Adaptive Backstepping Control of a Class of Uncertain Nonlinear Systems With Unknown Backlash-Like Hysteresis

Jing Zhou, Changyun Wen, and Ying Zhang

***Abstract*—In this note, we consider the same class of systems as in a previous paper, i.e., a class of uncertain dynamic nonlinear systems preceded by unknown backlash-like hysteresis nonlinearities, where the hysteresis is modeled by a differential equation, in the presence of bounded external disturbances. By using backstepping technique, robust adaptive backstepping control algorithms are developed. Unlike some existing control schemes for systems with hysteresis, the developed backstepping controllers do not require the uncertain parameters within known intervals. Also, no knowledge is assumed on the bound of the "disturbance-like" term, a combination of the external disturbances and a term separated from the hysteresis model. It is shown that the proposed controllers not only can guarantee global stability, but also transient performance.**

***Index Terms*—Adaptive control, backstepping, hysteresis, nonlinear system, robust control.**

I. Introduction

Hysteresis exists in a wide range of physical systems and devices, such as biology optics, electromagnetism, mechanical actuators, electronic relay circuits and other areas. Control of such systems is typically challenging. For backlash hysteresis, several adaptive control schemes have recently been proposed; see, for example, [1] and [2]. In [3]–[5], an inverse hysteresis nonlinearity was constructed. An adaptive hysteresis inverse cascaded with the plant was employed to cancel the effects of hysteresis. In [1], a dynamic hysteresis model is defined to pattern a backlash-like hysteresis rather than constructing an inverse model to mitigate the effects of the hysteresis. However, in [1], the term multiplying the control and the uncertain parameters of the system must be within known intervals and the "disturbance-like" term must be bounded with known bound. Projection was used to handle the "disturbance-like" term and unknown parameters. System stability was established and the tracking error was shown to converge to a residual.

In this note, we develop two simple backstepping adaptive control schemes for the same class of nonlinear systems as in [1], with bounded external disturbances included in our case. Besides showing global stability of the system, the transient performance in terms of L_2 norm of the tracking error is derived to be an explicit function of design parameters and thus our scheme allows designers to obtain the closed loop behavior by tuning design parameters in an explicit way. In the first scheme, a sign function is involved and this can ensure perfect tracking. To avoid possible chattering caused by the sign function, we propose an alternative smooth control law and the tracking error is still ensured to approach a prescribed bound in this case. In our design, the term multiplying the control and the system parameters are not assumed to be within known intervals. The bound of the "disturbance-like" term is not required. To handle such a term, an estimator is used to estimate its bound.

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This note is organized as follows. Section II states the problem of this note and assumptions on the nonlinear systems. Sections III presents the adaptive control design based on the backstepping technique and analyzes the stability and performance. Simulation results are presented in Section IV. Finally, Section V concludes this note.

II. Problem Statement

We consider the same class of systems as in [1]. For completeness, the system model is given as follows:

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i\left(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\right) = b\omega(v) + \bar{d}(t) \tag{1}$$

where Y_i are known continuous linear or nonlinear functions, $\bar{d}(t)$ denotes bounded external disturbances, parameters a_i are unknown constants and control gain b is unknown bounded constant, v is the control input, $\omega(v)$ denotes hysteresis type of nonlinearity described by

$$\frac{d\omega}{dt} = \alpha\left|\frac{dv}{dt}\right|(cv-\omega) + B_1\frac{dv}{dt} \tag{2}$$

where α, c, and B_1 are constants, $c > 0$ is the slope of the lines satisfying $c > B_1$. Based on the analysis in [1], this equation can be solved explicitly

$$\omega(t) = cv(t) + d_1(v) \tag{3}$$

$$d_1(v) = [\omega_0 - cv_0]e^{-\alpha(v-v_0)\operatorname{sgn}\dot{v}} + e^{-\alpha v \operatorname{sgn}\dot{v}}\int_{v_0}^{v}[B_1 - c]e^{\alpha\xi(\operatorname{sgn}\dot{v})}d\xi. \tag{4}$$

The solution indicates that dynamic (2) can be used to model a class of backlash-like hysteresis as shown in Fig. 1, where the parameters $\alpha = 1, c = 3.1635$, and $B_1 = 0.345$, the input signal $v(t) = 6.5\sin(2.3t)$ and the initial condition $\omega(0) = 0$. For $d_1(v)$, it is bounded as shown in [1].

