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AD HOC MODIFICATIONS OF QUANTUM ELECTRODYNAMICS

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A B S T R A C T

Ad hoc modifications of quantum electrodynamics are discussed with special reference to the restrictions imposed by charge conservation. It is shown that modifications in a charged particle propagator typically require, in addition to the one-photon vertex modification imposed by the Ward identity, the introduction of a possibly infinite number of multiphoton vertices. A procedure, based upon a generalization of the notion of minimal electromagnetic interaction, for constructing the required vertices is given. In the absence of closed loops, it is shown that such propagator modifications have no effect upon the theory. Finally a general procedure is given for characterizing any theory consistent with charge conservation in terms of a modified propagator and intrinsic multiphoton vertices. The actual vertices which appear in the theory are generated by the propagator and intrinsic vertices in accordance with a well-defined procedure. An example is discussed.

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I. INTRODUCTION

It has become conventional to interpret experiments on the electromagnetic properties of leptons in terms of ad hoc modifications introduced into the formulas of quantum electrodynamics [1),2)]. These modifications are characterized by parameters which serve to define momenta at which departures from conventional quantum electrodynamics would make their appearance. The original proposals had the advantage of extreme simplicity both in form and parametrization and therefore provided a convenient descriptive framework for discussing experimental results. On the other hand it was recognized by their proposers that these modifications were inconsistent with general theorems of field theory and were certainly not to be regarded as serious theoretical proposals. More recent proposals [3),4)] have been directed towards improving the theoretical consistency of the modifications, at the cost of increasing their complexity and arbitrariness. In particular, Low [3)] has suggested that modifications be described in terms of couplings to new particles, while Drell and McClure [4)] have directed their attention towards removing the most obvious theoretical deficiencies from ad hoc modifications in propagator functions. Because of the ad hoc nature of the modifications one might well ask why one should be particularly concerned with the general theorems of field theory. At the risk of belabouring the obvious we shall digress briefly to discuss this question. As long as one wishes to do no more than parameterize the degree to which the theory has been experimentally confirmed there is no special penalty associated with ignoring the general requirements. On the other hand one naturally asks whether a proposed modification makes predictions outside the context of the experimental situation which led to its introduction. The answer is, of course, affirmative and one usually pays for violation of the general theorems by finding that the most striking predictions cannot be believed.

Let us first consider the question of a cut-off photon propagator, characterized by

$$\frac{1}{q^2} \longrightarrow \frac{\Lambda^2}{q^2(q^2+\Lambda^2)} = \frac{1}{q^2} - \frac{1}{q^2+\Lambda^2} \qquad (1)^{5)}$$

The form (1) was suggested by the desire to remove the divergences of the theory and one notes that it is characterized by a single parameter Λ. While the modification reduces the effects of the interaction at large q^2, at q^2 near $-\Lambda^2$ its effects are greatly enhanced and, indeed, one predicts a resonance in positron-electron scattering which acquires a finite width on taking higher order effects into account. This resonance, together with the violation of the spectral condition [6] represented by the minus sign in (1), then leads to a large derivative of the phase shift with respect to the energy of the wrong sign, with causality difficulties as discussed by Wigner [7]. The fact that the pole remains on the physical sheet may have more serious consequences. As pointed out by Drell and McClure, this difficulty can be avoided simply by changing the sign. The results are then equivalent to introducing a new neutral vector boson of mass Λ into the theory (as advocated Low [3]). This change, of course, introduces an additional degree of arbitrariness in the form of the particle coupling constant. That is, once one has destroyed the convergence properties of the modification by choosing the coefficient of the second term positive, there is no reason to retain the value unity.

We next consider the case of charged particle propagator modifications. In addition to the question of spectral conditions, such changes introduced without other modifications violate the requirements imposed by charge conservation. This particular modification of the theory leads to a failure of the classical limit, exemplified especially by an incorrect value for the Thomson cross-section. In addition, the non-linear electromagnetic properties of the vacuum depend upon the potentials rather than the field strength. We note that charge conservation also imposes requirements on vertex modifications, and the consequences of their violation are similar.

Experiments on wide angle electron-positron pair production were originally proposed as a means for testing the charged particle propagator and "off the mass shell" vertex in quantum electrodynamics, as this process is not affected significantly by modifications of the photon propagator [1]. Recently observed deviations [8] from the theoretical prediction for this process, would seem to endow the problem of charged propagator and vertex modifications an increased interest.

This work is devoted to the establishment of a general procedure for satisfying the requirements of charge conservation in such modifications, followed by an analysis of their consequences. Investigation of this problem has been initiated by Drell and McClure [4]. These authors concern themselves particularly with the requirements of the Ward-Takahashi [9] identity, which takes the form

$$i(p_\mu - q_\mu)\Gamma_\mu(p,q) = S^{-1}(\underline{p}) - S^{-1}(\underline{q}) \qquad (2)$$

where $\underline{p} = i\,\gamma_\mu p_\mu$, $S(\underline{p})$ is the postulated charged particle propagator, and $\Gamma_\mu(p,q)$ is the associated one-photon vertex function. It evidently requires that any modification in the propagator be associated with some modification in the vertex function. Therefore the question of separating the effect of propagator modifications from that of "bona fide" modifications of the vertex function arises. There is no unique answer to this question, but Drell and McClure propose a natural and plausible definition of the vertex to be associated with a propagator modification. It can be written in an especially convenient and suggestive way in analogy with the Feynman identity [10].

$$\gamma_\mu = (\underline{p}+m)\frac{\underline{p}\gamma_\mu + \gamma_\mu \underline{q}}{q^2-p^2} - \frac{\underline{p}\gamma_\mu + \gamma_\mu \underline{q}}{q^2-p^2}(\underline{q}+m) \qquad (3)$$

in the form

$$\Gamma_\mu(p,q) = S^{-1}(\underline{p})\frac{\underline{p}\gamma_\mu + \gamma_\mu \underline{q}}{q^2-p^2} - \frac{\underline{p}\gamma_\mu + \gamma_\mu \underline{q}}{q^2-p^2}S^{-1}(\underline{q}) \qquad (4)$$

On application of expression (4) to the evaluation of the simplest process in which it occurs, one finds that the result is in general unsatisfactory. Compton scattering does not yield the Thomson limit, and second order radiative corrections to the vertex function do not satisfy the Ward-Takahashi identity. Drell and McClure correct both difficulties by adding a two-photon vertex to their theory, using only the criterion that this addition remove the obvious deficiencies of the expressions which they compute.

The difficulty described in the above paragraph is a manifestation of the fact that the Ward-Takahashi identity takes account of only a small part of the restriction imposed by charge conservation. Kazes [11)] has given an extensive generalization of this identity described as generalized current conservation. Chang and Mani [12)] have given a set of consequences of Kazes' results which are generalizations of (1) to "proper" (to be defined later) multiphoton vertices. As a consequence of these Chang-Mani identities, one is required to associate with any propagator modification a certain number of multiphoton vertices, the minimum number being immediately determined from the form of the modified propagator. One is therefore faced with the problem of constructing a large, and in typical cases even infinite, set of multiphoton vertices consistent with the Chang-Mani identities.

