Invited Paper

Adaptive Actuator Failure Compensation Designs for Linear Systems

Shuhao Chen, Gang Tao*, and Suresh M. Joshi

Abstract: This paper surveys some existing direct adaptive feedback control schemes for linear time-invariant systems with actuator failures characterized by the failure pattern that some inputs are stuck at some unknown fixed or varying values at unknown time instants, and applications of those schemes to aircraft flight control system models. Controller structures, plant-model matching conditions, and adaptive laws to update controller parameters are investigated for the following cases for continuous-time systems: state tracking using state feedback, output tracking using state feedback, and output tracking using output feedback. In addition, a discrete-time output tracking design using output feedback is presented. Robustness of this design with respect to unmodeled dynamics and disturbances is addressed using a modified robust adaptive law.

Keywords: Actuator failure, adaptive control, failure compensation, robust performance.

1. INTRODUCTION

Actuator failures can be uncertain, that is, it is not known when, in what manner, and how many actuators fail. For example, some unknown inputs may be stuck at some unknown values at unknown time instants. A number of aircraft accidents were caused by actuator failures, such as the horizontal stabilizer or the rudder being stuck in an unknown position, leading to catastrophic failures. Actuator failure compensation is an important and challenging problem for control systems research with both theoretical and practical significance.

1.1. Literature overview

In recent years, the actuator failure compensation problem has been studied via several different approaches. There have been a number of results in the literature on control of systems with failures. Typical design methods include: multiple-model, switching, and tuning designs, adaptive designs, fault detection and diagnosis designs, and robust control designs.

1.1.1 Multiple-model, switching, and tuning

For control of systems with component failures, one class of designs is based on multiple-model, switching, and tuning and has been applied to reconfigurable flight control [5, 9, 17, 57]. The basic idea of multiple-model, switching, and tuning designs is the assumption that the controlled system (plant) belongs to a set of plant models. For each model, a controller is designed to achieve the control objective. During system operation, these models run in parallel with the plant, and if one actuator fails, the switching mechanism will find the best matched model and switch to the appropriate controller. The multiple-model, switching, and tuning design has several forms [35]: one based on all-fixed plant models, one based on all-adaptive plant models, one based on fixed models and one adaptive model, and one based on fixed models with one free-tuning and one reinitialized adaptive model.

1.1.2 Adaptive designs

Another type of control designs are indirect or direct adaptive control based schemes [1, 4-6, 8, 34]. In [1], an indirect adaptive LQ controller is used to accommodate failures in the pitch control channel or the horizontal stabilizer, leading to performance improvement. In [4], several indirect and direct adaptive control algorithms are presented for control of aircraft with a failure characterized by a locked left horizontal tail surface. An adaptive controller is used to accommodate the system dynamics change caused

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by such a failure. In [5, 6], indirect adaptive control schemes are used for compensation of loss of effectiveness of control surfaces. In [8], an adaptive algorithm is used for control of a dynamic system with known dynamics but unknown actuator failures. The control law for the known dynamics is based on a model matching design, while the compensation for actuator failure is based on an adaptive tuning of actuation parameter matrices. A model-following adaptive design for failure compensation was presented in [34], which achieves output tracking for some multi-output systems.

Designs based on *indirect* adaptive control first estimate the system and failure parameters and then implement control law reconfiguration employing the functioning actuators. *Direct* adaptive control based designs do not explicitly involve system and failure parameter estimation, and instead they adaptively update control reconfiguration parameters online.

In this paper, we present a general framework for direct adaptive control of systems with both uncertain parameters and uncertain actuator failures, and demonstrate more adaptive schemes for different control designs and performance requirements.

Adaptive reconfigurable flight control designs using neural networks have been developed for aircraft systems with failures [10, 22, 27, 28, 39, 51]. Unlike the neural networks based adaptive designs, our adaptive failure compensation control designs are model-based, that is, the nominal system structural information is incorporated into adaptive failure compensation designs, to analytically ensure system *stability and tracking* properties. For applications, these two adaptive approaches can be further combined to achieve desired system performance.

1.1.3 Fault detection and diagnosis

The fault detection and diagnosis approach [11, 16, 19, 24, 26, 32, 38, 48-50] has also been used for control of systems with component failures. Related results also include those in [3, 53, 55, 56], using fault tolerant control designs, in [14, 23], using identification of multiplicative faults based on parameter estimation techniques, in [7], using function approximations for control and adaptive law design, in [2, 20, 25, 31, 33, 37, 50], using residual generation techniques for fault detection and diagnosis, and in [36, 52], using other design and analysis techniques.

1.1.4 Robust control designs

Robust control designs, which can deal with parameter variations and model uncertainties, have also been used to accommodate certain presumed component failures by treating them as uncertainties. As a result, system stability can be guaranteed and an acceptable closed-loop performance can be maintained in the presence of actuator failures. Typical robust control techniques used in the design of reliable control systems are H_∞ controller [47], linear quadratic regulator (LQR) [29, 46, 54], linear matrix inequality [18, 30], and eigenstructure assignment [58]. The robust control based fault tolerant designs use fixed parameter controllers which are for the worst case of failures and do not adapt to changes of system failure pattern and failure values.

1.2. Motivation for this research

Although there have been many advances in control of systems with unknown actuator failures, there are still many open and challenging problems. An effective adaptive actuator failure compensation approach is needed to handle both system parameter and actuator failure uncertainties.

Adaptive control designs are able to handle uncertainties in both system dynamics and actuator failures that can occur during system operation, using reduced amount of system knowledge needed for feedback control. Such failures are often uncertain in time, value and pattern, that is, when, how much and which actuators fail. Compared with multiple-model, switching, and tuning designs and fault diagnosis designs, adaptive failure compensation control designs have simpler controller structures. Only one adaptive controller is used to accommodate the system dynamics change caused by actuator failures. Adaptive actuator failure compensation designs adaptively adjust controller parameters using system response errors to achieve the desired performance. Adaptive actuator failure compensation schemes do not rely on the knowledge of actuator failures, while they also do not rely on the knowledge of the controlled system, as compared with robust control designs.

An important feature of adaptive failure compensation is that such a design is able to adapt to changes in system failure pattern and failure values, so that in addition to stability, asymptotic tracking of a reference signal is ensured, despite the system and failure uncertainties.

The key design task is to find the appropriate controller structure and adaptive laws such that under certain plant-model matching conditions, the adaptive controller can automatically adjust the remaining functional actuators to achieve a desired control objective despite the unknown failures of other actuators in the controlled system.

In this paper, we summarize some recent work on direct adaptive tracking control of linear time-invariant systems in the presence of unknown actuator failures characterized by the failure pattern that some inputs are stuck at some unknown fixed or varying values at unknown time instants. The paper is organized as follows. In Section 2, we formulate the adaptive failure compensation problems. In Sections 3, we present a state tracking design for the case when the plant parameters are known, as an introduction to adaptive actuator failure compensation. In Section 4, we present several state tracking designs for plants with unknown parameters, to formulate and solve some key issues in adaptive actuator failure compensation. We then give several state feedback designs for output tracking in Section 5 and output feedback designs for output tracking in Section 6. In Section 7, we present a discrete-time output feedback design for output tracking, and address its robustness with respect to unmodeled dynamics and output disturbances.