From the solution structure (3) of model (2), (1) becomes

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i\left(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\right) = \beta v(t) + d(t) \tag{5}$$

where $\beta = bc$ and $d(t) = bd_1(v(t)) + \bar{d}(t)$. The effect of $d(t)$ is due to both external disturbances and $bd_1(v(t))$. We call $d(t)$ a "disturbance-like" term for simplicity of presentation and use D to denote its bound.

Now, (5) is rewritten in the following form:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
&\vdots \\
\dot{x}_{n-1} &= x_n \\
\dot{x}_n &= -\sum_{i=1}^{r} a_i Y_i\left(x_1(t), x_2(t), \ldots, x_{(n-1)}(t)\right) \\
&\quad + \beta v(t) + d(t) \\
&= a^T Y + \beta v(t) + d(t)
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
x_1 &= x, x_2 = \dot{x}, \ldots, x_n = x^{(n-1)} \\
a &= [-a_1, -a_2, \ldots, -a_r]^T \\
Y &= [Y_1, Y_2, \ldots, Y_r]^T.
\end{aligned}$$

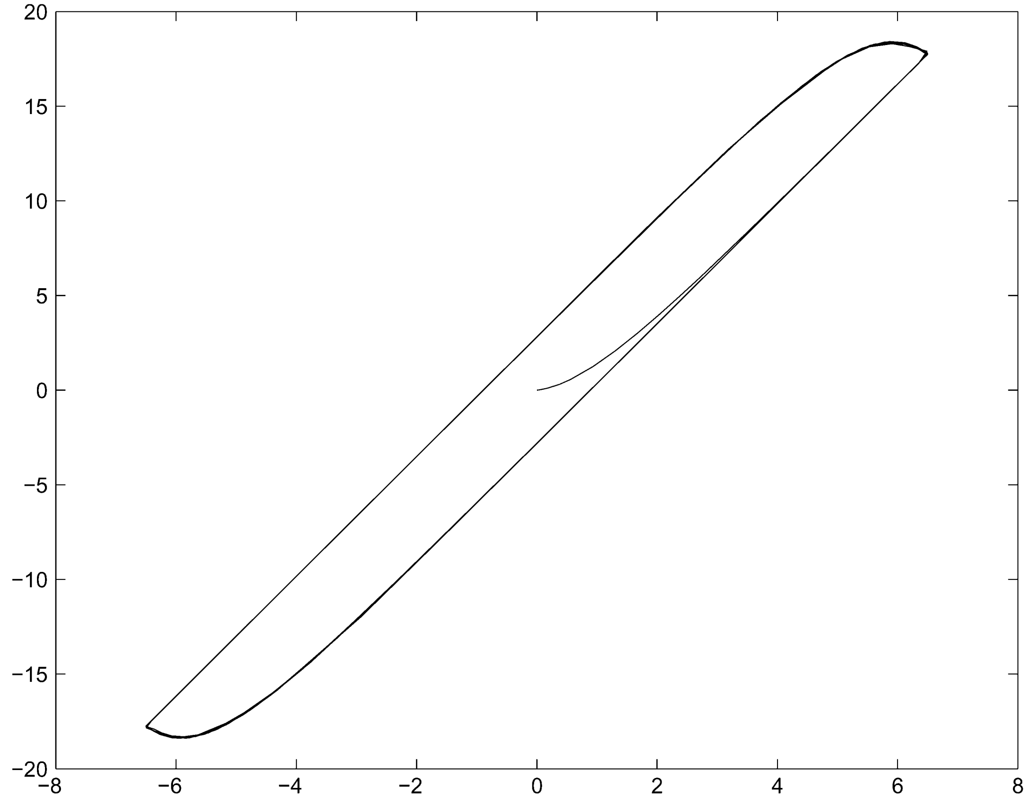

Fig. 1. Hysteresis curves.

