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Published on: 25 Mar 2015 - International Journal of Robust and Nonlinear Control (John Wiley & Sons, Ltd)

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SUMMARY

This paper presents an Adaptive Polytopic Observer (APO) design in order to develop an actuator fault estimation method dedicated to polytopic Linear Parameter Varying (LPV) descriptor systems. This paper extends a Fault Diagnosis (FD) method developed for regular LTI systems to polytopic LPV descriptor systems. Here, time-varying actuator faults are also considered whereas in many papers actuator faults are generally assumed to be constant. The design and convergence conditions of this APO are provided. The design is formulated through Linear Matrix Inequalities (LMI) techniques under equality constraints. The performances of the proposed actuator fault estimation scheme are illustrated using an electrical circuit.

1. INTRODUCTION

During the last three decades, both theoretical and experimental researches on fault detection have been intensively developed. For obvious economical reasons, systems in the real world have to work perfectly at all time under all conditions. That's why it is crucial to be able to detect and identify the possible faults that may affect a system as early as possible in order to prevent significant performance degradations or damages of the system [6], [34]. Fault Detection and Diagnosis (FDD) has been well studied for physical plants which were modeled by T-S fuzzy models. Recently Such efforts studded by Jiang B. et al. [17], have led to the development of a method for integrated robust fault estimation and accommodation for a class of discrete-time nonlinear systems described by a T-S fuzzy model. Also, dynamic output feedback fault-tolerant controllers have been developed by Zhang K. et al. [31] for T-S fuzzy systems with actuator faults in which a fuzzy augmented fault observer is proposed to yield fault estimates and, based on the information of on-line fault estimates, observer based output feedback fault-tolerant controllers are designed. Nevertheless for nonlinear systems it still remain a challenge [22].

By the way, some techniques have been developed to approximate nonlinear systems by Linear Parameter Varying (LPV) models that can be used efficiently to represent nonlinear systems as in [4], [25] for a winding machine, [26] for a twin rotor system. These methods allow to apply powerful linear design tools to complex nonlinear models. Based on this LPV representation

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of nonlinear systems, some researchers from the FDI community have developed model-based methods using LPV ordinary models as in [2], [5], and [11]. Similar result is found in [29] for linear parameter-varying (LPV) systems the presented method is based on multi-objective H_-/H_∞ fault detection observer design. LPV sliding mode observers have been largely introduced by [1] so as to reconstruct the states of the system. A particular class of LPV systems is the polytopic LPV form which allows to describe the system as a convex combination of sub-models defined by the vertices of a convex polytope [13], [15], [12]. These sub-models are then combined by convex weighting functions that yield to a global model.

Generally speaking, most of control and fault diagnosis methods for physical systems, use normal (or regular) models i.e. there is no algebraic relations between the system variables. However, Differential-Algebraic Equations (DAE) or implicit systems or singular systems or descriptor systems are of quite importance for the physical representation of some systems [20], [10]. Such systems appear for example in electrical circuits, mechanical systems with holonomic or non holonomic constraints, robotic systems with kinematical constraints and chemical systems [21]. Some practical problems must take into account physical constraints or algebraic relations and more generally impulsive behaviors caused by an improper transfer matrix: see the following books on singular systems [8], [9].

Various observer-based fault diagnosis methods for descriptor systems have been proposed but generally for Linear Time Invariant models [27]. The author in [19] studies an observer coming from a continuous nonlinear descriptor systems via a convex optimization. Recently, a fault diagnosis method to detect and estimate actuator faults for multi-models descriptor systems represented by using unknown input observers [15]. Therefore, in [3] the authors have presented a method of fault estimation for a particular class of discrete-time LPV descriptor systems. The authors in [16] and [14], have proposed a polytopic unknown inputs and proportional integral observers for LPV descriptor systems respectively. These observers are used to detect and isolate actuator faults. However, all these observers can only estimate constant faults which is a restrictive condition.

In order to estimate both constant and time varying faults, an Adaptive Polytopic Observer is proposed in this paper. In many applications where the parameters are unknown and states are not accessible, adaptive observer appears to be a valuable method in estimation of both parameters and states of the system. In [28] the authors have presented an adaptive fault diagnosis observer approach dedicated to regular LTI systems which can detect and estimate constant faults. In [32], the authors have designed an adaptive observer so as to estimate time-varying faults but only for regular LTI systems.

Thus, this paper proposes an Adaptive Polytopic Observer (APO) for a class of LPV descriptor systems. Fault Diagnosis for descriptor LPV system still remains a challenge and as far as authors knows there are very few papers for such systems that consider time-varying actuator fault estimation under disturbances. In literature, most of fault diagnosis approaches are designed only for regular systems and not for descriptor systems. To tackle such problem, the entire development of this paper is a real contribution for descriptor LPV systems with significant new results in terms of time-varying fault estimation in spite of disturbances. Both methods presented in [14] or in [3] can only estimate constant fault for descriptor LPV Systems; the real novelty in the paper consists in the ability of the approach to estimate time-varying actuator fault despite disturbances which is a real huge problem and a key problem to solve for some potential extensions to Fault Tolerant Control.

In comparison with the polytopic LPV proportional integral observer studied in [14] which can only estimate constant actuator fault, the main goals of this paper are: to extend existing results about fault estimation of LTI regular systems to polytopic LPV descriptor systems, and also to consider time-varying actuator faults estimation. Sufficient convergence conditions for the APO are given by solving a set of Linear matrix inequalities (LMI).

The outline of this paper is as follows. The structure of the LPV descriptor systems is formulated in Section 2. The Adaptive Polytopic Observer-based actuator fault estimation is presented in Section 3. Finally, in Section 4, a numerical example that considers a LPV descriptor systems is

used to assess the validity of the proposed approach.

In this paper, the notations are standard. $\mathbf{R}$ denotes the set of real numbers, $\mathcal{C}$ is the complex plane, A^+ denotes the generalized inverse of A; $Q > 0$ or $(Q < 0)$ indicates the symmetric matrix Q is positive (or negative) definite; $\| . \|$ denotes the standard norm symbol. Also, $\forall$ means "for all". An asterisk $*$ denotes the transposed element in the symmetric position.

