is that it represents the simplest closed-loop control that can be realized in practice. Consequently, many researchers have addressed the problem of static output feedback control design [9]. Fuller et al. [10] give a deep understanding and background of active isolation of vibration. Various control strategies are discussed including feedforward and feedback concepts for systems. When an active isolator is designed, two configurations are possible. The actuator can be placed either in series or in parallel with the passive mount. Beard et al. [11] investigated the first configuration by coupling a piezoelectric actuator in series with a passive mount. Due to the small deflection capacity of piezoelectric actuators, the use of such actuation is limited to the isolation of very small amplitude motion of base structure. In many positions, the base vibration doesn't exceed millimeters. As a result, an actuator with a longer fling, such as electromagnetic shaker, is required. An experimental study was conducted by Serrand and Elliott [12] on the active vibration isolation of a rigid equipment structure using two electromagnetic shakers, which were installed in parallel with two passive mounts.

An active isolator can be implemented using various feedback control strategies, among which independent velocity feedback control is one of the most popular. The absolute velocities of the equipment structure are measured at each mounting point and directly feedback to the actuators. Kim et al. [13] and Huang et al. [14] presented a theoretical and experimental investigation of an active vibration isolation system. In their studies, a velocity feedback control was employed, whereby each electrodynamics actuator is operated independently by feeding back the absolute velocity at the same location. Preumont et al. [15, 16] compared the force feedback and acceleration feedback implementation of a skyhook damper used to isolate a flexible structure from a disturbance source. The complete modeling of the active vibration isolation system is presented by Muller et al. [17]. However, their model was restricted to vertical motion of the system. Beadle et al. [18] derived the complete model and identified the parameters for the loaded as well as for the unloaded case. The isolation system is modeled by two rigid plates which are connected by four horizontal and four vertical actuators which have integrated accelerometers.

In addition, in optimization problems, we envisage many cases in which a problem can be formulated as a global optimization problem with very complex constraint, nonlinear and multi peaked characteristics. It is necessary, to find a global solution for this optimization problem. Some methods such as Genetic Algorithms, Simulated Annealing, Particle Swarm Optimization and so on, are mainly designed to be applied to these problems. In 1995, Eberhart and Kennedy [19] introduced Particle Swarm Optimization (PSO), a subset of Swarm Intelligence methods. This algorithm was inspired by collective behavior of insects and animals. Currently, PSO is a well known, effective approach for solving complex optimization problems [20, 21].

This paper is organized as follows. In Sect. 2, control model and the formulation of the problem are introduced. Active vibration isolation by using active mount to reduce vibrations transmitted from machinery to foundation and base as well as foundation to sensitive equipment is investigated. For this simulation, two different models are presented. In the first model, active isolation mount installed between vibrating machine and foundation is used to reduce

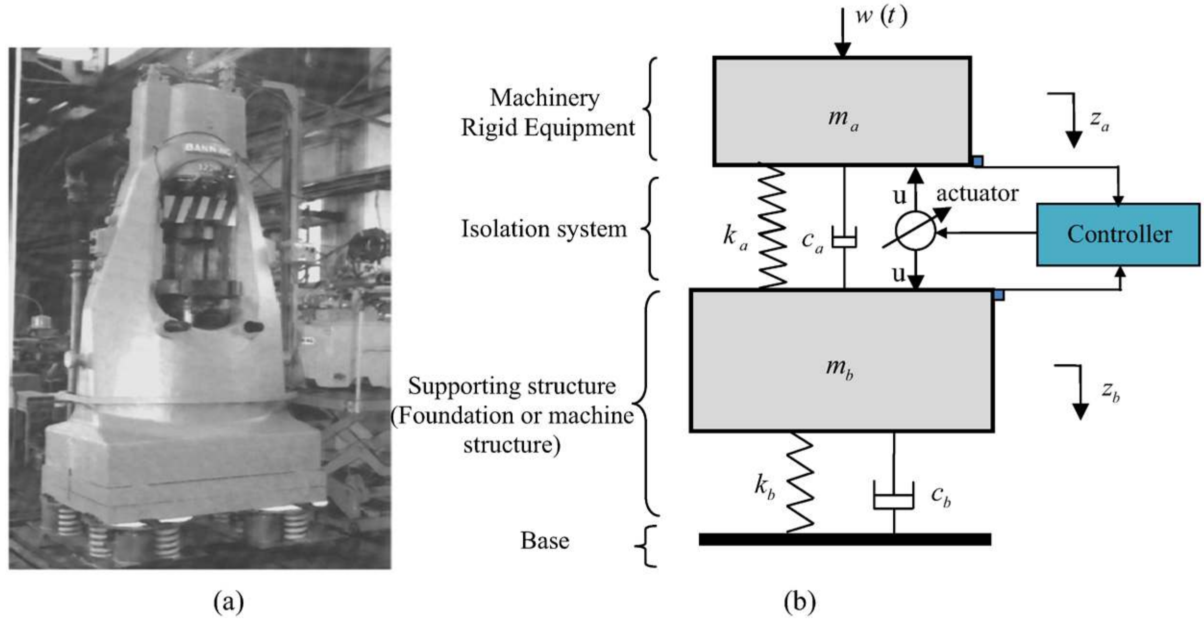

Fig. 1 Active isolation system to reduce transmission vibration from Machine to the base. (**a**) The example of forging hammer for this case, (**b**) the schematic of system model

transmitted vibrations from machinery to the foundation and base. In the second model, isolation system is used to reduce vibrations transmitted from vibrational environment to sensitive equipment. In this active anti-vibrational system, signals are obtained by sensitive vibrational detectors and feedback to the actuator through a feedback controller which is provided a compensation force to reduce transmitted of vibration. Controller in use is as static output feedback. The criterion considered in this paper to design controller and improve performance of isolation system is H_∞ control criterion. This criterion is obtained as cost function, and should be optimized. Particle Swarm Optimization (PSO) method is used to optimize this problem. The PSO algorithm is stated in Sect. 3. Section 4 presents the simulation results for two models and performance evaluations. Conclusions are given in Sect. 5.

2 Model description and problem statement

The main objective of the vibration isolation system is to reduce force and displacement transmitted from machinery to the foundation or from vibrating base to the equipment. This paper, in particular, investigates these two types of active isolation system. For this active isolation problem, in this section two different models are considered. These configurations can be used to represent many commonly encountered vibration isolation problems.

In the first model, active isolation control is used to reduce unwanted effects of vibrations caused by industrial machinery to the foundation and base. A good example of these cases is the forging hammer as depicted in Fig. 1(a). Hammer and press foundations are subjected to powerful dynamic effects during the operation of the supported equipment. These effects may extend to the surrounding and other sensitive machines or neighboring residential areas. The design objectives for foundations supporting equipment are to reduce the vibration amplitudes and the forces transmitted to the soil in order to minimize any disturbance to the neighborhood and surroundings. In order to achieve these objectives, active vibration isolation systems are used to support equipment. The mathematical model of these systems is shown in Fig. 1(b). In this model m_a is the mass of machinery and m_b is the mass of foundation or supporting structure. The spring with a stiffness k_a, and a damper with a damping c_a represent uncontrolled components of isolation system. The spring and damping constants k_b, and c_b represent the stiffness and damping of the foundation system.

Active part of the isolation system is an actuator with a physical force (u), usually applied by magnetic or hydraulic actuator to control system performance. Signals are measured and feedback to the actuator through a feedback controller which is provided compensation force to reduce the transmitted vibration. $w(t)$ is disturbance and vibration created in the system. In industrial machinery this disturbance is often created in reciprocating, rotating and impacting systems. Finally, Z_a and Z_b are machinery and foundation displacement, respectively.