2. PROBLEM STATEMENT

Consider a linear time-invariant plant

$$\dot{x}(t) = Ax(t) + Bu(t), \ \ y(t) = Cx(t), \qquad (1)$$

where $A \in R^{n\times n}$, $B \in R^{n\times m}$ denoted as $B = [b_1, \ldots, b_m]$ with $b_i \in R^n$, $i = 1, \ldots, m$, $C \in R^{1\times n}$ are unknown constant parameter matrices, $u(t) = [u_1, \ldots, u_m]^T \in R^m$ is the input vector whose components may fail during system operation, and $y(t) \in R^q$ is the plant output vector.

One type of actuator failure considered in this paper is modeled as

$$u_j(t) = \bar{u}_j, t \geq t_j, j \in \{1, 2, \ldots, m\}, \qquad (2)$$

where the constant value $\bar{u}_j$ and the failure time instant t_j are unknown.

In the presence of actuator failures, $u(t)$ can be expressed as

$$u(t) = v(t) + \sigma(\bar{u} - v(t)), \qquad (3)$$

where $v(t)$ is an applied control input to be designed, and

$$\bar{u} = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m]^T, \sigma = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_m\} \qquad (4)$$

$$\sigma_i = \begin{cases} 1 & \text{if the } i\text{th actuator fails, i.e., } u_i = \bar{u}_i \\ 0 & \text{otherwise.} \end{cases} \qquad (5)$$

More general types of actuator failures are

$$u_j(t) = \bar{u}_j + \bar{d}_j(t) + \bar{\delta}_j(t), \quad t \geq t_j, \qquad (6)$$

where the parameterizable time-varying failure components are

$$\bar{d}_j(t) = \sum_{l=1}^{n_d} \bar{d}_{jl} f_{jl}(t) \qquad (7)$$

for some unknown scalar constants $\bar{d}_{jl}$ and known scalar signals $f_{jl}(t)$, $j = 1, \ldots, m$, $l = 1, \ldots, n_d$, $n_d \geq 1$, and $\bar{\delta}_j(t)$ is an unknown and unparametrizable but bounded term. The actuator failure model (6) can be used to closely approximate a large class of practical failures, by a proper selection of these "basis" functions $f_{jl}(t)$, while parametrized by $\bar{d}_{jl}$.

The control problem considered in this paper is adaptive actuator failure compensation for any up to $m-q$ $(1 \leq q \leq m)$ actuator failures, with both plant parameters and failure parameters unknown. The basic assumption for the up to $m-q$ actuator failure compensation problems is

(A1) the system (1) is so constructed that for any up to $m-q$ actuator failures, the remaining actuators can still achieve the desired control objective, when implemented with known parameters.

The key task of adaptive control is to adjust the remaining controls to achieve the desired system performance when there are up to $m-q$ actuator failures whose parameters are unknown.

For state tracking, the reference state vector $x_m(t)$ is generated from the reference model

$$\dot{x}_m(t) = A_M x_m(t) + B_M r(t), \qquad (8)$$

where $A_M \in R^{n\times n}$, $B_M \in R^{n\times l}$ are known constant matrices such that all the eigenvalues of A_M are in the left-half complex plane, all columns of B_M are independent and $r(t) \in R^l$ is bounded and piecewise continuous.

For output tracking, the reference output $y_m(t)$ is generated from the reference model

$$y_m(t) = W_m(s)[r](t), \qquad (9)$$

where $W_m(s)$ is a stable rational matrix, and $r(t)$ is bounded and piecewise continuous.

The control objectives for the adaptive tracking problems can be stated as follows:

- design a *state feedback* control $v(t)$ to ensure *state tracking*: $\lim_{t\to\infty}(x(t) - x_m(t)) = 0$;
- design a *state feedback* control $v(t)$ to ensure *output tracking*: $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$; or
- design an *output feedback* control $v(t)$ to ensure *output tracking*: $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$;

in addition to closed-loop system signal boundedness.

The discrete-time and robustness problem formulation will be given in Section 7.

3. STATE TRACKING DESIGN WITH KNOWN PLANT PARAMETERS

An adaptive control scheme for state tracking with known plant parameters is given in [8] for the actuator failure model (2).

3.1. Nominal design

In [8], the plant (1) is further decomposed in the form:

$$\dot{x}_1 = A_1 x, \tag{10}$$

$$\dot{x}_2 = A_2 x + B_2 u, \tag{11}$$

where $x_1 \in R^{n-p}$, $x_2 \in R^p$, $B_2 \in R^{p\times m}$, $A_1 \in R^{(n-p)\ n}$, and $A_2 \in R^{p\times n}$.

The plant is subject to the following assumptions:

(A2a) $m > p$.

(A2b) $\det[\bar{B}_2] \neq 0$, for any $p \times p$ submatrix $\bar{B}_2$ of B_2.

Corresponding to the plant (10)–(11), the reference model (8) is also decomposed as

$$\dot{x}_{1m} = A_1 x_m, \tag{12}$$

$$\dot{x}_{2m} = A_m x_m + B_m r, \tag{13}$$

where $x_m = [x_{1m}^T, x_{2m}^T]$, $x_{1m} \in R^{n-p}$, $x_{2m} \in R^p$, $A_m \in R^{p\times n}$ is asymptotically stable, $B_m \in R^{p\times p}$, and $r \in R^p$ denotes the bounded piecewise continuous reference inputs. Furthermore, the matrix $A_0 = [A_1^T, A_m^T]^T$ is asymptotically stable.

As shown in this section, under Assumptions (A2a) and (A2b), an adaptive control scheme can be designed to achieve closed-loop stability and asymptotic tracking of $x_m(t)$ by $x(t)$, in the presence of any up to $m-p$ actuator failures.

In the case with no actuator failures, a (non-unique) nominal controller can be designed to achieve asymptotic tracking of $x_m(t)$ by $x(t)$. Among those controllers, the one of interest is that which minimizes the control effort $J = \frac{1}{2}u^T W_u u$, where $W_u = \mathrm{diag}[w_1, w_2, \ldots, w_m] > 0$. It can be verified that such a control law is given by

$$\begin{aligned} v &= v_0^* \\ &= W_u^{-1} B_2^T (B_2 W_u^{-1} B_2^T)^{-1} (-A_2 x + A_m x + B_m r), \end{aligned} \tag{14}$$

which is based on a control-mixing algorithm [21].