It is obvious that there can be no unique solution to this problem, and that one must therefore devise a "standard construction" for the multiphoton vertices. The palatability of this situation is perhaps improved when one recalls that this problem exists even in standard quantum electrodynamics. That is, the standard propagator imposes a restriction upon the form of the vertex function which one can introduce (i.e., the form of the particle-photon interaction Lagrangian) but does not determine it uniquely. The classification scheme which one uses to deal with this ambiguity goes under the name of "minimal electromagnetic interaction". We propose, therefore, to extend the notion of minimal electromagnetic interaction to modified propagators. The standard transformation,

$$\frac{\partial}{\partial x_\mu} \longrightarrow \frac{\partial}{\partial x_\mu} - ieA_\mu \, ,$$

suitably re-expressed in momentum space language and generalized to deal with non-polynomial functions of p_μ, provides a natural and convenient method of defining the minimal interaction associated with an essentially arbitrary form of the propagator. It results in the introduction of the minimum required number of multiphoton vertices, but no more.

An effective formalism for constructing the minimal interaction in accordance with the above prescription, and determining its consequences, is developed in Section III. It will be shown that the consequences can be discussed with great generality. One finds the striking result that the sum of all Feynman diagrams for any specified process of a given order in e, excluding closed loops, is totally independent of the propagator modification. That is to say, the result is the same as that of the unmodified theory. Since closed loops in lowest order simulate photon propagator modifications, effects distinguishable from photon propagator modifications appear first as sixth order corrections in the case of pair production, or in the case of the anomalous moment as corrections to the sixth order term. We conclude, therefore, that charged propagator modifications have no practical application whatever to the situations for which they were introduced.

Having dealt with the problem of propagator modifications, one is now in a position to give an unambiguous meaning to intrinsic one-photon ("intrinsic" in contrast to that "induced" by the propagator modification), as simply the difference between the actual vertex employed and the minimal vertex. It is an immediate consequence of (1) and this definition that $k_\mu \Gamma_\mu(\text{intrinsic}) = 0$. Intrinsic vertex modifications are of course also subject to the Chang-Mani identities and one must typically associate an additional set of multiphoton vertices with such modifications. The procedure introduced for the propagator can be applied directly to the intrinsic vertices and provides a natural definition for the associated multiphoton vertices. The details, and a simple example applicable to the pair production problem, are given in Section IV.

It will be shown that the procedure outlined can be repeated successively for two photon vertices, etc., and hence that intrinsic vertices can be defined for each rank. We carry this out largely to show that no generality has been lost by adopting the minimal construction. The ambiguity inherent in the situation has simply been systematized. The utility of the procedure is probably confined to intrinsic modifications of the one-photon vertex.

II. THE CHANG-MANI IDENTITIES AND THE MINIMUM NUMBER OF MULTIPHOTON VERTICES

We begin this Section by rederiving certain generalizations of the Ward-Takahashi identity. For the applications to be made it is desirable that they be established on as general a basis as possible [13], and for this purpose we follow a procedure based upon the work of Takahashi [9].

The scattering amplitudes involving n photons distributed between incoming and outgoing states and a charged particle are expressed in terms of the τ function [14]:

$$\tau(y, z; x_1, \cdots, x_n) = \langle Vac | T[\psi(y)\bar{\psi}(z) A_{\mu_1}(x_1) \cdots A_{\mu_n}(x_n)] | Vac \rangle \tag{5}$$

where tensor indices associated with the electromagnetic potentials, $A_\mu(x)$, and possible indices associated with the charged particle fields, ψ, $\bar{\psi}$, have been suppressed and will be exhibited only where essential. We shall also need $\tau(x_1, \ldots, x_n)$, the τ functions of the electromagnetic potentials alone.

From the conditions

$$\Box A_{\mu_1} = -j_\mu \tag{6}$$

$$\delta(x_{j0} - x_{k0})[A(x_j), A(x_k)] = 0 \tag{7}$$

$$\delta(y_0 - x_{j0})[\psi(y), A(x_j)] = \delta(z_0 - x_{j0})[\bar{\psi}(z), A(x_j)] = 0 \tag{8}$$

$$\delta(y_0 - x_{j0})\left[\psi(y), \frac{\partial A(x_j)}{\partial x_{j0}}\right] = \delta(z_0 - x_{j0})\left[\bar{\psi}(z), \frac{\partial A(x_j)}{\partial x_{j0}}\right] = 0 \tag{9}$$

and

$$\delta(x_{j0}-x_{k0})\left[\frac{\partial}{\partial x_{j0}}A(x_j),\ A(x_k)\right] = \text{C number} \tag{10}$$

we have

$$\Box_n \tau(y,z;x_1,\cdots x_n) = -\langle Vac|T[\psi(y)\bar{\psi}(z)A(x_1)\cdots A(x_{n-1})\,j(x_n)|Vac\rangle$$

$$-\sum_{i=1}^{n-1}\tau(y,z;x_1,\cdots,x_{n-1})_i\,\delta(x_{n0}-x_{i0})\left[\frac{\partial A(x_n)}{\partial x_{n0}},A(x_i)\right] \tag{11}$$

where the subscript i on the τ function means that the variable x_i is omitted from the set of arguments. Next, from the conditions

$$\delta(x_{n0}-y_0)[j_0(x_n),\psi(y)] = -e\,\delta_4(x_n-y)\psi(y) \tag{12}$$

$$\delta(x_{n0}-z_0)[j_0(x_n),\bar{\psi}(z)] = e\,\delta_4(x_n-z)\bar{\psi}(z) \tag{13}$$

$$\delta(x_{n0}-x_{i0})[j_0(x_n),A(x_i)] = 0 \tag{14}$$

$$\partial_n j(x_n) \equiv \frac{\partial j_{\mu_n}(x_n)}{\partial x_{\mu_n}} = 0 \tag{15}$$

we have

$$\partial_n\Box_n\tau(y,z;x_1,\cdots,x_n) = e\,\tau(y,z;x_1,\cdots,x_{n-1})[\delta(y-x_n)-\delta(z-x_n)]$$

$$+\sum_{i=1}^{n-1}\tau(y,z;x_1,\cdots,x_{n-1})_i\,\partial_n\Box_n\tau(x_i,x_n) \tag{16}$$

Similarly, one finds

$$\partial_n \Box_n \tau(x_1, \cdots, x_n) = \sum_{j=1}^{n-1} \tau(x_1, \cdots, x_{n-1})_i \, \partial_n \Box_n \tau(x_i, x_n) \tag{17}$$

Since the τ functions are symmetric in the x_i, similar relations hold for derivatives with respect to the other x_i.

We now introduce the connected τ functions, which we call φ functions. These are defined recursively by the formulae :

$$\begin{aligned} \varphi(y, z;) &= \tau(y, z;) \\ \varphi(y, z; x) &= \tau(y, z; x) \end{aligned} \tag{18}$$

$$\varphi(y, z; x_1, \cdots, x_n) = \tau(y, z, x_1, \cdots x_n) - \mathcal{P}_n \sum_{j=1}^{\nu} \frac{\varphi(y, z; x_{2j+1}, \cdots, x_n)\, \tau(x_1, \cdots, x_{2j})}{(n-2j)!\,(2j)!} \tag{19}$$

where $\nu = \frac{n}{2}$ or $\frac{n-1}{2}$ accordingly as n is even or odd and $\mathcal{P}_n$ means a summation over all permutations of the variables $x_1, \ldots, x_n$. From the symmetry of the φ functions implied by (19), we have

$$\begin{aligned} &\mathcal{P}_n \sum_{j=1}^{\nu} \frac{\varphi(y, z; x_{2j+1}, \cdots, x_n)\, \tau(x_1, \cdots, x_{2j})}{(n-2j)!\,(2j)!} \\ &= \mathcal{P}_{n-1} \left\{ \sum_{j=1}^{\nu'} \frac{\varphi(y, z; x_{2j+1}, \cdots, x_n)\, \tau(x_1, \cdots, x_{2j})}{(n-2j-1)!\,(2j)!} \right. \\ &\qquad \left. + \sum_{j=1}^{\nu} \frac{\varphi(y, z; x_{2j}, \cdots, x_{n-1})\, \tau(x_1, \cdots, x_{2j-1}, x_n)}{(n-2j)!\,(2j-1)!} \right\} \end{aligned} \tag{20}$$

where $\nu' = \frac{n}{2} - 1$ or $\frac{n-1}{2}$ for n even or odd.