The governing equations of motion are given by:

$$\begin{aligned} m_a\ddot{z}_a &= -k_a(z_a - z_b) - c_a(\dot{z}_a - \dot{z}_b) + w(t) - u(t), \\ m_b\ddot{z}_b &= k_a(z_a - z_b) + c_a(\dot{z}_a - \dot{z}_b) \\ &\quad - k_b z_b - c_b\dot{z}_b + u(t). \end{aligned} \tag{1}$$

Considering state variables as follows:

$$\begin{aligned} x_1 &= z_a - z_b, \quad x_2 = z_b, \\ x_3 &= \frac{dz_a}{dt}, \quad x_4 = \frac{dz_b}{dt}, \end{aligned} \tag{2}$$

where x_1, x_2, x_3 and x_4 are relative and absolute displacement of foundation, and velocity of machinery and foundation, respectively. So, state vector is defined as $x = [x_1\ x_2\ x_3\ x_4]^T$. State equation of system introduced by (1) is as follows:

$$\dot{x} = Ax + B_1 w + B_2 u, \tag{3}$$

where constant matrices A, B_1, and B_2 are defined as:

$$A = \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -k_a/m_a & 0 & -c_a/m_a & c_a/m_a \\ k_a/m_b & -k_b/m_b & c_a/m_b & -(c_a+c_b)/m_b \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0 \\ 0 \\ 1/m_a \\ 0 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 0 \\ 0 \\ -1/m_a \\ 1/m_b \end{pmatrix}.$$

The main objective of this active isolation system is to reduce transmitted forces through the foundation to soil and surrounding environment. Additionally, it is also important to ensure that displacement and acceleration of foundation as well as relative displacement between the machine and the foundation and machine acceleration are minimized as far as possible. Therefore, the controlled output, z, are defined as $F_b = (k_b z_b + c_b\dot{z}_b)$, z_b, $\ddot{z}_b$, $(z_a - z_b)$, and $\ddot{z}_a$. The control state equations of output are presented as:

$$z = C_1 x + D_{11} w + D_{12} u \tag{4}$$

where matrices C_1, D_{11}, and D_{12} are as follows:

$$C_1 = \begin{pmatrix} 0 & k_b & 0 & c_b \\ 0 & 1 & 0 & 0 \\ k_a/m_b & -k_b/m_b & c_a/m_b & -(c_a+c_b)/m_b \\ 1 & 0 & 0 & 0 \\ -k_a/m_a & 0 & -c_a/m_a & c_a/m_a \end{pmatrix},$$

$$D_{11} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1/m_a \end{pmatrix}, \qquad D_{12} = \begin{pmatrix} 0 \\ 0 \\ 1/m_b \\ 0 \\ -1/m_a \end{pmatrix}.$$

Minimizing the above mentioned outputs simultaneously is impossible and in most cases, decreasing one output causes the other one to increase. Constant $\alpha, \beta, \lambda, \zeta$, and γ are defined to desirable balance between outputs of the system. These weightings are used to control the trade-off between the control objectives. In this way, by weight considered for each output we identify the importance of that output. Matrices C_1, D_{11}, and D_{12} were rewritten as follows:

$$z = \begin{pmatrix} \alpha(k_b z_b + c_b\dot{z}_b) \\ \beta(z_a) \\ \lambda(\ddot{z}_a) \\ \zeta(z_a - z_b) \\ \gamma(\ddot{z}_a) \end{pmatrix} = C_1 x + D_{11} w + D_{12} u$$

$$C_1 = \begin{pmatrix} 0 & \alpha k_b & 0 & \alpha c_b \\ 0 & \beta & 0 & 0 \\ \lambda k_a/m_b & -\lambda k_b/m_b & \lambda c_a/m_b & -\lambda(c_a+c_b)/m_b \\ \zeta & 0 & 0 & 0 \\ -\gamma k_a/m_a & 0 & -\gamma c_a/m_a & \gamma c_a/m_a \end{pmatrix},$$

$$D_{11} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \gamma/m_a \end{pmatrix}, \qquad D_{12} = \begin{pmatrix} 0 \\ 0 \\ \lambda/m_b \\ 0 \\ -\gamma/m_a \end{pmatrix}. \tag{4a}$$

In this model we define all weighting coefficients equal to one.

Some measurable variables are used as controller input. In this system, relative displacement of machine and velocity of foundation are considered as measurable variables. The input controller equations can be expressed as the following:

$$y = C_2 x + D_{21} w + D_{22} u \tag{5}$$

where, constant matrices C_2, D_{21}, and D_{22} are as follows:

$$C_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$
$$D_{21} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad D_{22} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

We can express (3), (4a), and (5) in the simplified following form:

$$\begin{pmatrix} \dot{x} \\ z \\ y \end{pmatrix} = \left(\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right) \begin{pmatrix} x \\ w \\ u \end{pmatrix}. \tag{6}$$

Controller selected to improve system performance is a static output feedback as follows:

$$u = Ky = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} z_a - z_b \\ \dot{z}_b \end{bmatrix}. \tag{7}$$

Considering (5) and (7) we can write:

$$u = K(I - D_{22}K)^{-1}(C_2 x + D_{21} w). \tag{8}$$

Substituting (8) in (3) and (4), the state equations for close-loop system are obtained as:

$$\begin{pmatrix} \dot{x} \\ z \end{pmatrix} = \begin{pmatrix} A_{cl} & B_{cl} \\ C_{cl} & D_{cl} \end{pmatrix} \begin{pmatrix} x \\ w \end{pmatrix}. \tag{9}$$

So that:

$$\begin{aligned} A_{cl} &= F_l\left[\begin{pmatrix} A & B_2 \\ C_2 & D_{22} \end{pmatrix}, K\right], \\ B_{cl} &= F_l\left[\begin{pmatrix} B_1 & B_2 \\ D_{21} & D_{22} \end{pmatrix}, K\right], \\ C_{cl} &= F_l\left[\begin{pmatrix} C_1 & D_{12} \\ C_2 & D_{22} \end{pmatrix}, K\right], \\ D_{cl} &= F_l\left[\begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}, K\right], \end{aligned} \tag{10}$$

where:

$$F_l\left[\begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix}, Y\right] = X_{11} + X_{12}Y(I - X_{22}Y)^{-1}X_{21} \tag{11}$$

Close loop transfer function of the system will be as follows. Also, general structure of the closed loop feedback control of the system is presented in Fig. 2, whose simplified model is shown in Fig. 3.

$$T_{zw}(s) = D_{cl} + C_{cl}(sI - A_{cl})^{-1}B_{cl} \tag{12}$$

According to the H_∞ control theory, by minimizing $\|T_{zw}(j\omega)\|_\infty$ we can keep value of outputs near zero. Indeed, solving this problem means to find $K = [k_1\ k_2]$ in such a way that $\|T_{zw}(j\omega)\|_\infty$ will be the minimum value possible [22]. In this way, we can hope that undesirable input will have minimum effect on outputs of the system and independently of the value of w, so that we can keep outputs value near zero. Stability of the polynomial of poles of $T_{zw}(s)$ is the necessary condition for each controller and indeed, which is proposed as a constraint in optimization problem. On the other hand, there is some limitation on control gain $[K]$. In other words, we can't consider absolute value of vector components $[K]$ more than a given limit. Maximum absolute value of a vector component is equal to the infinite norm of that vector. So, this limitation present another constraint as $\|K\|_\infty \le K_{\max}$. Such a constraint is very complex and the common methods in mathematics are unable to satisfy it in objective function. One of the best options to solve such a problem is using artificial intelligence optimization methods. For the same reason, in this paper PSO method is used to solve this control problem. The PSO method for solving this optimization problem is presented in the next section.