For the development of control laws, the following assumptions are made.

Assumption 1: The uncertain parameters b and c are such that $\beta > 0$.

Assumption 2: The desired trajectory $y_r(t)$ and its $(n-1)$th-order derivatives are known and bounded.

The control objectives are to design backstepping adaptive control laws such that

- the closed loop is globally stable in sense that all the signals in the loop are uniformly ultimately bounded;
- the tracking error $x(t) - y_r(t)$ is adjustable during the transient period by an explicit choice of design parameters and $\lim_{t\to\infty} x(t) - y_r(t) = 0$ or $\lim_{t\to\infty} |x(t) - y_r(t)| \leq \delta_1$ for an arbitrary specified bound δ_1.

Remark 1: Compared with [1], the uncertain parameters β and a_i are not assumed inside known intervals. The bound D for $d(t)$ is not assumed to be known and it will be estimated by our adaptive controllers. Also the control objectives are not only to ensure global stability, but also transient performance.

III. Design of Adaptive Controllers

Before presenting the adaptive control design using the backstepping technique in [6] and [7] to achieve the desired control objectives, the following change of coordinates is made:

$$z_1 = x_1 - y_r \tag{7}$$

$$z_i = x_i - y_r^{(i-1)} - \alpha_{i-1}, \qquad i = 2, 3, \ldots, n \tag{8}$$

where α_{i-1} is the virtual control at the ith *step* and will be determined in later discussion. In the following, two control schemes are proposed.

A. Control Scheme I

To illustrate the backstepping procedures, only the last step of the design, i.e., *step* n, is elaborated in details.

- *Step* 1: We start with the equation for the tracking error z_1 obtained from (6) to (8)

$$\dot{z}_1 = z_2 + \alpha_1. \tag{9}$$

We design the virtual control law α_1 as

$$\alpha_1 = -c_1 z_1 \tag{10}$$

where c_1 is a positive design parameter. From (9) and (10), we have

$$z_1 \dot{z}_1 = -c_1 z_1^2 + z_1 z_2. \tag{11}$$

- *Step* $i (i = 2, \ldots, n-1)$: Choose

$$\alpha_i = -c_i z_i - z_{i-1} + \dot{\alpha}_{i-1}\left(x_1, \ldots, x_{i-1}, y_r, \ldots, y_r^{(i-1)}\right) \tag{12}$$

where $c_i, i = 2, \ldots, n-1$ are positive design parameters. From (8) and (12), we obtain

$$z_i \dot{z}_i = -z_{i-1} z_i - c_i z_i^2 + z_i z_{i+1}. \tag{13}$$

- *Step* n: From (6) and (8), we obtain

$$\dot{z}_n = \beta v(t) + a^T Y + d(t) - y_r^{(n)} - \dot{\alpha}_{n-1}. \tag{14}$$

Then, the adaptive control law is designed as follows:

$$v = \hat{e}\bar{v} \tag{15}$$

$$\bar{v} = -c_n z_n - z_{n-1} - \hat{a}^T Y - \operatorname{sgn}(z_n)\hat{D} + y_r^{(n)} + \dot{\alpha}_{n-1} \tag{16}$$

