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Adaptive Algorithmic Behavior for Solving Mixed Integer Programs Using Bandit Algorithms

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ZIB-Report (Print) ISSN 1438-0064
ZIB-Report (Internet) ISSN 2192-7782

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July 20, 2018

Abstract

State-of-the-art solvers for mixed integer programs (MIP) govern a variety of algorithmic components. Ideally, the solver adaptively learns to concentrate its computational budget on those components that perform well on a particular problem, especially if they are time consuming. We focus on three such algorithms, namely the classes of large neighborhood search and diving heuristics as well as Simplex pricing strategies. For each class we propose a selection strategy that is updated based on the observed runtime behavior, aiming to ultimately select only the best algorithms for a given instance. We review several common strategies for such a selection scenario under uncertainty, also known as Multi Armed Bandit Problem. In order to apply those bandit strategies, we carefully design reward functions to rank and compare each individual heuristic or pricing algorithm within its respective class. Finally, we discuss the computational benefits of using the proposed adaptive selection within the **SCIP** Optimization Suite on publicly available MIP instances.

1 Introduction

Most modern MIP solvers employ the LP-based branch-and-bound method. At each node of the search tree, a relaxation of the problem is solved by the dual Simplex algorithm to obtain both improving solutions and lower bounds on the problem. Many primal heuristics have been proposed (see [4] for an overview and further references) to aid the branch-and-bound procedure in finding good feasible solutions quickly. Two important subclasses of primal heuristics for MIP are large neighborhood search (LNS) and diving heuristics (Sec. 3.2 and 3.3). In total, **SCIP** [8] in version 5.0 features 12 diving heuristics, 10 pure LNS heuristics, further LNS heuristics for MINLP as well as certain primal heuristics that solve auxiliary problems in special cases. As there is no theoretical reason why certain heuristics work better for a MIP instance, tuning them individually can be a tedious task. Similarly, a large amount of the total solving time is

spent during LP re-optimizations, so the choice of an efficient pricing method can be crucial (Sec. 3.1).

In all three cases, it is desirable to "learn" the best performing algorithms during the solving process. To this end, we investigate computational benefits of adaptive algorithmic behavior, governed by algorithms for the multi-armed bandit problem (Sec. 2). Such bandit algorithms try to balance their selection carefully between exploration among the available set of algorithms and exploitation of the best performing ones. We discuss suitable reward functions to measure the success of an algorithm for each class, and present promising results in a computational study.

2 Bandit Selection Strategies

The selection among a set of actions under uncertain payoffs appears as *multi-armed bandit problem* (MAB)[5]. In each *round* $t > 0$, a player selects an *action* a_t out of a set of actions \mathcal{A} . In turn for playing a_t , the player observes a *reward* $r_{a_t,t} \in [0, 1]$ with the goal to maximize the *total reward* $\sum_t r_{a_t,t}$. We call algorithms for MAB *selection strategies*. MAB distinguishes two scenarios. In the *stochastic scenario*, rewards are drawn from a reward distribution R_a that is independent of t . As the expected rewards are not known to the player beforehand, a good selection strategy carefully balances exploration between the actions and exploitation of the best single action. A common strategy to address stochastic scenarios is based on *upper confidence bounds* (UCB). Let $T_{a,t}$ denote the number of times that action $a \in \mathcal{A}$ has been selected until round t , and let $\bar{r}_{a,t}$ denote the mean reward of action a after a has been selected once ($T_{a,t} > 0$). After playing every action once, UCB selects the action that maximizes an upper confidence bound

$$a_t := \operatorname{argmax}_{a \in \mathcal{A}} \left\{ \bar{r}_{a,t-1} + \sqrt{\alpha \cdot \frac{\ln(1+t)}{T_{a,t-1}}} \right\}$$

on the expected reward. The confidence band around a mean reward increases with every round in which an action is not played, forcing the selection of inferior actions from time to time. Its width is controlled by a parameter $\alpha \geq 0$. Note that a UCB strategy acts entirely greedy for $\alpha = 0$.

The second MAB scenario is the *adversarial scenario*, in which the rewards are picked by an opponent aiming at maximizing the player's total regret. Selection strategies for this scenario usually involve weighted sampling from an incrementally updated probability distribution $p_{a,t} = \mathbb{P}(a_t = a)$, starting from a uniform distribution $p_{a,1} = \frac{1}{|\mathcal{A}|}$. A special variant of weighted sampling is the Exp.3 selection strategy [5], which scales an observed reward with the current selection probability of action a_t . Therefore, if an action with small selection probability yields a high reward, this action will have a much higher chance to be selected in subsequent rounds.

3 Adaptive Algorithmic Behavior

We give three examples of branch-and-bound solving components suitable for an adaptive selection strategy. At the end of each section, we show the individual

impact on the academic MIP solver **SCIP** and LP solver **SoPlex** [8]. All experiments are based on a pre-release version of **SCIP** 6.0 and **SoPlex** 3.1.1. The experiments have been performed on a test set **MMMC** of 496 instances combining the benchmark sets **MILPLIB 3**, **MILPLIB 2003**, **MILPLIB 2010**, and **COR@L** (see [10] and [6]). Each experiment has been conducted on a cluster with identical machines to ensure comparable running time measurements.

3.1 Pricing for the Dual Simplex Algorithm

The dual Simplex algorithm is one of the most important techniques for LP problems and key for the LP-based branch-and-bound approach. Among the few algorithmic choices within the Simplex algorithm, one is the determination of the direction to search for a new basic solution, called *pricing step*. In this paper, we consider three well-known and practically proven methods called *devex pricing* [9], *steepest edge pricing*, and *quick start steepest edge* [7]. All methods try to select a direction that is steepest in regard to the dual objective improvement, thereby balancing accuracy of the decision and computational overhead per iteration. Devex pricing requires the least work per iteration but may lead to a higher number of total iterations. On the other hand, steepest edge computes accurate improvement measures that often lead to a considerable smaller iteration count, and an initialization step that can be expensive to compute, depending on the starting basis. While this is less relevant for pure LP solving, in the branch-and-bound context many LP re-optimizations are performed that start from an advanced basis. Here, quick start steepest edge sacrifices accuracy for a faster initialization. We refer to the literature for an in-depth description of these pricing techniques.

In our computational study we compare average LP throughput (LPs per seconds) and running time of three fixed pricers and three bandit selection variants after the root node has been processed, see Table 1. The selection strategies have to select from the set of available actions $\mathcal{A} = \{\text{devex}, \text{qsteep}, \text{steep}\}$. As the UCB strategy (cf. Sec. 2), requires a reward within the interval $[0, 1]$, we scale the measured running time $\tau_{a,t}$ of pricer a at time step t as $\frac{1}{1 + \tau_{a,t}/\bar{\tau}_t}$, where $\bar{\tau}_t$ denotes the average running time of all LP resolves so far, independently of the selected pricer. A value of $\alpha = 2$ is used for UCB. We also test a **greedy** strategy that always selects the pricer with minimum modified average running time $\bar{\tau}_{a,t}^\sigma = \sum_{t':a_t=a} \tau_{a,t'}/(T_{a,t} + \sigma_a)$, using shift values of $\sigma_a = 100$ for **devex** and $\sigma_a = 50$ for the other two. The favorite pricer **devex** is also kept if its LP iteration count stays below 20 on average. The use of the shift values encourages more exploration among the available pricers at the beginning. As a last variant (**weighted**), we use the modified means of the **greedy** selection method as input for a weighted sampling. We initialize the sampling probabilities as $p_{a,t} \propto (\bar{\tau}_{a,t}^\sigma + 10^{-4})^{-1}$. Here, the symbol \propto expresses "proportional to", that means up to a scaling constant. Among the fixed pricers, **devex** is clearly the one with the highest LP throughput. The throughput can be increased by 6 % when using UCB, and even 14 % when using **greedy**. In contrast, **weighted** does not yield an improved LP throughput. While the **greedy** strategy even yields a 3 % time improvement, the positive result of UCB for the LP throughput is still too marginal to make **SCIP** consistently solve problems faster on average.

Table 1: Results for LP pricers. Columns: shifted geom. mean LP throughput (**LPthpt**, shift: 1), time in seconds (**time**, shift: 1), and respective quotients (**LPthpt_Q**, **time_Q**). 105 instances, 4 LP seeds, 900 sec. time limit

Pricer	solved	LPthpt	LPthpt _Q	time	time _Q
<code>devex</code>	64	74.24	1.000	91.82	1.000
<code>steep</code>	65	62.66	0.844	99.41	1.083
<code>qsteep</code>	60	58.00	0.781	101.13	1.101
UCB	63	79.02	1.064	92.80	1.011
<code>weighted</code>	65	71.93	0.969	93.95	1.023
<code>greedy</code>	65	85.06	1.146	89.11	0.970

3.2 Large Neighborhood Search Heuristics

The first class of algorithms that we studied in the context of adaptive behavior are the Large Neighborhood Search (LNS) heuristics. Briefly, an LNS heuristic solves an auxiliary MIP under strict working limits, which is derived from the original MIP by fixing variables, adding constraints, and/or changing the objective function. In total, **SCIP** features 10 LNS heuristics, eight of which we integrated into a framework called Adaptive Large Neighborhood Search (ALNS). ALNS adapts the selection of the next LNS heuristic that should be executed based on the average reward observed so far. Besides the adaptive selection procedure, ALNS features more techniques such as a dynamic target fixing rate and a generic variable fixing procedure. ALNS has been first released with **SCIP** 5.0, and further improved for **SCIP** 6.0. Its reward function has been designed to prefer LNS heuristics that find improving solutions with a small computational effort. To this end, it convexly combines a simple indicator function whether an improving solution has been found with the obtained gap that has been closed. The obtained score is then scaled by the involved effort, as a function of the fixing rate and the number of nodes spent inside the sub-MIP.

On the test set used for the present work, the individual parameters for the selection strategies have first been optimized by a separate simulation procedure. With those parameters, ALNS has been called on a total of 445 problem instances. With an `Exp.3` strategy, it could solve two more instances than the **SCIP** default, and yielded a speed-up of 2.3 %, and even 4.6 % on the subset of MIPLIB 2010 benchmark instances, solving one additional instance. Details about the simulation procedure as well as the results will be presented in an own technical report about ALNS that is currently under preparation.