2. PROBLEM STATEMENT

Let us consider the following LPV descriptor system in fault-free case subject to disturbance:

$$\begin{cases} E\dot{x}(t) = \widetilde{A}(\theta(t))x(t) + \widetilde{B}(\theta(t))(u(t) + f(t)) + \widetilde{R}(\theta(t))d(t) \\ y(t) = Cx(t) + Hd(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^p$ is the inputs vector, $d(t) \in \mathbf{R}^q$ is a bounded unknown input vector i.e., $\|d(t)\| \leq \beta$ which represents disturbances and $y(t) \in \mathbf{R}^m$ represents the measured outputs vector. $f(t) \in \mathbf{R}^p$ represents the actuator fault vector by an additive external signal as in [6], [14]. Sometimes in FDI, these malfunctions can also be represented by a multiplicative external signal as in [33], [24] by the following faulty control input $u_f(t) = (I_p - \gamma)u(t)$ which is equivalent to an additive signal such that: $u(t) + f(t)$ where $f(t) = -\gamma u(t)$ with

$$\gamma \triangleq diag[\gamma^1, \gamma^2, \dots, \gamma^p], \ 0 \leq \gamma^k \leq 1 \text{ such that}$$
$$\begin{cases} \gamma^k = 1 \rightarrow \text{a total failure of the } k^{th} \text{ actuator } k \in [1, \dots, p] \\ \gamma^k = 0 \rightarrow \text{the } k^{th} \text{ healthy actuator} \end{cases} \tag{2}$$

where the term γ^k represents the loss of effectiveness of kth actuator. Disturbances and faults are supposed to be unknown. Matrix $E \in \mathbf{R}^{n \times n}$ may be singular and $rank(E) = r < n$. $\widetilde{A}(\cdot), \widetilde{B}(\cdot)$ and $\widetilde{R}(\cdot)$ are continuous functions which depend affinely on the time-varying parameter vector $\theta(t) \in \mathbf{R}^l$. Matrices C and H are fixed. It is also assumed that each component $\theta_i(t), i \in [1, \dots, l]$ of this time-varying parameter vector $\theta(t)$ is bounded and lies into a hypercube such that [30]:

$$\theta(t) \in \Upsilon = \{\theta_i \mid \underline{\theta}_i(t) \leq \theta_i(t) \leq \overline{\theta}_i(t)\}, \quad \forall i \in [1, \dots, l], \forall t \geq 0 \tag{3}$$

The matrices $\widetilde{A}(\theta(t)), \widetilde{B}(\theta(t)), \widetilde{R}(\theta(t))$ of the LPV descriptor system (1) with the affine parameter dependence (3) are represented such that $\forall \theta(t) \in \Upsilon$:

$$\widetilde{M}(\theta(t)) = \widetilde{M}_0 + \sum_{i=1}^{l} \theta_i(t)\widetilde{M}_i \tag{4}$$

where $\widetilde{M}$ stands for matrices $\widetilde{A}, \widetilde{B}$ and $\widetilde{R}$. The system (1) can be transformed into a convex interpolation of the vertices of Υ where the vertices S_i of the polytope are defined such that [23]: $S_i = \begin{bmatrix} A_i & B_i & R_i & C & H \end{bmatrix}, \forall i \in [1, \dots, h]$ where $h = 2^l$. The polytopic coordinates are denoted $\rho(\theta(t))$ and vary into the convex set Ω:

$$\Omega = \left\{ \rho(\theta(t)) \in \mathbf{R}^h, \rho(\theta(t)) = [\rho_1(\theta(t)), ..., \rho_h(\theta(t))]^T, \ \rho_i(\theta(t)) \geq 0, \forall i, \sum_{i=1}^{h} \rho_i(\theta(t)) = 1 \right\} \tag{5}$$

Then, the usual assumptions related to LPV systems are: $\rho(\theta(t))$ is bounded, it is assumed to be fault-free and it is available. To ease the presentation, it is assumed that the matrices $\widetilde{A}(\cdot), \widetilde{B}(\cdot)$ and $\widetilde{R}(\cdot)$ are given by convex combinations $\forall t \geq 0$ and the polytopic LPV descriptor system with the time-varying parameter vector $\rho(\theta(t)) \in \Omega$ given by:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{h} \rho_i(\theta(t))(A_i x(t) + B_i u(t) + F_i f(t) + R_i d(t)) \\ y(t) = Cx(t) + Hd(t) \end{cases} \tag{6}$$

where $A_i \in \mathbf{R}^{n \times n}$, $B_i = F_i \in \mathbf{R}^{n \times p}$, $R_i \in \mathbf{R}^{n \times q}$, $C \in \mathbf{R}^{m \times n}$ and $H \in \mathbf{R}^{m \times q}$ are time invariant matrices defined for the ith model.

Before starting the design of the APO, we assume that:

A1.
$$rank(CF_i) = rank(F_i) = p \quad \forall i = [1, \ldots, h]$$

A2. The triple matrix (E, A_i, C) is observable, i.e. [8], [9]

$$rank \begin{bmatrix} sE - A_i \\ C \end{bmatrix} = n, \ \forall s \in \mathcal{C}, \quad \forall i = [1, \ldots, h] \tag{7}$$

and

$$rank \begin{bmatrix} E \\ C \end{bmatrix} = n \tag{8}$$

A3. The fault $f(t)$ satisfies $\|f(t)\| \leq \alpha_1$ and the derivative of $f(t)$ with respect to time is norm bounded i.e. $\|\dot{f}(t)\| \leq \alpha_2$ and $0 \leq \alpha_1, \alpha_2 < \infty$.

A4.
$$p + q \leq m$$

A5.
$$rank \begin{bmatrix} E & 0 \\ C & H \end{bmatrix} = n + rank(H)$$

Note that $A3$ is a quite general assumption in the literature [6], [18]. Assumption $A4$ has to be verified to be able to provide actuator fault estimation.