To handle actuator failures, the following modified nominal controller is suggested in [8]:

$$v = v^* = W_u^{-1} B_2^T (B_2 W_u^{-1} B_2^T)^{-1} [\Theta^* \eta + \xi^*], \tag{15}$$

where $\Theta^* \in R^{p\times p}$ and $\xi^* \in R^p$ and

$$\eta = -A_2 x + A_m x + B_m r. \tag{16}$$

The nominal plant-model matching parameters are chosen as

$$\Theta^* = D^{-1},\ \xi^* = -D^{-1} B \sigma \bar{u}, \tag{17}$$

where

$$D = D_o W_o, \tag{18}$$

$$D_o = B_2 (I - \sigma) W_u^{-1} B_2^T, \tag{19}$$

$$W_o = (B_2 W_u^{-1} B_2^T)^{-1}. \tag{20}$$

Under Assumption (A2b), the inverse of D in (18) exists for any up to $m-p$ actuator failures, that is, for any up to $m-p$ elements of σ in (4) equal to 1.

In the presence of actuator failures, in view of (3), (15)–(20), we have

$$\begin{aligned} B_2 u(t) &= B_2 (v^*(t) + \sigma(\bar{u} - v^*(t))) \\ &= -A_2 x + A_m x + B_m r, \end{aligned} \tag{21}$$

which leads to desired closed-loop stability and asymptotic tracking of x_m by x.

3.2. Adaptive design

When failure parameters are unknown, the adaptive version of the control law (15) is

$$v = W_u^{-1} B_2^T (B_2 W_u^{-1} B_2^T)^{-1} [\Theta \eta + \xi], \tag{22}$$

where $\Theta \in R^{p\times p}$ and $\xi \in R^p$ are the estimates of Θ^* and ξ^*, respectively, updated from

$$\dot{\Theta} = -\gamma_\theta W_0^{-1} \bar{P}^T e \eta^T, \tag{23}$$

$$\dot{\xi} = -\gamma_\xi W_0^{-1} \bar{P} e, \tag{24}$$

where $e(t) = x(t) - x_m(t)$ is the state tracking error, $\gamma_\theta > 0$, $\gamma_\xi > 0$ are the adaptive gains, and $\bar{P} = P B_0$, $B_0 = [0_{p\times(n-p)}, I_{p\times p}]^T$, P is the solution of the Lyapunov matrix equation

$$A_0^T P + P A_0 = -Q, \tag{25}$$

where $A_0 = [A_1^T, A_m^T]^T$, and $Q = Q^T \in R^{n\times n}$ is positive definite.

The controller (22) with the adaptive law (23)–(24), ensures that all closed-loop system signals are bounded and $\lim_{t\to\infty} e(t) = 0$, for any up to $m-p$ actuator failures described by (2) [8].

4. STATE TRACKING DESIGNS WITH UNKNOWN PLANT PARAMETERS

The adaptive actuator failure compensation schemes of state feedback for state tracking, based on the assumption that some plant parameters are unknown, are presented in [43] for the case of up to $m-1$ actuator failures and in [12] for a more general case of up to $m-q$ $(q \ge 1)$ actuator failures.

4.1. Matching conditions

When both the plant and failure parameters are known, the controller structure used in [12] is

$$v(t) = v^*(t) = K_1^{*T} x(t) + K_2^{*T} r(t) + k_3^*, \tag{26}$$

where $K_1^* = [k_{11}^*, \dots, k_{1m}^*] \in R^{n\times m}$ and $K_2^* = [k_{21}^*, \dots, k_{2m}^*] \in R^{l\times m}$ are to be defined for plant-model matching, and $k_3^* = [k_{31}^*, \dots, k_{3m}^*]^T \in R^m$ is to be chosen for compensation of the actuation error $u - v = \sigma(\bar{u} - v)$ (here we first consider the failure model (2)).

The plant-model matching conditions for up to $m-q$ actuator failures are that for every $B_a \in R^{n\times q}$ consisting of q columns of B, there exist $K_{1a}^* \in R^{n\times q}$, $K_{2a}^* \in R^{l\times q}$, and $k_{3a}^* \in R^q$ such that

$$B_a K_{1a}^{*T} = A_M - A, \tag{27}$$

$$B_a K_{2a}^{*T} = B_M, \tag{28}$$

$$B_a k_{3a}^* = -B_f \bar{u}_f, \tag{29}$$

where $B_f \in R^{n\times(m-q)}$ consists of the other $m-q$ columns of B and $\bar{u}_f \in R^{m-q}$ is the failure value vector, whose entries are the failure values of the $m-q$ failed actuators.

Necessary and sufficient conditions for the existence of such K_{1a}^*, K_{2a}^*, and k_{3a}^* are that for every $B_a \in R^{n\times q}$ consisting of q columns of B,

$$\operatorname{rank}(B_a) = \operatorname{rank}([B_a, A_M - A]), \tag{30}$$

$$\operatorname{rank}(B_a) = \operatorname{rank}([B_a, B_M]), \tag{31}$$

$$\operatorname{rank}(B_a) = \operatorname{rank}(B). \tag{32}$$

The parameters K_1^*, K_2^*, and k_3^* are piecewise constant, i.e., they change their values at the time instants when actuator failures occur.

For the special case of $q = 1$, that is, for the actuator failure compensation problem considered in [43] where there are up to $m-1$ actuator failures, conditions (31)–(32) become

$$\operatorname{rank}(b_i) = \operatorname{rank}([b_i, B_M]), \tag{33}$$

$$\operatorname{rank}(b_i) = \operatorname{rank}(B), \tag{34}$$

for any $i = 1, 2, \dots, m$. Conditions (33) and (34) imply that all the columns of B_M and B are parallel to each other. This is the conclusion made in [43], and means that for the reference model (8), $l = 1$ is the nontrivial choice, and $K_2^{*T} = k_2^* \in R^m$.

4.2. Adaptive designs

When the plant and failure parameters are unknown, the adaptive controller structure used in [12] is

$$v(t) = K_1^T(t)x(t) + K_2^T(t)r(t) + k_3(t), \tag{35}$$

where K_1, K_2, and k_3 are the estimates of K_1^*, K_2^*, and k_3^*, respectively.

To derive the adaptive laws updating the parameter estimates, the following assumption is needed:

(A3) The matrix B is known.

With this assumption, the adaptive laws that update the controller parameters are

$$\dot{k}_{1j}(t) = -\Gamma_{1j} x(t) e^T(t) P b_j, \tag{36}$$

$$\dot{k}_{2j}(t) = -\Gamma_{2j} r(t) e^T(t) P b_j, \tag{37}$$

$$\dot{k}_{3j}(t) = -\gamma_{3j} e^T(t) P b_j \tag{38}$$

for $j = 1, \dots, m$, where $e(t) = x(t) - x_m(t)$ is the tracking error, and $P \in R^{n\times n}$, $P = P^T > 0$ such that

$$P A_M + A_M^T P = -Q \tag{39}$$

for some constant $Q \in R^{n\times n}$ such that $Q = Q^T > 0$, $\Gamma_{1j} \in R^{n\times n}$ and $\Gamma_{2j} \in R^{l\times l}$ are constant such that $\Gamma_{1j} = \Gamma_{1j}^T > 0$, $\Gamma_{2j} = \Gamma_{2j}^T > 0$, $\gamma_{3j} > 0$ is constant, $j = 1, \dots, m$.