The relation

$$\partial_n \Box_n \varphi(y,z;x_1,\cdots,x_n) = e\varphi(y,z;x_1,\cdots,x_{n-1}) \cdot [\delta(x_n-y)-\delta(x_n-z)] \tag{21}$$

holds for $n=1$ from (16) and (18): and can thence be demonstrated for all n by applying induction to (19), using (16) and (20).

Equation (21) is a generalized Ward identity relating improper n photon and $n-1$ photon vertices. In momentum space it can be written

$$-ik_n k_n^2 \tilde{\varphi}(q;k_1,\cdots,k_n) = e[\tilde{\varphi}(q+k_n;k_1,\cdots,k_{n-1}) - \tilde{\varphi}(q;k_1,\cdots,k_{n-1})] \tag{22}$$

In (22), $\tilde{\varphi}$ is the Fourier transform of φ apart from an over-all energy momentum conserving δ function, q the momentum of the incoming charged particle and k_i the incoming momenta of the photons. The Kazes identities are expressed in terms of quantities Γ_n, which we call semiproper n photon vertices, defined by

$$\tilde{\varphi}(q;k_1,\cdots,k_n) = e^n S(q+\sum_{i=1}^{n} k_i)\,\Gamma_n(q;k_1,\cdots,k_n)\,S(q) \cdot D(k_1)\cdots D(k_n) \quad , \tag{23}$$

where S and D are the charged particle and photon propagator, respectively. Using [15]

$$k_\mu k^2 D_{\mu\nu}(k^2) = k_\nu \tag{24}$$

and factoring out the remaining D functions, yields [16]

$$ik_n \Gamma_n(q;k_1,\cdots,k_n) = S(q+\sum^{n-1} k_i)\,\Gamma_n(q;k_1,\cdots,k_{n-1})\,S(q) - S(q+k_n+\sum^{n-1} k_i)\,\Gamma_n(q+k_n;k_1,\cdots,k_{n-1})\,S(q+k_n) \tag{25}$$

In diagram language, the semiproper vertices differ from the improper vertices in the removal of all external photon and charged particle lines together with their associated selfenergy parts. The Chang-Mani identities give similar relations for proper n photon vertices. These are represented by $V_n(q;k_1,\ldots,k_n)$ and are defined diagrammatically by the requirement that they be non-disconnectable by cutting a single charge bearing line. Algebraically, they are defined by the recursion formula

$$V_n(q;k_1,\cdots,k_n) = \Gamma_n(q;k_1,\cdots,k_n) - P_n \sum_{j=1}^{n-1} \frac{V_j(q_j,k_{j+1},\cdots,k_n)\,\Gamma_{n-j}(q;k_1,\cdots,k_j)}{j!\,(n-j)!} \tag{26}$$

with $V_1 = \Gamma_1$, $q_j = k_j + q_{j-1}$, and $q_1 = k_1 + q$.

The Chang-Mani identities

$$i k_n V_n(q;k_1,\cdots,k_n) = V_{n-1}(q;k_1,\cdots k_{n-1}) - V_{n-1}(q+k_n;k_1,\cdots,k_{n-1}) \tag{27}$$

are easily demonstrated inductively, using (25) and (26). Equation (2) can be included in (27) by identifying V_0 with $-S^{-1}$.

The differential identities

$$V_{\mu_1\cdots\mu_{n-1}\lambda}(q;k_1,\cdots,k_{n-1},0) = i\frac{\partial}{\partial q_\lambda} V_{\mu_1\cdots\mu_{n-1}}(q;k_1,\cdots,k_{n-1}) \tag{28}$$

follow from Eq. (27).

We note that the only new relations required to derive (27) beyond those needed for (2) are Eqs. (7), (10) and (14), and hence conclude that it would be difficult to justify respecting (2) while ignoring (27). Furthermore, for the derivation of (28), which is sufficient to determine the minimum required number of multiphoton vertices, the locality of the charge density imposed by (12), (13) and (14) can be relaxed [17]. We also mention that the V_n are symmetric functions of the photon momenta and tensor indices, so that (27) and (28) hold for arbitrary permutations of the photon variables.

Equations (27) and (28) were derived as relations between renormalized quantities. Apart from questions of infinite coefficients, they hold also between unrenormalized quantities and should also hold term by term in a perturbation series expansion. They should therefore also hold for the zero order functions on the basis of which one computes Feynman diagrams. We therefore require that any theory with a modified propagator and/or modified vertex contain the minimum number of multiphoton vertices, $e_o^n V_n$, required to satisfy (28). It follows from

$$V_n(q;0,\cdots,0) = -(i)^n \frac{\partial}{\partial q_{\mu_1}} \cdots \frac{\partial}{\partial q_{\mu_n}} S^{-1}(q) \tag{29}$$

$$V_n(q;k_1,0,\cdots,0) = i^{n-1} \frac{\partial}{\partial q_{\mu_2}} \cdots \frac{\partial}{\partial q_{\mu_n}} V_1(q;k_1) \tag{30}$$

that the number is infinite unless S^{-1} and V_1 are polynomials. If S^{-1} and V_1 are polynomials, the minimum required number is n_o-1, where n_o is the smallest value of n for which (29) and (30) give zero.

III. CHARGED PROPAGATOR MODIFICATIONS

For definiteness, we discuss the most important case, namely spin $\frac{1}{2}$ electrodynamics. The discussion is easily generalized. For the spin $\frac{1}{2}$ case the propagator can always be expressed as a function of $\underset{\sim}{p} = i\,\gamma_\mu p_\mu$. We assume the modified propagator, $S(\underset{\sim}{p})$, is such that $S(\underset{\sim}{p})-S_m(\underset{\sim}{p})$ is regular at $\underset{\sim}{p}=-m$, where

$$S_m(\underset{\sim}{p}) = \frac{1}{\underset{\sim}{p}+m}$$

If $S^{-1}(\underset{\sim}{p})$ is a polynomial in $\underset{\sim}{p}$ of degree n, then Eq. (29) and the discussion following it show that multiphoton vertices up to and including rank n are required. Otherwise the required number is infinite. In order to systematize the ambiguity involved in the notion of propagator modification, we seek a generalization of the idea "minimal electromagnetic interaction" to modified propagators. That is to say, we wish to define the "minimal electromagnetic interaction generated by the propagator". Our principal criterion will be that (27) be satisfied with the minimum n, and that the resultant interaction be minimized in a sense which we can specify completely only <u>a posteriori</u>. As a partial expression of minimality, however, we would, in agreement with McClure and Drell [4], wish to impose

$$\bar{\psi}(q+k)\,\Gamma_\mu(\underset{\sim}{q};k)\,\psi(q) = \bar{\psi}(q+k)\,\gamma_\mu\,\psi(q) \tag{31}$$

The construction should also be as simple and convenient as is consistent with (27) and the form of S.

