Adaptive Antenna Arrays for OFDM Systems With Cochannel Interference

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***Abstract*—Orthogonal frequency-division multiplexing (OFDM) is one of the promising techniques for future mobile wireless data systems. For OFDM systems with cochannel interference, adaptive antenna arrays can be used for interference suppression. This paper focuses on a key issue for adaptive antenna arrays, that is, parameter estimation for the *minimum mean square error* (MMSE) *diversity combiner* (DC). Using the instantaneous correlation estimation approach developed in the paper, an *original parameter estimator* for the MMSE-DC is derived. Based on the original estimator, we propose an *enhanced parameter estimator*. Extensive computer simulation demonstrates that the MMSE-DC using the proposed parameter estimators can effectively suppress both synchronous and asynchronous interference in OFDM systems for packet and continuous data transmission.**

***Index Terms*—Interference suppression, MMSE diversity combiner, mobile wireless channel, parameter estimation.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) [1]–[5] is one of the promising techniques for achieving the high-speed data rate required in future wireless data systems. OFDM increases the symbol duration by dividing the entire channel into many narrow subchannels and transmitting data in parallel. Therefore, it is one of the most effective techniques for combatting multipath delay spread over mobile wireless channels. For OFDM systems with cochannel interference, adaptive antenna arrays are desirable. These require that the parameters for the *minimum mean square error* (MMSE) diversity combiner (DC) be estimated.

For OFDM systems without cochannel interference, channel parameter estimation [3], [6]–[9] has been investigated to improve system performance by allowing for coherent demodulation. Moreover, for systems with receiver diversity, the maximum-ratio (MR) DC, which is equivalent to the MMSE-DC in this case, can be obtained using estimated channel parameters. In [7] and [8], a channel estimator for OFDM systems has been developed based on the singular-value-decomposition or frequency-domain filtering. Time-domain filtering has been proposed in [3] and [6] to further improve the performance of channel estimators. In [9], we have studied a robust channel estimator for OFDM systems based on both the time- and frequency-domain filtering. This estimator is not as sensitive to the channel statistics compared Wiener or MMSE estimator.

Adaptive antenna arrays [10]–[14] have been successfully used in TDMA mobile wireless systems to mitigate rapid dispersive fading, suppress cochannel interference, and, therefore, improve communication capacity. For systems with flat fading, the direct matrix inversion (DMI) [10], [11] or the diagonal loading DMI (DMI/DL) [12] algorithm for antenna diversity can be used to enhance desired signal reception and suppress interference effectively. The DMI/DL algorithm [13], [14] can be also used for spatial-temporal equalization in TDMA systems to suppress both intersymbol and cochannel interference.

In this paper, we study the use of adaptive antenna arrays in the OFDM systems to suppress cochannel interference. The difficulty of adaptive antenna arrays for OFDM systems stems from the fast change of parameters for the MMSE-DC because OFDM systems have much longer symbol duration than that of single carrier or TDMA systems. Hence, the parameter estimation approaches for TDMA systems [12]–[14] are not applicable to OFDM systems. Our investigation here, therefore, emphasizes the parameter estimation for the MMSE-DC for both packet and continuous data transmission.

The rest of the paper is organized as follows. Section II describes adaptive antenna arrays for OFDM systems with cochannel interference. Section III introduces a basic approach to estimate the instantaneous correlation required to calculate the parameters of the MMSE-DC. Next, Section IV develops improved approaches for channel parameter and instantaneous correlation estimations, which, therefore, enhance the parameter estimator for the MMSE-DC. Finally, Section V presents extensive computer simulation results to demonstrate the effectiveness of adaptive antenna arrays for OFDM systems.

II. OFDM Systems with Adaptive Antenna Arrays

In this section, we first introduce the mathematical model of OFDM systems with receiver diversity and then describe adaptive antenna arrays for systems with cochannel interference.

A. OFDM Systems with Receiver Diversity

The OFDM system with receiver diversity considered in this paper is shown in Fig. 1. $b[n, k]$'s, binary data to be transmitted, are coded into $a[n, k]$'s across tones of each OFDM block using the Reed–Solomon (R–S) code to correct the burst errors resulting from frequency-selective fading. $a[n, k]$'s are then modulated into $s_0[n, k]$'s using PSK modulation. Since the

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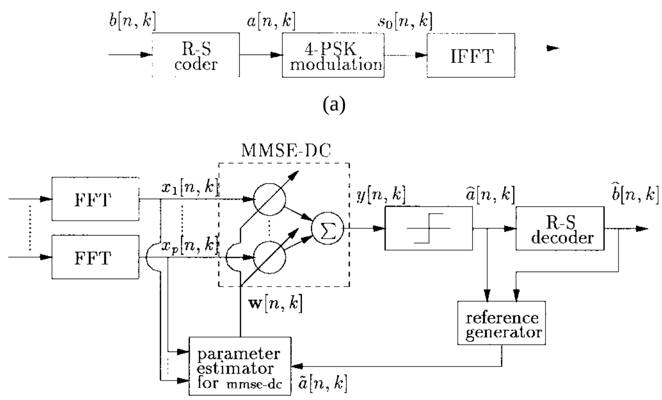


Fig. 1. OFDM system with the MMSE diversity combiner.

phase shift of each subchannel can be recovered by means of the MMSE-DC, coherent phase-shift keying (PSK) modulation is used in the system to exploit an about 3-dB SNR advantage over differential PSK (DPSK).

Let the entire channel bandwidth be divided into K subchannels and the OFDM block length be T_s. For p-branch receiver diversity systems with cochannel interference, the received signal from the mth antenna at the kth tone of the nth block can be expressed as

$$x_m[n, k] = H_m^{(0)}[n, k]s_0[n, k] + v_m[n, k] \tag{1}$$

for all n and $k = 0, 1, \cdots, K-1$, where $s_0[n, k]$ is the desired data from the transmitter at the corresponding block and tone, $H_m^{(0)}[n, k]$ is the frequency response for the desired signal from the mth antenna at the corresponding block and tone, and $v_m[n, k]$ includes additive complex white Gaussian noise and cochannel interference.

If an OFDM system has L cochannel interferers, then $v_m[n, k]$ can be expressed as

$$v_m[n, k] = \sum_{l=1}^{L} H_m^{(l)}[n, k]s_l[n, k] + n_m[n, k] \tag{2}$$

where $H_m^{(l)}[n, k]$'s for $l = 1, 2, \cdots, L$ are the frequency responses corresponding to the lth cochannel interferer at the mth antenna at the corresponding block and tone, $s_l[n, k]$'s for $l = 1, 2, \cdots, L$ are the complex data from the lth cochannel interferer, and $n_m[n, k]$ is the additive complex white Gaussian noise, with zero-mean and variance σ^2, from the mth antenna.

Note that, in the above discussions, we have assumed that cochannel interferers and the desired signal are synchronized to simplify the analysis, even though it is not necessarily true for wireless networks. However, the effects of synchronous and asynchronous interference on the system are similar, as shown by the simulation result in Section V-E. We have also ignored the effects of timing and frequency offsets on the system by assuming that they have been well taken care of by using timing and frequency estimation.

In this paper, we assume that both the desired and interfering data are *independent, identically distributed* (i.i.d.) complex random variables with zero-mean and unit variance. We also assume that $H_m^{(l)}[n, k]$'s for different m's or l's are independent, stationary, and complex Gaussian with zero-mean, but different variance σ_l^2. Hence, for $i, j = 1, \cdots, p$ and $l_1, l = 0, \cdots, L$

$$E\Big\{H_i^{(l_1)}[n+n_1, k+k_1]H_j^{(l)^*}[n_1, k_1]\Big\} = \begin{cases} \sigma_l^2 r_H[n, k], & \text{if } i = j \text{ ; } l_1 = l, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

It has been demonstrated in [9] that for time-varying dispersive, Rayleigh fading channels

$$r_H[n, k] = r_t[n]r_f[k] \tag{4}$$

that is, the correlation function of the channel frequency responses can be separated into the multiplication of a *time*-domain correlation $r_t[n]$ and a *frequency*-domain correlation $r_f[k]$. $r_t[n]$ is dependent on the vehicle speed or, equivalently, the Doppler frequency [15], while $r_f[k]$ depends on the delay profile of the wireless channel. With this *separation property*, we are able to simplify our instantaneous correlation estimator described in the next section.

B. Adaptive Antenna Arrays for OFDM Systems

For OFDM systems without cochannel interference [9], the MR-DC can be obtained with knowledge of the channel parameters only, which is equivalent to the MMSE-DC. However, for OFDM systems with cochannel interference, both the *instantaneous* correlation of the received signals and the channel parameters for the desired signal have to be known to obtain the MMSE-DC.