The adaptive controller (35), with the adaptive law (36)–(38), applied to the system (1) with actuator failures (2), guarantees that all closed-loop signals are bounded and the tracking error $e(t)$ goes to zero as t goes to infinity.

The performance of the adaptive scheme has been verified by simulation results of the lateral motion control of a Boeing 747 aircraft model [12] and the longitudinal motion control of a Boeing 737 aircraft model [13].

For the case of up to $m-1$ actuator failures, [43] gives another adaptive control scheme without the need of Assumption (A3). The plant-model matching controller structure used in this scheme with both known plant and actuator failure parameters is

$$v^*(t) = K_1^{*T}(t)x(t) + k_2^*(t)r(t) + k_3^*(t), \tag{40}$$

where $K_1^*(t) \in R^{n\times m}$, $k_2^*(t) \in R^m$, and $k_3^*(t) \in R^m$. It is shown in [43] that when the plant-model matching conditions for up to $m-1$ actuator failures stated in the above subsection are satisfied, there exist such K_1^*, k_2^*, and k_3^* for plant-model matching. In this case, the nontrivial choice is $l=1$, $B_M = b_M \in R^n$, and $b_j k_{s2j}^* = b_M$, for some constant $k_{s2j}^* \in R$, $j = 1, \ldots, m$.

When both plant and failure parameters are unknown, the adaptive version of (40) is

$$v(t) = K_1^T(t)x(t) + k_2(t)r(t) + k_3(t), \tag{41}$$

where $K_1(t)$, $k_2(t)$, and $k_3(t)$ are adaptive estimates of the unknown parameters K_1^*, k_2^* and k_3^*.

For $j = 1, \ldots, m$, the adaptive laws that update the controller parameters are

$$\dot{k}_{1j}(t) = -\text{sign}[k_{s2j}^*]\Gamma_{1j}x(t)e^T(t)Pb_M, \tag{42}$$

$$\dot{k}_{2j}(t) = -\text{sign}[k_{s2j}^*]\gamma_{2j}r(t)e^T(t)Pb_M, \tag{43}$$

$$\dot{k}_{3j}(t) = -\text{sign}[k_{s2j}^*]\gamma_j e^T(t)Pb_M, \tag{44}$$

where $e(t) = x(t) - x_m(t)$ is the state tracking error, the signs of k_{s2j}^*, $j = 1, \ldots, m$, are assumed to be known, $P \in R^{n\times n}$, $P = P^T > 0$ such that

$$PA_M + A_M^T P = -Q \tag{45}$$

for any constant $Q \in R^{n\times n}$ such that $Q = Q^T > 0$, $\Gamma_{1j} \in R^{n\times n}$ is constant such that $\Gamma_{1j} = \Gamma_{1j}^T > 0$, $\gamma_{2j} > 0$ and $\gamma_j > 0$ are constant, $j = 1, \ldots, m$.

The adaptive controller (41), with the adaptive law (42)–(44), applied to the system (1) with actuator failures (2), guarantees that all closed-loop signals are bounded and the tracking error $e(t)$ goes to zero as t goes to infinity.

Remark 1: The adaptive control schemes presented in this section do not use the knowledge of A in the controller and the adaptive laws. However, some knowledge of A is desirable for obtaining a suitable design of A_M such that the matching condition (30) can be satisfied.
Furthermore, if A is known, the matching parameter K_1^* in (26) and (40) can be exactly known for the no-failure case, this may reduce the level of system uncertainty, thus facilitate the adaptive control design. How to use the information of A is a topic of future research.

Remark 2: For the general actuator failure model (6), adaptive failure compensation designs are given in [43] for the case of up to $m-1$ failures, and may also be developed for the case of up to $m-q$ failures, with $1 \le q \le m-1$.

5. STATE FEEDBACK DESIGNS FOR OUTPUT TRACKING

For the plant (1) with $y(t) \in R$, adaptive actuator failure compensation schemes for output tracking are developed in [40] for the actuator failure model (2) and in [45] for the failure model (6).

5.1. Matching conditions

When both plant and failure parameters are known, a nominal controller structure is

$$v(t) = v^*(t) = K_1^{*T}x(t) + k_2^* r(t) + k_3^*, \tag{46}$$

where $K_1^* = [k_{11}^*, \ldots, k_{1m}^*] \in R^{n\times m}$, $k_2^* = [k_{21}^*, \ldots, k_{2m}^*]^T \in R^m$, and $k_3^* = [k_{31}^*, \ldots, k_{3m}^*]^T \in R^m$.

For plant-model matching in the presence of up to $m-1$ actuator failures modeled in (6), i.e., $\bar{u}_j$ may be time-varying, it is required that for any failure pattern that there are p actuator failures, that is, $u_j(t) = \bar{u}_j$, $j = j_1, \ldots, j_p$, $1 \le p \le m-1$, there exist some K_1^*, k_2^*, and k_3^* that satisfy the matching conditions

$$C(sI - A - \sum_{i\ne j_1,\ldots,j_p} b_i k_{1i}^*)^{-1} \sum_{i\ne j_1,\ldots,j_p} b_i k_{2i}^* = W_m(s) \tag{47}$$

$$C(sI - A - \sum_{i\ne j_1,\ldots,j_p} b_i k_{1i}^*)^{-1} \cdot (\sum_{i\ne j_1,\ldots,j_p} b_i k_{3i}^* + \sum_{j=j_1,\ldots,j_p} b_j \bar{u}_j) = 0, \tag{48}$$

in addition to internal system stability, where, for a scalar output $y(t) \in R$, $W_m(s) = P_m^{-1}(s)$ for a stable polynomial of degree n^*.

A necessary and sufficient condition for the matching equations (47) and (48) is that there exist some

$k_{1i}^* \in R^n$ and $\alpha_{ij} \in R$, $i, j = 1, \ldots, m$, such that

$$C(sI - A - b_i k_{1i}^{*T})^{-1} b_j = \alpha_{ij} W_m(s). \quad (49)$$

Assume (C, A, b_i) is controllable, $i = 1, 2, \ldots, m$, the necessary and sufficient condition to meet (49) is that $C(sI - A)^{-1} b_i$, $i = 1, 2, \ldots, m$, all have relative degrees equal to that of $W_m(s)$ [44, 45]. The internal stability condition is that all zeros of $C(sI - A)^{-1} b_i$, $i = 1, 2, \ldots, m$, are stable.