3.3 Diving Heuristics

Another class of heuristics are *diving heuristics*. Starting from a fractional LP solution, diving heuristics explore an auxiliary search tree in a depth-first fashion. The branching rules used in diving heuristics usually tend towards feasibility. In contrast to that, branching rules of the main search process, e.g., reliability branching [2], focus on a good subdivision of the problem. For an overview of the diving heuristics available in **SCIP**, we refer to [3]. In **SCIP**, diving heuristics also provide useful search information. For example, domain propagation is applied after rounding variables, to reduce variable domains or

Table 2: Aggregated results for adaptive diving over three random seeds. Columns: shifted geom. mean of generated nodes (**nodes**, shift: 100), solving time in seconds (**time**, shift: 1), and respective quotients (**nodes_Q** and **time_Q**).

	instances	default			adaptivediving		
		solved	nodes	time	solved	nodes _Q	time _Q
all	491	320	2550	152	327	0.938	0.958
affected	284	274	1120	46	281	0.939	0.945
[10,tilim]	245	210	2693	158	217	0.899	0.922
[100,tilim]	144	109	5821	526	116	0.904	0.909
MIPLIB 2010	86	67	5064	301	70	0.930	0.969

even detect infeasibility. The latter can be analyzed by conflict analysis techniques, e.g., [1, 11], to derive additional global information.

For our computational experiments, we have extended **SCIP** by a new primal heuristic plugin that selects one out of nine available diving heuristics at each call. A weighted sampling is used as selection strategy, where the sample weight of a diving heuristic (action) a is computed as

$$p_{a,t} \propto \left(\frac{\sum_{t'} b_{a,t'} + 100}{\sum_{t'} c_{a,t'} + 100} + 10^{-4} \right)^{-1}$$

with $b_{a,t'}$ denoting the number of backtracks performed by a at round t' , and $c_{a,t'}$ denoting the number of conflict constraints generated. Both values are 0 if a has not been selected in round t' .

Table 2 compares the performance of **SCIP** in its standard configuration (**default**) and with adaptive diving selection (**adaptivediving**) on the MMMC test set with a time limit of one hour. An instance is called “solved” only if it has been solved consistently with each of three tested random seeds. Using **adaptivediving**, **SCIP** could solve seven more instances consistently within the time limit. On non-trivial instances where at least one configuration needs 10 or more seconds, it leads to speed up of almost 8 %. On the state-of-the-art benchmark set MIPLIB 2010, **adaptivediving** leads to a slight performance improvement by 3 % and three more solved instances.

4 Conclusion

We have proposed adaptive control mechanisms for three algorithm classes within **SCIP**. For each class, we introduce a suitable reward function to rank the different algorithms. The adaptive LP pricing intuitively has the nature of a stochastic bandit scenario. This intuition is confirmed by our results, in which the greedy and UCB strategy yield higher LP throughputs than any fixed pricer. On the contrary, for primal heuristics, it is sufficient to select successful ones more often, which is why a weighted sampling selection strategy can be preferred. In every case, we obtain considerable performance improvements on a diverse set of general MIP instances.

While the work on the two heuristic frameworks is almost completed, the adaptive LP pricing is still prototypical. Future work on this requires to replace

the measured solving time by a deterministic reward criterion. For the primal heuristics, it is interesting to compare the obtained results with new selection principles that are not based on past rewards.

Acknowledgements We thank Tobias Achterberg for useful comments and hints, especially with regard to Sec. 3.1. The work for this article has been partly conducted within the *Research Campus MODAL* funded by the German Federal Ministry of Education and Research (BMBF grant number 05M14ZAM).

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Table 3: Computational results over three random seeds on MMMc test set comparing SCIP in its default configuration (**default**), without diving heuristics at all (**nodiving**), and extended by the new adaptive diving heuristics (**adaptivediving**). The table shows the absolute and relative numbers of nodes (**nodes**, **nodes_Q**) and solving time (**time**, **time_Q**). In addition, the average number of feasible solutions found by **adaptivediving** is shown (**nsols**). The right-most column (**impr.sols**) indicates whether at least one improving solution was found by **adaptivediving** with at least one seed (✓). Relative changes by at least 5% are highlighted in bold and blue (**improvement**) or italic and red (**deterioration**).

Instance	default		nodiving				adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
10teams	212.7	14.0	471.8	2.22	15.9	1.14	58.1	0.27	9.2	0.66	—	✓
22433	3.0	2.0	3.7	1.23	2.2	1.08	3.0	1.01	1.6	0.78	1.0	✓
23588	542.7	4.2	529.6	0.98	4.2	0.99	489.2	0.90	4.2	0.99	—	✓
30n20b8	43.7	153.6	131.9	3.02	246.2	1.60	33.3	0.76	161.5	1.05	1.0	✓
Test3	1.9	4.2	1.9	1.00	3.9	0.93	1.3	0.69	3.5	0.84	0.3	✓
a1c1s1	90230.8	3600.0	88560.4	0.98	3600.0	1.00	88181.4	0.98	3600.0	1.00	7.3	✓
acc-tight5	569.2	99.4	512.9	0.90	72.7	0.73	817.3	1.44	123.7	1.24	—	✓
aflow30a	533.1	16.1	846.5	1.59	19.5	1.21	678.2	1.27	16.8	1.04	0.7	✓
aflow40b	20770.6	561.1	13399.8	0.65	521.8	0.93	16235.3	0.78	577.8	1.03	—	✓
air03	2.0	2.1	2.0	1.00	2.0	0.96	2.0	1.00	1.9	0.91	1.3	✓
air04	77.5	52.3	91.1	1.18	43.3	0.83	48.6	0.63	45.6	0.87	0.7	✓
air05	462.1	29.8	363.8	0.79	27.2	0.91	308.2	0.67	29.2	0.98	—	✓
aligninq	623.6	25.9	1509.8	2.42	25.9	1.00	1658.0	2.66	24.3	0.94	—	✓
app1-2	19.8	771.0	36.6	1.85	753.4	0.98	23.6	1.19	741.1	0.96	—	✓
arki001	632928.1	3600.0	569989.2	0.90	3600.0	1.00	538136.2	0.85	3600.0	1.00	4.0	✓
ash608gpia-3col	3.6	28.1	7.8	2.15	26.8	0.95	3.0	0.82	27.9	0.99	—	✓
atlanta-ip	6341.3	3600.0	5933.9	0.94	3600.0	1.00	4965.6	0.78	3600.0	1.00	—	✓
bab5	40843.0	3600.0	28727.2	0.70	3600.0	1.00	41647.2	1.02	3600.0	1.00	1.7	✓
bc	15073.0	989.1	16115.8	1.07	1034.9	1.05	17845.8	1.18	1086.5	1.10	0.3	✓
bc1	2074.6	186.1	4620.7	2.23	209.4	1.12	2391.8	1.15	163.5	0.88	0.3	✓
beasleyC3	27.4	28.6	119.6	4.37	34.9	1.22	123.5	4.51	41.9	1.46	14.7	✓
bell3a	2470.4	0.9	1651.6	0.67	0.8	0.81	1307.7	0.53	0.6	0.68	26.0	✓
bell5	367.0	0.5	465.5	1.27	0.5	1.00	605.8	1.65	0.5	1.05	43.3	✓
biella1	4831.4	1027.1	2463.6	0.51	686.8	0.67	3007.6	0.62	744.5	0.72	1.0	✓
bienst1	13512.7	115.3	11775.7	0.87	100.8	0.87	14638.0	1.08	119.5	1.04	4.7	✓
bienst2	70278.9	569.3	53181.0	0.76	434.3	0.76	55481.9	0.79	451.8	0.79	2.0	✓
binkar10_1	2453.0	31.5	2530.8	1.03	28.2	0.90	3255.5	1.33	40.1	1.27	1.3	✓
blend2	1355.4	1.4	742.1	0.55	0.7	0.52	1166.8	0.86	1.2	0.87	—	✓
bley_xl1	9.6	182.8	8.5	0.89	181.2	0.99	1.7	0.17	168.7	0.92	1.0	✓
bnatt350	9721.7	963.2	4776.2	0.49	467.1	0.48	4615.4	0.47	542.8	0.56	—	✓

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Table 3: Computational results over three random seeds on MMMc test set comparing SCIP in its default configuration (**default**), without diving heuristics at all (**nodiving**), and extended by the new adaptive diving heuristics (**adaptivediving**). The table shows the absolute and relative numbers of nodes (**nodes**, **nodes_Q**) and solving time (**time**, **time_Q**). In addition, the average number of feasible solutions found by **adaptivediving** is shown (**nsols**). The right-most column (**impr.sols**) indicates whether at least one improving solution was found by **adaptivediving** with at least one seed (✓). Relative changes by at least 5% are highlighted in bold and blue (**improvement**) or italic and red (**deterioration**).