Let $r_{ij}[n, k]$ be the instantaneous correlation of the received signals from the ith and the jth antennas corresponding to the same block and tone, defined as

$$\begin{aligned} r_{ij}[n, k] &\triangleq E_c\{x_i[n, k]x_j^*[n, k]\} \\ &= \sum_{l=0}^{L} H_i^{(l)}[n, k]H_j^{(l)^*}[n, k] + \sigma^2\delta[i-j] \end{aligned} \tag{5}$$

where E_c is the *conditional expectation* given the channel parameters corresponding to both the desired signal and interference. With $r_{ij}[n, k]$'s and $H_i^{(0)}[n, k]$'s, the parameters of the MMSE-DC can be calculated by the DMI/DL algorithm [12] as

$$\mathbf{w}[n, k] = (\mathbf{R}[n, k] + \gamma\mathbf{I})^{-1}\mathbf{H}^{(0)}[n, k] \tag{6}$$

where $\mathbf{I}$ is a $p \times p$ identity matrix, $\mathbf{R}[n, k]$ is a $p \times p$ matrix defined as

$$\mathbf{R}[n, k] \triangleq \begin{pmatrix} r_{11}[n, k] & r_{12}[n, k] & \cdots & r_{1p}[n, k] \\ r_{21}[n, k] & r_{22}[n, k] & \cdots & r_{2p}[n, k] \\ \vdots & \vdots & \cdots & \vdots \\ r_{p1}[n, k] & r_{p2}[n, k] & \cdots & r_{pp}[n, k] \end{pmatrix} \tag{7}$$

$\mathbf{H}^{(0)}[n, k]$ is a $p \times 1$ vector defined as

$$\mathbf{H}^{(0)}[n, k] \triangleq \left(H_1^{(0)}[n, k], \cdots, H_p^{(0)}[n, k]\right)^T \tag{8}$$

and γ in (5) is a diagonal loading factor that can be determined by the strategies discussed in [12]–[14]. As indicated in [12], diagonal loading here will prevent the singularity due to the matrix inversion and improve the performance of adaptive antenna arrays.

With the parameter vector $\mathbf{w}[n, k]$, the desired signal can be estimated as

$$y[n, k] = \mathbf{w}^H[n, k]\mathbf{x}[n, k] \tag{9}$$

where $\mathbf{x}[n, k]$ is the received signal vector defined as

$$\mathbf{x}[n, k] \triangleq (x_1[n, k], x_2[n, k], \cdots, x_p[n, k])^T. \tag{10}$$

III. Instantaneous Correlation Estimation

As indicated in Section II-B, to obtain the parameters for the MMSE-DC in OFDM systems, both the channel parameters and instantaneous correlations of the received signals have to be estimated. We have already developed a robust channel parameter estimator in [9]. The robust channel parameter estimator makes full use of the time- and frequency-domain correlations of channel parameters, and therefore, it is able to estimate channel parameters under low signal-to-noise ratio. Hence, we focus on the instantaneous correlation estimation here.

A. Configuration of Estimator

Since $r_{ij}[n, k]$'s are correlated for different blocks and tones, the MMSE estimator for $r_{ij}[n, k]$ can be constructed by

$$\hat{r}_{ij}[n, k] = \sum_{m=-\infty}^{+\infty} \sum_{l=-(K-k)}^{k-1} f_{ij}[m, l, k]\tilde{r}_{ij}[n-m, k-l] \tag{11}$$

where $f_{ij}[m, l, k]$'s are selected to minimize

$$\text{MSE}(\{f[m, l, k]\}) = E|\hat{r}_{ij}[n, k] - r_{ij}[n, k]|^2 \tag{12}$$

and $\tilde{r}_{ij}$ is the temporal estimation of the instantaneous correlation between the signals from the ith and the jth antennas defined as

$$\tilde{r}_{ij}[n, k] \triangleq x_i[n, k]x_j^*[n, k]. \tag{13}$$

Using the *orthogonality principle* [16], the $f[m, l, k]$'s are determined by

$$E\{(\hat{r}_{ij}[n, k] - r_{ij}[n, k])\tilde{r}_{ij}^*[n-m, k-l]\} = 0 \tag{14}$$

for $l = -(K-k), \cdots, (k-1)$ and $m = 0, \pm1, \pm2, \cdots$, or equivalently

$$\sum_{m_1=-\infty}^{+\infty} \sum_{l_1=-(K-k)}^{k-1} f_{ij}[m_1, l_1, k]o_{ij}[m-m_1, l-l_1] = p_{ij}[m, l] \tag{15}$$

where

$$o_{ij}[m, l] \triangleq E(\tilde{r}_{ij}[n+m, k+l]\tilde{r}_{ij}^*[n, k]) \tag{16}$$

and

$$p_{ij}[m, l] \triangleq E(r_{ij}[n+m, k+l]\tilde{r}_{ij}^*[n, k]). \tag{17}$$

From the Appendix, (16) and (17) can be written as

$$p_{ij}[m, l] = c_0 s_t[m]s_f[l] + c_1\delta[i-j] \tag{18}$$

and

$$o_{ij}[m, l] = c_0 s_t[m]s_f[l] + (c_1 - c_0)\delta[m, l] + c_1\delta[i-j] \tag{19}$$

where

$$s_t[m] \triangleq |r_t[m]|^2 \qquad s_f[l] \triangleq |r_f[l]|^2 \tag{20}$$

$$c_0 = \sum_{l=0}^{L} \sigma_l^4 \quad \text{and} \quad c_1 = \left(\sum_{l=0}^{L} \sigma_l^2 + \sigma^2\right)^2. \tag{21}$$

Equation (15) can be written in matrix form as

$$\sum_{m_1=-\infty}^{\infty} \mathbf{O}_{ij}[m-m_1]\mathbf{F}_{ij}[m_1] = \mathbf{P}_{ij}[m] \tag{22}$$

where $\mathbf{F}_{ij}[m]$, $\mathbf{O}_{ij}[m]$, and $\mathbf{P}_{ij}[m]$ are as shown in (23)–(25) at the bottom of the next page, where $\mathbf{I}$ is a $K \times K$ identity matrix, $\mathbf{E}$ is a $K \times K$ matrix with all elements being one, and $\mathbf{S}_f$ is a $K \times K$ matrix defined as shown in (26) at the bottom of the next page.

Using the discussions in [7]–[9] and the property of the discrete Fourier transform (DFT) [17]

$$\mathbf{S}_f = \mathbf{WAW}^H \tag{27}$$

where $\mathbf{W}$ is a DFT matrix defined as

$$\mathbf{W} \triangleq \frac{1}{\sqrt{K}} \cdot \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j(2\pi/K)} & \cdots & e^{-j((2\pi(K-1))/K)} \\ \vdots & \cdots & \cdots & \vdots \\ 1 & e^{-j((2\pi(K-1))/K)} & \cdots & e^{-j((2\pi(K-1)(K-1))/K)} \end{pmatrix} \tag{28}$$

and $\mathbf{A}$ is a diagonal matrix with elements

$$a_k = \frac{1}{K} d_k \otimes d_{-k}^* \tag{29}$$

with $\otimes$ denoting the circular convolution with modulo K, d_k's being the eigenvalues of the nonnegative definite matrix $\mathbf{R}_f = (r_f[i-j])_{ij=1}^K$. Note that

$$\sum_{k=0}^{K-1} a_k = \text{Tr}\{\mathbf{S}_f\} = \sum_{k=0}^{K-1} |r_f[0]|^2 = K \tag{30}$$

where $\text{Tr}\{\mathbf{X}\}$ is the trace of the square matrix $\mathbf{X}$, defined as the summation of its diagonal elements. According to [9], for a channel with a maximum delay spread t_d, $d_k = 0$ for $k \geq K_o$ $(=\lceil (t_d/T_s)K \rceil)$, where $\lceil x \rceil$ denotes the smallest integer larger than x. Hence, $a_k = 0$ for $K_o \leq k \leq K - K_o$.

Thus, $\mathbf{P}_{ij}[m]$'s and $\mathbf{O}_{ij}[m]$'s for $m = 0, \pm1, \pm2, \cdots$ can be simultaneously diagonalized by $\mathbf{W}$ into

$$\mathbf{P}_{ij}[m] = \mathbf{WD}_p[m]\mathbf{W}^H \tag{31}$$

and

$$\mathbf{O}_{ij}[m] = \mathbf{W}\mathbf{D}_o[m]\mathbf{W}^H \tag{32}$$

where $\mathbf{D}_p[m]$ and $\mathbf{D}_o[m]$ are diagonal matrices with

$$d_{p,k}[m] = a_k c_0 s_t[m] + K c_1 \delta[i-j]\delta[k] \tag{33}$$
$$d_{o,k}[m] = a_k c_0 s_t[m] + (c_1 - c_0)\delta[m] + K c_1 \delta[i-j]\delta[k] \tag{34}$$

respectively.

Equation (22) can be also expressed in the frequency domain as

$$\mathbf{O}_{ij}(\omega)\mathbf{F}_{ij}(\omega) = \mathbf{P}_{ij}(\omega) \tag{35}$$

where

$$\mathbf{F}_{ij}(\omega) \triangleq \sum_{m=-\infty}^{\infty} \mathbf{F}_{ij}[m]e^{-jm\omega} \tag{36}$$
$$\mathbf{P}_{ij}(\omega) \triangleq \sum_{m=-\infty}^{\infty} \mathbf{P}_{ij}[m]e^{-jm\omega} = \mathbf{W}\mathbf{D}_p(\omega)\mathbf{W}^H \tag{37}$$
$$\mathbf{O}_{ij}(\omega) \triangleq \sum_{m=-\infty}^{\infty} \mathbf{O}_{ij}[m]e^{-jm\omega} = \mathbf{W}\mathbf{D}_o(\omega)\mathbf{W}^H. \tag{38}$$

$\mathbf{D}_p(\omega)$ and $\mathbf{D}_o(\omega)$ are diagonal matrices with diagonal elements

$$\begin{aligned} d_{p,k}(\omega) &\triangleq \mathcal{F}\{d_{p,k}[m]\} \\ &= a_k c_0 s_t(\omega) + 2\pi K c_1 \delta[i-j]\delta[k]\delta(\omega) \end{aligned} \tag{39}$$

and

$$\begin{aligned} d_{o,k}(\omega) &\triangleq \mathcal{F}\{d_{o,k}[m]\} \\ &= a_k c_0 s_t(\omega) + (c_1 - c_0) + 2\pi K c_1 \delta[i-j]\delta[k]\delta(\omega) \end{aligned} \tag{40}$$

respectively.