The main goal of the paper is to estimate time-varying actuator fault for polytopic LPV descriptor systems. In [32], the authors have performed an adaptive observer so as to estimate time-varying faults but only for regular LTI systems. The authors in [14] have developed a proportional integral observer for actuator fault estimation but with the following constraint $\dot{f}(t) = 0$. Moreover, the same constraint has been considered in [3] for discrete-time descriptor systems where the main goal was also to estimate faults. So, in this paper, the main contribution consists in designing an Adaptive Polytopic Observer for LPV descriptor systems which is able to deal with time-varying actuator faults and by the way to tackle the previous restrictive constraints of the above mentioned papers. The following section is dedicated to the design of the Adaptive Polytopic Observer.

3. ACTUATOR FAULT ESTIMATION FOR POLYTOPIC LPV DESCRIPTOR SYSTEM

3.1. Adaptive Polytopic Observer Design

The proposed Adaptive Polytopic Observer (APO) has the following structure:

$$\begin{cases} \dot{z}(t) = \sum\limits_{i=1}^{h} \rho_i(\theta(t))(N_i z(t) + G_i u(t) + L_i y(t) + F_i \hat{f}(t)) \\ \hat{x}(t) = z(t) + T_2 y(t) \\ \hat{y}(t) = C\hat{x}(t) \\ r(t) = S(y(t) - \hat{y}(t)) \\ \dot{\hat{f}}(t) = \Gamma \sum\limits_{i=1}^{h} \rho_i(\theta(t))U_i(\dot{r}(t) + \sigma r(t)) \end{cases} \tag{9}$$

where $z(t) \in \mathbf{R}^n$ is the observer state vector, $\hat{x}(t) \in \mathbf{R}^n$ is the estimated state vector, $r(t) \in \mathbf{R}^m$ is the residual vector, $\hat{y}(t) \in \mathbf{R}^n$ is the estimated output vector and $\hat{f}(t) \in \mathbf{R}^p$ is the estimated actuator fault of $f(t)$. N_i, G_i, L_i, T_2, U_i, S are unknown matrices of appropriate dimensions to be determined, $\Gamma \in \mathbf{R}^{p \times p}$ is a learning rate symmetric positive definite matrix and $\sigma \in \mathbf{R}$ is a positive

scalar. The state estimation error $e(t)$ is defined as:

$$e(t) = x(t) - \hat{x}(t)$$

Then, it follows from (6) and (9) that:

$$e(t) = (I_n - T_2C)x(t) - z(t) - T_2Hd(t) \tag{10}$$

Under assumption that $rank \begin{bmatrix} E \\ C \end{bmatrix} = n$, there exists nonsingular matrices $T_1 \in \mathbf{R}^{n \times n}$ and $T_2 \in \mathbf{R}^{n \times m}$ such that [10]:

$$T_1E + T_2C = I_n \tag{11}$$

Furthermore, the fault estimation error $e_f(t)$ can be expressed as:

$$e_f(t) = f(t) - \hat{f}(t) \tag{12}$$

From (10), the state estimation error dynamic with the relation (11) is given by:

$$\dot{e}(t) = T_1E\dot{x}(t) - \dot{z}(t) - T_2H\dot{d}(t) \tag{13}$$

Using (6) and (9), the residual $r(t)$ can be expressed as:

$$r(t) = SCe(t) + SHd(t) \tag{14}$$

Then, if the following conditions hold true $\forall\, i = 1, ..., h$:

$$T_1A_i - L_iC - N_iT_1E = 0 \tag{15}$$
$$T_1B_i - G_i = 0 \tag{16}$$
$$T_1E = I_n - T_2C \tag{17}$$
$$T_1R_i + N_iT_2H - L_iH = 0 \tag{18}$$
$$T_2H = 0 \tag{19}$$
$$SH = 0 \tag{20}$$

By taking into account (6), (9) and (13), the estimation error dynamic $\dot{e}(t)$ and the residual $r(t)$ can be written such that:

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i(\theta(t))[N_ie(t) + M_if(t) + F_ie_f(t)] \tag{21}$$
$$r(t) = SCe(t) \tag{22}$$

$$\text{with} \qquad M_i = (T_1 - I_n)F_i \tag{23}$$

The substitution of (17) into (15) yields to:

$$N_i = T_1A_i + (N_iT_2 - L_i)C \tag{24}$$
$$N_i = T_1A_i + K_iC \tag{25}$$

$$\text{where} \quad K_i = N_iT_2 - L_i \tag{26}$$

To find simultaneously matrices T_1 and T_2 from equations (17) and (19), one can define the following augmented matrix:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} E & 0 \\ C & H \end{bmatrix} = \begin{bmatrix} I_n & 0 \end{bmatrix} \tag{27}$$

Under assumption $A5$, [19], the solution of (27) can be expressed such that:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} = \begin{bmatrix} I_n & 0 \end{bmatrix} \begin{bmatrix} E & 0 \\ C & H \end{bmatrix}^+ \tag{28}$$

Matrix S can be determined by solving equation (20) by considering assumption $A4$ i.e. the number of disturbances and faults can not be larger than the number of measurements, a general solution for (20) is given by (See Chapter 4 of [6]):

$$S = \Pi_1 [I_m - H(H^T H)^{-1} H^T] \tag{29}$$

where $\Pi_1 \in \mathbb{R}^{m \times m}$ is an arbitrary design matrix.

3.2. Convergence analysis

Let us consider the following Lemma:

Lemma 1
Given a scalar $\mu > 0$ and a symmetric positive definite matrix P_1, the following inequality holds [32]:

$$2x^T y \leq \frac{1}{\mu} x^T P_1 x + \mu y^T P_1^{-1} y \qquad x, y \in \mathbf{R}^n \tag{30}$$

■

In contrast to the proportional integral observer [14], here time-varying faults are considered. Then, it follows that $\dot{f}(t) \neq 0$ and by consequence, the dynamic of fault estimation error is expressed as follows:

$$\dot{e}_f(t) = \dot{f}(t) - \dot{\hat{f}}(t) \tag{31}$$

By using (25), the dynamic of the state estimation error (21) becomes:

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) [\underbrace{(T_1 A_i + K_i C)}_{N_i} e(t) + M_i f(t) + F_i e_f(t)] \tag{32}$$

The convergence of the state estimation error (32) can be verified by the following Theorem.