Instance	default			nodiving			adaptivediving						
	nodes	time		nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
cap6000	2021.9	3.3		2195.9	<i>1.09</i>	3.0	0.89	1937.3	0.96	3.2	0.96	50.7	
core2536-691	164.5	196.3		149.9	0.91	137.3	0.70	83.2	0.51	130.1	0.66	0.3	✓
cov1075	37461.2	108.7		26781.9	0.71	69.8	0.64	21022.7	0.56	71.1	0.65	8.7	✓
csched010	227915.0	3600.0		207252.2	0.91	3600.0	1.00	204593.6	0.90	3505.2	0.97	0.3	✓
d10200	484293.5	3600.0		612265.4	<i>1.26</i>	3600.0	1.00	401617.9	0.83	3600.0	1.00	0.7	✓
d20200	61761.8	3600.0		164217.2	2.66	3600.0	1.00	73368.6	<i>1.19</i>	3600.0	1.00	1.7	✓
dano3_3	12.4	112.9		14.8	<i>1.19</i>	112.7	1.00	6.6	0.54	116.8	1.03	0.7	✓
dano3_4	10.4	136.9		10.2	0.97	134.8	0.98	9.0	0.86	150.6	<i>1.10</i>	1.0	✓
dano3_5	200.7	330.6		192.1	0.96	284.6	0.86	145.8	0.73	275.2	0.83	0.7	✓
dano3mip	460.6	3600.0		1042.9	<i>2.26</i>	3600.0	1.00	495.6	<i>1.08</i>	3600.0	1.00	—	
danooint	1181425.3	3558.1	1125317.4	0.95	3122.3	0.88	1113895.2	0.94	3231.1	0.91	1.7	✓	
dcmulti	86.0	2.4	55.9	0.65	2.3	0.97	99.5	<i>1.16</i>	2.0	0.85	1.3	✓	
dfn-gwin-UUM	22916.1	99.2	24228.9	<i>1.06</i>	99.8	1.01	23843.1	1.04	102.0	1.03	140.7	✓	
disctom	1.0	5.9	1.0	1.00	5.9	1.00	1.0	1.00	5.9	1.00	—		
ds	594.5	3600.0	1019.6	<i>1.72</i>	3600.0	1.00	454.2	0.76	3600.0	1.00	58.3	✓	
dsbmip	9.6	1.6	16.8	<i>1.76</i>	1.6	1.02	4.8	0.51	0.9	0.56	1.3	✓	
egout	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	1.0	✓	
eil33-2	631.9	67.2	679.0	<i>1.07</i>	73.8	<i>1.10</i>	729.4	<i>1.15</i>	73.8	<i>1.10</i>	1.3	✓	
eilB101	10544.5	239.0	9813.6	0.93	218.8	0.92	17189.5	<i>1.63</i>	336.4	<i>1.41</i>	6.3	✓	
enigma	935.3	0.5	431.0	0.46	0.5	0.96	326.1	0.35	0.5	0.96	—		
enlight13	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—		
enlight14	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—		
ex9	1.0	27.5	1.0	1.00	28.1	1.02	1.0	1.00	28.2	1.02	—		
fast0507	1025.5	235.9	803.7	0.78	175.9	0.75	745.5	0.73	189.5	0.80	7.7	✓	
fiball	5028.8	2113.6	9336.7	<i>1.86</i>	3491.6	<i>1.65</i>	4384.4	0.87	1847.5	0.87	1.0	✓	
fiber	4.0	1.7	4.6	<i>1.16</i>	1.9	<i>1.12</i>	4.0	1.00	1.7	0.98	3.3	✓	
fixnet6	3.3	4.7	3.0	0.90	4.6	0.97	3.3	1.00	5.1	<i>1.08</i>	10.0	✓	
flugpl	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	1.0	✓	
gen	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—		
germanrr	1325.5	3600.0	2893.7	<i>2.18</i>	3600.0	1.00	906.8	0.68	3600.0	1.00	3.3	✓	

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Table 3: Computational results over three random seeds on MMMc test set comparing SCIP in its default configuration (**default**), without diving heuristics at all (**nodiving**), and extended by the new adaptive diving heuristics (**adaptivediving**). The table shows the absolute and relative numbers of nodes (**nodes**, **nodes_Q**) and solving time (**time**, **time_Q**). In addition, the average number of feasible solutions found by **adaptivediving** is shown (**nsols**). The right-most column (**impr.sols**) indicates whether at least one improving solution was found by **adaptivediving** with at least one seed (✓). Relative changes by at least 5% are highlighted in bold and blue (**improvement**) or italic and red (**deterioration**).

Instance	default			nodiving				adaptivediving					
	nodes	time		nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
gesa2	1.3	0.5		1.3	1.00	0.5	1.00	1.7	<i>1.25</i>	0.5	1.00	1.0	✓
gesa2-o	2.6	0.9		2.6	1.00	0.8	0.94	2.0	0.76	0.7	0.78	1.0	✓
gesa3	8.8	3.4		6.9	0.79	3.2	0.95	5.9	0.68	3.9	<i>1.15</i>	1.3	✓
gesa3_o	6.6	2.8		7.3	<i>1.10</i>	4.0	<i>1.44</i>	6.0	0.90	3.3	<i>1.18</i>	0.3	✓
glass4	2077361.5	2920.8		2458448.3	1.18	3600.0	<i>1.23</i>	1030344.4	0.50	1401.2	0.48	6.0	✓
gmu-35-40	1461071.3	3600.0		3247969.2	<i>2.22</i>	3600.0	1.00	1687277.3	<i>1.16</i>	3600.0	1.00	2.0	✓
gt2	1.0	0.5		1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
haprp	1.0	0.5		1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
harp2	2203367.5	1079.0		2492591.7	<i>1.13</i>	1142.8	<i>1.06</i>	1644638.6	0.75	767.6	0.71	22.7	✓
iis-100-0-cov	86589.8	532.6		88130.1	1.02	511.1	0.96	86035.9	0.99	521.9	0.98	9.0	✓
iis-bupa-cov	174929.6	2006.0		158889.8	0.91	1725.9	0.86	160776.5	0.92	1924.5	0.96	10.3	✓
iis-pima-cov	7264.1	311.6		6351.2	0.87	270.2	0.87	6105.1	0.84	272.3	0.87	8.3	✓
khb05250	3.2	0.5		2.0	0.62	0.5	0.99	3.2	1.00	0.5	0.99	2.3	✓
l152lav	32.4	2.2		35.4	<i>1.09</i>	2.5	<i>1.12</i>	56.2	<i>1.74</i>	3.3	<i>1.49</i>	0.7	✓
lectsched-4-obj	1473.1	33.3		243.9	0.17	19.5	0.59	38.6	0.03	12.0	0.36	0.3	✓
leo1	45925.9	3600.0		56743.9	<i>1.24</i>	3600.0	1.00	57568.8	<i>1.25</i>	3600.0	1.00	0.3	✓
leo2	61837.4	3600.0		63021.1	1.02	3600.0	1.00	62135.3	1.00	3600.0	1.00	—	
liu	828433.0	3600.0		931693.0	<i>1.12</i>	3600.0	1.00	665812.4	0.80	3600.0	1.00	31.3	✓
lrn	1134.1	3600.0		1402.7	<i>1.24</i>	3600.0	1.00	1535.3	<i>1.35</i>	3600.0	1.00	3.0	✓
lseu	58.1	0.5		95.4	<i>1.64</i>	0.5	1.05	82.6	<i>1.42</i>	0.6	<i>1.13</i>	0.3	✓
m100n500k4r1	2235185.7	3600.0		2537595.1	<i>1.14</i>	3600.0	1.00	2329569.4	1.04	3600.0	1.00	8.0	✓
macrophage	9683.9	243.4		6345.3	0.66	146.3	0.60	7636.8	0.79	165.3	0.68	1.0	✓
manna81	1.0	0.5		1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
map18	266.1	261.6		271.4	1.02	222.6	0.85	256.2	0.96	332.9	<i>1.27</i>	0.7	✓
map20	291.9	254.2		265.4	0.91	186.2	0.73	236.9	0.81	250.4	0.98	0.3	✓
markshare1	19663456.6	3600.0		25464037.1	<i>1.29</i>	3600.0	1.00	20584380.8	1.05	3600.0	1.00	16.3	✓
markshare2	4423023.0	3600.0		10510623.5	<i>2.38</i>	3600.0	1.00	4377939.3	0.99	3600.0	1.00	13.0	✓
mas74	6338096.1	1695.5		4807477.7	0.76	1257.8	0.74	6424904.2	1.01	1644.7	0.97	2.0	✓
mas76	215133.6	89.3		203671.4	0.95	71.9	0.81	265165.3	<i>1.23</i>	97.8	<i>1.09</i>	0.7	✓
mcsched	9526.9	223.0		6921.6	0.73	171.3	0.77	8695.2	0.91	204.5	0.92	1.0	✓

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Instance	default			nodiving				adaptivediving					
	nodes	time		nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
mik-250-1-100-1	17563.0	54.7		30966.9	1.76	82.2	1.50	23661.6	1.35	67.8	1.24	20.7	
mine-166-5	1653.8	60.1		1005.9	0.61	48.6	0.81	570.9	0.34	71.3	1.19	0.7	✓
mine-90-10	25782.5	173.9		21434.8	0.83	188.9	1.09	31446.8	1.22	201.6	1.16	2.3	✓
misc03	31.0	0.7		17.8	0.57	0.7	0.88	25.2	0.81	0.7	0.97	0.3	✓
misc06	3.3	0.6		3.0	0.90	0.5	0.85	3.0	0.90	0.6	0.97	6.7	✓
misc07	8546.6	13.1		5314.6	0.62	9.7	0.73	4884.8	0.57	10.1	0.77	0.3	✓
mitre	1.0	10.2		1.0	1.00	10.2	1.00	1.0	1.00	10.2	1.00	—	
mkc	387333.6	3600.0		384365.8	0.99	3600.0	1.00	435964.5	1.13	3600.0	1.00	0.3	✓
mkc1	28036.1	203.7		320870.5	11.45	1125.8	5.53	455465.8	16.25	1398.1	6.86	1.0	✓
mod008	2.0	0.5		2.0	1.00	0.5	1.00	2.0	1.00	0.5	1.00	9.7	
mod010	2.0	0.5		2.0	1.00	0.5	1.00	2.0	1.00	0.5	1.00	1.0	✓
mod011	643.0	373.1		777.3	1.21	402.4	1.08	615.4	0.96	360.7	0.97	1.3	✓
modglob	2.0	0.5		2.0	1.00	0.5	1.00	2.0	1.00	0.5	1.00	4.3	✓
momentum1	7017.3	3600.0		11426.1	1.63	3600.0	1.00	12250.4	1.75	3600.0	1.00	—	
momentum2	40632.1	3600.0		36985.8	0.91	3600.0	1.00	33270.2	0.82	3600.0	1.00	1.3	✓
momentum3	134.8	3600.0		141.6	1.05	3600.0	1.00	77.3	0.57	3600.0	1.00	—	
msc98-ip	3522.6	3600.0		3635.1	1.03	3600.0	1.00	1850.0	0.53	3600.0	1.00	0.3	✓
mspp16	3.0	428.1		4.3	1.43	402.8	0.94	4.9	1.64	452.7	1.06	—	
mzzv11	1704.6	331.3		1538.3	0.90	305.4	0.92	958.2	0.56	264.1	0.80	1.0	✓
mzzv42z	177.3	161.4		276.4	1.56	167.8	1.04	66.8	0.38	181.0	1.12	1.0	✓
n3div36	88449.9	3600.0		91767.9	1.04	3600.0	1.00	89341.7	1.01	3600.0	1.00	—	
n3seq24	151.2	3600.0		459.2	3.04	3600.0	1.00	74.9	0.49	3600.0	1.00	—	
n4-3	2835.7	178.6		2449.8	0.86	159.7	0.89	3467.0	1.22	220.1	1.23	73.0	✓
nag	31388.4	3600.0		34045.5	1.08	3600.0	1.00	29353.0	0.94	3600.0	1.00	3.7	✓
neos-1053234	418752.8	2172.9		922222.1	2.20	2872.2	1.32	400453.6	0.96	2070.5	0.95	—	
neos-1053591	2377.9	3.8		2540.2	1.07	3.7	0.98	2128.5	0.90	3.3	0.87	1.0	✓
neos-1056905	4956678.8	3235.8		8191129.7	1.65	3600.0	1.11	5087261.1	1.03	2688.0	0.83	3.7	✓
neos-1058477	1.0	0.5		1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
neos-1061020	869.7	199.6		1145.4	1.32	221.1	1.11	866.2	1.00	205.0	1.03	—	
neos-1062641	1.0	0.5		1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	