For the case of the normal propagator, one can generate the minimal electromagnetic interaction by means of the substitution $\gamma_\mu(\partial/\partial x_\mu) \to \gamma_\mu(\partial/\partial x_\mu) - ie\,\gamma_\mu A_\mu(x)$ in the expression $\bar{\psi}(x)S_m^{-1}(\gamma_\mu(\partial/\partial x_\mu))\psi(x)$. For polynomial S^{-1}, such a substitution in this expression with S_m^{-1} replaced by S^{-1} clearly yields the minimum number of multiphoton interactions, and constitutes a suggestive candidate for the minimal electromagnetic interaction. We shall see that the formulation of this substitution is particularly simple in momentum space, as well as its generalization to non-polynomial S^{-1}: and that it satisfies all of our criteria for the minimal interaction.

a) Operator Calculus

The operator calculus developed below turns out to be a very useful tool for carrying out the procedure described above, and for evaluating its consequences.

Let $F(\underset{\sim}{q})$ be some Dirac matrix function of the vector q_μ, expressed explicitly in terms of $\underset{\sim}{q}$. On account of the identity, $2iq_\mu = \underset{\sim}{q}\gamma_\mu + \gamma_\mu \underset{\sim}{q}$, this is always possible. We define the linear operator $d_\mu(k)$ on F by

$$n_\mu d_\mu(k) F(\underset{\sim}{q}) = \lim_{\varepsilon \to 0} U_k \left[\frac{F(\underset{\sim}{q} + \varepsilon \underset{\sim}{n} U_k^{-1}) - F(\underset{\sim}{q})}{i\varepsilon} \right] \tag{32}$$

where n_μ is a unit four vector and U_k is a displacement operator on q, defined by $U_k q_\mu U_k^{-1} = q_\mu + k_\mu$. Because $\underset{\sim}{q}$ and $\underset{\sim}{n} U_k^{-1}$ do not commute, the explicit realization of the operation is not always trivial. It can, however, obviously be carried out for polynomials.

We shall wish to require that $d_\mu(k) F(\underset{\sim}{q})$ be a Dirac matrix function of $\underset{\sim}{q}$ also, and in particular that it not be a non-commuting operator in q_μ space. That is to say, we wish

$$\left[q_\nu, (d_\mu(k) F(\underset{\sim}{q}) \right] = 0 \tag{33}$$

Equation (33) obviously holds for all polynomials and probably holds also for all cases in which (32) is defined simply because one then expects

$$U_k \left[F(\underset{\sim}{q} + \varepsilon \underset{\sim}{n} U_k^{-1}) - F(\underset{\sim}{q}) \right]$$

for small ε to be a sum of terms of the form

$$\varepsilon U_k F_1(\underset{\sim}{q}) U_k^{-1} \underset{\sim}{n} F_2(\underset{\sim}{q}) = \varepsilon F_1(\underset{\sim}{q} + \underset{\sim}{k}) \underset{\sim}{n} F_2(\underset{\sim}{q})$$

We shall not pursue this further because it will be sufficient for our purposes to confine our attention to a class of functions R, defined by the requirement that the limit in (32) exists when applied to its elements and (33) holds.

If F and G belong to R, one easily shows that FG belong to R and

$$d_\mu(k) F(\underset{\sim}{q}) G(\underset{\sim}{q}) = [d_\mu(k) F(\underset{\sim}{q})] G(\underset{\sim}{q}) + F(\underset{\sim}{q}+\underset{\sim}{k}) d_\mu(k) G(\underset{\sim}{q}) \tag{34}$$

Furthermore

$$d_\mu(k) F^{-1}(\underset{\sim}{q}) = -F^{-1}(\underset{\sim}{q}+\underset{\sim}{k})\left(d_\mu(k) F(\underset{\sim}{q})\right) F^{-1}(\underset{\sim}{q}) \tag{35}$$

Equations (34) and (35) provide a convenient means of realizing the operation $d_\mu(k)$ for a very large class of functions. For applications to quantum field theory, rational functions, and functions with spectral representations are presumably sufficient, and are clearly included.

We now define a subset of R, called R_0 whose elements satisfy

$$i k_\mu d_\mu(k) F(\underset{\sim}{q}) = F(\underset{\sim}{q}+\underset{\sim}{k}) - F(\underset{\sim}{q}) \tag{36}$$

$$d_\mu(k_1) d_\nu(k_2) F(\underset{\sim}{q}) = d_\nu(k_2) d_\mu(k_1) F(\underset{\sim}{q}) \tag{37}$$

It follows by straightforward computation, using (34) and (35), that if F and G are in R_0, so is F+G, FG and F^{-1}. Hence from the trivial observation that (36) and (37) hold for the product of q with an arbitrary q independent Dirac matrix, one sees that the class R_0 is also large, and sufficiently large for our applications to quantum electrodynamics.

b) The Minimal Electromagnetic Interaction

Let the assumed form for a proposed modified propagator, $S(q)$, be a function in R_0. Then S^{-1} is also in R_0. We define the minimal electromagnetic interaction associated with S by the recursion relation [18]

$$V_n(\underset{\sim}{q};k_1,\cdots,k_n) = -d(k_n)\,V_{n-1}(\underset{\sim}{q};k_1,\cdots,k_{n-1}) \qquad (38)$$

with $V_0 = -S^{-1}$. We have resumed our practice of suppressing tensor indices.

From Eq. (36) one sees that the V_n satisfy the Chang-Mani identities. From Eq. (37) it follows that they are symmetric functions of the photon variables. It follows from Eq. (32) that

$$d_\mu(k)[\underset{\sim}{q}A+B] = \gamma_\mu A \qquad (39)$$

where A and B are arbitrary Dirac matrices independent of q. From (39) and (34) it follows that $d(k)$ reduces the degree of any polynomial function of $\underset{\sim}{q}$ by one and yields zero when applied to a constant. Hence if S^{-1} is a polynomial of degree n, $V_\nu = 0$ for $\nu > n$, and thus only the minimum required number of multiphoton vertices has been introduced. Evidently, the V_n defined in Eq. (38) satisfy all of the <u>a priori</u> requirements. It is not difficult to see that they correspond to the construction based upon $\frac{\partial}{\partial x_\mu} \rightarrow \frac{\partial}{\partial x_\mu} - ieA_\mu$ when S^{-1} is a polynomial. Since we regard this connection as interesting rather than essential, we omit a detailed demonstration. The sense in which the resultant interaction is minimal will be exhibited in the next subsection.

We note that for S^{-1} in R_0; $\Gamma_1 = V_1$ defined by Eq. (38) is the same as that given by Eq. (4) and thus is in agreement with the choice of Drell and McClure. This follows from Eq. (3) and the fact that the operation in Eq. (4) satisfies (34), (35) and (36). It does not, however, satisfy (37) or (39) and hence is not suitable for defining higher rank V_n.

The fact that our Γ_1 agrees with that of McClure and Drell of course guarantees Eq. (31). There is some ambiguity in the meaning of $(\partial/\partial x_\mu) \rightarrow (\partial/\partial x_\mu) - ieA_\mu$ which we have resolved by our requirement that q^2 always be written $-\underset{\sim}{q}\underset{\sim}{q}$ in defining the d operation. As may be inferred from a study of Eqs. (40) and (41) below, it is because of this choice that Eq. (31) is satisfied.