Theorem 1
Under Assumptions A_1 to A_5, given scalars $\sigma, \mu > 0$, if there exists symmetric positive definite matrices Q, P_1, P_2, P_3 and matrices $W_i = QK_i$ and U_i such that, $\forall i \in [1, \ldots, h]$:

$$\begin{bmatrix} \Psi_i & * \\ -\frac{1}{\sigma} F_i^T(QT_1 A_i + W_i C) & -\frac{2}{\sigma} F_i^T Q F_i + \frac{1}{\sigma\mu} P_2 + \frac{1}{\sigma\mu} P_3 \end{bmatrix} < 0 \tag{33}$$

s.t.

$$W_i H + QT_1 R_i = 0 \tag{34}$$
$$F_i^T Q - U_i S C = 0 \tag{35}$$

where

$$\Psi_i = (T_1 A_i)^T Q + Q(T_1 A_i) + \frac{1}{\mu} P_1 + C^T W_i^T + W_i C$$

Then, the following adaptive fault estimation algorithm

$$\dot{\hat{f}}(t) = \Gamma \sum_{i=1}^{h} \rho_i(\theta(t)) U_i(\dot{r}(t) + \sigma r(t)) \tag{36}$$

can realize $e(t)$ and $e_f(t)$ uniformly bounded, where $\Gamma \in \mathbf{R}^{p \times p}$ is a symmetric positive definite learning rate matrix. ■

Proof 1

Consider the Lyapunov function with the following quadratic form:

$$V(e(t), e_f(t)) = e^T(t)Qe(t) + \frac{1}{\sigma}e_f^T(t)\Gamma^{-1}e_f(t) \tag{37}$$

Convergence condition for the estimation error yields that the time derivative of the Lyapunov function (37) should be negative definite. The derivative of $V(e(t), e_f(t))$ with respect to time evaluated on trajectories of the estimation error equation (21) is:

$$\dot{V}(e(t), e_f(t)) = \sum_{i=1}^{h} \rho_i(\theta(t)) \big\{ e^T(t)[N_i^T Q + QN_i]e(t) + 2e^T(t)QM_i f(t) \\ + 2e^T(t)QF_i e_f(t) + \frac{1}{\sigma}\dot{e}_f^T(t)\Gamma^{-1}e_f(t) + \frac{1}{\sigma}e_f^T(t)\Gamma^{-1}\dot{e}_f(t) \big\} \tag{38}$$

From (22) and (9), the equation (36) can be expressed as:

$$\dot{\hat{f}}(t) = \Gamma \sum_{i=1}^{h} \rho_i(\theta(t)) U_i S \big(\dot{y}(t) - \dot{\hat{y}}(t) + \sigma(y(t) - \hat{y}(t)) \big) \tag{39}$$

and under $e(t) = x(t) - \hat{x}(t)$, (39) is equivalent to:

$$\dot{\hat{f}}(t) = \Gamma \sum_{i=1}^{h} \rho_i(\theta(t)) U_i SC \big(\dot{e}(t) + \sigma e(t) \big) \tag{40}$$

By using (31) and substituting (40) into equation (38), one can obtain:

$$\dot{V}(e(t), e_f(t)) = \sum_{i=1}^{h} \rho_i(\theta(t)) \big\{ e^T(t)[N_i^T Q + QN_i]e(t) + 2e^T(t)QM_i f(t) + 2e^T(t)QF_i e_f(t) \\ + \frac{1}{\sigma}\dot{e}_f^T(t)\Gamma^{-1}e_f(t) + \frac{1}{\sigma}e_f^T(t)\Gamma^{-1}[\dot{f}(t) - \Gamma U_i SC(\dot{e}(t) + \sigma e(t))] \big\} \tag{41}$$

Substituting (21) into (41) and using (35), it follows that:

$$\dot{V}(e(t), e_f(t)) = \sum_{i=1}^{h} \rho_i(\theta(t)) \big\{ e^T(t)[N_i^T Q + QN_i]e(t) - \frac{2}{\sigma}e_f^T(t)F_i^T QN_i e(t) - \frac{2}{\sigma}e_f^T(t)F_i^T QF_i e_f(t) \\ - \frac{2}{\sigma}e_f^T(t)F_i^T QM_i f(t) + 2e^T(t)QM_i f(t) + \frac{2}{\sigma}e_f^T \Gamma^{-1}\dot{f}(t) \big\} \tag{42}$$

By taking into account assumption A3, for a positive scalar μ and symmetric matrices $P_1 > 0$, $P_2 > 0$ and $P_3 > 0$ and by using Lemma 1, we can obtain:

$$2e^T(t)QM_i f(t) \le \frac{1}{\mu}e^T(t)P_1 e(t) + \mu f^T(t)M_i^T Q P_1^{-1} QM_i f(t)$$

$$\le \frac{1}{\mu}e^T(t)P_1 e(t) + \mu\alpha_1^2\lambda_{max}(M_i^T Q^T P_1^{-1} QM_i)$$

$$\frac{2}{\sigma}e_f^T(t)\Gamma^{-1}\dot{f}(t) \le \frac{1}{\mu\sigma}e_f^T(t)P_2 e_f(t) + \frac{\mu}{\sigma}\dot{f}^T(t)\Gamma^{-T}P_2^{-1}\Gamma^{-1}\dot{f}(t) \tag{43}$$

$$\le \frac{1}{\mu\sigma}e_f^T(t)P_2 e_f(t) + \frac{\mu}{\sigma}\alpha_2^2\lambda_{max}(\Gamma^{-1}P_2^{-1}\Gamma^{-1})$$

$$\frac{-2}{\sigma}e_f^T(t)F_i^T QM_i f(t) \le \frac{1}{\mu\sigma}e_f^T(t)P_3 e_f(t) + \frac{\mu}{\sigma}f^T(t)M_i^T QF_i P_3^{-1}F_i^T QM_i f(t)$$

$$\le \frac{1}{\mu\sigma}e_f^T(t)P_3 e_f(t) + + \frac{\mu}{\sigma}\alpha_1^2\lambda_{max}(M_i^T Q^T F_i P_3^{-1}F_i^T QM_i)$$