In the second model objective is reducing the effects of unwanted vibrations and disturbance transmitted from the vibrating foundation and environment to sensitive machinery and equipment. A good example of these systems is the Scanning Electron Microscope (SEM) as depicted in Fig. 4(a). Electron microscopes are designed to resolve features of materials down to a few nanometers in size. The final image is built up from the number of electrons emitted from each spot on the sample. Micro-vibrations can generate internal relative motion along a beam path that either blurs

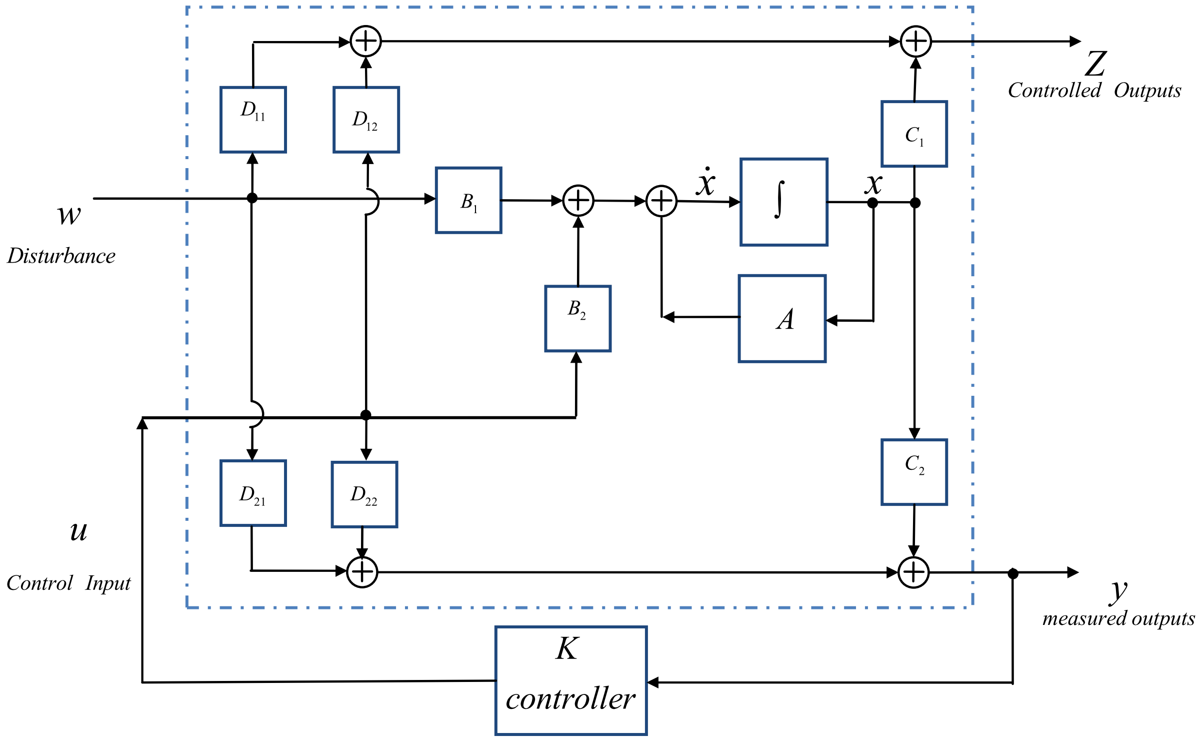

Fig. 2 General structure of control system

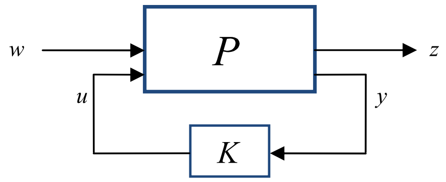

Fig. 3 Simplified model of static output feedback controller

an optical image or causes an electron beam to deviate from its intended path. Therefore, it is essential to isolate electron microscopes from unwanted vibrations. These systems can be modeled using a two-mass model as shown in Fig. 4(b). In this case m_a is the mass of equipment. m_b is the mass of foundation and supporting base. Inactive components of suspension are a spring with stiffness k_a, and a damper with damping c_a. Equivalent damping and stiffness coefficients of foundation are also defined as c_b and k_b, respectively. Z_a and Z_b are equipment and foundation displacement, respectively. Active part of the isolation system is an actuator with a physical force (u). $w(t)$ is disturbances created in the system.

Similarly, for the second model, we calculate matrices $\dot{x}$, z, and y. Governing equations are expressed as follows:

$$\begin{aligned} m_a\ddot{z}_a &= -k_a(z_a - z_b) - c_a(\dot{z}_a - \dot{z}_b) + u(t), \\ m_b\ddot{z}_b &= k_a(z_a - z_b) + c_a(\dot{z}_a - \dot{z}_b) \\ &\quad - k_b(z_b) - c_b(\dot{z}_b) - u(t) + w. \end{aligned} \tag{13}$$

Considering a set of state variables as follows:

$$\begin{aligned} x_1 &= z_a - z_b, \quad x_2 = z_b, \\ x_3 &= \dot{z}_a, \quad x_4 = \dot{z}_b, \end{aligned} \tag{14}$$

where x_1, x_2, x_3 and x_4 are relative displacement, absolute displacement of foundation, and velocity of equipment and foundation, respectively. State vector is defined as $x = [x_1\ x_2\ x_3\ x_4]^T$. State equation of system introduced by (13) is as follows:

$$\dot{x} = Ax + B_1w + B_2u. \tag{15}$$

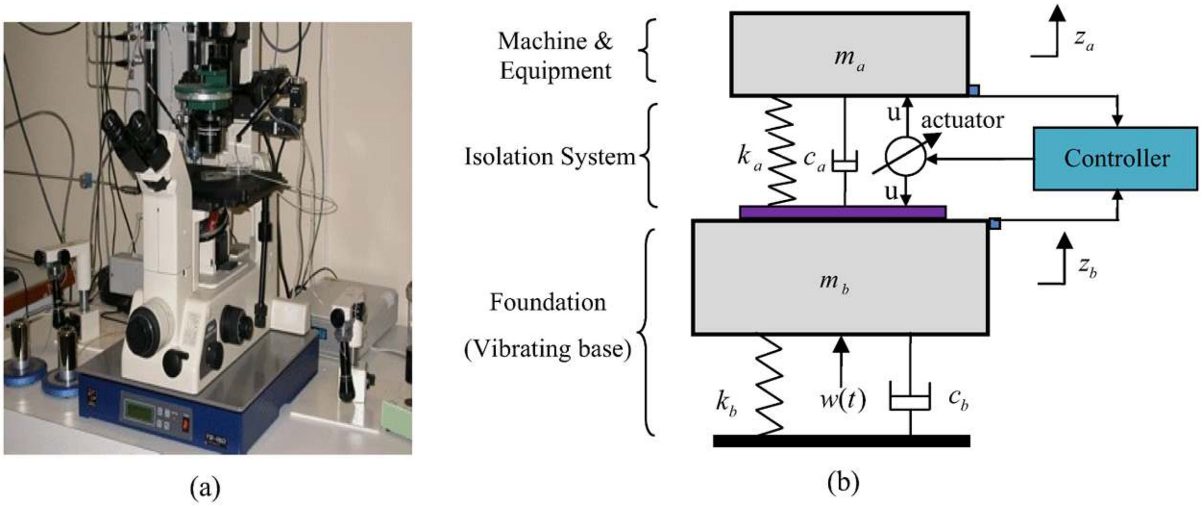

Fig. 4 Active isolation system to reduce transmission vibration from surrounding environment (vibrating base) to the equipment. (**a**) The example of SEM, (**b**) the schematic of system model

Where, constant matrices A, B_1, and B_2 are defined as the following:

$$A = \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -k_a/m_a & 0 & -c_a/m_a & c_a/m_a \\ k_a/m_b & -k_b/m_b & c_a/m_b & -(c_a+c_b)/m_b \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1/m_a \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 0 \\ 0 \\ 1/m_a \\ -1/m_b \end{pmatrix}.$$

General objective of controlling above system is reducing displacement and acceleration of machinery and sensitive equipment. Therefore, outputs of the controlled system are defined as z_a and $\ddot{z}_a$, respectively. Output equation is presented as the following:

$$z = C_1 x + D_{11} w + D_{12} u. \tag{16}$$

We have matrices C_1, D_{11}, and D_{12} as follows:

$$C_1 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -k_a/m_a & 0 & -c_a/m_a & c_a/m_a \end{pmatrix},$$

$$D_{11} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad D_{12} = \begin{pmatrix} 0 \\ 1/m_a \end{pmatrix}.$$

In this system, measurable variables are considered as relative displacement of equipment respected to vibrational foundation and velocity of foundation, which can be expressed as the following equations:

$$y = C_2 x + D_{21} w + D_{22} u. \tag{17}$$

Where, constant matrices C_2, D_{21}, and D_{22} are as follows:

$$C_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$D_{21} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad D_{22} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Similarly to first model, equations (15), (16), and (17) are expressed in the simplified form of (6). Considering static output feedback controller, the state equations and close loop transfer function of the system are obtained in general form of (9) and (12). According to the H_∞ control theory, minimizing $\|T_{zw}(j\omega)\|_\infty$ and considering constraint in optimization problem can satisfy the performance requirement. In this paper the PSO method is used to solve control problem. In the next section, the PSO method is presented briefly.