Some examples may be of interest. The simplest example of the type of modification that has been considered in the past is

$$S(\underset{\sim}{q}) = \frac{M-m}{(\underset{\sim}{q}+m)(\underset{\sim}{q}+M)} = \frac{1}{\underset{\sim}{q}+m} - \frac{1}{\underset{\sim}{q}+M} \tag{40}$$

The minimal interaction is

$$V_1 = \gamma_\mu + \frac{(\underset{\sim}{q}+\underset{\sim}{k}+m)\gamma_\mu + \gamma_\mu(\underset{\sim}{q}+m)}{M-m}$$

$$V_2 = \frac{-2\delta_{\mu\nu}}{M-m} \tag{41}$$

Another example, of interest especially because of its applicability to spectral representations, is

$$S^{-1}(\underset{\sim}{q}) = \frac{(\underset{\sim}{q}+m)(M_0-m)}{\underset{\sim}{q}+M_0} \tag{42}$$

The minimal interaction is

$$V_1 = (M_0-m)^2 \frac{1}{\underset{\sim}{q}_1+M_0}\gamma_1\frac{1}{\underset{\sim}{q}+M_0} \tag{43}$$

$$\vdots$$

$$V_n = (-1)^{n-1}(M_0-m)^2 P_n \frac{1}{\underset{\sim}{q}_n+M_0}\gamma_n \cdots \gamma_1 \frac{1}{\underset{\sim}{q}+M_0}$$

$$\vdots$$

c) Properties of the Minimal Electromagnetic Interaction for Open Charged Propagator Lines

We consider the interaction of n photons with an open charged propagator. These are represented by $S(q_n)\Gamma_n(q, k_1, \ldots, k_n)S(q)$, where the Γ_n written here are the zero coupling limit of the Γ_n defined in Eq. (23). For example, we have

$$\begin{aligned}\Gamma_2(q; k_1, k_2) &= V_1(q+k_2; k_1)\, S(q+k_2)\, V_1(q; k_2) \\ &\quad + V_1(q+k_1; k_2)\, S(q+k_1)\, V_1(q; k_1) + V_2(q; k_1, k_2)\end{aligned} \tag{44}$$

The Γ_n and V_n defined in this Section are related in the same way [Eq. (26)] as those defined in Section II. We rewrite Eq. (26) in the form

$$\begin{aligned}&S(q_n)\Gamma_n(q; k_1, \ldots, k_n)S(q) \\ &= \mathscr{P}_n \sum_{j=1}^{n} \frac{S(q_n)V_j(q_j; k_{j+1}, \ldots, k_n)S(q_j)\Gamma_{n-j}(q; k_1, \ldots, k_j)S(q)}{j!\,(n-j)!}\end{aligned} \tag{45}$$

We now prove that

$$S(q_n)\Gamma_n(q; k_1, \ldots, k_n)S(q) = (-1)^n d(k_1)\cdots d(k_n) S(q) \tag{46}$$

For $n=1$, Eq. (46) holds on account of Eq. (38) and (35). Hence it is true, provided

$$\begin{aligned}&S(q_n)\Gamma_n(q; k_1, \ldots, k_n)S(q) \\ &= -d(k_n)S(q_{n-1})\Gamma_{n-1}(q; k_1, \ldots, k_{n-1})S(q)\end{aligned} \tag{47}$$

The result follows inductively from Eq. (45). Applying $-d$ to Eq. (45) with n replaced by $n-1$, using Eq. (34) and the inductive assumption (47) for $\nu < n-1$, we find

$$-d(k_n)\, S\Gamma_{n-1}S = \mathcal{P}_{n-1}\left\{\sum_{j=1}^{n-1}\frac{1}{j!\,(n-j-1)!}\right.$$

$$\cdot\left[S V_1^{(n)} S V_j S \Gamma_{n-j-1} S\right.$$

$$\left.\left. + S V_{j+1}^{(n)} S \Gamma_{n-j-1} S + S V_j S \Gamma_{n-j}^{(n)} S\right]\right\} \qquad (48)$$

In writing Eq. (48) we have suppressed all arguments. Equation (34) guarantees that the proper momentum q_j appears in the argument of each S, V, or Γ. The superscript (n) indicates the quantity in which k_n is located. Rearranging (48), using (45) on the first term, we find

$$-d(k_n)\, S\Gamma_{n-1}S = \mathcal{P}_{n-1}\left\{\sum_{j=1}^{n-1}\left[\frac{S V_j^{(n)} S \Gamma_{n-j} S}{(j-1)!\,(n-j)!}\right.\right.$$

$$\left.\left. + \frac{S V_j S \Gamma_{n-j}^{(n)} S}{j!\,(n-j-1)!}\right] + \frac{S V_n^{(n)} S}{(n-1)!}\right\} \qquad (49)$$

From the symmetry of V_n we have

$$\mathcal{P}_{n-1}\frac{S V_n S}{(n-1)!} = \mathcal{P}_n \frac{S V_n S}{n!} \qquad (50)$$

Fixing attention on a particular value of j, we observe that

$$\mathcal{P}_{n-1}\left[\frac{S V_j S \Gamma_{n-j}^{(n)} S}{j!\,(n-j-1)!} + \frac{S V_j^{(n)} S \Gamma_{n-j} S}{(j-1)!\,(n-j)!}\right]$$

$$= \bar{\mathcal{P}}_{n-1}\left[S V_j S \Gamma_{n-j}^{(n)} S + S V_j^{(n)} S \Gamma_{n-j} S\right] \qquad (51)$$

$$= \bar{\mathcal{P}}_n\left[S V_j S \Gamma_{n-j} S\right]$$

$$= \mathcal{P}_n \frac{S V_j S \Gamma_{n-j} S}{j!\,(n-j)!}$$

The symbol $\tilde{\mathcal{P}}$ means a sum over inequivalent permutations. That is, the sum is to contain one and only one representative of the set of permutations differing only by a rearrangement of variables within the symmetric functions [e.g., V and Γ in Eq. (51)]. For its application in this Section it is equivalent to specify that each possible argument of the central S function occurs once and only once. Equations (51), (50) and (49) then yield (47), so the proof is complete.

We now discuss the interaction of n photons with an incoming or outgoing charged particle. The relevant quantities are

$$S(q_n)\Gamma_n(q;k_1,\cdots,k_n)\psi(q) \tag{52}$$

and

$$\bar{\psi}(q_n)\Gamma_n(q;k_1,\cdots,k_n)S(q) \tag{53}$$

From the regularity of $S-S_m$ at $q=-m$ it follows that we may write $S \equiv S_m S_1$ (thus defining S_1) with

$$S_1(q)\psi(q) = \psi(q) \tag{54}$$

$$\bar{\psi}(q_n)S_1(q_n) = \bar{\psi}(q_n) \tag{55}$$

Furthermore, because

$$S^{-1}(q)\psi(q) = \bar{\psi}(q_n)S^{-1}(q_n) = 0 \tag{56}$$

and because $S(q)-S_m(q)$ is non-singular at $q=-m$, we have

$$[d(k_1)\cdots d(k_n)(S(q)-S_m(q))]\,S^{-1}(q)\psi(q) = 0$$

$$\tag{57}$$

$$\bar{\psi}(q_n)S^{-1}(q_n)\,[d(k_1)\cdots d(k_n)(S(q)-S_m(q))] = 0$$

From (54), (55), (57), and (46) we have, finally,

$$S(q_n)\Gamma_n(q;k_1,\cdots,k_n)\psi(q) = [(-1)^n d(k_1)\cdots d(k_n)\, S_m(q)]\, S_m^{-1}(q)\,\psi(q) \tag{58}$$

$$\bar{\psi}(q_n)\Gamma_n(q;k_1,\cdots,k_n)S(q) = \bar{\psi}(q_n)\, S_m^{-1}(q_n)[(-1)^n d(k_1)\cdots d(k_n)\, S_m(q)]$$

Note that because of the singularity of $S_m(q)$ at $q=-m$ the right-hand sides of (58) do not vanish in spite of (56). Equation (58), together with (54) and (55), shows tha. for all process to all orders in perturbation theory, excluding closed charge particle loops, all minimally interacting propagators give the same result.