Using (43), the time derivative of (42) can be bounded as follows:

$$\dot{V}(e(t), e_f(t)) \le \sum_{i=1}^{h} \rho_i(\theta(t)) \big\{ e^T(t)[N_i^T Q + QN_i]e(t) + \frac{1}{\mu}e^T(t)P_1 e(t) - \frac{2}{\sigma}e_f^T(t)F_i^T QN_i e(t) \\ - \frac{2}{\sigma}e_f^T(t)F_i^T QF_i e_f(t) + \frac{1}{\sigma\mu}e_f^T(t)P_2 e_f(t) + \frac{1}{\sigma\mu}e_f^T(t)P_3 e_f(t) + \delta \big\} \tag{44}$$

where
$$\delta = \max_i \left[\mu\alpha_1^2 \lambda_{max}(M_i^T Q^T P_1^{-1} Q M_i) + \frac{\mu}{\sigma}\alpha_2^2 \lambda_{max}(\Gamma^{-1} P_2^{-1}\Gamma^{-1}) + \frac{\mu}{\sigma}\alpha_1^2 \lambda_{max}(M_i^T Q^T F_i P_3^{-1} F_i^T Q M_i) \right]$$

Using (25) and for $W_i = QK_i$, the inequality (44) can be written as:

$$\dot{V}(e(t), e_f(t)) \le \sum_{i=1}^{h} \rho_i(\theta(t)) \{ \begin{bmatrix} e^T(t) & e_f^T(t) \end{bmatrix} \Xi_i \begin{bmatrix} e(t) \\ e_f(t) \end{bmatrix} + \delta \} \tag{45}$$

with

$$\Xi_i = \begin{bmatrix} \Psi_i & * \\ -\frac{1}{\sigma}F_i^T(QT_1 A_i + W_i C) & -\frac{2}{\sigma}F_i^T Q F_i + \frac{1}{\sigma\mu}P_2 + \frac{1}{\sigma\mu}P_3 \end{bmatrix}$$

and

$$\Psi_i = (T_1 A_i)^T Q + Q(T_1 A_i) + \frac{1}{\mu}P_1 + C^T W_i^T + W_i C$$

Considering that $\sum_{i=1}^{h} \rho_i(\theta(t)) = 1$ and $\rho_i(\theta(t)) \ge 0$ and because F_i is of full column rank $\forall i \in [1, ..., h]$ (see assumption $A1$), when $\Xi_i < 0$ one can obtain that:

$$\dot{V}(e(t), e_f(t)) \le -\varepsilon \left\| \begin{bmatrix} e(t) \\ e_f(t) \end{bmatrix} \right\|^2 + \delta \tag{46}$$

where $\varepsilon = \min_i(\lambda_{min}(-\Xi_i))$. Then, $\dot{V}(e(t), e_f(t)) < 0$ for $\varepsilon \left\| \begin{bmatrix} e(t) \\ e_f(t) \end{bmatrix} \right\|^2 > \delta, \forall t \ge 0$ which means that $e(t), e_f(t)$ converges to a small set according to Lyapunov stability theory. Now, the obtained gains matrices K_i must satisfied also the constraint (18) $\forall i \in [1, \ldots, h]$. Then, the equation (18) can be written as follows:

$$(N_i T_2 - L_i)H + T_1 R_i = 0 \tag{47}$$

and under equation (25), it can be expressed as:

$$K_i H + T_1 R_i = 0 \tag{48}$$

For $K_i = Q^{-1}W_i \ \forall i \in [1, \ldots, h]$, equation (48) becomes:

$$W_i H + QT_1 R_i = 0 \tag{49}$$

Then, gains matrices K_i will be obtained by solving LMIs (33) under constraints (34) and (35). □

Therefore, Theorem (1) implies that error $(e(t), e_f(t))$ are uniformly bounded. Now, its easy to show that the estimated fault can be deduced from the expression (36) as follows :

$$\hat{f}(t) = \Gamma \sum_{i=1}^{h} \rho_i(\theta(t)) U_i \left(r(t) + \sigma \int_{t_f}^{t} r(\tau)d\tau \right) \tag{50}$$

where t_f is the time of fault occurring. The estimated fault expression (50) combines a proportional term with an integral one. The proportional term have an influence on the rapidity of fault estimation [32]. The proposed APO makes possible to reconstruct the state of the system whatever the presence of the unknown inputs and actuator faults and generate a residual signal which can indicates that a fault occurs.

Remark 1: It is easy to solve inequality (33) of Theorem 1, but the solving difficulty is added due to the presence of equality constraints (34) and (35). However, it is possible to transform (34) and (35) in Theorem 1 into the following optimization problem [7]:

$$\begin{aligned} \min \quad & \eta \\ \text{subject to (33)} \end{aligned} \tag{51}$$

$$\left[\begin{array}{cc} \eta I & W_i H + Q T_1 R_i \\ * & \eta I \end{array} \right] < 0 \tag{52}$$

$$\left[\begin{array}{cc} \eta I & F_i^T Q - U_i S C \\ * & \eta I \end{array} \right] < 0 \tag{53}$$

*Remark 2:*Note that, after solving the LMI problem under equalities constraints given in Theorem 1, the input-to-state convergence condition given in (46) is satisfied. Thus, in the case of time-varying faults with a bounded first time derivative, the state estimation error $e(t)$ and the fault estimation error $e_f(t)$ converge to a ball, centered at the origin, defined by the terms δ and ε. The radius of the ball in which $e(t)$ converges can be minimized by a choice of the parameter Γ that minimizes δ without changing ε (that does not depend on Γ). It thus improves the accuracy of the estimation.

Moreover, the APO studied in this paper can be considered as an improvement of the classical PI observer, in the sense that the convergence of the state estimation error and fault estimation error is proved (in a ball centered at the origin) even in a nonconstant fault case, whereas the assumption of a constant fault is needed to prove the convergence of the state estimation error when using a PI observer [14], [3].