3 The Particle Swarm Optimization (PSO) algorithm

PSO algorithm, which is one of random optimization methods available among artificial intelligence ones, belongs particularly to a smaller set of algorithms and methods called swarm intelligence. This algorithm was inspired by social behavior of animals and insects such as bird flocking and fish schooling. For example, when a bird among others obtains a better opportunity to feed, other birds tend to follow it. This algorithm was introduced by Kennedy and Eberhart in 1995 and today has many applications in solving numerous problems from different branches of science. The PSO algorithm employs a swarm of multiple particles, each of which has their own position and velocity (transfer vector). All of the particles share information obtained from the other particles, and interaction among the particles makes the search efficient. Although only simple operations compose the PSO algorithm, the PSO algorithm can solve complex optimization problems efficiently [19–21].

The algorithm starts by generating an initial population. Particles are distributed in search space, whose function is to be optimized. Each particle has two main properties of position and speed (transfer vector). Position of each particle is a point (or vector) from search space and speed of particle motion is also considered as a vector. In initial stage of algorithm, points are created in space randomly and have random (but limited) speeds. In each stage, speed and position of each particle are changed. Each particle has its own position and computes the value of the objective function in position of space where it lies. Then using combination of information of its current position and the best position where already it has been as well as information of one or more particles of the best ones available in population, it selects a direction to move. During different stages of solving algorithm, at last, particles converge to the best feasible solution.

Transfer vector and the positions of the i^{th} particle in $j+1^{\text{th}}$ stage of the particle are listed to the following equations:

$$v_{j+1}^{i} = wv_{j}^{i} + c_1 r_1 (x_j^{i,best} - x_j^{i}) + c_2 r_2 (x_j^{gbest} - x_j^{i}), \tag{18}$$

$$x_{j+1}^{i} = x_j^{i} + v_{j+1}^{i}. \tag{19}$$

x_j^i and v_j^i are position and speed of i^{th} particle in j^{th} stage of solving algorithm, respectively. Constant coefficient w called inertia coefficient is a number in interval [0, 1]. This coefficient has an important role in detecting better solutions by algorithm. Also $x_j^{i,best}$ is the best position experienced by i^{th} particle to j^{th} stage and x_j^{gbest} is the best position experienced by all particles to j^{th} iteration. c_1 and c_2 are constant coefficients called learning coefficients whose sum is usually less than 4 and often both of them are selected as 2. r_1 and r_2 are also random numbers in interval [0, 1] with uniform distribution [23–27].

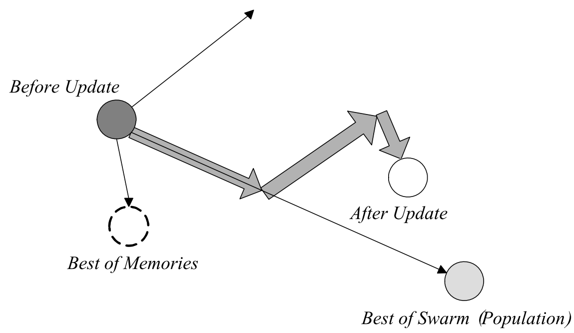

Fig. 5 Structure of the PSO algorithm

As we can see from Fig. 5, direction of the particle motion is a linear combination and, of course, with random coefficient from vectors which at last result in particle position being improved. In this way, a good search is performed in the space and appropriate solutions are evolved for optimization problem.

In some case, information of the best neighbor is also used. In this way, relation (19) is rewrite as follows:

$$v_{j+1}^{i} = wv_{j}^{i} + c_1 r_1 (x_j^{i,best} - x_j^{i}) + c_2 r_2 (x_j^{gbest} - x_j^{i}) + c_3 r_3 (x_j^{i,nbest} - x_j^{i}), \tag{20}$$

where $x_j^{i,nbest}$ is the position of the best neighbor of i^{th} particle in j^{th} stage of solving algorithm. c_3 and r_3 have also conditions similar to those of previous coefficients.

In the next section simulation results of the performance of PSO in solving active control problem are reported.

4 Simulation results and performance evaluation

In order to demonstrate the effectiveness of this method two different models, which was shown in Fig. 1 and Fig. 4, has been considered. The performance of the active models is compared with those of

Table 1 Parameter values for simulated models

Model one: Active isolation model to reduce transfer of vibration from machinery to the foundation and base (Fig. 1)		
Mass of machinery	$m_a = 560$ kg	
Mass of foundation or base	$m_b = 1000$ kg	
Damping coefficient of the isolation system	$c_a = 10$ N/ms^{-1}	
Stiffness of the isolation system	$k_a = 1.5e4$ N/m	
Damping coefficient of foundation	$c_b = 100$ N/ms^{-1}	
Stiffness of foundation	$k_b = 2.5e5$ N/m	
Model two: Active isolation model to reduce transfer of vibrations from vibrating base to the sensitive equipment and machine (Fig. 4)		
	Case (a)	Case (b)
Mass of foundation or structure	$m_b = 560$ kg	$m_b = 560$ kg
Mass of machinery or sensitive equipment	$m_a = 10$ kg	$m_a = 100$ kg
Damping coefficient of foundation	$c_b = 10$ N/ms^{-1}	$c_b = 100$ N/ms^{-1}
Stiffness of foundation	$k_b = 1.5e5$ N/m	$k_b = 1.5e5$ N/m
Damping coefficient of the isolation system	$c_a = 100$ N/ms^{-1}	$c_a = 10$ N/ms^{-1}
Stiffness of the isolation system	$k_a = 2.5e4$ N/m	$k_a = 2.5e4$ N/m

its passive isolation. Moreover, in order to verify the isolation effectiveness, two structural responses, i.e. the frequency and time response, are chosen as the performance indices to be compared in the following discussions. The parameter values used in the simulation models are summarized in Table 1. In addition, parameters considered for PSO algorithm are $w = 0.99^j w_0$, $w_0 = 1$, $c_1 = 3$, and $c_2 = 1$. Moreover, it is assumed $N_{pop} = 250$ and $T_{\max} = 200$ where, N_{pop} is the total number of particles available in PSO and $T_{\max}$ is maximum iteration number and is used for checking termination criterion in this algorithm. As we can see, inertia coefficient has been determined in such a way that it is decreased. Also, there are some limitations on interest in control $[K]$ used in feedback direction. Maximum absolute value of a vector component is equal to the infinite norm of that vector. Limitation present on components $[K]$ is applied on algorithm as $\|K\|_\infty \le K_{\max}$. On the other hand, cost function used in algorithm, which obtains its optimum value, is defined as follows:

$$f(k_1, k_2) = \begin{cases} \|T_{zw}(j\omega | K = [k_1\ k_2])\|_\infty, \\ \quad T_{zw}(s) \text{ is stable}, \\ \infty, \quad \text{otherwise}. \end{cases} \tag{21}$$

For some values of $[K]$ which cause instability of the system, value of the cost function has been considered as a penalty in the form of infinity (∞) to ensure that controller found by algorithm doesn't cause instability of the system and closed looped system is stable.