Therefore, the parameters $f_{ij}[m, l, k]$'s for the MMSE estimation of $r_{ij}[n, k]$ are determined by

$$\mathbf{F}_{ij}(\omega) = \mathbf{O}_{ij}^{-1}(\omega)\mathbf{P}_{ij}(\omega) = \mathbf{W}\Psi(\omega)\mathbf{W}^H \tag{41}$$

where $\Psi(\omega)$ is a diagonal matrix with

$$\psi_k(\omega) = \frac{a_k c_0 s_t(\omega) + 2\pi K c_1 \delta(\omega)\delta[i-j]\delta[k]}{a_k c_0 s_t(\omega) + (c_1 - c_0) + 2\pi K c_1 \delta(\omega)\delta[i-j]\delta[k]}. \tag{42}$$

From (33) and (34), there are dc components contained in both $d_{p,k}[m]$ and $d_{o,k}[m]$ when $i = j$, which causes $\psi_1(\omega)$ in (42) to be discontinuous at $\omega = 0$. If the discontinuity of $\psi(\omega)$ at $\omega = 0$ is ignored by letting $\psi(0) = \lim_{\omega \to 0} \psi(\omega)$, then the dc component in $r_{ii}[n, k]$ is affected. However, the diagonal loading algorithm for diversity combining can compensate for the lost dc component. Hence, in the following discussion, we let

$$\psi_k(\omega) = \frac{a_k c_0 s_t(\omega)}{a_k c_0 s_t(\omega) + (c_1 - c_0)} \tag{43}$$

for $i, j = 1, 2, \cdots, p$ and $k = 0, 1, \cdots, K-1$.

The above discussion suggests the configuration of the instantaneous correlation estimator, shown in Fig. 2.

$$\mathbf{F}_{ij}[m] \triangleq \begin{pmatrix} f_{ij}[m, -(K-1), 1] & f_{ij}[m, -(K-2), 2] & \cdots & f_{ij}[m, 0, K] \\ f_{ij}[m, -(K-2), 1] & f_{ij}[m, -(K-3), 2] & \cdots & f_{ij}[m, 1, K] \\ \vdots & \vdots & \vdots & \vdots \\ f_{ij}[m, 0, 1] & f_{ij}[m, 1, 2] & \cdots & f_{ij}[m, K-1, K] \end{pmatrix} \tag{23}$$

$$\mathbf{O}_{ij}[m] \triangleq \begin{pmatrix} o_{ij}[m, 0] & o_{ij}[m, 1] & \cdots & o_{ij}[m, K-1] \\ o_{ij}[m, -1] & o_{ij}[m, 0] & \ddots & o_{ij}[m, K-2] \\ \vdots & \ddots & \ddots & \vdots \\ o_{ij}[m, -(K-1)] & o_{ij}[m, -(K-2)] & \ddots & o_{ij}[m, 0] \end{pmatrix} = c_0 s_t[m]\mathbf{S}_f + (c_1 - c_0)\delta[m]\mathbf{I} + c_1\delta[i-j]\mathbf{E} \tag{24}$$

and

$$\mathbf{P}_{ij}[m] \triangleq \begin{pmatrix} p_{ij}[m, 0] & p_{ij}[m, 1] & \cdots & p_{ij}[m, K-1] \\ p_{ij}[m, -1] & p_{ij}[m, 0] & \ddots & p_{ij}[m, K-2] \\ \vdots & \ddots & \ddots & \vdots \\ p_{ij}[m, -(K-1)] & p_{ij}[m, -(K-2)] & \ddots & p_{ij}[m, 0] \end{pmatrix} = c_0 s_t[m]\mathbf{S}_f + c_1\delta[i-j]\mathbf{E} \tag{25}$$

$$\mathbf{S}_f = \begin{pmatrix} |r_f[0]|^2 & |r_f[1]|^2 & \ddots & |r_f[K-1]|^2 \\ |r_f[-1]|^2 & |r_f[0]|^2 & \ddots & |r_f[K-2]|^2 \\ \vdots & \ddots & \ddots & \vdots \\ |r_f[-(K-1)]|^2 & |r_f[-(K-2)]|^2 & \ddots & |r_f[0]|^2 \end{pmatrix} \tag{26}$$

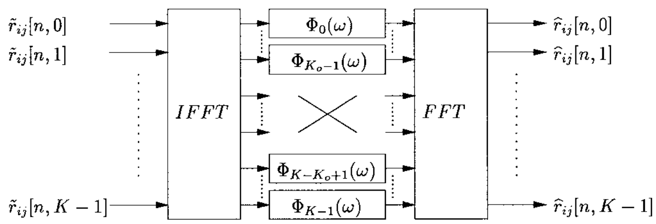


Fig. 2. Instantaneous correlation estimator for the MMSE-DC.

B. Robust Estimation

From (12), the average MSE of the parameter estimation can be expressed in the frequency domain as

$$\overline{\text{MSE}} \triangleq \frac{1}{K}\sum_{k=0}^{K-1} E|\hat{r}_{ij}[n,k] - r_{ij}[n,k]|^2 = \text{Tr}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}[\mathbf{F}(\omega)\mathbf{O}(\omega)\mathbf{F}^H(\omega) - 2\mathbf{F}(\omega)\mathbf{P}(\omega) + \mathbf{P}(\omega)]\,d\omega\right\} \tag{44}$$

where $\text{Tr}\{*\}$ denotes the trace of a matrix defined as the summation of its diagonal elements. For any estimator (not necessarily the MMSE estimator) with parameters

$$\overline{\mathbf{F}}(\omega) = \mathbf{W}\overline{\Psi}(\omega)\mathbf{W}^H$$

and

$$\overline{\Psi}(\omega) = \text{diag}\{\overline{\psi}_0(\omega), \overline{\psi}_1(\omega), \cdots, \overline{\psi}_{K-1}(\omega)\}$$

the average MSE of the estimator can be further simplified into

$$\overline{\text{MSE}} = \frac{1}{K}\sum_{k=0}^{K-1}\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[|\overline{\psi}_k(\omega)|^2 d_{o,k}(\omega) - 2\overline{\psi}_k(\omega)d_{p,k}(\omega) + d_{p,k}(\omega)\right]d\omega. \tag{45}$$

For an ideal MMSE estimator, the estimator parameters are selected to match the channel statistics, i.e.,

$$\overline{\psi}_k(\omega) = \psi_k(\omega) \tag{46}$$

then

$$\overline{\text{MSE}}_o = \frac{1}{K}\sum_{n=0}^{K-1}\frac{1}{2\pi}\int_{-2\omega_d}^{2\omega_d}\frac{a_k c_0 s_t(\omega)(c_1 - c_0)}{a_k c_0 s_t(\omega) + c_1 - c_0}\,d\omega = \frac{\sigma^4}{K}\sum_{n=0}^{K-1}\frac{1}{2\pi}\int_{-2\omega_d}^{2\omega_d}\frac{a_k\alpha_0\alpha_1 s_t(\omega)}{a_k\alpha_0 s_t(\omega) + \alpha_1}\,d\omega \tag{47}$$

where

$$\alpha_0 \triangleq \frac{\sum_{l=0}^{L}\sigma_l^4}{\sigma^4}$$

and

$$\alpha_1 \triangleq \frac{\left(\sum_{l=0}^{L}\sigma_l^2 + \sigma^2\right)^2 - \sum_{l=0}^{L}\sigma_l^4}{\sigma^4}. \tag{48}$$

TABLE I
$\overline{\text{NMSE}}$'s FOR DIFFERENT ESTIMATORS

SNR(dB)	SIR(dB)	f_d(Hz)	t_d(μsec.)	$\overline{\text{NMSE}}_o$(dB)	$\overline{\text{NMSE}}_h$(dB)	$\overline{\text{NMSE}}_r$(dB)
5	5	40	20	-19.5973	-19.5957	-9.4898
10	5	40	20	-11.7321	-11.7307	-1.6124
15	5	40	20	-2.7058	-2.7044	7.4185
20	5	40	20	6.9345	6.9359	17.0603
10	0	40	20	-7.4532	-7.4516	2.6551
10	5	40	20	-11.7321	-11.7307	-1.6124
10	10	40	20	-15.0197	-15.0184	-4.8883
10	20	40	20	-17.7878	-17.7867	-7.6508
10	5	10	20	-17.7503	-17.7491	-7.6127
10	5	40	20	-11.7321	-11.7307	-1.6124
10	5	100	20	-7.7577	-7.7557	2.3267
10	5	200	20	-4.7556	-4.7526	5.2706
10	5	40	5	-11.7321	-11.7307	-8.0541
10	5	40	10	-11.7321	-11.7307	-4.7512
10	5	40	20	-11.7321	-11.7307	-1.6124
10	5	40	40	-11.7321	-11.7307	1.4396

Usually, $\psi_k(\omega)$ is unknown since the Doppler frequency of mobile systems is not available, therefore, the estimator is designed to match $p_{2\omega_d}(\omega)$ defined as

$$p_{2\omega_d}(\omega) = \begin{cases} \frac{\pi}{2\omega_d}, & \text{if } |\omega| \le 2\omega_d \\ 0, & \text{otherwise} \end{cases} \tag{49}$$

then

$$\overline{\psi}_k(\omega) = \frac{a_k\alpha_0\alpha_1 p_{2\omega_d}(\omega)}{a_k\alpha_0 p_{2\omega_d}(\omega) + \alpha_1} \tag{50}$$

and

$$\overline{\text{MSE}}_h = \frac{\sigma^4}{K}\sum_{n=0}^{K-1}\frac{a_k\alpha_1}{\frac{\pi}{2\omega_d}a_k\alpha_0 + \alpha_1}. \tag{51}$$

Similar to [9], it can be proven that, for any channel with Doppler frequency less than ω_d, the average MSE of the estimator matching $p_{2\omega_d}(\omega)$ is $\overline{\text{MSE}}_h$.