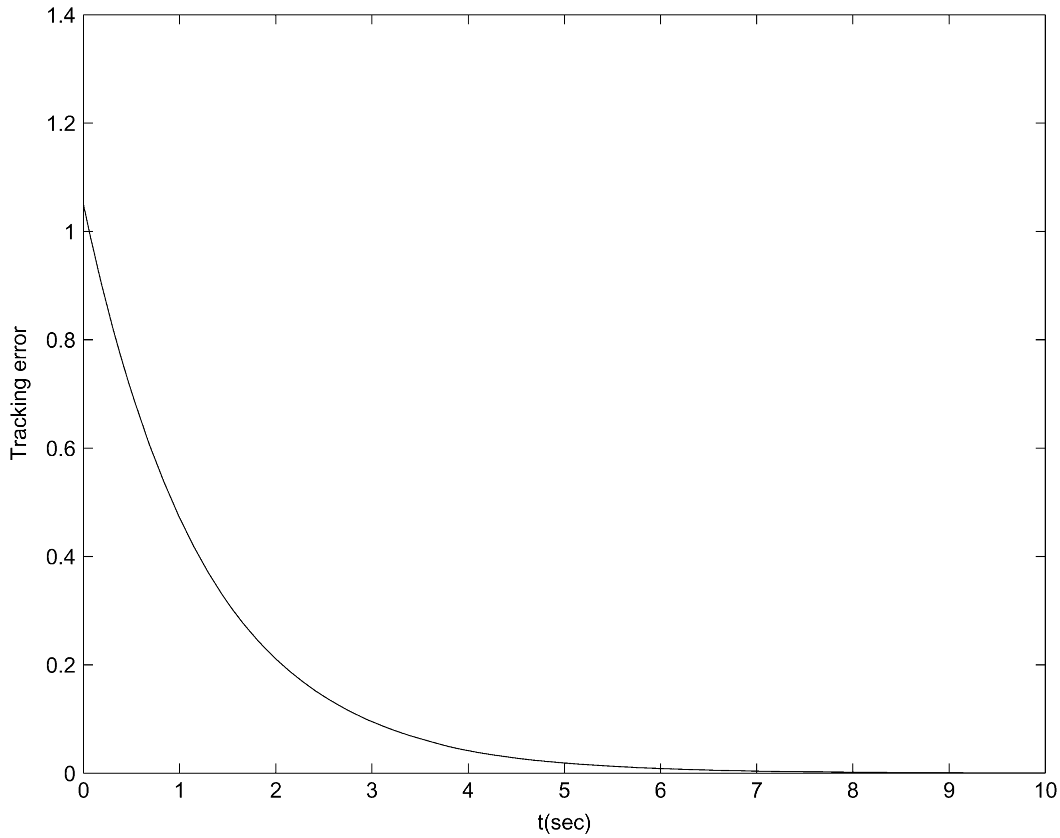

Fig. 2. Tracking error-Scheme I.

$$\dot{\hat{e}} = -\gamma \bar{v} z_n \tag{17}$$

$$\dot{\hat{a}} = \Gamma Y z_n \tag{18}$$

$$\dot{\hat{D}} = \eta |z_n| \tag{19}$$

where c_n, γ, and η are three positive design parameters, Γ is a positive–definite matrix, $\hat{e}, \hat{a}$, and $\hat{D}$ are estimates of $e = 1/\beta, a$, and D. Let $\tilde{e} = e - \hat{e}, \tilde{a} = a - \hat{a}$, and $\tilde{D} = D - \hat{D}$. Note that $\beta v(t)$ in (14) can be expressed as

$$\beta v = \beta \hat{e} \bar{v} = \bar{v} - \beta \tilde{e} \bar{v}. \tag{20}$$

From (14), (16), and (20), we obtain

$$\dot{z}_n = -c_n z_n - z_{n-1} + \tilde{a}^T Y - \operatorname{sgn}(z_n)\hat{D} + d(t) - \beta \tilde{e} \bar{v}. \tag{21}$$

We define Lyapunov function as

$$V = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2} \tilde{a}^T \Gamma^{-1} \tilde{a} + \frac{\beta}{2\gamma} \tilde{e}^2 + \frac{1}{2\eta} \tilde{D}^2. \tag{22}$$