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Instance	default			nodiving			adaptivediving						
	nodes	time		nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
neos-1067731	75928.2	3600.0		104378.6	<i>1.38</i>	3600.0	1.00	82587.7	<i>1.09</i>	3600.0	1.00	1.3	✓
neos-1096528	3942.2	1766.6		2716.6	0.69	1684.0	0.95	5263.6	<i>1.33</i>	2148.4	<i>1.22</i>	0.3	✓
neos-1109824	107.3	13.5		163.2	<i>1.52</i>	14.1	1.05	435.1	<i>4.05</i>	22.5	<i>1.67</i>	0.3	✓
neos-1112782	907703.2	3600.0		867034.3	0.95	3600.0	1.00	947311.6	1.04	3600.0	1.00	—	
neos-1112787	401700.9	3600.0		423880.7	<i>1.05</i>	3600.0	1.00	409889.8	1.02	3600.0	1.00	—	
neos-1120495	16.2	11.7		10.3	0.64	4.9	0.42	7.4	0.46	5.9	0.50	1.0	✓
neos-1121679	19581278.4	3600.0		25447433.5	<i>1.30</i>	3600.0	1.00	20612095.3	<i>1.05</i>	3600.0	1.00	16.3	✓
neos-1122047	1.0	6.0		1.0	1.00	6.0	1.00	1.0	1.00	6.0	1.01	—	
neos-1126860	4590.7	561.7		4983.3	<i>1.09</i>	479.4	0.85	4643.7	1.01	561.5	1.00	1.0	✓
neos-1140050	549.7	3600.0		407.3	0.74	3600.0	1.00	600.9	<i>1.09</i>	3600.0	1.00	—	
neos-1151496	29.8	10.3		94.0	<i>3.16</i>	19.1	<i>1.85</i>	59.5	<i>2.00</i>	16.7	<i>1.62</i>	—	
neos-1171448	15.1	19.6		79.1	<i>5.25</i>	52.2	<i>2.67</i>	1.0	0.07	13.9	0.71	0.7	✓
neos-1171692	170.2	33.0		4526.5	<i>26.60</i>	2672.2	<i>81.01</i>	8.3	0.05	10.7	0.32	0.7	✓
neos-1171737	1761.3	3600.0		2519.8	<i>1.43</i>	3600.0	1.00	2756.3	<i>1.56</i>	3600.0	1.00	0.3	✓
neos-1173026	1.0	0.5		1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	0.7	✓
neos-1200887	5173.7	10.6		5985.8	<i>1.16</i>	10.8	1.02	3588.3	0.69	9.0	0.85	1.0	✓
neos-1208069	1180.1	50.1		1305.2	<i>1.11</i>	53.3	<i>1.06</i>	824.9	0.70	59.0	<i>1.18</i>	—	
neos-1208135	3058.6	291.4		3717.6	<i>1.22</i>	182.2	0.62	2013.0	0.66	132.8	0.46	1.0	✓
neos-1211578	7614.2	4.4		5431.7	0.71	3.3	0.75	5667.8	0.74	3.4	0.79	1.0	✓
neos-1215259	1029.6	39.6		911.4	0.89	36.1	0.91	1141.0	<i>1.11</i>	49.1	<i>1.24</i>	—	
neos-1215891	3055.5	309.3		7638.3	<i>2.50</i>	533.9	<i>1.73</i>	2312.4	0.76	210.7	0.68	—	
neos-1223462	153.5	152.1		236.6	<i>1.54</i>	172.2	<i>1.13</i>	363.3	<i>2.37</i>	182.4	<i>1.20</i>	0.3	✓
neos-1224597	12.2	11.2		24.6	<i>2.02</i>	11.2	1.00	12.2	1.00	10.4	0.93	—	
neos-1225589	3.0	0.5		7.3	<i>2.44</i>	0.5	0.97	7.9	<i>2.64</i>	0.5	1.02	33.7	✓
neos-1228986	24206.5	12.4		45629.4	<i>1.89</i>	18.0	<i>1.45</i>	23074.2	0.95	11.1	0.89	1.0	✓
neos-1281048	41.6	7.2		39.8	0.96	5.7	0.79	35.1	0.84	6.2	0.86	0.3	✓
neos-1311124	7494756.8	3600.0		7226683.0	0.96	3600.0	1.00	7537735.7	1.01	3600.0	1.00	1.0	✓
neos-1324574	17185.2	1457.0		43359.1	<i>2.52</i>	3401.8	<i>2.33</i>	16764.4	0.98	1226.0	0.84	—	
neos-1330346	182663.5	3451.6		146965.9	0.81	3600.0	1.04	198130.5	<i>1.08</i>	2984.5	0.86	—	
neos-1330635	1.0	0.5		1.0	1.00	0.5	1.00	1.3	<i>1.32</i>	0.5	1.00	0.3	✓

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Instance	default		nodiving				adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
neos-1337307	354039.9	3600.0	339961.0	0.96	3600.0	1.00	350512.4	0.99	3600.0	1.00	0.3	✓
neos-1346382	6883767.2	3600.0	7740568.6	1.12	3600.0	1.00	5950992.0	0.86	3600.0	1.00	1.0	✓
neos-1354092	213.7	3600.0	258.0	1.21	3600.0	1.00	144.5	0.68	3600.0	1.00	—	
neos-1367061	1.0	25.9	1.0	1.00	26.1	1.00	1.0	1.00	26.1	1.01	—	
neos-1396125	11385.0	93.1	14679.1	1.29	92.7	1.00	20326.1	1.78	119.4	1.28	0.3	✓
neos-1407044	33.6	3600.0	41.9	1.25	3600.0	1.00	33.6	1.00	3600.0	1.00	—	
neos-1413153	2.3	3.2	2.3	1.00	3.2	1.00	2.9	1.26	3.4	1.06	1.3	✓
neos-1415183	1.7	4.5	1.7	1.00	4.5	1.00	1.7	1.00	4.8	1.08	0.7	✓
neos-1417043	1.0	1141.4	1.0	1.00	1140.7	1.00	1.0	1.00	1135.6	0.99	—	
neos-1420205	6094.2	4.0	10093.1	1.66	5.7	1.42	14549.0	2.39	8.4	2.08	0.3	✓
neos-1420546	836.1	3600.0	1342.0	1.60	3600.0	1.00	686.0	0.82	3600.0	1.00	4.7	✓
neos-1420790	73156.0	3600.0	93275.3	1.27	3600.0	1.00	48949.8	0.67	3600.0	1.00	7.3	✓
neos-1423785	16498.6	3600.0	10194.9	0.62	3600.0	1.00	22543.0	1.37	3600.0	1.00	6.0	✓
neos-1425699	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
neos-1426662	2592408.2	3600.0	3174356.0	1.22	3600.0	1.00	3082790.9	1.19	3600.0	1.00	1.0	✓
neos-1427181	1764466.7	1815.7	2450802.5	1.39	3600.0	1.98	958984.3	0.54	1018.7	0.56	1.0	✓
neos-1427261	560816.0	3600.0	1597704.9	2.85	3600.0	1.00	606443.5	1.08	3600.0	1.00	2.3	✓
neos-1429185	2500985.2	3600.0	3368842.0	1.35	3600.0	1.00	2608021.7	1.04	3600.0	1.00	1.3	✓
neos-1429212	1125.7	3600.0	1599.3	1.42	3600.0	1.00	966.5	0.86	3600.0	1.00	—	
neos-1429461	4009112.2	3600.0	4776886.7	1.19	3600.0	1.00	4324122.2	1.08	3600.0	1.00	1.3	✓
neos-1430701	35061.4	27.0	55174.3	1.57	33.5	1.24	43063.8	1.23	30.6	1.13	1.7	✓
neos-1430811	261.0	3600.0	905.8	3.47	3600.0	1.00	380.5	1.46	3600.0	1.00	—	
neos-1436709	2055406.2	3600.0	2410935.7	1.17	3600.0	1.00	2096401.2	1.02	3600.0	1.00	1.0	✓
neos-1436713	356491.2	3600.0	1013352.9	2.84	3600.0	1.00	450314.8	1.26	3600.0	1.00	1.7	✓
neos-1437164	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
neos-1439395	322812.0	179.9	609646.7	1.89	297.7	1.66	247735.4	0.77	135.0	0.75	1.0	✓
neos-1440225	2972.8	54.6	5014.9	1.69	83.0	1.52	1873.4	0.63	37.2	0.68	—	
neos-1440447	3342.2	3.9	4243.4	1.27	4.3	1.09	3604.4	1.08	4.6	1.18	1.0	✓
neos-1440457	1869959.8	3600.0	1930525.2	1.03	3600.0	1.00	1743319.3	0.93	3600.0	1.00	1.7	✓
neos-1440460	3845061.5	2496.2	5447207.8	1.42	3600.0	1.44	4703501.0	1.22	3600.0	1.44	1.3	✓

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Table 3: Computational results over three random seeds on MMMc test set comparing SCIP in its default configuration (**default**), without diving heuristics at all (**nodiving**), and extended by the new adaptive diving heuristics (**adaptivediving**). The table shows the absolute and relative numbers of nodes (**nodes**, **nodes_Q**) and solving time (**time**, **time_Q**). In addition, the average number of feasible solutions found by **adaptivediving** is shown (**nsols**). The right-most column (**impr.sols**) indicates whether at least one improving solution was found by **adaptivediving** with at least one seed (✓). Relative changes by at least 5% are highlighted in bold and blue (**improvement**) or italic and red (**deterioration**).