For the first model, simulation are done with applying two conditions as $\|K\|_\infty \le 14000$ or $\|K\|_\infty \le 50000$. For these two states, the optimum values of feedback matrix gain are obtained as follows:

$$K_{opt,1} = \begin{bmatrix} -14000 & 107.8 \end{bmatrix} \quad \text{s.t.} \quad \|K\|_\infty \le 14000,$$
$$K_{opt,2} = \begin{bmatrix} -14999 & -327.4 \end{bmatrix} \quad \text{s.t.} \quad \|K\|_\infty \le 50000.$$

Optimum values are obtained for $\|T_{zw}(j\omega)\|_\infty$ in two states above are the followings:

$$K = \begin{bmatrix} -14000 & 107.8 \end{bmatrix} \quad \Rightarrow \quad \|T_{zw}(j\omega)\|_\infty = 72.667,$$
$$K = \begin{bmatrix} -14999 & -327.34 \end{bmatrix}$$
$$\quad \Rightarrow \quad \|T_{zw}(j\omega)\|_\infty = 3.5596.$$

It is necessary to note that PSO algorithm has found the best feasible solutions under conditions above for the problem and has converged to the universal optimum of it. Convergence trend of algorithm and reduction of the cost function for constraint $\|K\|_\infty \le 50000$ can be seen in Fig. 6. Apart from physical limitations, universal minimum of the above cost function, from a mathematical point of view, is in $K = [-14999\ -327.34]$ for which it will be $\|T_{zw}(j\omega)\|_\infty = 3.5596$.

The behavior of a control system usually can be characterized by its frequency response.

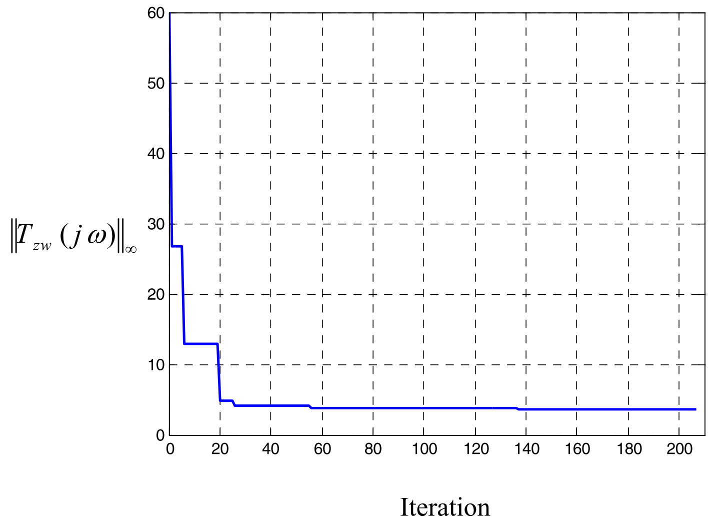

Fig. 6 Downward trends the cost function in different stages of solving PSO algorithm

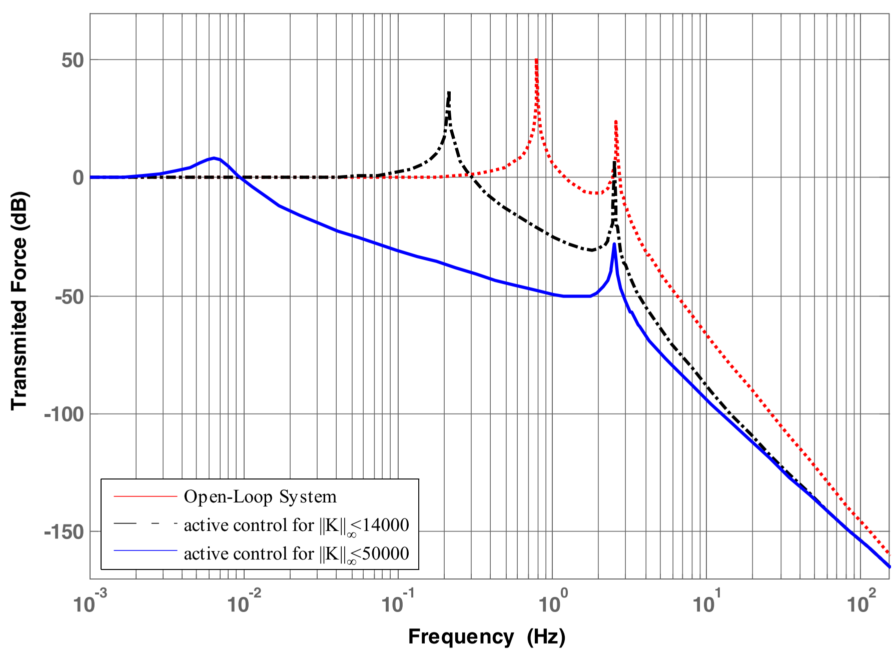

Fig. 7 Frequency response of transmitted forces through the foundation (supporting structure) to soil and surrounding environment due to disturbance input

Figure 7 compares the frequency response of the transmitted force using active isolation for two above control gain with those of the passive isolation system.

The figure shows that the transmitted force is reduced at the active control system within the frequency range and it has very good performance. It is obvious in the frequency region including the resonance frequency.

Figures 8, 9 and 10 show time response of transmitted force at the frequency of 2.59 and 0.79 Hz (close to the resonant frequency) and 0.48 Hz, respectively. These figures indicate that the transmitted force

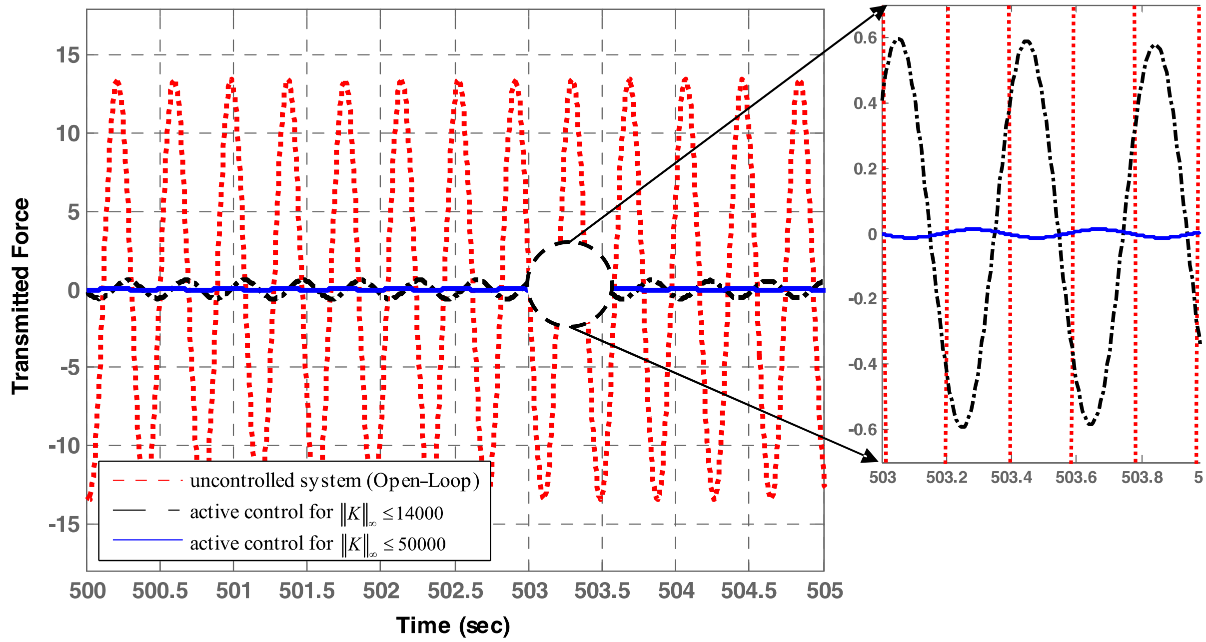

Fig. 8 Time response of transmitted forces through the foundation to surrounding environment due to disturbance input for frequency of 2.59 Hz