Then, the derivative of V along with (6) and (15) to (19) is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n} z_i \dot{z}_i + \tilde{a}^T \Gamma^{-1} \dot{\tilde{a}} + \frac{\beta}{\gamma} \tilde{e} \dot{\tilde{e}} + \frac{1}{\eta} \tilde{D} \dot{\tilde{D}} \\ &\le -\sum_{i=1}^{n} c_i z_i^2 + \tilde{a}^T \Gamma^{-1} (\Gamma Y z_n - \dot{\hat{a}}) \\ &\quad - \frac{\beta}{\gamma} \tilde{e} (\gamma \bar{v} z_n + \dot{\hat{e}}) + \frac{1}{\eta} \tilde{D} (\eta |z_n| - \dot{\hat{D}}) \qquad (23) \\ &= -\sum_{i=1}^{n} c_i z_i^2 \qquad (24) \end{aligned}$$

where we have used (11), (13), (21), and the fact that $z_n d(t) \le |z_n| D$ to obtain (52).

We then have the following stability and performance results based on this scheme.

Theorem 1: Consider the uncertain nonlinear system (1) satisfying Assumptions 1–2. With the application of controller (15) and the parameter update laws (17)–(19), the following statements hold.

- The resulting closed-loop system is globally stable.
- The asymptotic tracking is achieved, i.e.,

$$\lim_{t \to \infty} [x(t) - y_r(t)] = 0. \tag{25}$$

- The transient tracking error performance is given by

$$\| x(t) - y_r(t) \|_2 \le \frac{1}{\sqrt{c_1}} \left(\frac{1}{2} \tilde{a}(0)^T \Gamma^{-1} \tilde{a}(0) + \frac{\beta}{2\gamma} \tilde{e}(0)^2 + \frac{1}{2\eta} \tilde{D}(0)^2 \right)^{1/2}. \tag{26}$$

Proof: From (24), we established that V is non increasing. Hence, $z_i, i = 1, \ldots, n, \hat{e}, \hat{a}, \hat{D}$ are bounded. By applying the LaSalle–Yoshizawa theorem in [7] to (24), it further follows that $z_i(t) \to 0, i = 1, \ldots, n$ as $t \to \infty$, which implies that $\lim_{t \to \infty} [x(t) - y_r(t)] = 0$.

From (24), we also have that

$$\begin{aligned} \| z_1 \|_2^2 = \int_0^\infty |z_1(\tau)|^2 d\tau &\le \frac{1}{c_1} (V(0) - V(\infty)) \\ &\le \frac{1}{c_1} V(0) \end{aligned} \tag{27}$$