Since a_k depends on the channel delay profile, which is usually unknown, the robust correlation estimator should match

$$\hat{a}_k = \begin{cases} \frac{K}{2K_o - 1}, & \text{if } 0 \le k \le K_o - 1 \text{ or } K - K_o + 1 \le k \le K - 1 \\ 0, & \text{otherwise.} \end{cases} \tag{52}$$

In this case,

$$\overline{\text{MSE}}_r = \sigma^4\frac{\alpha_0\alpha_1}{\frac{\pi K}{2\omega_d(2K_o - 1)}\alpha_0 + \alpha_1} \tag{53}$$

which is also the average MSE of the instantaneous correlation estimator for those OFDM systems with time-varying dispersive, Rayleigh fading channels with Doppler frequency less than $2\omega_d$ and delay spread less than $(K_o/K)T_s$.

Table I illustrates the normalized average MSE's (defined as $\overline{\text{NMSE}}/\sigma^4$) of the above three estimators for a two-ray channel with one cochannel interferer. From the table, there is only a slight degradation if $p_{2\omega_d}(\omega)$ substitutes for $\psi_k(\omega)$. However, robust design in the frequency-domain results in a significant degradation.

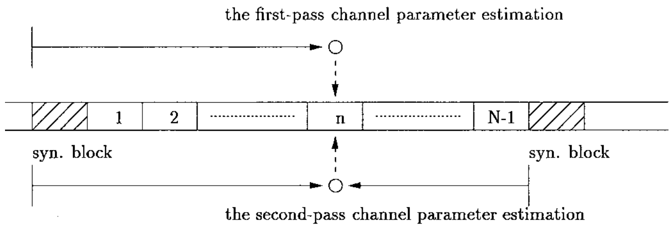


Fig. 3. Enhanced channel parameter estimation.

With the instantaneous correlation estimation approach developed in this section, together with the channel estimation approach in [9], we are able to estimate the parameters of the MMSE-DC by (6). That is called the *original estimator* for the parameters of the MMSE-DC to distinguish it from the *enhanced estimator* developed in the next section.

IV. Enhanced Approach for Adaptive Antenna Array

In this section, we investigate an enhanced approach for adaptive antenna arrays in OFDM systems by improving both the channel parameter and the instantaneous correlation estimations.

A. Enhanced Channel Estimation

We have introduced several reference generation approaches in [9]; however, there is no approach to generate the future reference at the present time. Therefore, at the present time n, channel estimation can be only based on the temporal channel estimation up to time n for the first-pass channel estimation in Fig. 3.

However, after the first-pass channel estimation, we can obtain the temporal channel estimation at all times. As shown in Fig. 3, if the second-pass channel estimation is applied, then improved channel estimation at time n can be obtained by exploiting the past, current, and future temporal channel estimations. Assume that the ideal reference, that is, the true transmitted signal $s_0[n, k]$, is used in the first-pass channel estimation. Then, the average MSE for the second-pass robust channel estimation is

$$\overline{\text{MSE}}_s = \frac{1}{\dfrac{\pi K}{\omega_d K_o \rho} + 1} \tag{54}$$

where

$$\rho \triangleq \frac{\sigma^2 + \sum_{l=1}^{L} \sigma_l^2}{\sigma_0^2}. \tag{55}$$

Recall from [9] that the average MSE for the first-pass robust channel estimation is

$$\overline{\text{MSE}}_f = \frac{K_o \rho}{K}\left(1 - \frac{1}{\left[\dfrac{\pi K}{K_o \rho \omega_d} + 1\right]^{\omega_d/\pi}}\right) \tag{56}$$

which is larger than $\overline{\text{MSE}}_s$.

B. Enhanced Parameter Estimation

From the definition of the correlation matrix in (7), we have

$$\mathbf{R}[n, k] = \mathbf{H}^{(0)}[n, k]\mathbf{H}^{(0)^H}[n, k] + \mathbf{R}_v[n, k] \tag{57}$$

where $\mathbf{R}_v[n, k] \triangleq (E\{v_i[n, k]v_j^*[n, k]\})_{ij=1}^{p}$. Using the *matrix inversion lemma* [18], we obtain the equation shown at the bottom of the page. Hence,

$$\begin{aligned}\mathbf{w}[n, k] &= (\mathbf{R}[n, k] + \gamma\mathbf{I})^{-1}\mathbf{H}^{(0)}[n, k] \\ &= \frac{1}{1 + \mathbf{H}^{(0)^H}[n, k](\mathbf{R}_v[n, k] + \gamma\mathbf{I})^{-1}\mathbf{H}^{(0)}[n, k]} \\ &\quad \cdot \hat{\mathbf{w}}[n, k]\end{aligned} \tag{58}$$

where $\hat{\mathbf{w}}[n, k]$ is defined as

$$\hat{\mathbf{w}}[n, k] \triangleq (\mathbf{R}_v[n, k] + \gamma\mathbf{I})^{-1}\mathbf{H}^{(0)}[n, k]. \tag{59}$$

Hence, there is only an amplitude difference between $\mathbf{w}[n, k]$ and $\hat{\mathbf{w}}[n, k]$.

Recall that the OFDM system here uses PSK, which carries information through the phases of tones. Therefore, $\hat{\mathbf{w}}[n, k]$ may substitute for $\mathbf{w}[n, k]$ and $\mathbf{R}_v[n, k]$ can be used instead of $\mathbf{R}[n, k]$.

When the channel parameters and the (desired) transmitted data are known, then $v_m[n, k]$ can be obtained by subtracting the desired signal components from the received signals $x_m[n, k]$. Let $\hat{H}_m^{(0)}[n, k]$ be the estimated channel parameters using the enhanced approach in Section IV-A, then, $v_m[n, k]$ can be estimated as

$$\hat{v}_m[n, k] = v_m[n, k] + \left(H_m^{(0)}[n, k] - \hat{H}_m^{(0)}[n, k]\right)s_0[n, k]. \tag{60}$$

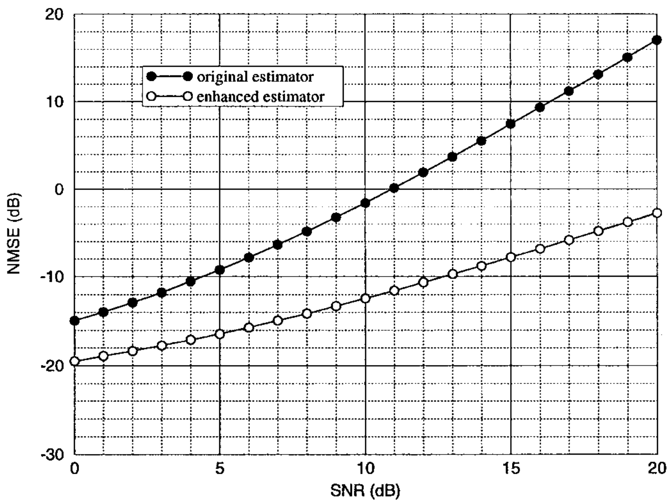
From $\hat{v}_m[n, k]$, $\mathbf{R}_v[n, k]$ can be estimated using the approach developed in Section III. The average MSE for $\mathbf{R}_v[n, k]$ estimation is

$$\overline{\text{MSE}}_r = \sigma^4 \frac{\hat{\alpha}_0\hat{\alpha}_1}{\dfrac{\pi K}{2\omega_d(2K_o - 1)}\hat{\alpha}_0 + \hat{\alpha}_1} \tag{61}$$

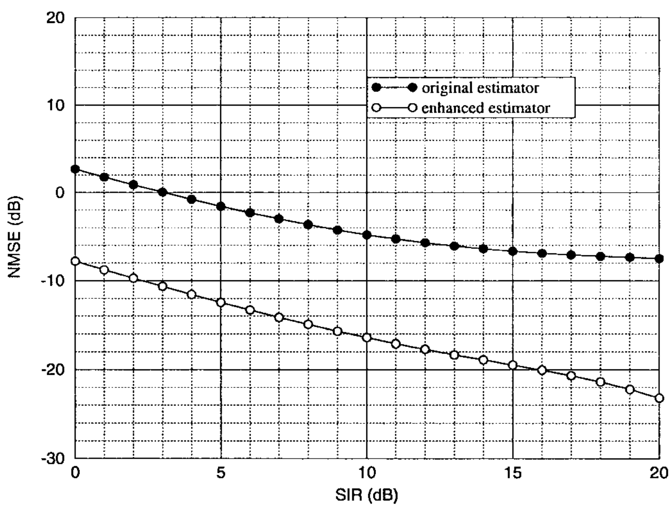
where $\hat{\alpha}_0$ and $\hat{\alpha}_1$ are defined as

$$\hat{\alpha}_0 \triangleq \frac{\sum_{l=1}^{L} \sigma_l^4 + \hat{\sigma}_0^4}{\sigma^4} \tag{62}$$

$$(\mathbf{R}[n, k] + \gamma\mathbf{I})^{-1} = (\mathbf{R}_v[n, k] + \gamma\mathbf{I})^{-1} - \frac{\left(\mathbf{R}_v[n, k] + \gamma\mathbf{I})^{-1}\mathbf{H}^{(0)}[n, k]\mathbf{H}^{(0)^H}[n, k](\mathbf{R}_v[n, k] + \gamma\mathbf{I}\right)^{-1}}{1 + \mathbf{H}^{(0)^H}[n, k](\mathbf{R}_v[n, k] + \gamma\mathbf{I})^{-1}\mathbf{H}^{(0)}[n, k]}$$



(a)



(b)

Fig. 4. Comparison of the enhanced correlation estimator with the original one: (a) NMSE versus SNR when SIR $= 5$ dB, $f_d = 40$ Hz, and $t_d = 20$ μs and (b) NMSE versus SIR when SNR $= 10$ dB, $f_d = 40$ Hz, and $t_d = 20$ μs.

and

$$\hat{\alpha}_1 \triangleq \frac{\left(\sum_{l=1}^{L} \sigma_l^2 + \hat{\sigma}_0^2 + \sigma^2\right)^2 - \left(\sum_{l=1}^{L} \sigma_l^4 + \hat{\sigma}_0^4\right)}{\sigma^4} \tag{63}$$

respectively, with

$$\hat{\sigma}_0^2 = \frac{\sigma_0^2}{\dfrac{\pi K}{\omega_d K_o \rho} + 1}. \tag{64}$$

Fig. 4 compares the $\overline{\text{NMSE}}$ of the original instantaneous correlation estimation and the enhanced instantaneous correlation estimation. From the figure, the enhanced instantaneous correlation estimation has much better performance than the original one.