Some elucidation of the significance of the minimal interaction may be obtained by considering the special case of a propagator of the form

$$S(q) = S_m(q) + \sum \frac{\lambda_i}{q + M_i} \tag{59}$$

which may be thought of as describing a particle with a number of mass states. If one introduces (59) as an ad hoc modification of quantum electrodynamics without changing the vertex interaction and without introducing any multiphoton interactions, then the resultant Feynman diagrams contain factors of the form

$$\frac{\lambda_i}{k + q + M_i}\,\gamma_\mu\,\frac{\lambda_j}{q + M_j} \tag{60}$$

corresponding to transitions between the different mass states. It is well known that the form of the transition given by (60) violates gauge invariance. The minimal interaction for this propagator yields, according to (46),

$$S\Gamma_n S = S_m\Gamma_n^{(m)}S_m + \sum_i \lambda_i S_{M_i}\Gamma_n^{(M_i)}S_{M_i} \tag{61}$$

in an obvious notation, and shows that the role played by the minimal vertex modifications is simply to remove all of the gauge violating couplings (60) between the different mass states without introducing any extra, gauge invariant, transition couplings between them. Connecting (61) to a real incoming or outgoing particle then eliminates the extra mass states altogether just because no transitions between them have been introduced. This interpretation can be regarded as providing a justification of our characterization of the interaction introduced in this section as minimal.

d) Properties of the Minimal Interaction for Closed Loops

We show in this subsection that the simple result obtained for open charge lines no longer holds for closed loops. Indeed, on the basis of Eq. (46) and the discussion of the above paragraph, there is no reason to expect that it should since the higher mass states can be produced and annihilated in pairs by the minimal interaction. The actual result for the closed loops, while not as simple as the previous sentence might suggest, is nevertheless simple enough to be interesting, and we therefore present it below.

We shall be concerned with the interaction of n (n even) photons with a single closed charged line. We note first of all that for unmodified spin $\frac{1}{2}$ quantum electrodynamics, this quantity is proportional to

$$\Pi_n(m; k_1 \cdots k_n) = \int d_4 q \, \mathrm{Tr}\left\{ \Gamma_n^{(m)}(\underset{\sim}{q}; k_1, \cdots, k_n)\, S_m(\underset{\sim}{q}) \right\} \tag{62}$$
$$\equiv \Pi_n(m)$$

where

$$S_m(\underset{\sim}{q}_n)\, \Gamma_n^{(m)}(\underset{\sim}{q}, k_1, \cdots, k_n)\, S_m(\underset{\sim}{q}) \tag{63}$$
$$= (-1)^n d(k_1) \cdots d(k_n)\, S_m(\underset{\sim}{q})$$

For a theory which includes multiphoton vertices (as well as for spin $\frac{1}{2}$ electrodynamics in higher order) (62), with $\Gamma_n^{(m)}$ and S_m replaced by Γ_n and S, no longer holds. The reason is that diagrams which differ only by a cyclic permutation of the vertices together with their attached photon momenta about the loop are not to be counted as distinct. One may deal with this problem by exhibiting explicitly the vertex at which a particular photon (say the n^{th}) interacts, and suppressing its permutations. Thus we write

$$\Pi_n(k_1,\cdots;k_n) = n\int d_4q\,\mathrm{Tr}\sum_{j=0}^{n-1} P_{n-1}\frac{V_{n-j}(q_j;k_{j+1},\cdots k_n)S(q_j)\Gamma_j(q;k_1,\cdots,k_j)S(q)}{(n-j-1)!\,j!}$$

$$= n\int d_4q\,\mathrm{Tr}\sum_{j=0}^{n-1}\bar{P}_{n-1}\left[V_{n-j}(q_j;k_{j+1},\cdots;k_n)S(q_j)\Gamma_j(q;k_1,\cdots,k_j)S(q)\right] \tag{64}$$

with $\Gamma_0 = 1$.

On account of the factor n, (64) reduces to (62) for an unmodified theory. Taking account of the fact that there is nothing special about k_n, i.e., that Π_n is symmetric in all $k_1,\ldots,k_n$, one can readily see that

$$\Pi_n(k_1,\cdots,k_n) = \int d_4q\,\mathrm{Tr}\sum_{j=0}\bar{P}_n(n-j)V_{n-j}S\Gamma_jS$$

$$= -\int d_4q\,\mathrm{Tr}\sum_{j=0}^{n-1}P_n\frac{[d(k_{j+1})\cdots d(k_n)S^{-1}(q_j)][d(k_1)\cdots d(k_j)S(q)]}{(n-j-1)!\,j!} \tag{65}$$

using (46) and (38).

We shall discuss further the reduction of Π_n under the restrictions [19]

$$S = S_m + \int_{-\infty}^{\infty} f(\kappa) S_\kappa \, d\kappa = \int_{-\infty}^{\infty} f_1(\kappa) S_\kappa \, d\kappa \tag{66}$$

with

$$0 < 1 + \int_{-\infty}^{\infty} f(\kappa)\, d\kappa < \infty \tag{67}$$

and

$$S^{-1} = \alpha(\underset{\sim}{q} + m) + \beta + \int_{-\infty}^{\infty} g(\mu) S_\mu \, d\mu \tag{68}$$

where f and g have no singularities worse than δ functions. We assume the discrete points in the spectra to be isolated from the continuum. Equation (68) implies that the continuous spectrum of g coincides with that of f, and, indeed

$$g(\mu) = \frac{-f(\mu)}{S_+(-\mu)\, S_-(-\mu)} \tag{69}$$

It is evident from Eq. (43) that the individual terms of (65) contain a factor of the general form

$$\int d_4 q \, \mathrm{Tr}\left\{ [\gamma S_\mu \gamma \cdots \gamma S_\mu \gamma] S_\mu(\underset{\sim}{q}_r) S_\kappa(\underset{\sim}{q}_r) [\gamma S_\kappa \gamma \cdots \gamma S_\kappa \gamma] S_\kappa(\underset{\sim}{q}_s) S_\mu(\underset{\sim}{q}_s) \right\} \tag{70}$$

where we have made use of the freedom to permute cyclically within the trace and the fact that momentum conservation guarantees $q_s = q_{s+n}$. Separating the adjacent factors $S_\mu S_\kappa$ by partial fractions, and performing a straightforward but tedious combinatorial analysis which we omit here, we find

$$\Pi_n = \alpha\int f_1(\kappa)\Pi_n(\kappa)d\kappa + \int g(\mu)f_1(\kappa)\frac{\partial}{\partial\mu}\left[\frac{1}{(\mu-\kappa)}\left(\Pi_n(\kappa)-\Pi_n(\mu)\right)\right]d\mu d\kappa \tag{71}$$

Just as in the case of open charge lines, cross terms between different "mass states" have disappeared, but in addition to a contribution from the spectrum of S, there is also a contribution from the spectrum of S^{-1}. This can be made more explicit by carrying out the integration over one of the variables in (71) and separating out the contributions from the discrete points. The result is

$$\Pi_n = \Pi_n(m) + \sum_i \Pi_n(\kappa_i) - \sum_j \Pi_n(\mu_j) + \int_{\text{continuum}} d\mu\, g(\mu)\left[S_R(-\mu)\frac{\partial\Pi_n(\mu)}{\partial\mu} + 2\Pi_n(\mu)\bar{S}(-\mu)\frac{\partial}{\partial\mu}\frac{S_R(-\mu)}{\bar{S}(-\mu)}\right] \tag{72}$$

where $\bar{S} = (S_+S_-)^{\frac{1}{2}}$, $S_R = \frac{1}{2}(S_++S_-)$.