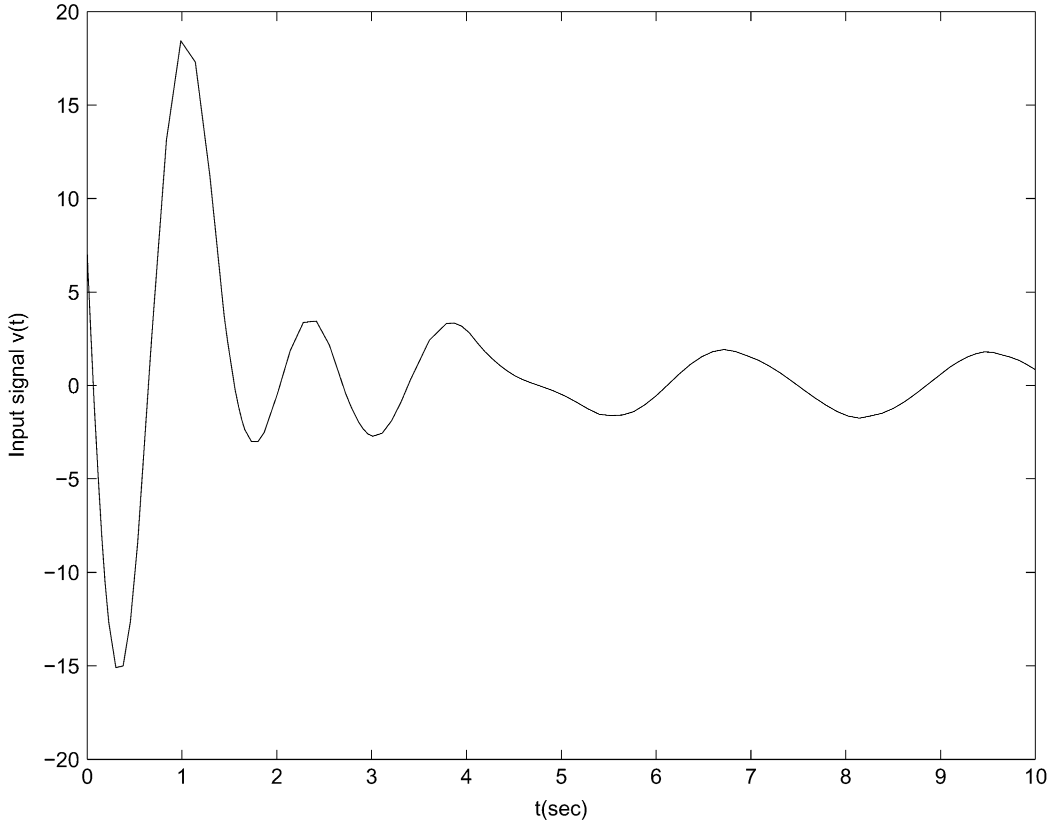

Fig. 3. Control signal $v(t)$-Scheme I.

Thus, by setting $z_i(0) = 0, i = 1, \ldots, n$, we obtain

$$V(0) = \frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) + \frac{\beta}{2\gamma}\tilde{e}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2 \tag{28}$$

a decreasing function of γ, η, and Γ, independent of c_1. This means that the bound resulting from (27) and (28) is

$$\| z_1 \|_2 \leq \frac{1}{\sqrt{c_1}}\left(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) + \frac{\beta}{2\gamma}\tilde{e}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\right)^{1/2}. \tag{29}$$

$\triangle\triangle\triangle$

Remark 2: From Theorem 1, the following conclusions can be obtained.

- The transient performance depends on the initial estimate errors $\tilde{e}(0), \tilde{a}(0), \tilde{D}(0)$, and the explicit design parameters. The closer the initial estimates $\hat{e}(0), \hat{a}(0)$, and $\hat{D}(0)$ to the true values e, a, and D, the better the transient performance.
- The bound for $\| x(t) - y_r(t) \|_2$ is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains γ, η, and Γ.
- To improve the tracking error performance we can also increase the gain c_1. However, increasing c_1 will influence other performance such as $\| \dot{x} - \dot{y}_r \|_2$ as shown later.

Since $\dot{V} \leq 0$, immediately from (22) we know

$$V(t) = \sum_{i=1}^{n}\frac{1}{2}z_i^2 + \frac{1}{2}\tilde{a}^T\Gamma^{-1}\tilde{a} + \frac{\beta}{2\gamma}\tilde{e}^2 + \frac{1}{2\eta}\tilde{D}^2 \leq V(0).$$

Then

$$\| z_i \|_\infty \leq \sqrt{2V(0)}, \qquad i = 1, \ldots, n \tag{30}$$

$$\| \tilde{a} \|_\infty \leq \sqrt{\bar{\lambda}(\Gamma)}\sqrt{2V(0)} \tag{31}$$

From (7), (8) for $i = 2$, and (10), we get

$$\begin{aligned}\| \dot{x} - \dot{y}_r \|_2 &= \| z_2 - c_1 z_1 \|_2 \\ &\leq \| z_2 \|_2 + c_1 \| z_1 \|_2 .\end{aligned} \tag{32}$$

Similar to the proof of (29), we can get $\| z_2 \|_2 \leq \sqrt{V(0)}/\sqrt{c_2}$ and, thus

$$\| \dot{x} - \dot{y}_r \|_2 \leq \left(\frac{1}{\sqrt{c_2}} + \sqrt{c_1}\right)\sqrt{V(0)} \tag{33}$$

From (33), we can see that increasing c_1 also increase the error $\| \dot{x} - \dot{y}_r \|_2$. This suggests fixing the gain c_1 to some acceptable value and adjust the other gains such as γ, η, and Γ.