It should be noted that we have used the ideal data in (60). In practical systems, only the data from the reference generator are available; that causes a slight performance degradation for the enhanced correlation approach.

V. Computer Simulation

In this section, we demonstrate, through extensive computer simulation, the performance of adaptive antenna arrays for OFDM systems with cochannel interference.

The OFDM system used in our simulation is similar to the one in [2] and [9], except that it has a cochannel interferer with the same statistics as the desired signal. The entire channel bandwidth (800 kHz) is divided into 128 subchannels. The four subchannels on each end are used as guard tones, and the rest (120 tones) are used to transmit data. To make the tones orthogonal to each other, the symbol period is $T_s = 160$ μs. An additional 40-μs guard interval is used to protect the OFDM block from intersymbol interference due to delay spread, which results in the total block length $T_f = 200$ μs and symbol rate $r_b = 5$ kb. QPSK modulation is used with coherent modulation. A (40, 20) R–S code, with each code symbol consisting of three QPSK symbols grouped in frequency, is used in the system. Hence, each OFDM block forms an R–S codeword. The R–S decoder erases ten symbols based on signal strength and corrects five additional random errors. Hence, the simulated system can transmit data at 600 kb/s over an 800-kHz channel. To suppress error propagation, 10% of the OFDM blocks are periodically inserted as training blocks in the data stream. For the enhanced estimation with two passes, the second pass uses information contained within ten OFDM blocks using only one synchronization block which is appropriate for packet transmission.

The *undecoded/decoded dual-mode reference* [9] is used for the channel estimation, which generates references from the decoded data if the R–S decoder can successfully correct all errors in an OFDM block; otherwise, it uses the decided (sliced) undecoded symbols.

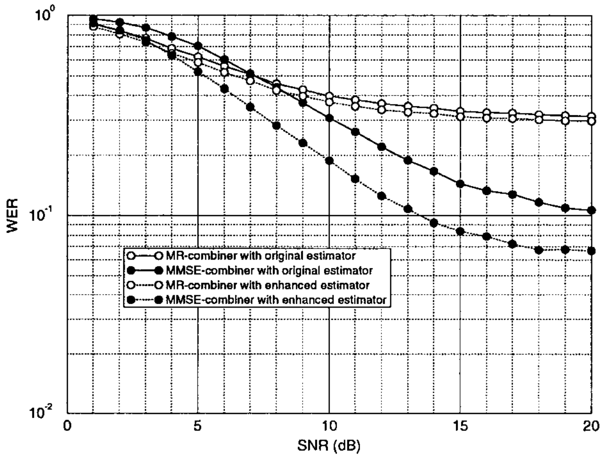
To gain insights into the average behavior of adaptive antenna arrays for OFDM systems, we have averaged the performance over 10 000 blocks. First, we introduce the simulation results for OFDM systems with two antenna diversity and a two-ray channel model.

A. Two-Branch Diversity with a Two-Ray Channel Model

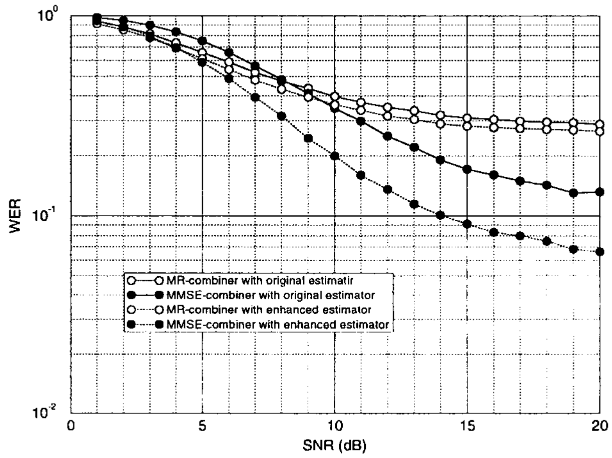
A two-path Rayleigh fading channel model [13], [14] with different delay spreads and Doppler frequencies is used in the simulations in this section. The channels corresponding to different receivers have the same statistics. Two receiver antennas are used for diversity. The cochannel interferer is assumed to be synchronous with and have the same statistics as the desired signals.

Figs. 5–7 show the word error rate (WER) of the original and the enhanced estimators for adaptive antenna arrays in OFDM systems under different channel conditions.

Fig. 5 compares the WER's of the MMSE-DC and the MR-DC for channels with SIR $= 5$ dB and different SNR's, f_d's, and t_d's. Note that without cochannel interference, the MR-DC is equivalent to the MMSE-DC. However, when cochannel interference exists, the OFDM system with the MMSE-DC has much better performance than the one with the MR-DC.



(a)



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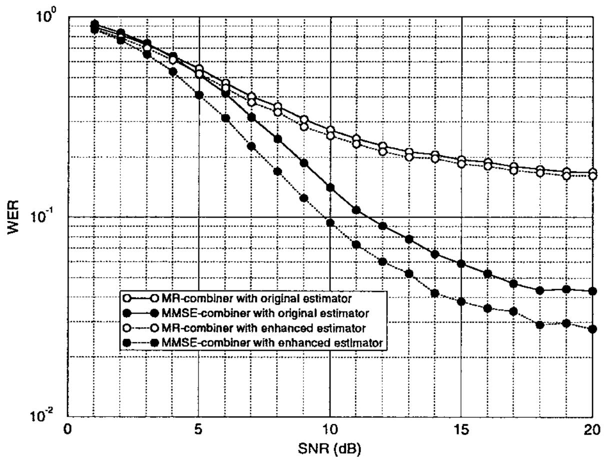
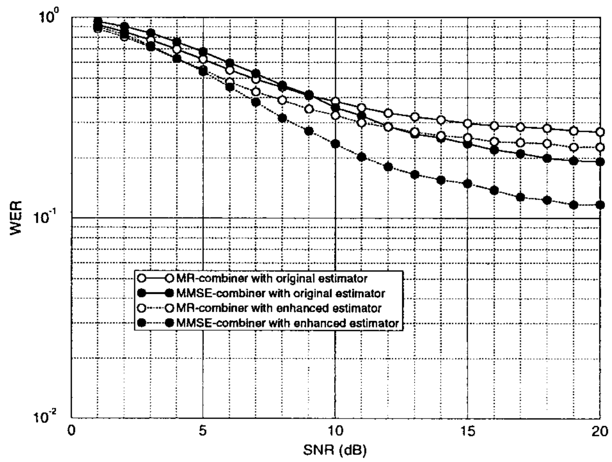


(c)



(d)

Fig. 5. WER versus SNR for the MMSE-DC's and the MR-DC's with different parameter estimators and channels with SIR $= 5$ dB and (a) $f_d = 10$ Hz, $t_d = 20$ μs; (b) $f_d = 40$ Hz, $t_d = 20$ μs; (c) $f_d = 40$ Hz, $t_d = 5$ μs; and (d) $f_d = 200$ Hz, $t_d = 5$ μs.

In particular, the MMSE-DC with the enhanced parameter estimator has better performance than the one with the original estimator. When $f_d = 10$ Hz and $t_d = 20$ μs, the required SNR for 10% WER is 13 dB for the enhanced estimator and 20 dB for the original one. When $f_d = 40$ Hz and $t_d = 5$ μs, the required SNR for the enhanced estimator is about 1.5 dB less than for the original one.

Fig. 6 shows the WER versus SIR for the channels with different f_d's, t_d's, and SNR's. When $f_d = 10$ Hz, $t_d = 20$ μs, and SNR $= 20$ dB, the required SIR for 10% WER is as low as 3.5 dB and the required SIR for 1% WER is about 9 dB. With an increase of f_d or t_d, the system performance becomes worse. For channels with $t_d = 20$ μs and SNR $= 20$ dB, the required SIR for 10% WER increases from 3.5 to 4 dB when f_d increases from 10 to 40 Hz; for channels with $t_d = 5$ μs and SNR $= 20$ dB, the required SIR for 10% WER increases from about 1.8 to 5.5 dB when f_d increases from 40 to 200 Hz. Similarly, for channels with $f_d = 40$ Hz and SNR $= 20$ dB, the required SIR for 10% WER increases from 1.8 to 4 dB when t_d increases from 5 to 20 μs.

We have also tested the robustness of the estimators by fixing the matching f_d and t_d while changing the delay spread and Doppler frequency of the channels.