Instance	default		nodiving				adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
neos-1441553	1.3	1.9	1.3	1.00	1.6	0.84	1.0	0.76	1.4	0.73	0.3	✓
neos-1442119	1631305.1	3600.0	1950306.8	1.20	3600.0	1.00	1568703.4	0.96	3600.0	1.00	1.0	✓
neos-1442657	2337818.1	3600.0	2716061.2	1.16	3600.0	1.00	2305938.3	0.99	3600.0	1.00	1.0	✓
neos-1445532	1244.9	3600.0	2259.8	1.81	3600.0	1.00	2323.8	1.87	3600.0	1.00	1.0	✓
neos-1445738	8033.9	3600.0	7220.6	0.90	3600.0	1.00	9033.8	1.12	3600.0	1.00	4.3	✓
neos-1445743	51.1	44.5	19.3	0.38	51.6	1.16	3.3	0.07	52.2	1.17	4.3	✓
neos-1445755	36.2	45.4	67.0	1.85	49.5	1.09	156.8	4.33	43.2	0.95	5.0	✓
neos-1445765	147.9	41.8	124.1	0.84	40.1	0.96	256.5	1.74	44.7	1.07	6.0	✓
neos-1451294	2755.2	1154.5	4767.2	1.73	1701.7	1.47	1673.6	0.61	850.4	0.74	1.0	✓
neos-1456979	26421.6	3600.0	19765.3	0.75	3600.0	1.00	28764.2	1.09	3600.0	1.00	1.3	✓
neos-1460246	1564054.1	3600.0	3073044.6	1.97	3600.0	1.00	1673384.3	1.07	3600.0	1.00	—	
neos-1460265	113.6	5.5	127.9	1.13	5.9	1.06	99.6	0.88	4.6	0.84	1.0	✓
neos-1460543	4967.7	3600.0	10189.0	2.05	3600.0	1.00	5543.7	1.12	3600.0	1.00	3.0	✓
neos-1460641	318140.4	3600.0	248512.6	0.78	3600.0	1.00	295331.9	0.93	3600.0	1.00	2.7	✓
neos-1461051	2596.5	26.0	2541.2	0.98	25.7	0.99	2017.5	0.78	22.9	0.88	—	
neos-1464762	441568.6	3600.0	427882.2	0.97	3600.0	1.00	257345.7	0.58	3600.0	1.00	1.7	✓
neos-1467067	7214231.6	3600.0	7437876.4	1.03	3600.0	1.00	6794105.7	0.94	3600.0	1.00	1.0	✓
neos-1467371	427774.6	3600.0	325660.7	0.76	3600.0	1.00	377995.4	0.88	3600.0	1.00	0.7	✓
neos-1467467	70309.6	3600.0	89027.4	1.27	3600.0	1.00	38093.5	0.54	3600.0	1.00	0.3	✓
neos-1480121	127.7	0.5	330.4	2.59	5.5	10.97	119.9	0.94	4.2	8.43	1.7	✓
neos-1489999	27.9	2.8	26.9	0.96	2.6	0.94	38.7	1.39	3.7	1.33	0.3	✓
neos-1516309	1.0	0.5	1.0	1.00	0.5	1.01	1.0	1.00	0.5	1.00	—	
neos-1582420	860.1	44.1	486.0	0.56	30.8	0.70	215.7	0.25	29.7	0.67	0.3	✓
neos-1593097	27619.9	3600.0	24674.4	0.89	3600.0	1.00	25109.3	0.91	3600.0	1.00	1.7	✓
neos-1595230	34795.5	237.8	38402.7	1.10	198.6	0.83	24084.1	0.69	177.1	0.74	—	
neos-1597104	17.6	227.6	5.0	0.28	180.6	0.79	11.1	0.63	212.4	0.93	—	
neos-1599274	1.0	0.9	1.0	1.00	0.9	0.96	1.0	1.00	0.9	0.94	1.0	✓
neos-1601936	2893.9	2650.5	2107.6	0.73	2859.4	1.08	962.1	0.33	1534.9	0.58	1.0	✓
neos-1603512	13.0	2.0	13.7	1.05	1.9	0.96	8.8	0.67	1.9	0.92	—	
neos-1603518	20.6	5.8	28.9	1.40	6.4	1.10	26.0	1.26	6.1	1.06	—	

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Instance	default			nodiving			adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
neos-1603965	32040.5	3600.0	46357.4	1.45	3600.0	1.00	40267.8	1.26	3600.0	1.00	1.0	✓
neos-1605061	435.8	3600.0	428.3	0.98	3600.0	1.00	611.6	1.40	3600.0	1.00	—	
neos-1605075	1656.5	3600.0	1200.3	0.72	2733.4	0.76	1028.8	0.62	3600.0	1.00	1.7	✓
neos-1616732	1220610.5	3191.9	1238740.1	1.01	3383.6	1.06	1233346.4	1.01	3199.1	1.00	7.3	✓
neos-1620770	592673.2	3600.0	1065280.6	1.80	3600.0	1.00	610626.9	1.03	3600.0	1.00	—	
neos-1620807	1027.9	5.0	1562.3	1.52	5.7	1.14	2123.4	2.07	7.7	1.53	—	
neos-1622252	884058.2	3227.6	928864.0	1.05	2502.4	0.78	836096.5	0.95	3600.0	1.11	—	
neos-430149	32846.4	30.9	33951.8	1.03	30.3	0.98	23787.7	0.72	23.6	0.76	0.3	✓
neos-476283	419.3	140.1	500.9	1.20	138.8	0.99	638.7	1.52	156.5	1.12	1.3	✓
neos-480878	11810.1	46.3	9748.7	0.82	34.6	0.75	8826.7	0.75	38.3	0.83	29.0	✓
neos-494568	4.9	16.5	6.3	1.28	21.4	1.29	4.1	0.84	17.3	1.05	0.3	✓
neos-495307	71326.2	3600.0	104866.4	1.47	3600.0	1.00	57285.4	0.80	3600.0	1.00	134.7	
neos-498623	40.5	45.5	55.5	1.37	49.0	1.07	6.3	0.16	35.2	0.77	0.3	✓
neos-501453	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
neos-501474	2.0	0.5	2.0	1.00	0.5	1.00	2.0	1.00	0.5	1.00	—	
neos-503737	24214.7	536.5	14562.5	0.60	330.4	0.62	4680.7	0.19	225.8	0.42	1.3	✓
neos-504674	6176.5	42.6	5774.2	0.94	38.9	0.91	6190.0	1.00	44.8	1.05	1.0	✓
neos-504815	2257.0	14.4	2934.0	1.30	18.2	1.26	2245.2	0.99	14.8	1.03	1.0	✓
neos-506422	5285.9	48.6	3985.5	0.75	37.6	0.77	2743.3	0.52	35.3	0.72	2.0	✓
neos-506428	265.5	3516.6	186.3	0.70	3600.0	1.02	155.2	0.58	3600.0	1.02	0.7	✓
neos-512201	3177.8	28.3	2450.2	0.77	24.9	0.88	3421.7	1.08	29.0	1.02	1.7	✓
neos-522351	1.0	1.2	1.0	1.00	1.3	1.02	1.0	1.00	1.2	0.99	—	
neos-525149	1.0	3.3	7.8	7.85	4.4	1.35	1.0	1.00	2.6	0.80	1.0	✓
neos-530627	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
neos-538867	32928.4	82.0	34850.1	1.06	102.7	1.25	25291.6	0.77	64.8	0.79	1.0	✓
neos-538916	6947.7	36.3	5593.1	0.81	27.9	0.77	7852.7	1.13	33.8	0.93	1.0	✓
neos-544324	13.5	39.5	72.6	5.37	39.7	1.00	15.3	1.13	27.8	0.70	0.3	✓
neos-547911	221.4	28.6	191.7	0.87	19.7	0.69	41.2	0.19	16.9	0.59	—	
neos-548047	13599.1	3600.0	24630.6	1.81	3600.0	1.00	15773.8	1.16	3600.0	1.00	0.3	✓
neos-548251	789491.1	3600.0	2042422.3	2.59	3600.0	1.00	568904.1	0.72	3600.0	1.00	1.7	✓