4. ILLUSTRATIVE EXAMPLE

The proposed example, considers an electrical network as shown in Figure (1) where R_j, $j = 1, \ldots, 8$ and L_1 and L_2 stand for the resistors and inductors, respectively. $e_1(t)$ and $e_2(t)$ are the voltage sources which are taken as the control inputs. We denote by $i_1(t), i_2(t), i_3(t)$ and $i_4(t)$ the amperage of the currents.
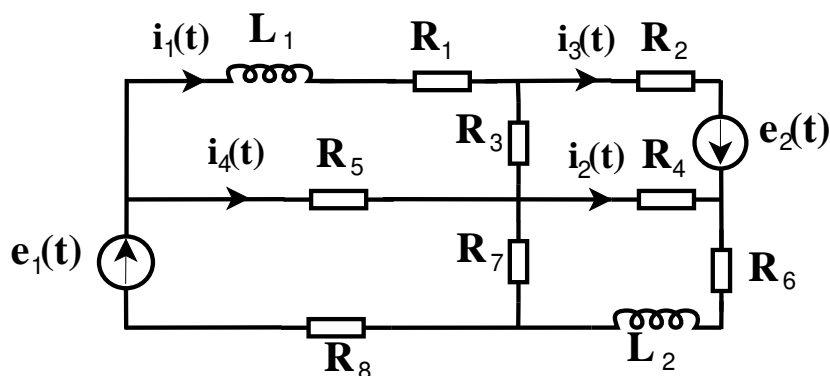

Figure 1. An electrical circuit

According to the basic circuit theory and the Kirchoff's laws, we get the following Differential-Algebraic Equations (DAE) which describe the system:

$$L_1 \frac{di_1(t)}{dt} = -(R_1 + R_3 + R_5)i_1(t) + R_3 i_3(t) + R_5 i_4(t)$$
$$L_2 \frac{di_2(t)}{dt} = -(R_4 + R_6 + R_7)i_2(t) + R_4 i_3(t) + R_7 i_4(t)$$
$$0 = -(R_2 + R_3 + R_4)i_3(t) + R_4 i_2(t) + R_3 i_1(t) + e_1(t)$$
$$0 = -(R_5 + R_7 + R_8)i_4(t) + R_7 i_2(t) + R_5 i_1(t) + e_2(t)$$
$$y_1(t) = i_1(t) + i_4(t)$$
$$y_2(t) = i_2(t) + i_3(t)$$
$$y_3(t) = i_4(t)$$

Let denote $[\ x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)\ ]^T = [\ i_1(t) \quad i_2(t) \quad i_3(t) \quad i_4(t)\ ]^T$ the state vector, $u(t) = [e1(t) \quad e2(t)]^T$ the control inputs, $y_1(t)$, $y_2(t)$ and $y_3(t)$ the outputs signals, R_1 and R_6 two variable resistors. The previous set of DAE can be represented as a LPV descriptor systems as follows:

$$\begin{cases} E\dot{x}(t) = \widetilde{A}(\theta(t))x(t) + \widetilde{B}(\theta(t))u(t) + \widetilde{R}(\theta(t))d(t) + \widetilde{F}(\theta(t))f(t) \\ y(t) = Cx(t) + Hd(t) \end{cases} \tag{54}$$

The resulting matrices of the LPV descriptor system are given by:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tilde{B}(\theta(t)) = B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \tilde{F}(\theta(t)) = F = B, C = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, H = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$$\tilde{A}(\theta(t)) = \begin{bmatrix} -\frac{R_{11}+\theta_1(t)}{L_1} & 0 & \frac{R_{13}}{L_1} & \frac{R_{14}}{L_1} \\ 0 & -\frac{R_{22}+\theta_2(t)}{L_2} & \frac{R_{23}}{L_2} & \frac{R_{24}}{L_2} \\ R_{31} & R_{32} & -R_{33} & 0 \\ R_{41} & R_{42} & 0 & -R_{44} \end{bmatrix} \text{ and } \tilde{R}(\theta(t)) = \begin{bmatrix} 0 \\ 0 \\ 0.7 + \theta_1(t) \\ 0.2 + \theta_2(t) \end{bmatrix}$$

where

$$R_{11} = R_1 + R_3 + R_5 = 15\Omega, \ R_{22} = R_4 + R_6 + R_7 = 40\Omega,$$
$$R_{33} = R_2 + R_3 + R_4 = 25\Omega, \ R_{44} = R_5 + R_7 + R_8 = 20\Omega,$$
$$R_{13} = R_{31} = R_3 = 3\Omega, \ R_{14} = R_{41} = R_5 = 2\Omega,$$
$$R_{23} = R_{32} = R_4 = 5\Omega, \ R_{24} = R_{42} = R_7 = 8\Omega$$
$$and \ L_1 = 0.3H, \ L_2 = 0.65H.$$

$\tilde{R}$ is a disturbance distribution matrix that affects the system where $d(t)$ is a random vector uniformly distributed in $[-1, 1]$ which acts like additive noise. H is also a disturbance distribution matrix that affects the outputs. $\theta_1(t)$ and $\theta_2(t)$ are two time-varying parameters which vary according to $\theta_1(t) \in [\ -0.5, \quad 0.5\ ]$ and $\theta_2(t) \in [\ -1, \quad 1\ ]$. The inputs control are described by

$$\begin{cases} u_1(t) = 12\sin(2.5t) \\ u_2(t) = 5 \end{cases}$$

It is assumed that the LPV descriptor system (54) is affected by two actuator faults which followed the assumption A_3 and that are defined by:

$$\begin{cases} f_1(t) = 0.35u_1(t) \text{ occurs in } u_1(t) \text{ when } 8 \leq t \leq 15s \\ f_2(t) = 0.25u_2(t) \text{ occurs in } u_2(t) \text{ when } 20 \leq t \leq 30s \end{cases}$$

Note that different actuator fault can appear in voltage source: an offset for bias or temporal variation for transient fault. Such fault can appear on real source voltage with various temporal duration and different shapes.