Fig. 7(a) shows the WER versus the channel's f_d. From the figure, if the channel's f_d is less than 50 Hz, the estimator matching a 40-Hz Doppler frequency has slightly better performance than the one matching 200 Hz. However, when the channel's f_d is larger than 50 Hz, the estimator matching 200-Hz Doppler frequency is much better than the other one. Hence, when designing a instantaneous correlation estimator for adaptive antenna arrays, we should let the estimator match the largest possible Doppler frequency of the system to obtain the best performance. A similar phenomenon is observed in Fig. 7(b) when the channel's delay spread varies and the estimator matches 20 or 40 μs, respectively. From the figure, if the channel's t_d is less than 20 μs, then the estimator matching 20 μs has much better performance than the one matching 40 μs. However, when the channel's t_d is larger than 20 μs, the estimator matching 20 μs does not work at all. Since the performance of the estimator is very sensitive to its matching delay spread, the performance can be significantly improved if the channel delay spread is adaptively estimated.

B. Four-Branch Diversity with a Two-Ray Channel Model

The channel model used here is the same as the one in the previous section, except that four-branch diversity is

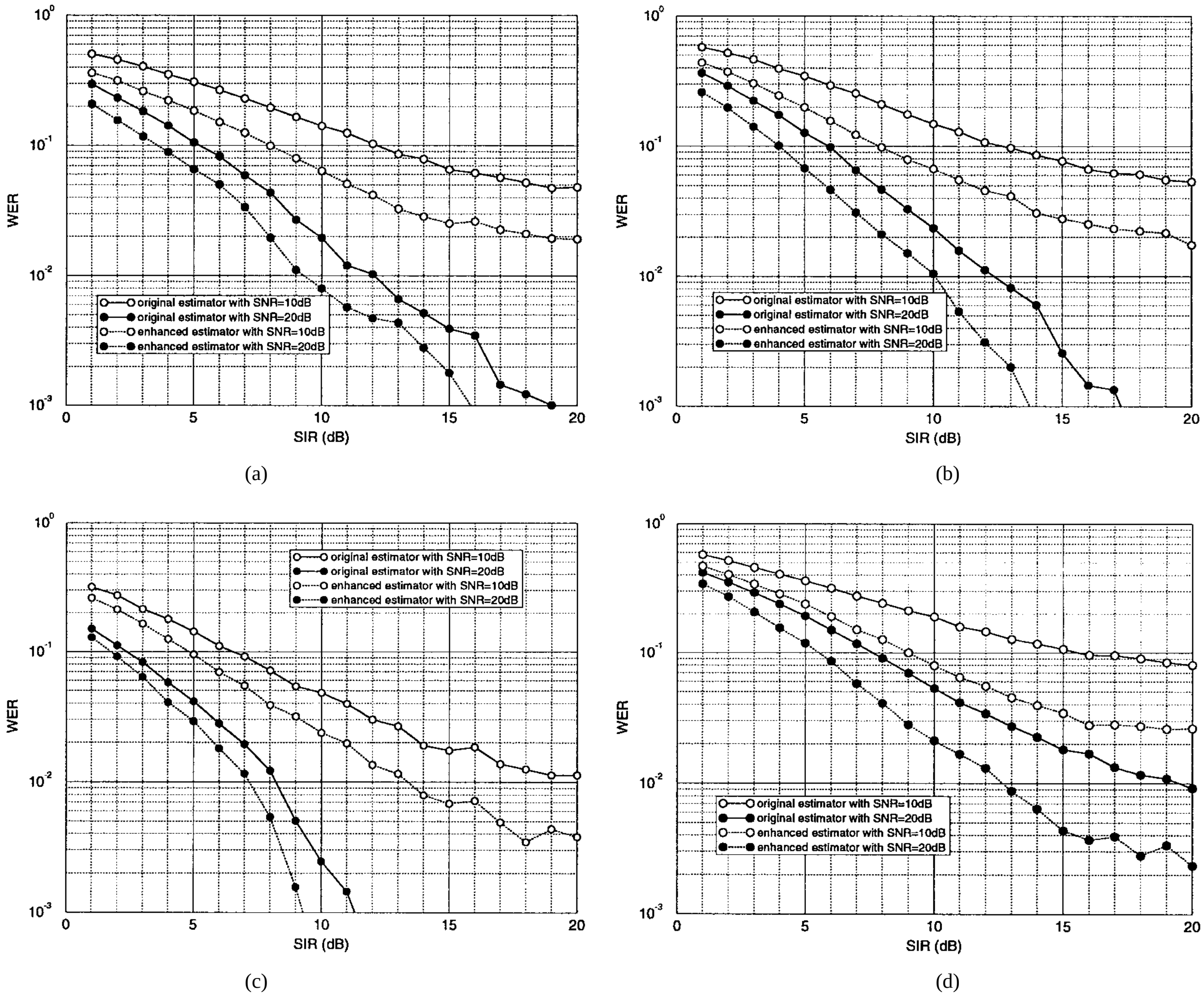


Fig. 6. WER of the MMSE-DC versus SIR for channels with different SNR's and (a) $f_d = 10$ Hz, $t_d = 20$ μs; (b) $f_d = 40$ Hz, $t_d = 20$ μs; (c) $f_d = 40$ Hz, $t_d = 5$ μs; and (d) $f_d = 200$ Hz, $t_d = 5$ μs.

employed. Fig. 8 shows the WER of the OFDM system with an adaptive antenna array for channels with different SNR's, SIR's, f_d's, and t_d's. Fig. 8(a) compares the performance of the MMSE-DC with that of the MR-DC when SIR $= 0$ dB, $f_d = 40$ Hz, and $t_d = 5$ μs. Similar to the two-branch case, the MR-DC does not work well when the system has cochannel interference. On the other hand, for the MMSE-DC, the required SNR for 1% WER is only 10 dB when cochannel interference is as large as SIR $= 0$ dB. When SNR $= 20$ dB, the required SIR for the MMSE-DC is as low as -1.6 dB.

C. Two-Branch Diversity with Different Channel Models

Here, we compare the performance of the MMSE-DC parameter estimator for channels with the two-ray, typical-urban (TU), and hilly-terrain (HT) [19] models, respectively. Note that for the TU or HT model, the delay of each ray is not sample-spaced, hence, there will be *delay leakage*. However, from the simulation result in Fig. 9, the system performance for different channel models is very close, which implies that *delay leakage* of TU and HT models has only negligible effect on the performance of the adaptive antenna arrays for OFDM systems.

D. Adaptive Delay Spread Estimation

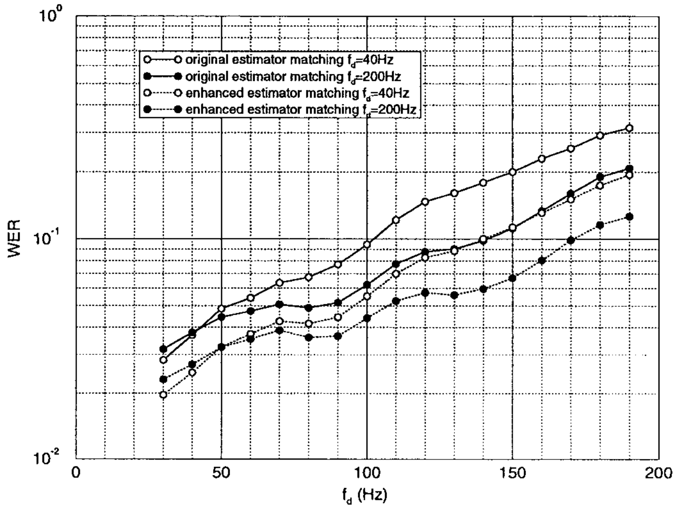
As indicated in Section V-A, the performance of the parameter estimator for the MMSE-DC is very sensitive to the estimator's matching delay spread. Hence, we need to study adaptive delay spread estimation for the parameter estimator.

From the discussion in Section III-A, the eigenvalues (a_k's) of $\mathbf{S}_f$ related to K_o by $a_k = 0$ for all $K_o \leq k \leq K - K_o$. Using this principle, the channel delay spread can be estimated by observing the average energy of the IFFT output of the instantaneous correlation estimator in Fig. 2.

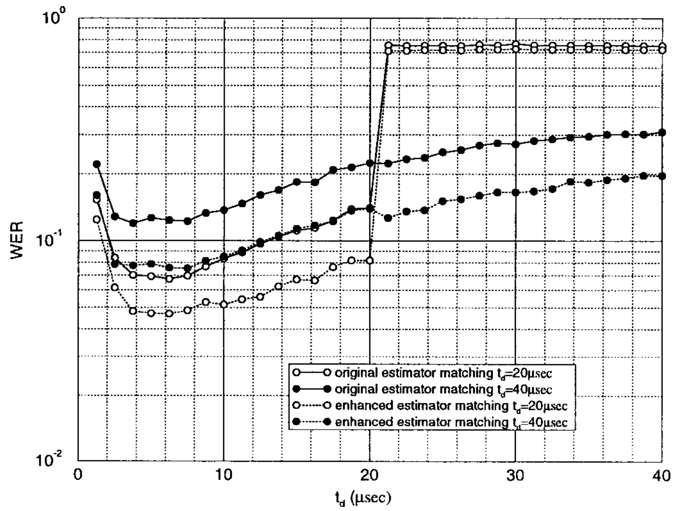
Fig. 10 compares the performance of the estimators that match estimated delay spread, true delay spread, 20- and 40-μs delay spreads, respectively. From the figure, the performance of the estimator with estimated matching delay spread is close to the one with matching true delay spread and much better than the one matching 40-μs delay spread when the channel's t_d is larger than 10 μs. Hence, adaptive delay spread estimation can be used effectively in adaptive antenna arrays in OFDM systems to improve the performance.