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Instance	default			nodiving				adaptivediving						
	nodes	time		nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols	
neos-551991	671.0	139.5		1793.7	2.67	220.4	1.58	609.1	0.91	131.9	0.95	–		
neos-555001	19.8	2.4		1.0	0.05	1.0	0.40	9.0	0.46	1.6	0.64	–		
neos-555298	639.3	65.0		2261.2	3.54	77.9	1.20	134.5	0.21	42.7	0.66	1.3	✓	
neos-555343	315086.5	2328.1		276719.5	0.88	2421.1	1.04	273288.3	0.87	2198.4	0.94	1.0	✓	
neos-555424	114209.2	1065.6		374022.3	3.27	3600.0	3.38	98479.8	0.86	1043.1	0.98	1.7	✓	
neos-555694	16.9	9.0		16.2	0.96	5.1	0.57	1.7	0.10	2.4	0.26	0.7	✓	
neos-555771	24.1	6.3		6.3	0.26	2.9	0.45	5.5	0.23	3.4	0.53	1.3	✓	
neos-555884	252762.8	3600.0		359810.0	1.42	3600.0	1.00	235609.8	0.93	3600.0	1.00	1.3	✓	
neos-555927	1115513.2	3600.0		1156971.4	1.04	3600.0	1.00	1141441.7	1.02	3600.0	1.00	3.3	✓	
neos-565672	8.3	3486.6		6.0	0.72	3486.8	1.00	6.3	0.76	3522.9	1.01	1.3	✓	
neos-565815	1.0	10.2		1.0	1.00	9.5	0.93	1.0	1.00	8.4	0.82	–		
neos-570431	257.1	9.7		326.3	1.27	10.5	1.08	276.7	1.08	10.4	1.08	0.3	✓	
neos-574665	4377366.4	3600.0		4252382.8	0.97	3600.0	1.00	3708991.3	0.85	3600.0	1.00	1.0	✓	
neos-578379	1.6	179.6		1.6	1.00	179.6	1.00	1.9	1.18	177.2	0.99	–		
neos-582605	501081.5	3600.0		607130.3	1.21	3600.0	1.00	443537.2	0.89	3600.0	1.00	–		
neos-583731	16.7	6.6		16.7	1.00	6.6	1.00	20.7	1.24	7.1	1.07	–		
neos-584146	858994.0	3600.0		966703.3	1.12	3600.0	1.00	1057800.0	1.23	3600.0	1.00	–		
neos-584851	8.9	6.9		6.9	0.77	7.1	1.03	7.8	0.87	7.3	1.06	0.3	✓	
neos-584866	112403.3	3600.0		113208.1	1.01	3600.0	1.00	99188.6	0.88	3600.0	1.00	0.3	✓	
neos-585192	749.0	21.2		737.4	0.98	20.9	0.99	785.0	1.05	20.1	0.95	0.7	✓	
neos-585467	79.5	8.6		62.2	0.78	8.2	0.95	113.7	1.43	9.4	1.09	1.0	✓	
neos-593853	28543.2	71.4		12670.0	0.44	31.6	0.44	57948.9	2.03	120.1	1.68	0.7	✓	
neos-595904	2.7	16.2		3.6	1.37	15.3	0.94	3.3	1.24	17.7	1.09	0.3	✓	
neos-595905	2.6	2.8		3.0	1.13	2.9	1.03	2.0	0.76	2.6	0.91	1.0	✓	
neos-595925	27.6	11.4		252.6	9.15	16.1	1.42	27.6	1.00	12.8	1.13	0.3	✓	
neos-598183	5.0	5.9		4.0	0.80	4.8	0.82	6.0	1.21	5.7	0.97	–		
neos-603073	231538.1	1099.6		81134.4	0.35	324.7	0.29	129468.2	0.56	479.2	0.44	1.3	✓	
neos-611135	74338.8	3600.0		77543.2	1.04	3600.0	1.00	89741.7	1.21	3600.0	1.00	2.0	✓	
neos-611838	713.9	19.1		670.9	0.94	15.9	0.83	668.3	0.94	18.4	0.97	4.0		
neos-612125	310.9	14.1		340.3	1.09	13.4	0.95	280.4	0.90	11.0	0.78	3.7	✓	

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Instance	default			nodiving			adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
neos-612143	702.0	17.5	726.5	1.03	15.6	0.89	602.0	0.86	17.8	1.01	3.7	✓
neos-612162	631.0	15.0	606.0	0.96	14.9	0.99	605.5	0.96	16.1	<i>1.07</i>	3.7	✓
neos-619167	65.7	3600.0	65.2	0.99	3600.0	1.00	37.9	0.58	3600.0	1.00	—	—
neos-631164	188430.0	3600.0	449023.0	<i>2.38</i>	3600.0	1.00	117213.5	0.62	3600.0	1.00	13.0	✓
neos-631517	167948.0	3600.0	504179.0	<i>3.00</i>	3600.0	1.00	118475.0	0.70	3600.0	1.00	11.7	✓
neos-631694	541659.9	3600.0	161942.9	0.30	1696.0	0.47	12411.9	0.02	629.4	0.17	0.3	✓
neos-631709	763.3	3600.0	694.0	0.91	3600.0	1.00	369.7	0.48	3600.0	1.00	—	—
neos-631710	1.0	3600.0	1.0	1.00	3600.0	1.00	1.0	1.00	3600.0	1.00	—	—
neos-631784	177973.5	3600.0	18259.6	0.10	650.1	0.18	143580.3	0.81	2975.3	0.83	0.3	✓
neos-632335	193.0	10.4	193.0	1.00	10.4	1.00	193.0	1.00	10.4	1.00	—	—
neos-633273	259.0	11.1	259.0	1.00	11.0	0.99	259.0	1.00	11.1	1.00	—	—
neos-655508	1.0	1.4	1.0	1.00	1.4	1.01	1.0	1.00	1.4	1.01	—	—
neos-662469	19081.9	3600.0	9208.4	0.48	1484.4	0.41	26618.9	<i>1.40</i>	3600.0	1.00	3.3	✓
neos-686190	7938.0	107.4	4368.0	0.55	72.2	0.67	7759.5	0.98	107.6	1.00	—	—
neos-691058	3096.9	3600.0	3505.3	<i>1.13</i>	3600.0	1.00	2878.3	0.93	3600.0	1.00	1.0	✓
neos-691073	6801.0	3600.0	7842.4	<i>1.15</i>	3600.0	1.00	5827.3	0.86	3600.0	1.00	0.3	✓
neos-693347	19879.9	2418.9	14909.3	0.75	1811.6	0.75	15391.8	0.77	2426.3	1.00	0.7	✓
neos-702280	1031.5	3600.0	1228.0	<i>1.19</i>	3600.0	1.00	898.9	0.87	3600.0	1.00	9.0	—
neos-709469	189.7	0.7	153.0	0.81	0.7	1.04	48.7	0.26	0.5	0.78	—	—
neos-717614	8855.5	26.6	1414.6	0.16	7.4	0.28	4045.8	0.46	17.1	0.64	0.3	✓
neos-738098	396.1	3600.0	1052.4	<i>2.66</i>	3600.0	1.00	1011.9	<i>2.55</i>	3600.0	1.00	—	—
neos-775946	4.5	8.0	13.4	<i>2.99</i>	8.9	<i>1.11</i>	2.7	0.59	6.7	0.84	1.7	✓
neos-780889	1.0	96.2	1.9	<i>1.92</i>	122.6	<i>1.27</i>	1.0	1.00	90.5	0.94	0.7	✓
neos-785899	28.3	2.3	32.1	<i>1.14</i>	3.0	<i>1.33</i>	15.5	0.55	4.1	<i>1.79</i>	—	—
neos-785912	203.2	41.0	271.8	<i>1.34</i>	37.2	0.91	274.1	<i>1.35</i>	57.5	<i>1.40</i>	—	—
neos-785914	13.0	8.3	23.8	<i>1.84</i>	9.4	<i>1.13</i>	12.6	0.97	7.8	0.94	—	—
neos-787933	1.0	1.6	1.0	1.00	1.6	1.00	1.0	1.00	1.6	1.00	—	—
neos-791021	36.1	367.7	195.0	<i>5.41</i>	648.1	<i>1.76</i>	35.6	0.99	343.1	0.93	1.0	✓
neos-796608	141.9	2.1	21041.2	<i>148.30</i>	61.8	<i>29.40</i>	2.7	0.02	0.5	0.24	0.7	✓
neos-799838	1.0	32.0	3.2	<i>3.20</i>	47.1	<i>1.47</i>	1.0	1.00	33.1	1.03	0.7	✓

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Table 3: Computational results over three random seeds on MMMc test set comparing SCIP in its default configuration (**default**), without diving heuristics at all (**nodiving**), and extended by the new adaptive diving heuristics (**adaptivediving**). The table shows the absolute and relative numbers of nodes (**nodes**, **nodes_Q**) and solving time (**time**, **time_Q**). In addition, the average number of feasible solutions found by **adaptivediving** is shown (**nsols**). The right-most column (**impr.sols**) indicates whether at least one improving solution was found by **adaptivediving** with at least one seed (✓). Relative changes by at least 5% are highlighted in bold and blue (**improvement**) or italic and red (**deterioration**).

Instance	default			nodiving			adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
neos-801834	241.0	42.2	179.6	0.74	36.6	0.86	155.7	0.65	39.9	0.94	1.0	✓
neos-803219	12618.8	35.7	14849.1	<i>1.18</i>	34.6	0.97	12244.3	0.97	33.5	0.94	1.7	✓
neos-803220	44266.4	78.7	51057.4	<i>1.15</i>	80.2	1.02	42513.6	0.96	75.5	0.96	2.3	✓
neos-806323	7280.4	32.2	6827.6	0.94	30.2	0.94	7238.3	0.99	34.2	<i>1.06</i>	1.0	✓
neos-807454	1.0	2.0	1.0	1.00	2.0	0.99	1.0	1.00	2.0	0.99	—	
neos-807639	2506.2	15.9	3034.8	<i>1.21</i>	17.4	<i>1.10</i>	2528.7	1.01	15.9	1.00	1.3	✓
neos-807705	3331.6	26.8	5277.8	<i>1.58</i>	30.7	<i>1.15</i>	3326.6	1.00	27.5	1.02	3.7	✓
neos-808072	97.2	19.3	80.9	0.83	16.2	0.84	186.3	<i>1.92</i>	25.7	<i>1.33</i>	—	
neos-808214	1148.6	22.9	1546.0	<i>1.35</i>	24.6	<i>1.08</i>	795.5	0.69	18.8	0.82	—	
neos-810286	132.6	75.8	50.8	0.38	64.9	0.86	81.2	0.61	60.5	0.80	—	
neos-810326	823.9	39.8	513.1	0.62	28.8	0.72	1350.1	<i>1.64</i>	55.5	<i>1.39</i>	0.7	✓
neos-820146	1505505.0	3600.0	1554994.4	1.03	3600.0	1.00	1054142.6	0.70	3600.0	1.00	—	
neos-820157	1180763.2	3600.0	1343696.6	<i>1.14</i>	3600.0	1.00	1080849.3	0.92	3600.0	1.00	—	
neos-820879	256.1	52.8	396.4	<i>1.55</i>	48.3	0.92	178.4	0.70	66.5	<i>1.26</i>	0.7	✓
neos-824661	43.1	1298.1	212.3	<i>4.93</i>	1254.5	0.97	44.7	1.04	1243.0	0.96	—	
neos-824695	8.5	184.3	201.8	<i>23.85</i>	613.8	<i>3.33</i>	20.4	<i>2.42</i>	269.8	<i>1.46</i>	0.7	✓
neos-825075	4.4	1.9	3.8	0.85	1.8	0.94	1.7	0.37	1.7	0.88	0.7	✓
neos-826224	1.0	49.0	22.4	<i>22.45</i>	169.9	<i>3.46</i>	1.0	1.00	50.1	1.02	0.7	✓
neos-826250	7.1	137.9	13.5	<i>1.90</i>	127.5	0.93	1.3	0.19	80.9	0.59	0.3	✓
neos-826650	15642.6	3600.0	18816.8	<i>1.20</i>	3600.0	1.00	9255.5	0.59	3600.0	1.00	0.3	✓
neos-826694	6.5	174.5	39.3	<i>6.07</i>	272.9	<i>1.56</i>	1.0	0.15	102.5	0.59	—	
neos-826812	1.0	61.7	1.0	1.00	39.8	0.65	1.0	1.00	66.8	<i>1.08</i>	0.3	✓
neos-826841	82991.1	3600.0	77077.1	0.93	3600.0	1.00	71126.2	0.86	3600.0	1.00	0.7	✓
neos-827015	292.7	1138.5	269.8	0.92	932.6	0.82	291.3	0.99	1264.9	<i>1.11</i>	—	
neos-827175	1.0	12.1	1.0	1.00	9.1	0.75	1.0	1.00	12.3	1.01	0.7	✓
neos-829552	453.9	530.9	277.9	0.61	268.6	0.51	364.5	0.80	422.6	0.80	0.3	✓
neos-830439	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
neos-831188	2682.1	282.8	2837.9	<i>1.06</i>	262.7	0.93	2754.9	1.03	280.2	0.99	—	
neos-839838	23347.8	858.9	31126.9	<i>1.33</i>	1065.9	<i>1.24</i>	24314.9	1.04	873.0	1.02	1.3	✓
neos-839859	3254.8	47.7	2838.9	0.87	45.0	0.94	3342.6	1.03	51.3	<i>1.08</i>	0.3	✓