For the situation that all the m actuators have similar physical characteristics (for example, they are segments of a multiple-segment rudder or elevator for an aircraft, or they are heating devices for an oven), a meaningful design of actuation is to use a proportional-actuation scheme, that is,

$$v_1(t) = \alpha_2 v_2(t) = \cdots = \alpha_m v_m(t) \quad (50)$$

for some chosen constant $\alpha_i > 0$, $i = 2, 3, \ldots, m$, or, simply, the equal-actuation scheme

$$v_1(t) = v_2(t) = \cdots = v_m(t). \quad (51)$$

The nominal controller based on this actuation scheme is

$$v_1^*(t) = \cdots = v_m^*(t) = k_{11}^{*T} x(t) + k_{21}^* r(t) + k_{31}^*, \quad (52)$$

where $k_{11}^* \in R^n$, $k_{21}^* \in R$, and $k_{31}^* \in R$, based on the following assumptions:

(A4a) $(A, \sum_{j \neq j_1, \ldots, j_p} b_j)$, $p \in \{0, \ldots, m-1\}$, are controllable;

(A4b) $(C, A, \sum_{j \neq j_1, \ldots, j_p} b_j)$, $p \in \{0, \ldots, m-1\}$, have the same relative degree n^*;

(A4c) $(C, A, \sum_{j \neq j_1, \ldots, j_p} b_j)$, $p \in \{0, \cdots, m-1\}$, are minimum phase; and

(A4d) $CA^{n^*-1} \sum_{j \neq j_1, \ldots, j_p} b_j$, $p \in \{0, \ldots, m-1\}$, have the same sign:

$$\text{sign}[k_{21}^*] = \text{sign}[CA^{n^*-1} \sum_{j \neq j_1, \ldots, j_p} b_j]. \quad (53)$$

5.2. Adaptive designs

While the general actuator failure (6) can be similarly handled, for the actuator failure model (2), the adaptive version [44] of the controller (46) is

$$v(t) = K_1^T(t) x(t) + k_2(t) r(t) + k_3(t), \quad (54)$$

where $K_1(t)$, $k_2(t)$, and $k_3(t)$ are the estimates of K_1^*, k_2^*, and k_3^*. Introduce the auxiliary signals

$$\omega(t) = [x^T(t), r(t), 1]^T, \quad (55)$$

$$\zeta(t) = W_m(s)[\omega](t), \quad (56)$$

$$\xi_i(t) = \theta_i^T(t)\zeta(t) - W_m[\theta_i^T \omega](t), \quad (57)$$

where

$$\theta_i = [k_{1i}^T, k_{2i}, k_{3i}]^T, \ i = 1, 2, \ldots, m \quad (58)$$

and define

$$\varepsilon(t) = e(t) + \rho^T(t)\xi(t), \quad (59)$$

where $e(t) = y(t) - y_m(t)$ is the output tracking error, $\xi(t) = [\xi_1(t), \ldots, \xi_m(t)]^T$, $\rho_j^* = \alpha_{ij}$, $j \neq j_1, \ldots, j_p$, $\rho_j^* = 0$, $j = j_1, \ldots, j_p$, $\rho^* = [\rho_1^*, \ldots, \rho_m^*]^T$, and ρ is the estimate of ρ^*.

The adaptive laws are

$$\dot{\theta}_i(t) = -\frac{\text{sign}[\rho_i^*]\Gamma_i \zeta(t)\varepsilon(t)}{1 + \zeta^T(t)\zeta(t) + \xi^T(t)\xi(t)}, \quad (60)$$

$$\dot{\rho}(t) = -\frac{\Gamma_\rho \xi(t)\varepsilon(t)}{1 + \zeta^T(t)\zeta(t) + \xi^T(t)\xi(t)}, \quad (61)$$

where $\Gamma_i = \Gamma_i^T > 0, i = 1, \ldots, m$, $\Gamma_\rho = \Gamma_\rho^T > 0$.

The adaptive law (60)–(61) is stable in the sense that $\theta_j(t)$, $j = 1, \ldots, m$, $\rho(t)$ are bounded, and $\frac{\varepsilon(t)}{N(t)} \in L^2 \cap L^\infty$, $\dot{\theta}_j(t) \in L^2 \cap L^\infty$, $j = 1, \ldots, m$, $\dot{\rho}(t) \in L^2 \cap L^\infty$, for $N(t) = \sqrt{1 + \zeta^T(t)\zeta(t) + \xi^T(t)\xi(t)}$.

For the failure model (2), the adaptive version of the controller (52) is

$$\begin{aligned} v_0(t) &= v_1(t) = v_2(t) = \cdots = v_m(t) \\ &= k_{11}^T(t)x(t) + k_{21}(t)r(t) + k_{31}(t), \end{aligned} \quad (62)$$

where $k_{11}(t)$, $k_{21}(t)$, and $k_1(t)$ are the estimates of the unknown parameters k_{11}^*, k_{21}^*, and k_{31}^*.

Introducing the auxiliary signals

$$\zeta(t) = W_m[\omega](t), \quad (63)$$

$$\xi(t) = \theta^T(t)\zeta(t) - W_m(s)[\theta^T \omega](t), \quad (64)$$

$$\varepsilon(t) = e(t) + \rho(t)\xi(t), \quad (65)$$

where $\theta = [k_{11}^T, k_{21}, k_1]^T$, $e(t) = y(t) - y_m(t)$ is the output tracking error, $\rho(t)$ is the estimate of $\rho^* = \frac{1}{k_{21}^*}$, $\omega(t)$ is defined in (55), the adaptive laws are

$$\dot{\theta}(t) = -\frac{\text{sign}[k_{21}^*]\Gamma\zeta(t)\varepsilon(t)}{1+\zeta^T\zeta+\xi^2}, \Gamma = \Gamma^T > 0 \quad (66)$$

$$\dot{\rho}(t) = -\frac{\gamma\xi(t)\varepsilon(t)}{1+\zeta^T\zeta+\xi^2}, \gamma > 0. \quad (67)$$

The adaptive controller (62), with the adaptive law (66)–(67), applied to the system (1) with actuator failures (2), guarantees that all closed-loop signals are bounded and the tracking error $e(t) = y(t) - y_m(t)$ goes to zero as t goes to infinity.

Simulation results with both the lateral motion control of a Boeing 747 aircraft [40] and the longitudinal motion control of a Boeing 737 aircraft [13] have demonstrated that the proposed control scheme ensures closed-loop stability and asymptotic output tracking.

The controller structure (62) can be further modified to handle the general failures (6) [45].

6. OUTPUT FEEDBACK DESIGN FOR OUTPUT TRACKING

An adaptive output feedback actuator failure compensation scheme for output tracking is presented in [42] for up to $m-1$ actuator failures modeled in (2).