The parameters of the polytopic LPV descriptor system evolve in a hyper-rectangle. Consequently, the matrices A_i and R_i of this system are given by:

$$A_1 = \begin{bmatrix} -51.66 & 0 & 10 & 6.66 \\ 0 & -63.07 & 7.69 & 12.3 \\ 3 & 5 & -25 & 0 \\ 2 & 8 & 0 & -20 \end{bmatrix}, A_2 = \begin{bmatrix} -51.66 & 0 & 10 & 6.66 \\ 0 & -60 & 7.69 & 12.3 \\ 3 & 5 & -25 & 0 \\ 2 & 8 & 0 & -20 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -48.33 & 0 & 10 & 6.66 \\ 0 & -63.07 & 7.69 & 12.3 \\ 3 & 5 & -25 & 0 \\ 2 & 8 & 0 & -20 \end{bmatrix}, A_4 = \begin{bmatrix} -48.33 & 0 & 10 & 6.66 \\ 0 & -60 & 7.69 & 12.3 \\ 3 & 5 & -25 & 0 \\ 2 & 8 & 0 & -20 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 0 \\ 0 \\ 0.2 \\ -0.8 \end{bmatrix}, R_2 = \begin{bmatrix} 0 \\ 0 \\ 0.2 \\ 1.2 \end{bmatrix}, R_3 = \begin{bmatrix} 0 \\ 0 \\ 1.2 \\ -0.8 \end{bmatrix} \text{ and } R_4 = \begin{bmatrix} 0 \\ 0 \\ 1.2 \\ 1.2 \end{bmatrix}$$

The weighting functions $\rho_i(\theta(t))$ are defined as combinations of θ_j and are given by [14]:

$$\rho_1(\theta(t)) = \frac{\theta_1(t) - \underline{\theta}_1}{\overline{\theta}_1 - \underline{\theta}_1} \frac{\theta_2(t) - \underline{\theta}_2}{\overline{\theta}_2 - \underline{\theta}_2} = \frac{(\theta_1(t) + 0.5)(\theta_2(t) + 1)}{2}$$

$$\rho_2(\theta(t)) = \frac{\theta_1(t) - \underline{\theta}_1}{\overline{\theta}_1 - \underline{\theta}_1} \frac{\overline{\theta}_2 - \theta_2(t)}{\overline{\theta}_2 - \underline{\theta}_2} = \frac{(\theta_1(t) + 0.5)(1 - \theta_2(t))}{2}$$

$$\rho_3(\theta(t)) = \frac{\overline{\theta}_1 - \theta_1(t)}{\overline{\theta}_1 - \underline{\theta}_1} \frac{\theta_2(t) - \underline{\theta}_2}{\overline{\theta}_2 - \underline{\theta}_2} = \frac{(0.5 - \theta_1(t))(\theta_2(t) + 1)}{2}$$

$$\rho_4(\theta(t)) = \frac{\overline{\theta}_1 - \theta_1(t)}{\overline{\theta}_1 - \underline{\theta}_1} \frac{\overline{\theta}_2 - \theta_2(t)}{\overline{\theta}_2 - \underline{\theta}_2} = \frac{(0.5 - \theta_1(t))(1 - \theta_2(t))}{2}$$

4.1. APO design and residual generation

To design the presented APO according to the proposed methodology, let's verify a necessary assumption A_4 to be able to provide actuator fault estimation with equation (50). After that, a matrix S (solution of and $SH = 0$) can be given by (29). Then, let's check the necessary assumption A_5. Then, matrices T_1 and T_2 can be computed from equation (28). After by checking A_1 and A_2, the gains matrices can be obtained by solving the LMIs (33) under constraints (34) and (35) via numerical approach within the LMI Toolbox. By choosing the scalars $\sigma = 2$, $\mu = 0.5$, these inequalities are fulfilled with:

$$K_1 = \begin{bmatrix} 21.1564 & -17.1376 & -25.1753 \\ -16.1420 & -0.9267 & 33.2106 \\ 38.9566 & -18.4702 & -59.4431 \\ 23.4193 & -16.5291 & -30.3096 \end{bmatrix}, K_2 = \begin{bmatrix} 1.7542 & 1.8862 & -5.3945 \\ -3.7885 & 13.0178 & -5.4409 \\ 5.4732 & -11.0272 & 0.0808 \\ 1.7234 & 1.8499 & -5.2967 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 1.4095 & 2.0748 & -4.8937 \\ -2.9091 & 12.5797 & -6.7615 \\ 4.2535 & -10.4436 & 1.9365 \\ 1.2742 & 2.0773 & -4.6257 \end{bmatrix}, K_4 = \begin{bmatrix} 1.2491 & 2.5999 & -5.0981 \\ -2.7452 & 13.0423 & -7.5518 \\ 3.9211 & -10.3360 & 2.4938 \\ 1.1244 & 2.5655 & -4.8143 \end{bmatrix}$$

The simulation results for time varying actuator faults detection are shown in Figures (2) and (3). From figures (2) and (3), one can see that the residuals are almost zero throughout the time simulation in fault-free case. The residual signals increase in magnitude considerably when actuators faults occur for $t \in [8, 15]s$ and $t \in [20, 30]s$. These two faults can be estimated by using the adaptive fault estimation algorithm as in (50).
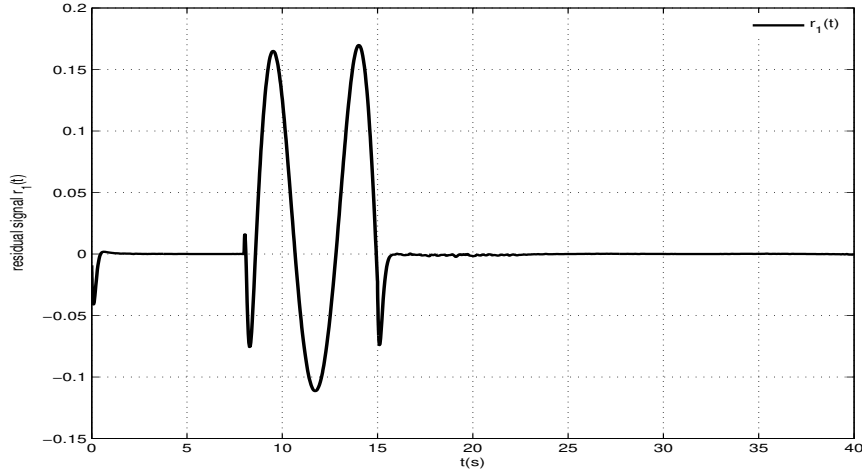
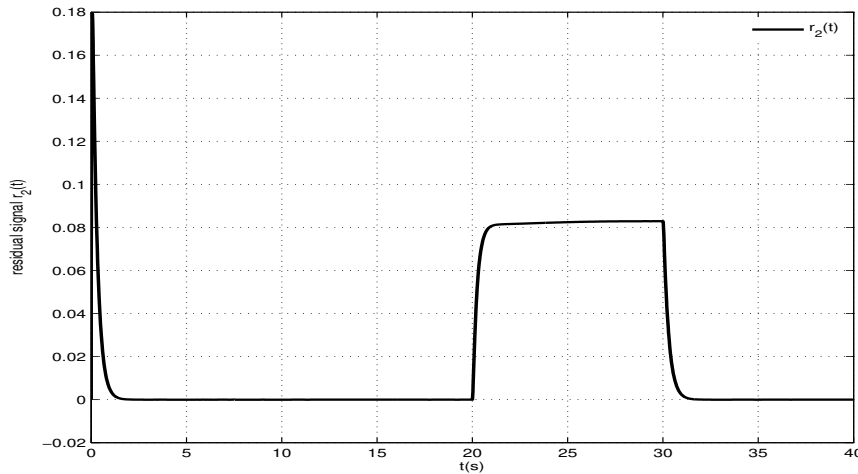Figure 2. Residual $r_1(t)$ signal when a fault occurs in $u_1(t)$