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Instance	default			nodiving			adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
neos-839894	277.1	3600.0	283.8	1.02	3600.0	1.00	206.5	0.74	3600.0	1.00	—	—
neos-841664	18450.7	3600.0	50623.4	2.74	3600.0	1.00	18012.5	0.98	3600.0	1.00	269.3	✓
neos-847302	154746.9	3600.0	104375.8	0.67	3600.0	1.00	195037.0	1.26	3600.0	1.00	1.7	✓
neos-848150	25.4	12.5	41.4	1.63	13.8	1.10	65.3	2.57	13.6	1.08	—	—
neos-848198	616.7	3600.0	2026.1	3.29	3600.0	1.00	516.6	0.84	3600.0	1.00	184.3	✓
neos-848589	104.3	3600.0	167.4	1.60	3600.0	1.00	101.2	0.97	3600.0	1.00	0.3	✓
neos-848845	2587.4	155.0	12070.0	4.67	245.9	1.59	14142.9	5.47	283.7	1.83	—	—
neos-849702	11836.6	268.5	6239.6	0.53	160.1	0.60	6346.0	0.54	175.8	0.66	—	—
neos-850681	1.3	4.2	6.0	4.53	6.4	1.53	1.3	1.00	4.8	1.14	—	—
neos-856059	190474.5	3117.3	210304.6	1.10	3389.9	1.09	72797.4	0.38	1405.9	0.45	9.0	✓
neos-859770	2.7	131.2	4.1	1.50	133.2	1.01	3.6	1.34	133.0	1.01	—	—
neos-860244	1.3	4.5	1.0	0.76	4.3	0.94	1.0	0.76	4.3	0.94	—	—
neos-860300	2.3	15.7	2.7	1.14	15.8	1.00	2.6	1.13	19.5	1.24	1.0	✓
neos-862348	86.7	9.3	87.0	1.00	9.1	0.98	38.8	0.45	9.9	1.07	1.0	✓
neos-863472	44960.0	48.0	38508.1	0.86	43.5	0.91	40379.5	0.90	43.8	0.91	—	—
neos-872648	15.9	3600.0	19.9	1.26	3600.0	1.00	14.6	0.92	3600.0	1.00	5.3	✓
neos-873061	27.6	3600.0	36.2	1.31	3600.0	1.00	40.1	1.45	3600.0	1.00	13.0	—
neos-876808	207.6	3600.0	459.8	2.21	3600.0	1.00	193.1	0.93	3600.0	1.00	0.7	✓
neos-880324	6.8	1.1	10.9	1.61	1.1	0.99	11.1	1.65	1.1	0.94	0.7	✓
neos-881765	17.9	2.0	17.9	1.00	2.0	0.99	55.8	3.11	2.4	1.20	—	—
neos-885086	274.8	3600.0	315.0	1.15	3600.0	1.00	129.3	0.47	1717.3	0.48	—	—
neos-885524	4585.4	3018.8	7955.3	1.74	1887.3	0.62	3002.9	0.66	1640.8	0.54	0.3	✓
neos-886822	178390.9	1976.9	116732.0	0.65	1470.8	0.74	177113.1	0.99	2174.5	1.10	0.3	✓
neos-892255	531.0	79.9	622.0	1.17	82.9	1.04	679.7	1.28	146.7	1.83	—	—
neos-905856	11281.3	134.5	5478.6	0.49	83.9	0.62	9109.0	0.81	128.8	0.96	—	—
neos-906865	16334.8	80.2	22002.9	1.35	116.4	1.45	18494.8	1.13	96.4	1.20	0.7	✓
neos-911880	1159496.9	1709.6	1733877.8	1.50	1959.2	1.15	2928989.1	2.53	3600.0	2.11	3.0	✓
neos-911970	3830083.0	3600.0	635148.8	0.17	1067.4	0.30	624932.8	0.16	938.3	0.26	1.0	✓
neos-912015	46.9	7.5	20.0	0.43	5.5	0.73	117.5	2.50	8.5	1.13	—	—
neos-912023	9.1	5.8	10.8	1.19	6.0	1.03	84.2	9.24	8.9	1.52	—	—

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Instance	default			nodiving				adaptivediving							
	nodes	time		nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols		
neos-913984	1.0	12.2		1.00	12.0	0.98		1.00	1.00	12.1	0.99	—			
neos-914441	1.0	5.0		1.00	5.0	1.00		1.7	1.66	6.6	1.31	1.3	✓		
neos-916173	13027.5	102.8		15257.2	1.17	108.5	1.06	16772.0	1.29	120.0	1.17	0.3	✓		
neos-916792	140610.8	1000.5		217878.2	1.55	1565.9	1.56	146682.8	1.04	1242.5	1.24	2.7	✓		
neos-930752	864.2	3600.0		2417.9	2.80	3600.0		1327.7	1.54	3600.0	1.00	1.7	✓		
neos-931517	874.7	3600.0		1237.9	1.42	3600.0	1.00	613.4	0.70	3600.0	1.00	1.3	✓		
neos-931538	5.0	99.6		5.0	1.00	70.0	0.70		3.5	0.70	70.3	0.71	0.3	✓	
neos-932721	18.3	22.1		19.5	1.06	18.8	0.85		2.2	0.12	12.2	0.55	—		
neos-932816	301.9	3600.0		673.4	2.23	3600.0	1.00	323.0	1.07	3600.0	1.00	2.0	✓		
neos-933364	17038.6	47.4		19695.6	1.16	57.4	1.21	7555.2	0.44	28.9	0.61	1.0	✓		
neos-933550	25.7	3.5		25.7	1.00	3.5	1.00	5.5	0.21	1.8	0.53	0.7	✓		
neos-933562	20014.7	3600.0		36440.8	1.82	3600.0	1.00	17020.4	0.85	3600.0	1.00	1.7	✓		
neos-933638	152.7	2862.4		477.2	3.13	3600.0	1.26	44.3	0.29	1752.6	0.61	—			
neos-933815	73373.1	89.8		32194.9	0.44	51.2	0.57	38306.8	0.52	92.9	1.03	1.3	✓		
neos-933966	133.1	2105.2		1296.0	9.73	2295.3	1.09	158.7	1.19	1558.6	0.74	0.7	✓		
neos-934278	220.7	3600.0		761.7	3.45	3600.0	1.00	244.1	1.11	3600.0	1.00	1.3	✓		
neos-934441	189.7	3600.0		446.9	2.36	3600.0	1.00	239.5	1.26	3600.0	1.00	1.0	✓		
neos-934531	3.6	111.2		4.0	1.12	82.5	0.74	6.5	1.79	94.5	0.85	—			
neos-935234	141.6	3600.0		285.8	2.02	3600.0	1.00	173.7	1.23	3600.0	1.00	1.3	✓		
neos-935348	76.6	3600.0		576.9	7.53	3600.0	1.00	291.6	3.81	3600.0	1.00	1.0	✓		
neos-935496	641435.5	3600.0		658733.2	1.03	3600.0	1.00	558638.0	0.87	3600.0	1.00	1.3	✓		
neos-935627	368.2	3600.0		472.8	1.28	3600.0	1.00	429.6	1.17	3600.0	1.00	0.7	✓		
neos-935674	610846.1	3600.0		661005.3	1.08	3600.0	1.00	459041.4	0.75	3600.0	1.00	1.7	✓		
neos-935769	420.5	3600.0		1824.5	4.34	3600.0	1.00	273.7	0.65	3600.0	1.00	0.7	✓		
neos-936660	511.2	3600.0		489.5	0.96	3600.0	1.00	317.0	0.62	3600.0	1.00	0.7	✓		
neos-937446	38.8	1229.1		1107.6	28.56	3600.0	2.93	81.1	2.09	2483.1	2.02	—			
neos-937511	268.2	3324.2		1158.3	4.32	3600.0	1.08	158.3	0.59	2158.6	0.65	2.3	✓		
neos-937815	390.6	3600.0		1750.0	4.48	3600.0	1.00	303.9	0.78	3600.0	1.00	2.0	✓		
neos-941262	396.1	3600.0		813.1	2.05	3600.0	1.00	195.6	0.49	3600.0	1.00	1.7	✓		
neos-941313	17.8	1731.1		18.8	1.06	1544.7	0.89	12.8	0.72	1554.0	0.90	—			