6.1. Matching conditions

Consider the case when all m actuators have similar physical characteristics such that the equal-actuation scheme can be used:

$$v_1(t) = \cdots = v_m(t) \triangleq v_0(t). \quad (68)$$

When both plant and failure parameters are known, the control input signal $v_0(t)$ is designed from the nominal controller structure [42]

$$\begin{aligned} v_0(t) &= v_0^*(t) \\ &= \theta_1^{*T}\omega_1(t) + \theta_2^{*T}\omega_2(t) + \theta_{20}^* y(t) + \theta_3^* r(t) + \theta_4^*, \end{aligned} \quad (69)$$

where $\theta_1^* \in R^{n-1}$, $\theta_2^* \in R^{n-1}$, $\theta_{20}^* \in R$, and $\theta_3^* \in R$ are parameters for plant-model output matching, $\theta_4^* \in R$ is a constant for compensation of the actuation error $u - v = \sigma(\bar{u} - v)$, and

$$\omega_1(t) = \frac{a(s)}{\Lambda(s)}[v_0](t), \ \omega_2(t) = \frac{a(s)}{\Lambda(s)}[y](t) \quad (70)$$

with $a(s) = [1, s, \cdots, s^{n-2}]^T$ and $\Lambda(s)$ being a monic stable polynomial of degree $n-1$. The controller (69) uses only the designed system input $v_0(t)$ and output $y(t)$ plus the given reference input $r(t)$, and not the internal system state variables $x(t)$.

The following assumptions (A5a) and (A5b) (that is, the assumptions (A4b) and (A4c)) are necessary and sufficient conditions for the existence of the matching parameters $\theta_1^*, \theta_2^*, \theta_{20}^*, \theta_3^*$, and θ_4^* in the presence of up to $m-1$ actuator failures such that $y(t)$ tracks $y_m(t)$ asymptotically.

(A5a) $(C, A, \sum_{j \neq j_1, \ldots, j_p} b_j)$, $p \in \{0, \ldots, m-1\}$,

have the same relative degree n^*; and

(A5b) $(C, A, \sum_{j \neq j_1, \ldots, j_p} b_j)$, $p \in \{0, \ldots, m-1\}$, are minimum phase.

6.2. Adaptive designs

The adaptive version of the plant-model matching controller (69) is

$$\begin{aligned} v_0(t) &= v_1(t) = v_2(t) = \cdots = v_m(t) \\ &= \theta_1^T\omega_1(t) + \theta_2^T\omega_2(t) + \theta_{20} y(t) + \theta_3 r(t) + \theta_4, \end{aligned} \quad (71)$$

where $\theta_1(t)$, $\theta_2(t)$, $\theta_{20}(t)$, $\theta_3(t)$, and $\theta_4(t)$ are the estimates of the unknown parameters θ_1^*, θ_2^*, θ_{20}^*, θ_3^*, and θ_4^*, respectively.

By defining

$$\theta(t) = [\theta_1^T(t), \theta_2^T(t), \theta_{20}(t), \theta_3(t), \theta_4(t)]^T, \quad (72)$$

$$\omega(t) = [\omega_1^T(t), \omega_2^T(t), y(t), r(t), 1]^T, \quad (73)$$

the adaptive laws for updating the parameter estimates are chosen as

$$\dot{\theta}(t) = -\frac{\text{sign}[\theta_3^*]\Gamma\zeta(t)\varepsilon(t)}{1+\zeta^T\zeta+\xi^2}, \Gamma = \Gamma^T > 0, \quad (74)$$

$$\dot{\rho}(t) = -\frac{\gamma\xi(t)\varepsilon(t)}{1+\zeta^T\zeta+\xi^2}, \gamma > 0 \quad (75)$$

with

$$\zeta(t) = W_m[\omega](t), \quad (76)$$

$$\xi(t) = \theta^T(t)\zeta(t) - W_m(s)[\theta^T\omega](t), \quad (77)$$

$$\varepsilon(t) = e(t) + \rho(t)\xi(t), \quad (78)$$

where $e(t) = y(t) - y_m(t)$ is the output tracking error and $\rho(t)$ is the estimate of $\rho^* = \frac{1}{\theta_3^*}$.

For the implement of adaptive laws (74)–(75), the following assumption is needed:

(A5c) $CA^{n^*-1}\sum_{j \neq j_1, \ldots, j_p} b_j$, $p \in \{0, \ldots, m-1\}$,

have the same sign:

$$k_p = \frac{1}{\theta_3^*},\ \text{sign}[\theta_3^*] = \text{sign}[k_p] = \text{sign}[CA^{n^*-1}\sum_{j\neq j_1,\ldots,j_p} b_j]. \tag{79}$$

The adaptive controller (71), with the adaptive law (74)–(75), applied to the system (1) with actuator failures (2), guarantees that all closed-loop signals are bounded and the tracking error $e(t) = y(t) - y_m(t)$ goes to zero as t goes to infinity.

The control scheme developed has been applied to a Boeing 747 lateral dynamics model and a Boeing 737 longitudinal dynamics model with different actuator failure patterns. All simulation results [13, 42] verified that the control scheme guarantees closed-loop stability as well as asymptotic output tracking in the presence of unknown actuator failures.

The controller structure (71) can be further modified to handle the general failures (6) [41].

7. DISCRETE-TIME ROBUST DESIGNS

In this section we demonstrate that adaptive actuator failure compensation can be designed in a discrete-time setting and can be made robust with respect to bounded output disturbances and additive and multiplicative stable unmodeled dynamics [15].

The controlled plant under consideration is described in the input-output form

$$y(k) = G(z)[u](k) + d(k), \tag{80}$$

where $y(k) \in R$ is the measured plant output, $u(k) \in R^m$ is the plant input vector whose components may fail during system operation, $d(k) \in R$ is a bounded external output disturbance, and $G(z)$ is a $1\times m$ transfer matrix. The symbol z is used to denote, as the case may be, the z-transform variable or the time-advance operator $z[x](k) = x(k+1)$. We denote the plant transfer matrix by

$$G(z) = G_0(z)(I + \mu\Delta_m(z)) + \mu\Delta_a(z),\ \mu \geq 0, \tag{81}$$

where $G_0(z) = [G_{01}(z), \ldots, G_{0j}(z)]$, $G_{0j}(z) = k_{pj}\frac{Z_j(z)}{P(z)}$, $j = 1,2,\ldots,m$, $\Delta_m(z) = \text{diag}\{\Delta_{m1}(z), \ldots, \Delta_{mm}(z)\}$, and $\Delta_a(z) = [\Delta_{a1}(z), \ldots, \Delta_{am}(z)]$. In (81), $G_0(z)$ is the nominal plant description, and $\mu\Delta_m(z)$ and $\mu\Delta_a(z)$ are multiplicative and additive unmodeled dynamics, respectively. As in Section 2, the type of actuator failures under consideration are modeled as

$$u_j(k) = \bar{u}_j,\ k \geq k_j,\ j \in \{1,2,\ldots,m\}, \tag{82}$$

where the constant value $\bar{u}_j$ and the failure time instant k_j are unknown. In this section, we consider the case that any up to $m-1$ actuators may fail during system operation. The more general cases when there are up to $m-q$ failures or the failures can be time-varying (as similar to that in (6)) can also be addressed using modified controller structures.