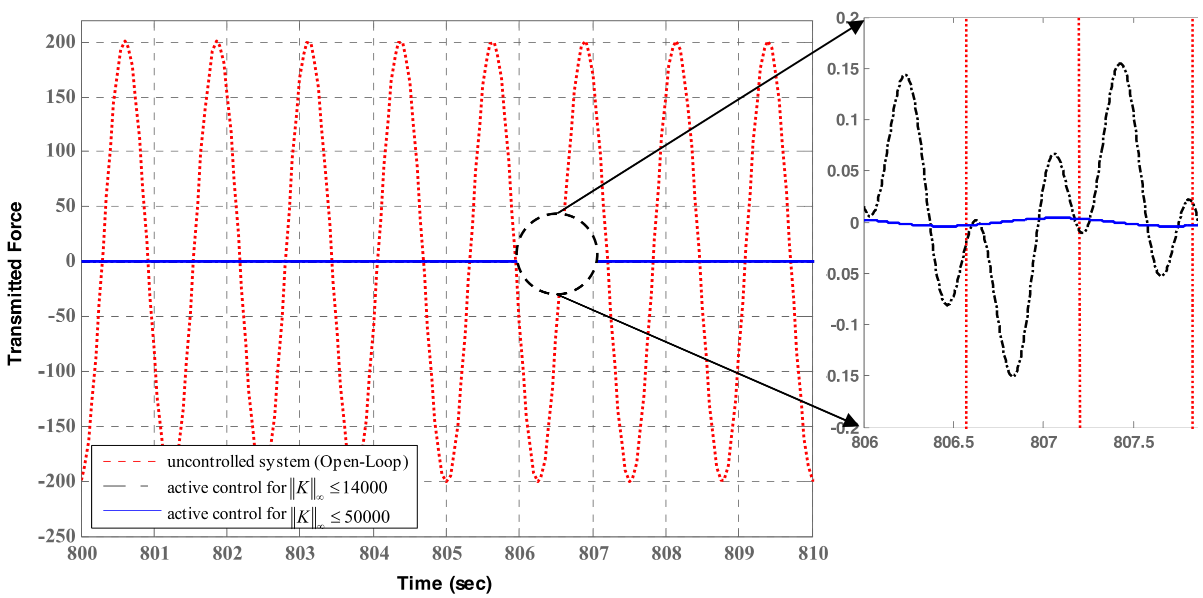

Fig. 9 Time response of transmitted forces through the foundation to surrounding environment due to disturbance input for frequency of 0.79 Hz

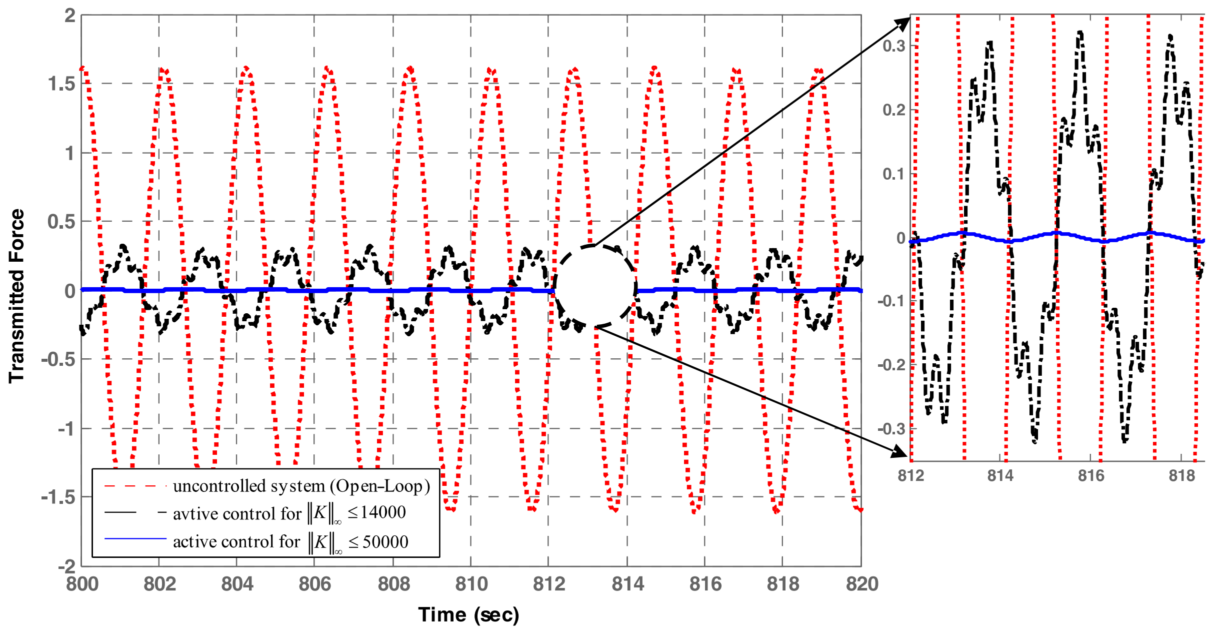

Fig. 10 Time response of transmitted forces through the foundation to surrounding environment due to disturbance input for frequency of 0.48 Hz

through the foundation due to disturbance input is reduced and responses of active isolation system are very better than the uncontrolled system.

On the other hand, the frequency and time response obtained from the active control with higher control condition ($\|K\|_\infty \le 50000$) show that is more effectively than the control condition as $\|K\|_\infty \le 14000$, because the optimal control gain is obtained for constraint $\|K\|_\infty \le 50000$ as $K_{opt} = [-14999\ -327.34]$. It is clear that increasing control condition more than a specific limit doesn't produce significant difference in quality of solutions. Therefore, increasing control condition is reasonable only to a specific limit and based on engineering justification.

Figures 11–14 show frequency response of foundation acceleration and displacement as well as machinery deflection and acceleration. It can be seen from these figures that the two H_∞ controllers systems satisfy the different performances and the effects of disturbance reduction by using active control. The foundation acceleration and displacement as well as machinery deflection and acceleration are very better than the uncontrolled system especially in the range of resonance. Therefore, this active control system can reduces the peaks in the frequency response.

Similar to the previous model, for the second model in case (a), simulation are done with applying two conditions as $\|K\|_\infty \le 10000$ and $\|K\|_\infty \le 100000$. For these two states, the optimum values of static output feedback controller are obtained as follows:

$$K_{opt,1} = \begin{bmatrix} 10000 & -2399.6 \end{bmatrix} \quad \text{s.t.} \quad \|K\|_\infty \le 10000,$$
$$K_{opt,2} = \begin{bmatrix} 24905 & -100.22 \end{bmatrix} \quad \text{s.t.} \quad \|K\|_\infty \le 100000.$$

Optimum values obtained for $\|T_{zw}(j\omega)\|_\infty$ in two states above are the followings:

$$K = \begin{bmatrix} 10000 & -2399.6 \end{bmatrix}$$
$$\Rightarrow \quad \|T_{zw}(j\omega)\|_\infty = 0.10082,$$
$$K = \begin{bmatrix} 24905 & -100.22 \end{bmatrix}$$
$$\Rightarrow \quad \|T_{zw}(j\omega)\|_\infty = 0.040421.$$

Also for case (b) simulation has been done with applying conditions as $\|K\|_\infty \le 30000$. For this state, the optimum value of feedback controller of the static output is obtained as follows:

$$K_{opt,1} = \begin{bmatrix} 24993 & -9.5837 \end{bmatrix} \quad \text{s.t.} \quad \|K\|_\infty \le 30000.$$

Optimum value obtained for $\|T_{zw}(j\omega)\|$ in this case is the following:

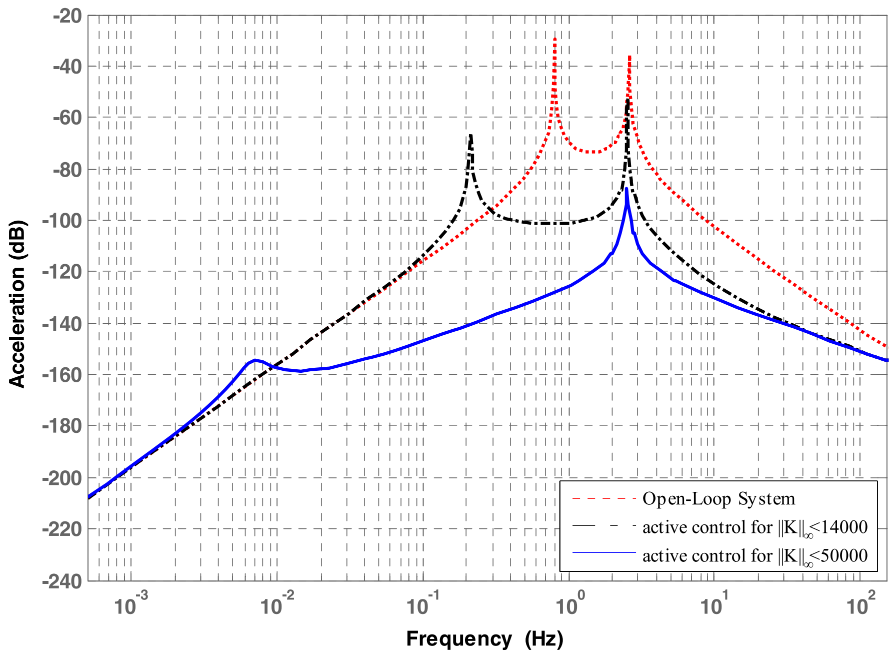

Fig. 11 Frequency response of foundation acceleration due to disturbance input

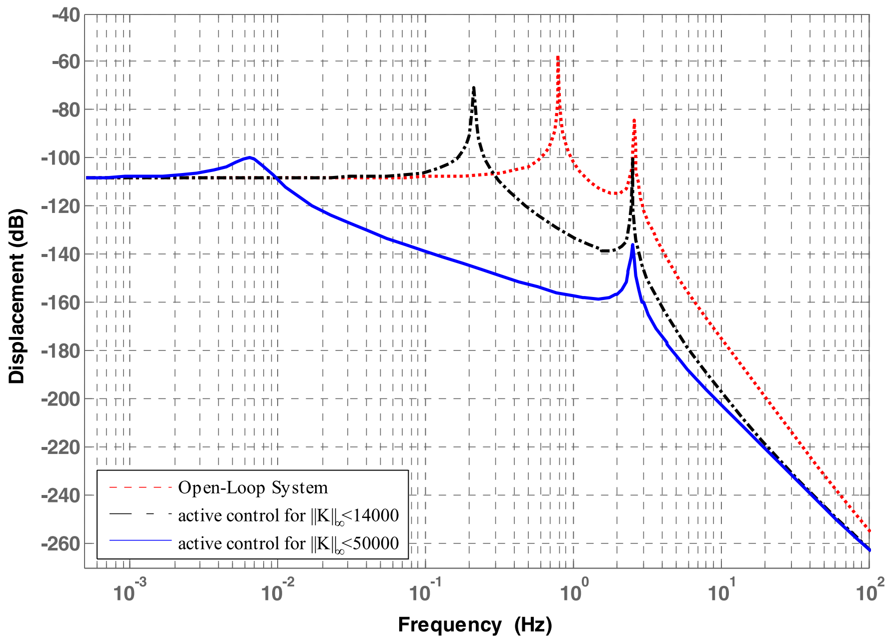

Fig. 12 Frequency response of foundation displacement due to disturbance input

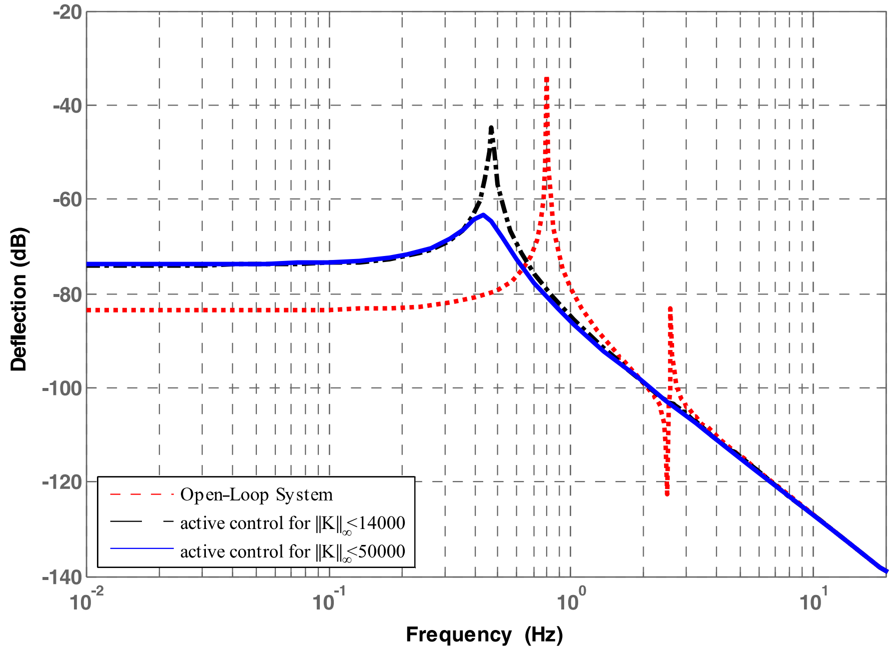

Fig. 13 Frequency response of relative displacement between the machine and the foundation due to disturbance input

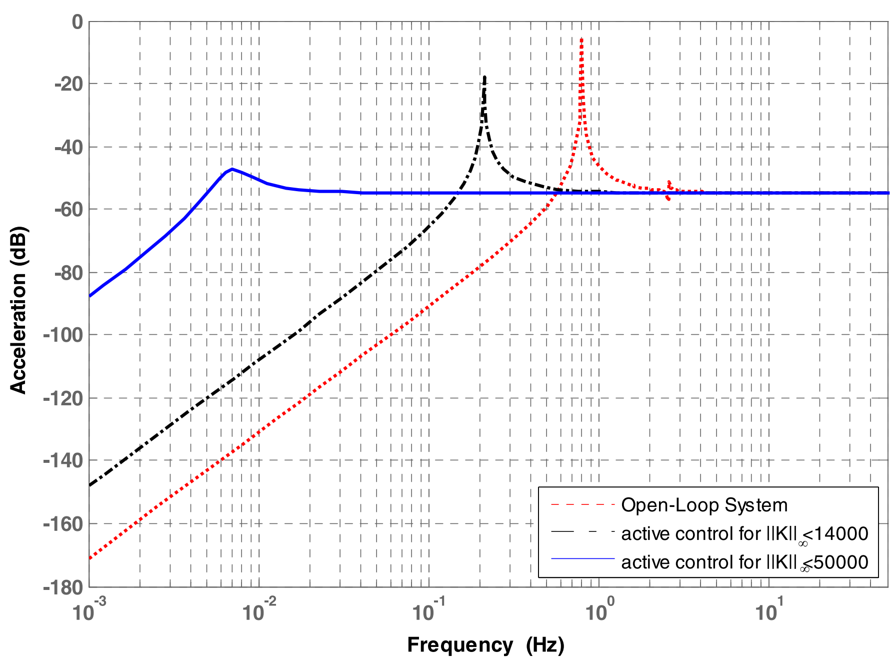

Fig. 14 Frequency response of machine acceleration due to disturbance input

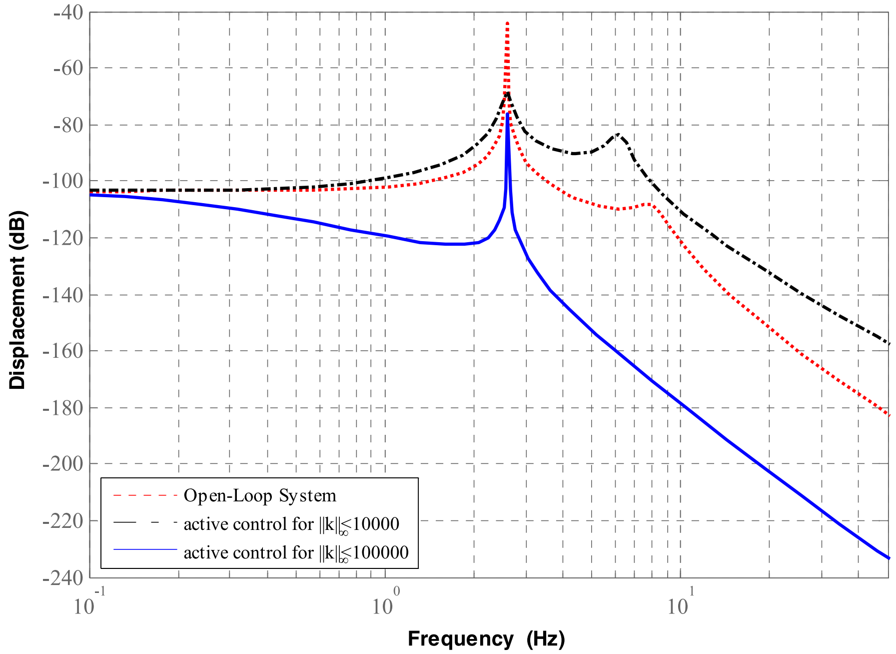

Fig. 15 Frequency response of equipment displacement due to disturbance input (case a)