Figure 3. Residual signal $r_2(t)$ when a fault occurs in $u_2(t)$

4.2. Actuator fault estimation

Let recall that $B_i = F_i = B$. Under assumption A_4 and by solving the LMIs (33) to (35), getting matrices $U_i = U = \begin{bmatrix} 0.8580 & 0.4280 & 0.4263 \\ -8.1460 & -4.0720 & -4.0729 \end{bmatrix}$ and choosing the learning rate matrix $\Gamma = diag(1, 2)$, the simulation results for time-varying actuator faults estimation using the adaptive algorithm given by (50) are mentioned in the following Figures.

Figures (4) and (5) show that estimated actuator faults by APO can converge to their real values which is more powerful than Proportional Integral Polytopic Observer (PIPO) presented in [14]. Then, it is shown that the PIPO makes possible to decouple the disturbances while it can only detect and estimate constant actuator faults as in Figure (5). Indeed, its actuator fault estimation does not match correctly the real one for the time-varying case in Figure (4). In contrast, the APO shows very good results for both time-varying and constant actuator faults estimations despite the
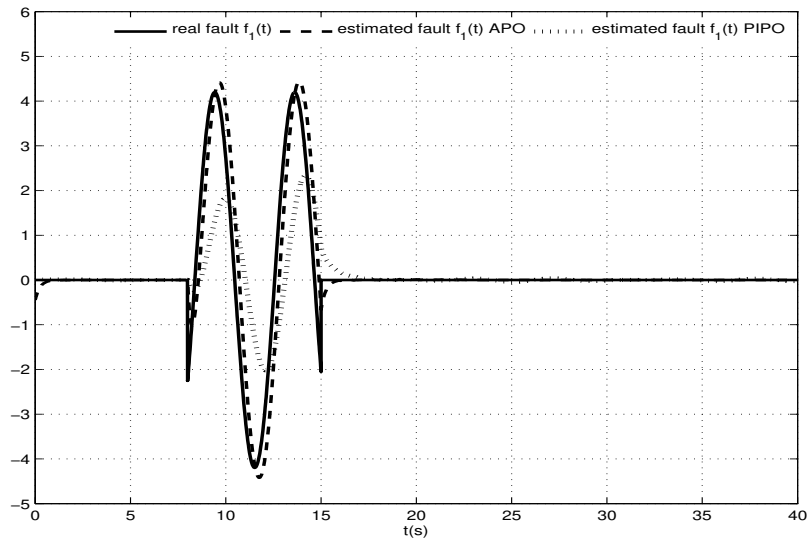
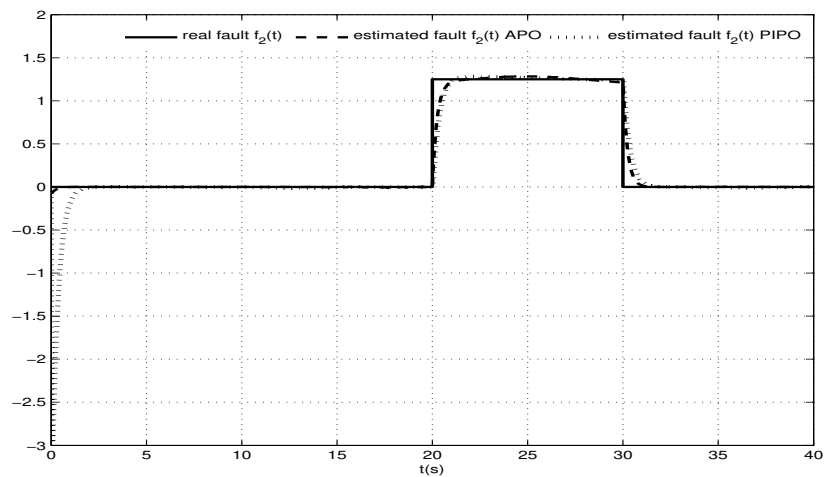Figure 4. Actuator fault $f_1(t)$ and its estimated $\hat{f}_1(t)$

Figure 5. Actuator fault $f_2(t)$ and its estimated $\hat{f}_2(t)$

presence of disturbance: it is a good improvement for such LPV descriptor systems.

5. CONCLUSION

In this paper, an actuator fault estimation scheme based on an Adaptive Polytopic Observer for LPV descriptor systems has been proposed. The developed strategy allows to consider not only constant faults but also time-varying actuator faults. The convergence conditions of this observer has been formulated and solved within a set of LMI under equalities constraints. The developed scheme has been applied in an electrical circuit so as to estimate both time-varying and constant actuator

faults for polytopic LPV descriptor systems. A comparison with previous works like PI observers, underlines the theoretical and practical improvements for time-varying actuator faut estimation for such LPV descriptor systems.

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