The basic assumption for adaptive actuator failure compensation is the same as (A1), that is,

(A6) the system (80) is so designed that for any up to $m-1$ actuator failures, the remaining actuators can still achieve a desired control objective.

The key task of adaptive control is to adjust the remaining controls to achieve the desired system performance when there are up to $m-1$ actuator failures whose parameters are unknown. As in (3), in the presence of actuator failures, $u(k)$ can be expressed as

$$u(k) = v(k) + \sigma(\bar{u} - v(k)), \tag{83}$$

where $v(k) = [v_1, \ldots, v_m]^T$ is an applied control input vector to be designed, and $\bar{u}$ and σ are defined in (4) and (5). The control objective is to design a feedback control $v(k)$ for the plant (80) unknown with actuator failures (82) unknown, and under Assumption (A6), such that, despite the presence of the control error $u - v = \sigma(\bar{u} - v)$, all closed-loop signals are bounded and the plant output $y(k)$ asymptotically tracks a given reference output $y_m(k)$ when $\mu = 0$ and $d(k) = 0$, or tracks $y_m(k)$ as close as possible when $\mu \neq 0$ and $d(k) \neq 0$.

The reference signal $y_m(k)$ may be generated from a reference model system

$$y_m(k) = W_m(z)[r](k), W_m(z) = \frac{1}{P_m(z)}, \tag{84}$$

where $P_m(z)$ is a stable monic polynomial of degree n^*, and $r(k)$ is a bounded signal.

7.1. Plant-model output matching

When there is no unmodeled dynamics $\mu\Delta_m(z)$, $\mu\Delta_a(z)$ and no disturbance $d(k)$, we use the controller structure

$$v_1(k) = \cdots = v_m(k) = v_0(k) \triangleq v_0^*(k) = \theta_1^{*T}\omega_1(k) + \theta_2^{*T}\omega_2(k) + \theta_{20}^* y(k) + \theta_3^* r(k) + \theta_4^*, \tag{85}$$

where $\theta_1^* \in R^{n-1}$, $\theta_2^* \in R^{n-1}$, $\theta_{20}^* \in R$, and $\theta_3^* \in R$ are to be defined for plant-model output matching,

$\theta_4^* \in R$ is to be chosen for compensation for the actuation error $u - v = \sigma(\bar{u} - v)$, and

$$\omega_1(k) = \frac{a(z)}{\Lambda(z)}[v_0](k),\ \omega_2(k) = \frac{a(z)}{\Lambda(z)}[y](k) \quad (86)$$

with $a(z) = [1, z, \ldots, z^{n-2}]^T$ and $\Lambda(z)$ being a monic stable polynomial of degree $n-1$, $\Lambda(z) = z^{n-1}$.

For the existence of the plant-model matching controller, we assume that for all failure patterns,

(A6a) $\sum_{j \neq j_1, \ldots j_p} \frac{k_{pj} Z_j(z)}{P(z)}$, $p \in \{0, \ldots, m-1\}$, have the same relative degree n^*; and

(A6b) $\sum_{j \neq j_1, \ldots j_p} \frac{k_{pj} Z_j(z)}{P(z)}$, $p \in \{0, \ldots, m-1\}$, are minimum phase.

7.2. Adaptive control design

For the plant (80) with unknown parameters, and with unknown actuator failures (82) when $\mu = 0$ and $d(k) = 0$, we use the adaptive version of the controller (85) as

$$\begin{aligned} v_1(k) &= \cdots = v_m(k) \triangleq v_0(k) \\ &= \theta_1^T \omega_1(k) + \theta_2^T \omega_2(k) + \theta_{20} y(k) + \theta_3 r(k) + \theta_4, \end{aligned} \quad (87)$$

where $\theta_1(k) \in R^{n-1}$, $\theta_2(k) \in R^{n-1}$, $\theta_{20}(k) \in R$, $\theta_3(k) \in R$, and $\theta_4(k) \in R$ are the estimates of the unknown parameters θ_1^*, θ_2^*, θ_{20}^*, θ_3^*, and θ_4^*, respectively.

Defining

$$\theta^* = [\theta_1^{*T}, \theta_2^{*T}, \theta_{20}^*, \theta_3^*, \theta_4^*]^T, \quad (88)$$

$$\theta(k) = [\theta_1^T(k), \theta_2^T(k), \theta_{20}(k), \theta_3(k), \theta_4(k)]^T, \quad (89)$$

$$\omega(k) = [\omega_1^T(k), \omega_2^T(k), y(k), r(k), 1]^T, \quad (90)$$

$$\tilde{\theta}(k) = \theta(k) - \theta^*, \quad (91)$$

and introducing the auxiliary signals

$$\zeta(k) = W_m(z)[\omega](k), \quad (92)$$

$$\xi(k) = \theta^T(k)\zeta(k) - W_m(z)[\theta^T \omega](k), \quad (93)$$

$$\varepsilon(k) = e(k) + \rho(k)\xi(k), \quad (94)$$

where $\rho(k)$ is the estimate of $\rho^* = \frac{1}{\theta_3^*} = \sum_{j \neq j_1, \ldots, j_p} k_{pj} \triangleq k_p$, we choose the adaptive laws as

$$\theta(k+1) - \theta(k) = -\frac{\text{sign}[k_p]\Gamma\zeta(k)\varepsilon(k)}{m^2(k)}, \quad (95)$$

$$\rho(k+1) - \rho(k) = -\frac{\gamma\xi(k)\varepsilon(k)}{m^2(k)}, \quad (96)$$

where $m^2(k) = 1 + \zeta^T(k)\zeta(k) + \xi^2(k)$, and the adaptation gains $\Gamma \in R^{(2n+1)\times(2n+1)}$ and $\gamma \in R$ are constant and satisfy $0 < \Gamma = \Gamma^T < \frac{\gamma_\theta}{k_p^0} I_{2n+1}$, $0 < \gamma_\theta < 2$, $0 < \gamma < 2$.

To implement (95), we need the following assumption

(A6c) $\text{sign}[k_p]$, the sign of k_p, is known, and $|k_p| \leq k_p^0$ for some known constant $k_p^0 > 0$.

The adaptive controller (87), with the adaptive law (95)–(96), applied to the system (80) with actuator failures (82), guarantees that all closed-loop signals are bounded and the tracking error $e(k) = y(k) - y_m(k)$ goes to zero as k goes to infinity.

In [15], this adaptive actuator failure compensation scheme has been applied to a discrete-time Boeing 747 lateral dynamics model with actuator failures. It is verified by the simulation results that both closed-loop system stability and asymptotic output tracking are achieved by the adaptive control scheme.

7.3. Robust adaptive compensation

The adaptive control scheme of Section 7.2 was designed for $\mu = 0$ and $d(k) = 0$ and may not ensure stability in the presence of unmodeled dynamics $\mu\Delta_m(z)$ and $\mu\Delta_a(z)$ for $\mu \neq 0$ and output disturbance $d(k) \neq 0$. In this subsection, we first introduce a robust nonadaptive control for $G_0(z)$ known, and then derive the robust adaptive laws for $G_0(z)$ unknown.