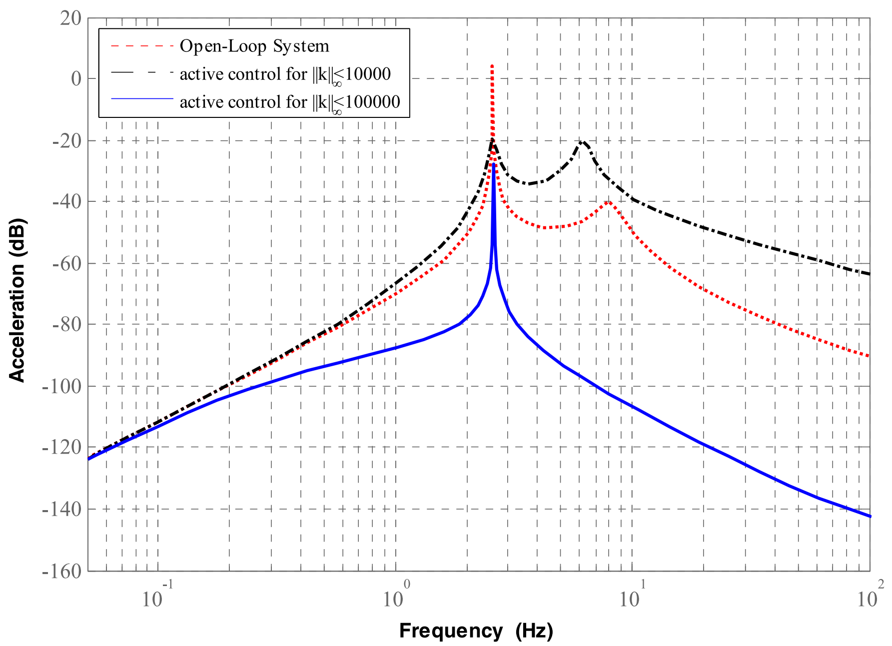

Fig. 16 Frequency response of equipment acceleration due to disturbance input (case a)

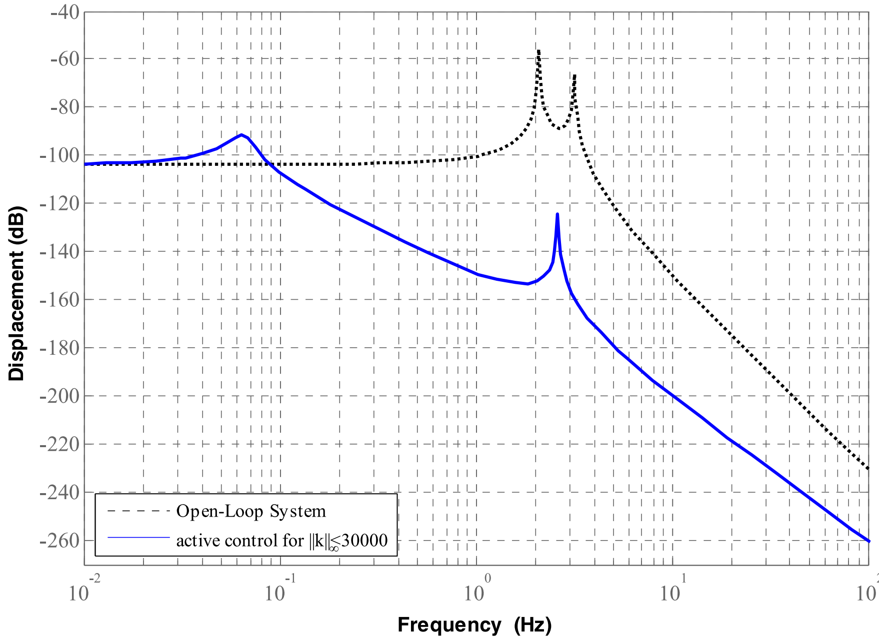

Fig. 17 Frequency response of equipment displacement due to disturbance input (case b)

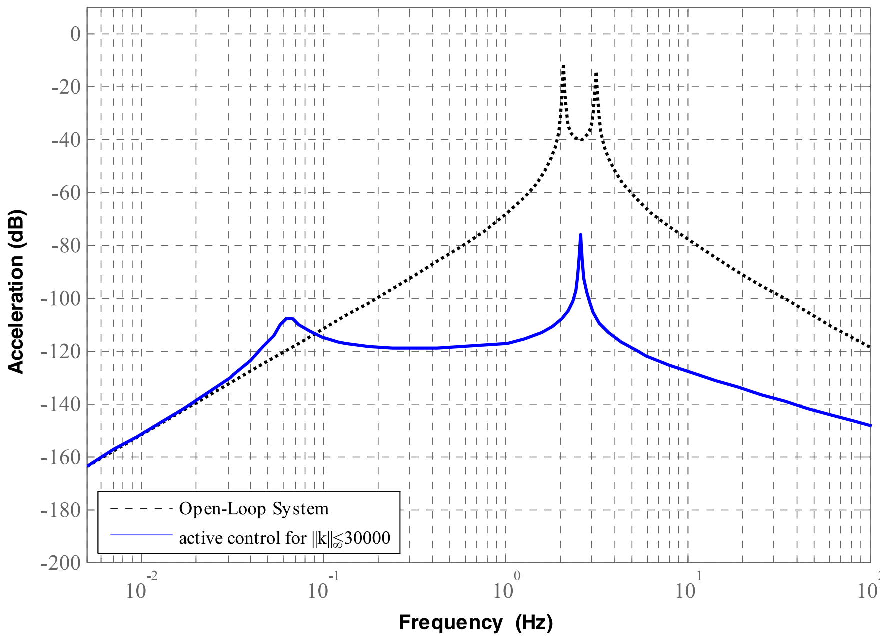

Fig. 18 Frequency response of equipment acceleration due to disturbance input (case b)

$$K = \begin{bmatrix} 24993 & -9.5837 \end{bmatrix}$$
$$\Rightarrow \quad \|T_{zw}(j\omega)\|_\infty = 0.00006088.$$

The frequency responses for above controllers from disturbance input are depicted in Figs. 15–18. It can be seen from these figures that the active vibration isolation system has good isolation performance against the disturbance acting on the system within the frequency range. These diagrams indicate the effects of disturbance reduction by using active control system. Therefore, this active control system can reduces the peaks in the frequency response.

5 Conclusions

Based on the solvability of PSO algorithm in optimization problem, the static output feedback H_∞ controller to design active vibration isolation system has been used. Two different models based on reducing transmission of vibration and disturbance from machinery to the foundation and also from foundation to the sensitive equipment has been considered. Active control based on static output feedback and H_∞ criterion was presented as a cost function and optimizing is performed by using PSO algorithm, which is effective approach to solve optimization problem. This approach was validated by numerical simulation and it was shown that the controller can cause the significant reduction in the resonance responses. The simulation results show the effectiveness of the presented approach.

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