E. Effect of Asynchronous Cochannel Interference

In the simulations of Sections V-A to V-D, the interferer and desired signal are assumed to be synchronized (time aligned).

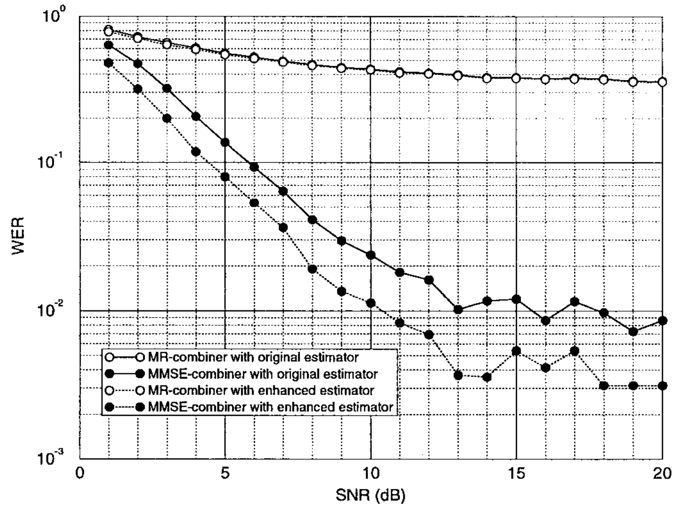


(a)

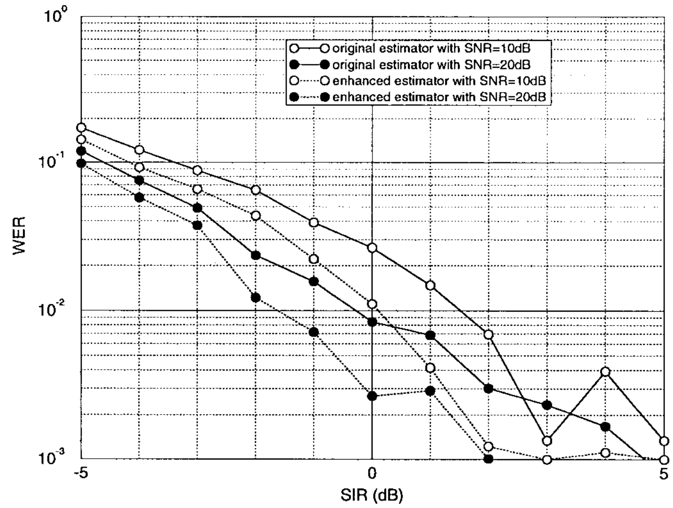


(b)

Fig. 7. (a) WER versus channel's f_d for channel with SNR $= 20$ dB, SIR $= 5$ dB, and $t_d = 5$ μs and parameter estimators matching $t_d = 5$ μs and different f_d's and (b) WER versus channel's t_d for channel with SNR $= 20$ dB, SIR $= 5$ dB, and $f_d = 40$ Hz and parameter estimators matching $f_d = 40$ Hz and different t_d's.



(a)



(b)

Fig. 8. Performance of parameter estimator for system with four antennas. (a) WER versus SNR for channel with SIR = 0 dB, $f_d = 40$ Hz, and $t_d = 5$ μs. (b) WER versus SIR for channel with SNR $= 10/20$ dB, $f_d = 40$ Hz, and $t_d = 5$ μs.

In practical systems, however, they are not necessarily synchronized. Here, we investigate the effect of an asynchronous cochannel interferer on adaptive antenna arrays in OFDM systems. Fig. 11 illustrates the WER versus the normalized time shift ($\Delta t/T_s$) between the desired signal and cochannel interferer for channels with SNR = 20 dB, SIR = 5 dB, $f_d = 40$ Hz, and $t_d = 20$ μs. From the figure, the time shift between the desired signal and interferer does not significantly affect the performance of adaptive antenna arrays in OFDM systems. Hence, the results of Sections V-A to V-D are also applicable to systems with asynchronous interference.

VI. Conclusions

In this paper, we have investigated adaptive antenna arrays for OFDM systems with cochannel interference. We have proposed both an original and an enhanced parameter estimator for the MMSE-DC in OFDM systems with cochannel interference. Computer simulation demonstrates that an adaptive antenna array in OFDM systems can suppress as much as 5-dB SIR cochannel synchronous/asynchronous interference for two-branch diversity. Hence, this represents a promising technique for future mobile data systems using OFDM.

Appendix
Statistics of Instantaneous Correlation

In this Appendix, we derive $p_{ij}[u, v]$ and $o_{ij}[u, v]$ in (18) and (19), respectively. Recall that they are respectively defined as

$$p_{ij}[u, v] = E\{r_{ij}[n+u, k+v]\tilde{r}_{ij}^*[n, k]\} \tag{A.1}$$

and

$$o_{ij}[u, v] = E\{\tilde{r}_{ij}[n+u, k+v]\tilde{r}_{ij}^*[n, k]\}. \tag{A.2}$$

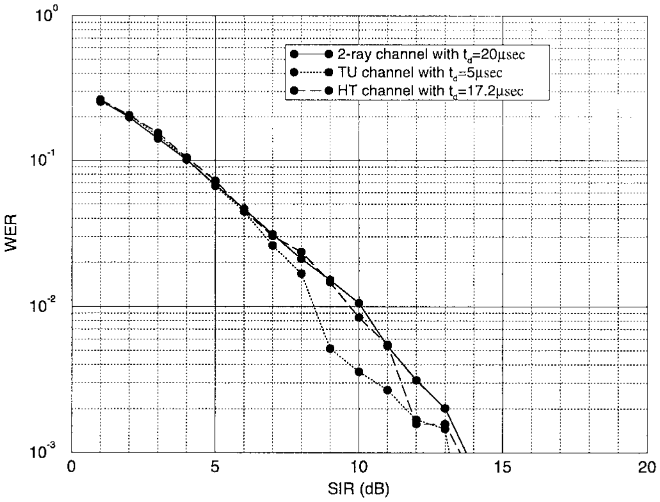


Fig. 9. Comparison of the MMSE-DC for two-ray, TU, and HT channel models with $f_d = 40$ Hz and SNR $= 20$ dB.

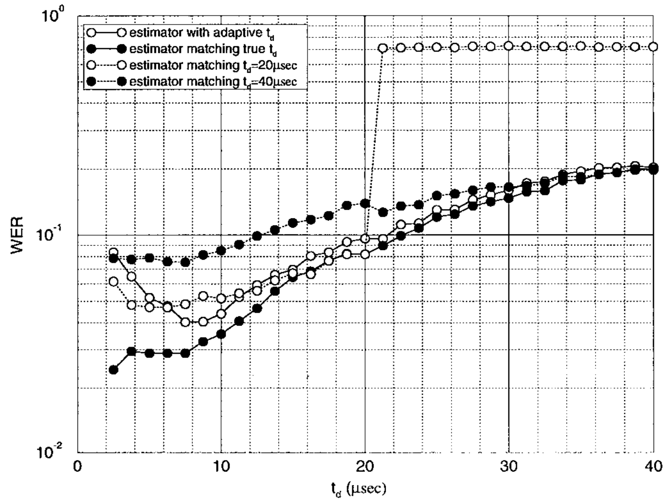


Fig. 10. WER versus t_d for the enhanced parameter estimator using different delay spread matching schemes when SNR $= 20$ dB, SIR $= 5$ dB, and $f_d = 40$ Hz.

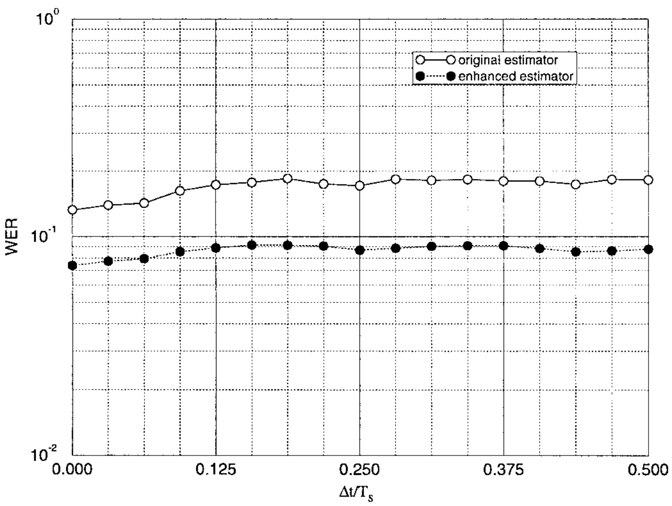


Fig. 11. The effect of asynchronous cochannel interferer to the MMSE-DC when SNR $= 20$ dB, SIR $= 5$ dB, $f_d = 40$ Hz, and $t_d = 20$ μs.