7.3.1 Robustness of plant-model matching

For a robust nonadaptive control for $G_0(z)$ known, we make the following assumptions on the unmodeled dynamics $\mu\Delta_m(z), \mu\Delta_a(z)$ and output disturbance $d(k)$:

(A6d) $\Delta_{aj}(z)$ and $\frac{\Delta_{mj}(z)}{P_m(z)}, j = 1, \ldots, m$, are proper rational functions;

(A6e) the impulse functions $h_{a0}(k)$, $h_{m0}(k)$ of $\Delta_a(z)$, $W_m(z)\Delta_m(z)$ satisfy

$$\sum_{k=0}^{\infty} |h_{a0}(k)| < c < \infty, \sum_{k=0}^{\infty} |h_{m0}(k)| < c < \infty \quad (97)$$

for some constant $c > 0$ independent of μ; and

(A6f) $d(k) \in L^\infty$.

Under the Assumptions (A6d), (A6e), and (A6f), there exists a $\mu_0 > 0$ such that for any $\mu \in [0, \mu_0)$, the controller (85) with $\theta_1 = \theta_1^*, \theta_2 = \theta_2^*, \theta_{20} = \theta_{20}^*, \theta_3 = \theta_3^*$, and $\theta_4 = \theta_4^*$, ensures that all signals in the closed-loop system are bounded and the tracking error $e(k) = y(k) - y_m(k)$ converges exponentially to the residual set

$$S_0 = \{e : |e| \le b_1 \mu \bar{r} + b_2 \bar{d}\} \tag{98}$$

for some constant $b_1 > 0$, $b_2 > 0$, where $\bar{r}$ and $\bar{d}$ are the upper bounds of $|r(k)|$ *and* $|d(k)|$.

7.3.2 Robust adaptive laws

For robust adaptive control, we change the Assumptions (A6d) and (A6e) as

(A6g) $\Delta_{aj}(z)$ and $\frac{\Delta_{mj}(z)}{P_m(z)}, j = 1, \ldots, m$, are strictly proper rational functions;

(A6h) $W_m(qz)\Delta_m(qz)(z+1)$ and $\Delta_a(qz)(z+1)$ are stable with a finite gain independent of μ for some constant $q \in (0,1)$, that is, the impulse functions $h_m(k)$ and $h_a(k)$ of $W_m(qz)\Delta_m(qz)(z+1)$ and $\Delta_a(qz)(z+1)$ satisfy

$$\sum_{k=0}^{\infty} |h_m(k)| < c < \infty, \sum_{k=0}^{\infty} |h_a(k)| < c < \infty \tag{99}$$

or some constant $c > 0$ independent of μ.

We still use the adaptive controller structure (87). To design the adaptive laws updating the controller parameters, we generate a new normalizing signal $m(k)$ from

$$m(k+1) = (1 - \delta_0) m(k) + \delta_1 (|u(k)| + |y(k)| + 1), \tag{100}$$

where $m(0) > 0$, $\delta_1 > 0$, $q < 1 - \delta_0 < 1$, with $q \in (0,1)$ defined in Assumption (A6h).

Introducing the auxiliary signals $\zeta(k)$, $\xi(k)$, and $\varepsilon(k)$ as in (92), (93), and (94), respectively, we choose the robust adaptive laws as

$$\theta(k+1) = \theta(k) - \frac{\text{sign}[k_p]\gamma_\theta \zeta(k)\varepsilon(k)}{m^2(k)} - \sigma_1(k)\theta(k), \tag{101}$$

$$\rho(k+1) = \rho(k) - \frac{\gamma \xi(k)\varepsilon(k)}{m^2(k)} - \sigma_2(k)\rho(k), \tag{102}$$

where the adaptation gains satisfy $0 < \gamma_\theta < \frac{1}{k_p^0}$, $0 < \gamma < 1$, for $k_p^0 \ge |k_p|$.

To handle the unmodeled dynamics $\mu\Delta_m(z), \mu\Delta_a(z)$ and the disturbance $d(k)$, we have introduced the switching signals $\sigma_1(k)$ and $\sigma_2(k)$:

$$\sigma_1(k) = \begin{cases} 0 & \text{if } \|\theta(k)\|_2 < 2M_1 \\ \sigma_0 & \text{if } \|\theta(k)\|_2 \ge 2M_1 \end{cases}, \tag{103}$$

$$\sigma_2(k) = \begin{cases} 0 & \text{if } |\rho(k)| < 2M_2 \\ \sigma_0 & \text{if } |\rho(k)| \ge 2M_2 \end{cases}, \tag{104}$$

where $0 < \sigma_0 < \frac{1}{2}(1 - \gamma_m)$, $\gamma_m = \max\{\gamma_\theta k_p^0, \gamma\} < 1$, and $M_1 > \|\theta^*\|_2$, $M_2 > |\rho^*|$. To implement $\sigma_1(k)$ and $\sigma_2(k)$, we need the knowledge of parameter bounds M_1 and M_2.

The adaptive controller (87), with the adaptive laws (101) and (102), applied to the system (80) with actuator failures (82), guarantees closed-loop signal boundedness, for all $\mu \in [0, \mu^*)$ and some $\mu^* > 0$.

8. CONCLUDING REMARKS

A survey of some direct adaptive feedback control schemes based on the model reference approach was presented for linear time-invariant systems with unknown actuator failures. The controller structures, plant-model matching conditions, and adaptive laws were presented. The adaptive control schemes ensure closed-loop signal boundedness and asymptotic tracking of the state vector or a scalar output.

For some applications, however, the controlled plant has multiple outputs, and these outputs are required to track a vector of reference outputs. In [41], we developed two adaptive control schemes using the model reference approach to compensate for unknown actuator failures in systems with q outputs. The controlled plant has q groups of actuators, each actuator group has more than one actuators to provide the needed redundancy, and actuators in a group have the same or similar physical characteristic. An equal (or proportional) actuation scheme can be used to design the control signals for actuators in each group. These adaptive actuator failure compensation schemes guarantee asymptotic multi-output tracking.

The output tracking designs presented are based on model reference adaptive control, which requires that the controlled plant is minimum phase for each actuator failure. There are many applications in which the controlled plants are nonminimum phase. An actuator failure compensation design based on adaptive pole placement control has been developed in [41]. In this scheme, the plant parameters and actuator failure parameters are first estimated online and the estimated parameters are then used to calculate the controller parameters. The developed control scheme is applicable to both minimum phase and nonminimum phase linear time-invariant plants, and it needs neither the *a priori* knowledge of the relative degree of the controlled plant, nor the condition that the controlled plant and its undamaged parts are of the same relative degree. However, the control singularity problem for this pole placement based design is still open.

Adaptive actuator failure compensation for nonlinear systems is an important area of research and some of the recent preliminary results have been presented in [41].

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