In obtaining (72) one makes use of the fact that the existence of a pole at $\underset{\sim}{q} = -\mu_i$ of S^{-1} implies

$$\int\frac{f_1(\kappa)}{\kappa-\mu_j}d\kappa = S(-\mu_j) = 0 \tag{73}$$

$$\int\frac{f_1(\kappa)}{(\kappa-\mu_j)^2}d\kappa = -\left.\frac{\partial S}{\partial \underset{\sim}{q}}\right|_{-\mu_j} = -\frac{1}{g_j} \tag{74}$$

where $g(\mu) = g_j\delta(\mu-\mu_j)$ for $\mu \approx \mu_j$, and, analogously,

$$\alpha - \int \frac{g(\mu)}{(\mu - \mu_i)^2}\, d\mu = \frac{1}{f_i} \tag{75}$$

Equation (69) has also been used. Equations (73), (74) and (75) are responsible for the absence of any explicit weight factors in the contribution from the discrete points [20].

Neither of the simple examples (40) and (42) satisfy Eq. (67). The closed loop contribution for both can be obtained as limiting cases from

$$S = \frac{1}{\underset{\sim}{q} + m} + \frac{f}{\underset{\sim}{q} + M} \tag{76}$$

which yields

$$S^{-1} = \frac{1}{1+f}(\underset{\sim}{q} + m) + \frac{(M-m)f}{(1+f)^2} - \frac{(M-m)^2}{(1+f)^3}\,\frac{1}{\underset{\sim}{q} + M_o}$$

$$= \frac{1}{1+f}(\underset{\sim}{q} + m) + \frac{f}{1-f}\,\frac{(\underset{\sim}{q} + m)(M_o - m)}{\underset{\sim}{q} + M_o} \tag{77}$$

with $M_o = \frac{M+mf}{1+f}$, and thus

$$\Pi_n = \Pi_n(m) + \Pi_n(M) - \Pi_n(M_o) \tag{78}$$

For the example of equation (40), $f \to -1$. Then $M_o \to \infty$ and

$$\Pi_n = \Pi_n(m) + \Pi_n(M) \tag{79}$$

For the example of Eq. (42), $f \to \infty$. Keeping M_0 fixed, $M \to \infty$ also and thus

$$\Pi_n = \Pi_n(m) - \Pi_n(M_0) \tag{80}$$

e) Discussion

We conclude from the results of the previous two subsections that charged propagator modifications in combination with their minimal electromagnetic interactions do not constitute a practical framework for the discussion of the breakdown of quantum electrodynamics in the current experimental context. Nevertheless, as the theory is not completely equivalent to the unmodified theory, it becomes of interest to enquire into its consistency.

It is easy to see that without further modification these theories frequently and perhaps always violate unitarity. The reason for this is that the polarization tensors contain threshold singularities at the threshold for the production of pairs of "excited mass states" and an attached two-particle branch cut. Furthermore, in the minimal theory the "excited mass states" are stable and hence the loss of probability corresponding to their production is not fed back by their decay into states of mass m and photons. Unitarity must always fail in this way if either the propagator or inverse propagator contain extra poles. The situation is probably often the same when only the continuum is present, but we see no reason to expect that there are not special choices of the spectral function that would retain unitarity.

The unitarity of the theory may, of course, be restored by making further modifications. We describe one such modification here. Assume that S consists of a sum of poles only at masses m and K_i with $i = 1, \ldots, n$. It is easy to see that if Eq. (67) is satisfied, S^{-1} will also have n poles μ_i, which we require to be real. The polarization tensors are then of course given by

$$\Pi_n = \Pi_n(m) + \sum_{i=1}^{n} \left[\Pi_n(\kappa_i) - \Pi_n(\mu_i)\right]$$

We now further modify the theory by introducing another particle of mass m^* different from all of the other masses. We associate the same absolute value of charge with both particles and associate with the new particle a modified propagator with the required pole at m^*, poles at each μ_i, zeros at each κ_i, and satisfying (67). This is always possible. The resultant polarization tensor is obviously

$$\Pi_n = \Pi_n(m) + \Pi_n(m^*)$$

and the theory is identical with ordinary quantum electrodynamics of two particles with mass m and m^*. Since the continuous spectrum can frequently be regarded as a limit of a distribution of poles, a similar construction may be possible for such cases as well. This example shows that it is possible to construct a modified Feynman diagram theory which is as consistent as the normal theory even though the propagator may violate the Lehmann spectral condition $(f(\kappa) \geq 0)$ [21]. The example is, of course, a trivial one, but suggests the possibility of non-trivial examples.

The argument for the general failure of unitarity can also be weakened by adding intrinsic vertices (a subject to be discussed in the next Section) to the theory. The addition of intrinsic vertices introduces transitions between the various "excited mass states" and hence makes them unstable. The probability expended in their production can be fed back via their decay products and unitarity thereby restored. We have, however, not investigated the question of whether such theories are actually unitary. It is clear that there is a need for general criteria to deal with this question. The simple examples given above show that the Lehmann spectral condition is neither necessary nor sufficient.

IV. INTRINSIC VERTEX MODIFICATIONS

We consider a modified quantum electrodynamics in which a modified propagator and all multiphoton vertices have been specified. We assume the theory to be consistent with charge conservation and therefore that the Chang-Mani identities, as well as symmetry conditions, are satisfied. We propose that such a theory be described as being a quantum electrodynamics with a modified charged propagator and a set of intrinsic modified multiphoton vertices. The intrinsic modified vertices are defined as follows. Let V_n stand for the specified vertices, W_n for the intrinsic vertex modifications. Then we write

$$W_n(\underset{\sim}{q};k_1,\cdots,k_n) \equiv \sum_{j=0}^{n} \mathcal{P}_n \frac{d(k_1)\cdots d(k_j)\, V_{n-j}(\underset{\sim}{q};k_{j+1},\cdots,k_n)}{j!\,(n-j)!} \tag{81}$$

We note that W_n is a symmetric function of its photon variables. We next show that it satisfies a transversality condition. Thus, rewriting W_n in the form

$$W_n = \sum_{j=0}^{n-1} \overline{\mathcal{P}}_{n-1} \Big[d(k_1)\cdots d(k_j)\, V_{n-j}(\underset{\sim}{q};k_{j+1},\cdots,k_n) + d(k_1)\cdots d(k_j)\, d(k_n)\, V_{n-j-1}(\underset{\sim}{q};k_{j+1},\cdots,k_{n-1}) \Big] \tag{82}$$

applying Eq. (27) to the first term and Eq. (36) to the second, one sees immediately

$$i k_n W_n = 0 \tag{83}$$

We have assumed that all of the V_j belong to the set of functions R_o.