From the definition of $\tilde{r}_{ij}[n, k]$, we have

$$\begin{aligned} p_{ij}[u, v] &= E\{r_{ij}[n+u, k+v]E_c(\tilde{r}^*_{ij}[n, k])\} \\ &= E\Bigg\{\Bigg(\sum_{l=0}^{L} H_i^{(l)}[n+u, k+v] \\ &\qquad \cdot H_j^{(l)^*}[n+u, k+v] + \sigma^2\delta[i-j]\Bigg) \\ &\qquad \cdot \Bigg(\sum_{l=0}^{L} H_i^{(l)}[n, k]H_j^{(l)^*}[n, k] + \sigma^2\delta[i-j]\Bigg)^*\Bigg\} \\ &= E\Bigg\{\sum_{l_1, l_2=0}^{L} H_i^{(l_1)}[n+u, k+v] \\ &\qquad \cdot H_j^{(l_1)^*}[n+u, k+v]H_i^{(l_2)^*}[n, k]H_j^{(l_2)}[n, k]\Bigg\} \\ &\quad + E\Bigg\{\sum_{l=0}^{L} \sigma^2 H_i^{(l)}[n+u, k+v] \\ &\qquad \cdot H_j^{(l)^*}[n+u, k+v] + \sigma^2 \\ &\qquad \cdot \Bigg(\sum_{l=0}^{L} H_i^{(l)}[n, k]H_j^{(l)^*}[n, k]\Bigg)^* + \sigma^4\Bigg\}\delta[i-j]. \end{aligned} \tag{A.3}$$

Since $H_i^{(l)}[n, k]$'s are stationary Gaussian processes, then

$$\begin{aligned} &E\Big\{H_i^{(l_1)}[n+u, k+v]H_j^{(l_1)^*}[n+u, k+v]H_i^{(l_2)^*}[n, k] \\ &\quad \cdot H_j^{(l_2)}[n, k]\Big\} \\ &\quad = \sigma_{l_1}^2\sigma_{l_2}^2\left(\delta[i-j] + \sigma_{l_1}^4|r_H[u, v]|^2\delta[l_1-l_2]\right). \end{aligned} \tag{A.4}$$

Thus, the first term in (A.3) is $(\sum_{l=0}^{L}\sigma_l^2)^2\delta[i-j] + \sum_{l=0}^{L}\sigma_l^4|r_H[u, v]|^2$ and the second term in (A.3) is $(2\sum_{l=0}^{L}\sigma_l^2\sigma^2 + \sigma^4)\delta[i-j]$. Therefore,

$$p_{ij}[u, v] = \left(\sum_{l=0}^{L}\sigma_l^4\right)|r_H[u, v]|^2 + \left(\sigma^2 + \sum_{l=0}^{L}\sigma_l^2\right)^2\delta[i-j]. \tag{A.5}$$

Using the moment and cumulant relation [20], we have

$$\begin{aligned} o_{ij}[u, v] &= E\{E_c(x_i[n+u, k+v]x_j^*[n+u, k+v] \\ &\qquad \cdot x_i^*[n, k]x_j[n, k])\} \\ &= E\{\mathrm{Cum}(x_i[n+u, k+v], x_j^*[n+u, k+v], \\ &\qquad x_i^*[n, k], x_j[n, k])\} \\ &\quad + E\{E_c(x_i[n+u, k+v], x_j^*[n+u, k+v]) \\ &\qquad \cdot E_c(x_i^*[n, k], x_j[n, k])\} \\ &\quad + E\{E_c(x_i[n+u, k+v], x_i^*[n, k]\} \\ &\qquad \cdot E_c\{x_j^*[n+u, k+v], x_j[n, k])\} \\ &\quad + E\{E_c(x_i[n+u, k+v], x_j[n, k]\} \\ &\qquad \cdot E_c\{x_j^*[n+u, k+v], x_i^*[n, k])\}. \end{aligned} \tag{A.6}$$

By means of the linearity of the cumulant,

$$\begin{aligned}&E\{\text{Cum}(x_i[n+u,\,k+v],\,x_j^*[n+u,\,k+v],\\&\quad x_i^*[n,\,k],\,x_j[n,\,k])\}\\&=E\Bigg\{\sum_{l_1,\,l_2,\,l_3,\,l_4=0}^{L} H_i^{(l_1)}[n+u,\,k+v]\\&\qquad\cdot H_j^{(l_2)^*}[n+u,\,k+v]H_i^{(l_3)^*}[n,\,k]H_j^{(l_4)}[n,\,k]\\&\qquad\cdot \text{Cum}\Big(a^{(l_1)}[n+u,\,k+v],\,a^{(l_2)^*}[n+u,\,k+v],\\&\qquad\qquad a^{(l_3)^*}[n,\,k],\,a^{(l_4)}[n,\,k]\Big)\Bigg\}. \qquad \text{(A.7)}\end{aligned}$$

Since $a^{(l)}[n,\,k]$'s are constant modulus and independent for different l, n, or k, then

$$\begin{aligned}&\text{Cum}\Big(a^{(l_1)}[n+u,\,k+v],\,a^{(l_2)^*}[n+u,\,k+v],\\&\quad a^{(l_3)^*}[n,\,k],\,a^{(l_4)}[n,\,k]\Big)\\&\qquad\qquad=-\delta[u,\,v]\delta[l_1-l_2,\,l_2-l_3,\,l_3-l_4].\end{aligned}$$

Therefore,

$$\begin{aligned}&E\{\text{Cum}(x_i[n+u,\,k+v],\,x_j^*[n+u,\,k+v],\\&\quad x_i^*[n,\,k],\,x_j[n,\,k])\}\\&=-E\left\{\sum_{l=0}^{L}\left|H_i^{(l)}[n,\,k]\right|^2\left|H_j^{(l)}[n,\,k]\right|^2\delta[u,\,v]\right\}\\&=-\sum_{l=0}^{L}\sigma_l^4(1+\delta[i-j])\delta[u,\,v]. \qquad \text{(A.8)}\end{aligned}$$

It can be shown

$$\begin{aligned}&E_c(x_i[n+u,\,k+v]x_j^*[n,\,k])\\&=\left(\sum_{l=0}^{L}H_i^{(l)}[n,\,k]H_j^{(l)^*}[n,\,k]+\sigma^2\delta[i-j]\right)\delta[u,\,v]. \qquad \text{(A.9)}\end{aligned}$$

Using the above identity, we have

$$\begin{aligned}&E\{E_c(x_i[n+u,\,k+v],\,x_j^*[n+u,\,k+v])\\&\quad\cdot E_c(x_i^*[n,\,k],\,x_j[n,\,k])\}\\&=E\Bigg\{\Bigg(\sum_{l=0}^{L}H_i^{(l)}[n+u,\,k+v]\\&\qquad\cdot H_j^{(l)^*}[n+u,\,k+v]+\sigma^2\delta[i-j]\Bigg)\\&\qquad\cdot\left(\sum_{l=0}^{L}H_i^{(l)}[n,\,k]H_j^{(l)^*}[n,\,k]+\sigma^2\delta[i-j]\right)^*\Bigg\}\\&=\left(\sum_{l=0}^{L}\sigma_l^4\right)|r_H[u,\,v]|^2+\left(\sum_{l=0}^{L}\sigma_l^2+\sigma^2\right)^2\delta[i-j] \qquad \text{(A.10)}\end{aligned}$$

and

$$\begin{aligned}&E\{E_c(x_i[n+u,\,k+v],\,x_i^*[n,\,k]\}\\&\quad\cdot E_c\{x_j^*[n+u,\,k+v],\,x_j[n,\,k])\}\\&=E\Bigg\{\left(\sum_{l=0}^{L}H_i^{(l)}[n,\,k]H_i^{(l)^*}[n,\,k]+\sigma^2\right)\\&\qquad\cdot\left(\sum_{l=0}^{L}H_j^{(l)}[n,\,k]H_j^{(l)^*}[n,\,k]+\sigma^2\right)^*\delta[u,\,v]\Bigg\}\\&=\left(\left|\sum_{l=0}^{L}\sigma_l^2+\sigma^2\right|^2+\sum_{l=0}^{L}\sigma_l^4\delta[i-j]\right)\delta[u,\,v]. \qquad \text{(A.11)}\end{aligned}$$

Notice that $E\{a_l^2[n,\,k]\}=0$, thus

$$\begin{aligned}&E\{E_c(x_i[n+u,\,k+v],\,x_j[n,\,k]\}\\&\qquad\cdot E_c\{x_j^*[n+u,\,k+v],\,x_i^*[n,\,k])\}=0. \qquad \text{(A.12)}\end{aligned}$$

Substituting (A.8) and (A.10)–(A.12) into (A.6), we have

$$\begin{aligned}o_{ij}[u,\,v]&=\sum_{l=0}^{L}\sigma_l^4|r_H[u,\,v]|^2\\&\quad+\left(\left|\sum_{l=0}^{L}\sigma_l^2+\sigma^2\right|^2-\sum_{l=0}^{L}\sigma_l^4\right)\delta[u,\,v]\\&\quad+\left|\sum_{l=0}^{L}\sigma_l^2+\sigma^2\right|^2\delta[i-j]. \qquad \text{(A.13)}\end{aligned}$$

Using the separation property [9] of the channel correlation function, $o_{ij}[u,\,v]$ and $p_{ij}[u,\,v]$ can be further simplified as

$$p_{ij}[u,\,v]=c_0s_t[u]s_f[v]+c_1\delta[i-j] \qquad \text{(A.14)}$$

and

$$o_{ij}[u,\,v]=c_0s_t[u]s_f[v]+(c_1-c_0)\delta[u,v]+c_1\delta[i-j] \qquad \text{(A.15)}$$

where we have used the definitions

$$s_t[u]\triangleq|r_t[u]|^2 \qquad s_f[u]\triangleq|r_f[v]|^2 \qquad \text{(A.16)}$$

and

$$c_0=\sum_{l=0}^{L}\sigma_l^4,\quad\text{and}\quad c_1=\left(\sum_{l=0}^{L}\sigma_l^2+\sigma^2\right)^2. \qquad \text{(A.17)}$$

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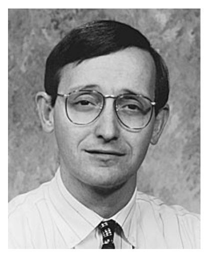

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