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Instance	default			nodiving			adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
neos-941698	174.4	13.2	167.0	0.96	12.9	0.98	42.9	0.25	7.3	0.55	—	✓
neos-941717	614427.9	3600.0	598685.0	0.97	3600.0	1.00	572084.3	0.93	3600.0	1.00	2.0	✓
neos-941782	817458.4	3600.0	694131.5	0.85	3600.0	1.00	704851.4	0.86	3600.0	1.00	1.3	✓
neos-942323	113.0	4.9	113.0	1.00	4.9	1.00	818.2	<i>7.24</i>	8.3	<i>1.69</i>	—	✓
neos-942830	324943.6	952.8	379201.9	<i>1.17</i>	1166.3	<i>1.22</i>	352690.1	1.08	974.1	1.02	0.3	✓
neos-942886	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	✓
neos-948126	298.9	3600.0	557.0	<i>1.86</i>	3600.0	1.00	223.7	0.75	3600.0	1.00	1.3	✓
neos-948268	1.0	12.1	1.0	1.00	12.1	1.00	1.0	1.00	12.1	1.00	—	✓
neos-948346	17.5	3600.0	45.7	<i>2.60</i>	3600.0	1.00	47.8	<i>2.73</i>	3600.0	1.00	0.3	✓
neos-950242	27.9	173.9	25.5	0.91	158.0	0.91	123.6	<i>4.43</i>	425.6	<i>2.45</i>	1.0	✓
neos-952987	1.0	3600.0	1.0	1.00	3600.0	1.00	1.0	1.00	3600.0	1.00	—	✓
neos-953928	62.3	400.4	69.7	<i>1.12</i>	373.3	0.93	17.8	0.29	272.9	0.68	—	✓
neos-954925	105.7	2173.4	570.1	<i>5.39</i>	3600.0	<i>1.66</i>	45.8	0.43	1218.4	0.56	—	✓
neos-955215	17478.0	18.7	17898.2	1.02	20.6	<i>1.10</i>	14791.5	0.85	15.4	0.83	1.0	✓
neos-955800	1250.8	45.5	1698.1	<i>1.36</i>	49.9	<i>1.10</i>	779.0	0.62	40.4	0.89	1.3	✓
neos-956971	141.5	2392.5	450.7	<i>3.18</i>	3269.7	<i>1.37</i>	43.1	0.30	1311.3	0.55	0.3	✓
neos-957143	129.2	2127.4	2126.6	<i>16.46</i>	3600.0	<i>1.69</i>	22.8	0.18	349.6	0.16	0.7	✓
neos-957270	1.0	2.1	1.0	1.00	2.1	1.00	1.0	1.00	2.1	1.00	—	✓
neos-957323	2.8	101.0	6.3	<i>2.22</i>	93.9	0.93	5.8	<i>2.06</i>	134.1	<i>1.33</i>	0.7	✓
neos-957389	1.0	12.5	1.0	1.00	12.3	0.99	1.0	1.00	12.4	0.99	—	✓
neos-960392	29.1	662.3	90.8	<i>3.12</i>	777.2	<i>1.17</i>	25.9	0.89	613.6	0.93	0.3	✓
neos-983171	247.0	3600.0	807.1	<i>3.27</i>	3600.0	1.00	148.2	0.60	3600.0	1.00	—	✓
neos-984165	343.3	3600.0	504.4	<i>1.47</i>	3600.0	1.00	276.6	0.81	3600.0	1.00	1.3	✓
neos13	18381.8	380.0	187287.8	<i>10.19</i>	2655.1	<i>6.99</i>	15453.4	0.84	350.6	0.92	6.3	✓
neos18	1300.3	18.5	1222.5	0.94	21.8	<i>1.18</i>	948.2	0.73	17.8	0.96	0.3	✓
net12	2375.9	1177.0	2112.9	0.89	818.2	0.69	2130.4	0.90	749.4	0.64	0.3	✓
netdiversion	29.2	1128.7	139.8	<i>4.79</i>	3091.9	<i>2.74</i>	12.7	0.43	1045.3	0.93	0.7	✓
newdano	720667.7	3600.0	642235.4	0.89	3600.0	1.00	730835.1	1.01	3600.0	1.00	1.3	✓
noswot	366545.4	90.8	916511.3	<i>2.50</i>	218.9	<i>2.41</i>	504470.5	<i>1.38</i>	124.7	<i>1.37</i>	0.3	✓
ns1208400	857.8	256.0	932.8	<i>1.09</i>	288.2	<i>1.13</i>	1379.0	1.61	283.5	<i>1.11</i>	0.3	✓

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Instance	default			nodiving			adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
ns1688347	790.2	71.2	834.8	<i>1.06</i>	68.8	0.97	1134.9	<i>1.44</i>	70.5	0.99	0.3	✓
ns1758913	1.3	2064.0	1.3	1.00	2051.7	0.99	1.0	0.76	1759.4	0.85	0.7	✓
ns1766074	891795.1	564.9	892001.5	1.00	562.6	1.00	899371.1	1.01	583.0	1.03	—	
ns1830653	5199.4	110.0	7136.5	<i>1.37</i>	134.2	<i>1.22</i>	5233.1	1.01	117.9	<i>1.07</i>	1.0	✓
nsa	207.8	1.9	208.5	1.00	1.3	0.71	211.9	1.02	1.9	1.01	—	
nsrand-ipx	54359.1	496.3	97719.9	<i>1.80</i>	846.3	<i>1.71</i>	36269.2	0.67	318.8	0.64	0.3	✓
nug08	1.7	63.2	2.0	<i>1.19</i>	52.4	0.83	1.3	0.80	58.4	0.93	0.7	✓
nw04	6.3	24.2	5.3	0.84	21.9	0.91	5.3	0.85	23.8	0.98	0.7	✓
opm2-z7-s2	2179.1	243.8	2734.0	<i>1.25</i>	304.6	<i>1.25</i>	2343.2	<i>1.07</i>	233.2	0.96	1.0	✓
opt1217	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
p0033	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
p0201	12.7	0.8	9.5	0.75	0.5	0.65	8.1	0.64	0.7	0.91	0.7	✓
p0282	2.3	0.5	2.3	1.00	0.5	0.99	2.3	1.00	0.5	0.94	3.3	✓
p0548	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
p2756	13.2	3.3	11.0	0.84	2.5	0.78	5.1	0.39	3.0	0.92	1.3	✓
p6b	323294.8	3600.0	219821.9	0.68	3600.0	1.00	304344.2	0.94	3600.0	1.00	43.0	✓
pg	393.2	17.4	322.1	0.82	13.6	0.78	346.9	0.88	16.0	0.92	1.3	✓
pg5_34	205199.5	2352.5	242974.3	<i>1.18</i>	2456.6	1.04	193454.2	0.94	2203.8	0.94	1.3	✓
pigeon-10	4019215.8	1619.4	11611815.7	<i>2.89</i>	3600.0	<i>2.22</i>	8387118.0	<i>2.09</i>	3046.2	<i>1.88</i>	0.3	✓
pk1	334752.2	105.4	370026.6	<i>1.10</i>	110.2	1.05	307638.2	0.92	99.4	0.94	2.3	✓
pp08a	207.8	1.4	200.2	0.96	1.4	0.99	212.1	1.02	1.5	<i>1.05</i>	10.3	✓
pp08aCUTS	122.2	1.8	133.7	<i>1.09</i>	1.9	<i>1.09</i>	118.3	0.97	1.9	1.04	9.0	✓
prod1	26945.2	18.2	47322.8	<i>1.76</i>	27.6	<i>1.52</i>	43144.3	<i>1.60</i>	29.5	<i>1.62</i>	2.3	✓
prod2	98039.7	104.4	122168.5	<i>1.25</i>	120.3	<i>1.15</i>	110835.1	<i>1.13</i>	111.4	<i>1.07</i>	9.3	
protfold	5453.4	3600.0	6804.1	<i>1.25</i>	3600.0	1.00	4773.2	0.88	3600.0	1.00	0.3	✓
pw-myciel4	299777.0	2220.1	162153.8	0.54	1018.3	0.46	266983.7	0.89	1849.0	0.83	1.0	✓
qap10	5.7	152.3	3.3	0.57	87.9	0.58	1.7	0.29	119.4	0.78	0.7	✓
qiu	2580.1	24.2	2512.8	0.97	20.9	0.86	3918.7	<i>1.52</i>	28.1	<i>1.16</i>	8.3	✓
qnet1	15.7	3.7	9.7	0.62	3.8	1.03	2.6	0.17	1.5	0.40	5.7	✓
qnet1_o	2.0	1.8	5.1	<i>2.54</i>	1.8	0.98	2.3	<i>1.16</i>	2.2	<i>1.23</i>	1.0	✓

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Instance	default		nodiving				adaptivediving					
	nodes	time	nodes	nodes _Q	time	time _Q	nodes	nodes _Q	time	time _Q	nsols	impr.sols
t1717	2003.0	3600.0	2313.9	<i>1.16</i>	3600.0	1.00	1277.3	0.64	3600.0	1.00	2.3	✓
tanglegram1	34.7	400.0	60.1	<i>1.73</i>	490.5	<i>1.23</i>	42.6	<i>1.23</i>	452.6	<i>1.13</i>	—	
tanglegram2	4.2	6.8	3.0	0.71	5.2	0.77	4.2	1.00	6.3	0.93	0.3	✓
timtab1	47140.5	55.1	30422.2	0.65	42.0	0.76	49461.7	1.05	56.3	1.02	4.0	✓
timtab2	1764544.1	3600.0	1688186.0	0.96	3600.0	1.00	1741605.1	0.99	3600.0	1.00	1.0	✓
tr12-30	384457.8	618.9	387094.5	1.01	603.7	0.97	390286.7	1.01	655.6	<i>1.06</i>	2.0	✓
triptim1	1.7	456.9	8.5	5.13	824.5	<i>1.80</i>	1.7	1.00	511.3	<i>1.12</i>	1.0	✓
unical_7	81.1	280.0	185.9	<i>2.29</i>	285.2	1.02	69.1	0.85	262.0	0.94	1.7	✓
vpm1	1.0	0.5	1.0	1.00	0.5	1.00	1.0	1.00	0.5	1.00	—	
vpm2	249.4	1.9	240.6	0.96	1.8	0.93	239.1	0.96	2.0	1.03	1.0	✓
vpphard	1079.3	3600.0	2446.2	<i>2.27</i>	3600.0	1.00	1294.4	<i>1.20</i>	3600.0	1.00	2.3	✓
zib54-UUE	87933.1	1734.3	76307.0	0.87	1363.8	0.79	62251.4	0.71	1382.7	0.80	43.0	✓
geom.	870.56	131.06	1066.52	1.23	132.64	1.01	763.29	0.88	125.22	0.96		
shgeom. (100/1)	2532.32	150.82	2901.74	1.15	152.81	1.01	2375.66	0.94	144.57	0.96		