Equation (81) can be inverted and the V_n expressed in terms of the W_n, thus, $(W_o = V_o = -S^{-1})$

$$V_n(\underset{\sim}{q};k_1,\cdots,k_n) = \sum_{j=0}^{n} \mathcal{P}_n \frac{d(k_{j+1})\cdots d(k_n)\, W_j(\underset{\sim}{q};k_1\cdots k_j)}{j!\,(n-j)!} (-1)^{n-j} \tag{84}$$

Equation (84) may be verified by substituting (81) into (84) and using the combinatorial formulae

$$\bar{\mathcal{P}}_n \, d(k_{j+1}) \cdots d(k_n) \left[\bar{\mathcal{P}}_j \, d(k_{i+1}) \cdots d(k_j) V_i(\underset{\sim}{q}; k_1, \cdots, k_i) \right]$$
$$= \frac{(n-i)!}{(j-i)!(n-j)!} \bar{\mathcal{P}}_n \, d(k_{i+1}) \cdots d(k_n) V_i(\underset{\sim}{q}; k_1, \cdots, k_i) \qquad (85)$$

and

$$\sum_{j=i}^{n} \frac{(n-i)!(-1)^{n-j}}{(j-i)!(n-j)!} = \sum_{r=0}^{n-i} \frac{(n-i)!(-1)^r}{r!(n-i-r)!} = (1-1)^{n-i} = 0 \qquad (86)$$
$$i < n$$

According to (84), V_n consists of its intrinsic part, plus the minimal interaction induced by the propagator, plus a sum of additional interactions induced by each of the lower rank intrinsic vertices. The form of the multiphoton photon vertex induced by a particular intrinsic vertex is a straightforward generalization of the minimal construction defined for the propagator. Equation (84) therefore provides a procedure for specifying a charge conserving modified quantum electrodynamics. One specifies the form of the propagator and the form of the intrinsic vertices W_n, which must be symmetric and satisfy the transversality condition (83). Equation (84) then provides the complete set of multiphoton vertices necessary to maintain charge conservation. Its form is such that it never provides more non-zero vertices than the minimum possible number required by the Chang-Mani identities and the specified form of the W_n. Equation (81) shows that no generality is lost by adopting this procedure.

For applications in the current experimental context, one is of course concerned with simple modifications of the theory. In the spirit of earlier attempts we would take the view that a simple modification is one in which either the charged propagator or the intrinsic one-photon vertex is modified, all other intrinsic vertices being set equal to zero. The effects of propagator modifications have been discussed in Section III. We turn now to a brief consideration of intrinsic one-photon vertex modifications.

We note first of all that from our point of view a Pauli moment is an intrinsic one-photon vertex, whose form is such that it induces no higher order multiphoton vertices. This is in keeping with the fact that a theory without intrinsic vertices is, by our definition, a minimal theory. A simple vertex modification which has appeared in the literature [2] in connection with discussions of the breakdown of quantum electrodynamics is

$$\gamma_\mu \to \gamma_\mu \frac{\Lambda^2}{\Lambda^2 + k^2} = \gamma_\mu - \frac{\gamma_\mu k^2}{\Lambda^2 + k^2} \tag{87}$$

In order to make (87) fit our definition of intrinsic, we should write

$$\frac{-\gamma_\mu k^2}{\Lambda^2 + k^2} \longrightarrow - \frac{\gamma_\mu k^2 + i k_\mu \underset{\sim}{k}}{\Lambda^2 + k^2} = W_{1\mu} \tag{88}$$

in which case (83) is satisfied. As is well known, however, the change in going from (87) to (88) has no effect on the theory. Furthermore, no multiphoton photon vertices are induced, so that the applications which have been given are consistent with charge conservation. For a quantum electrodynamics in which only one kind of charged particle is playing a substantial role, (87) is indistinguishable from a modification of the photon propagator. Expressions which are generalizations of (87) of course play an important role in the description of hadronic form factors, but there is no indication as yet for the need of such terms in leptonic interactions apart from the well-known terms generated by higher order perturbations within the framework of the ordinary theory.

The only indication for the need of any substantial modification in quantum electrodynamics has come from recent observations of electron-positron pair production. Without discounting the possibilities that the effect may have its origin in some incompletely understood normal process, we wish to consider here, for illustrative purposes, the simplest intrinsic vertex modification which could play a role in that process.

We propose for this purpose

$$V_{1\mu} = \gamma_\mu \pm \frac{g(k^2)}{\Lambda^2}\left[\sigma_{\mu\nu} k_\nu (\underset{\sim}{q}+m) + (\underset{\sim}{q}+\underset{\sim}{k}+m)\sigma_{\mu\nu} k_\nu\right] \tag{89}$$

$$= \gamma_\mu + W_{1\mu}$$

Only one induced vertex is required. It is given by

$$\begin{aligned} V_{2\mu_1\mu_2} &= d_{\mu_2}(k_2)\, W_{1\mu_1}(\underset{\sim}{q};k_1) + d_{\mu_1}(k_1)\, W_{1\mu_2}(\underset{\sim}{q};k_2) \\ &= \mp \frac{g(k_1^2)}{\Lambda^2}\left(\sigma_{\mu_1\lambda} k_{1\lambda}\gamma_{\mu_2} + \gamma_{\mu_2}\sigma_{\mu_1\lambda} k_{1\lambda}\right) \\ &\quad \mp \frac{g(k_2^2)}{\Lambda^2}\left(\sigma_{\mu_2\rho} k_{2\rho}\gamma_{\mu_1} + \gamma_{\mu_1}\sigma_{\mu_2\lambda} k_{2\lambda}\right) \end{aligned} \tag{90}$$

The modification corresponds to an "off the mass shell" magnetic moment. The parameter Λ has the dimensions of mass and g is a form factor set equal to unity at $k^2=0$. The Feynman diagram transition matrix for pair production given by (89) and (90) is

$$\begin{aligned} M_{pair} &= \underset{\sim}{v}\,\frac{1}{\underset{\sim}{k}-\underset{\sim}{p}_1+m}\,\underset{\sim}{\varepsilon} + \underset{\sim}{\varepsilon}\,\frac{1}{\underset{\sim}{p}_2-\underset{\sim}{k}+m}\,\underset{\sim}{v} \\ &\quad + \frac{g(q^2)}{\Lambda^4}\left(\underset{\sim}{\varepsilon}\,\underset{\sim}{k}\,(\underset{\sim}{p}_2+m)\,\underset{\sim}{q}\,\underset{\sim}{v} + \underset{\sim}{v}\,\underset{\sim}{q}\,(-\underset{\sim}{p}_1+m)\,\underset{\sim}{k}\,\underset{\sim}{\varepsilon}\right) \end{aligned} \tag{91}$$

where p_1, p_2 are the positron and electron momenta, k_μ, ε_μ the photon momentum and polarization, and q_μ, v_μ that of the external field. The $1/\Lambda^4$ dependence comes about because of a cancellation between the contribution from V_2 and the γ_μ, W_ν cross terms. It is because of this cancellation that the Low, Gell-Mann, Goldberger[22)] theorem on the low energy behaviour of the Compton effect is satisfied. The effect of the additional term on large angle pair production has been estimated. It gives rise to a fractional correction of order $(Q_F/\Lambda)^4$ from the interference term and $(Q_F/\Lambda)^8$ from the square of the anomalous amplitude, where $Q_F = -2p\cdot k$ is the virtual mass of the intermediate electron. It is clear that large deviations from the Bethe-Heitler formula will appear at $Q_F \sim \Lambda$. Since the dependence of the reported deviation on Q_F seems much less rapid than Q_F^4 or Q_F^8, it does not seem appropriate to pursue the consequences of (91) in greater detail at this time.

The effect of (89) and (91) on the second order magnetic moment of the electron is finite and of order $(m/\Lambda)^2$ provided

$$\int d_4 k \left(\frac{g(k^2)}{k^2 \Lambda^2} \pm \frac{g^2(k^2)}{2 \Lambda^4} \right) \tag{92}$$

converges at momenta of the order of Λ. It is clear from the form of (92) that the actual value is sensitive to the sign in Eq. (89) and the behaviour of g at high momenta, which is not the case for pair production. The question of the convergence of a theory based upon (89) and (90) has not been investigated, but it is clear that cancellations would be required to save the situation for closed loops. These forms could, however, be regarded as an intermediate energy approximation. That is, convergence factors involving q and $q+k$ could be added.

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FOOTNOTES AND REFERENCES

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