B. Control Scheme II

In the previous scheme, a discontinuous function $\mathrm{sgn}(z_n)$ is involved in the control and this may cause chattering. To avoid this, we now propose an alternative smooth control scheme.

First, we define a function $sg_i(z_i)$ as follows:

$$sg_i(z_i) = \begin{cases} \dfrac{z_i}{|z_i|}, & |z_i| \geq \delta_i \\ \dfrac{z_i}{(\delta_i^2 - z_i^2)^{n-i+2} + |z_i|}, & |z_i| < \delta_i \end{cases} \tag{34}$$

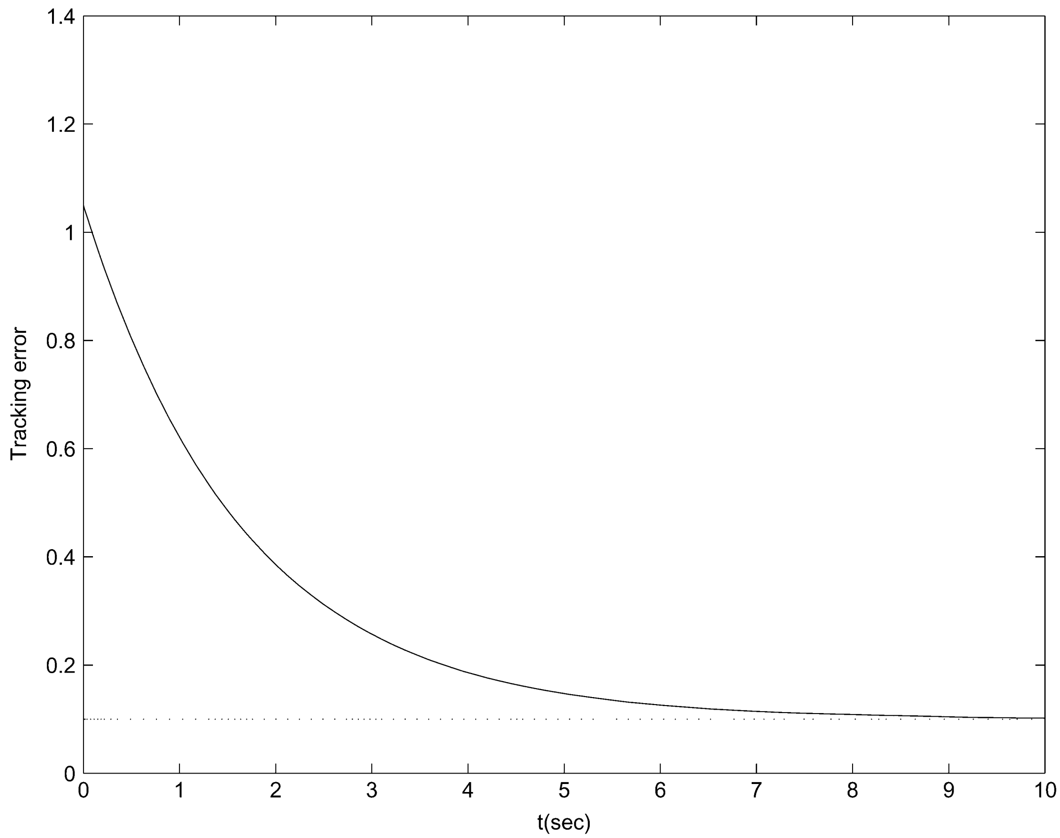

Fig. 4. Tracking error-Scheme II.

where $\delta_i (i = 1, \ldots, n)$ is a positive design parameter. It can be shown that $sg_i(z_i)$ is $(n - i + 2)$th-order differentiable. We also design a function $f_i(z_i)$ as

$$f_i(z_i) = \begin{cases} 1, & |z_i| \geq \delta_i \\ 0, & |z_i| < \delta_i. \end{cases} \tag{35}$$

Then, we can get

$$sg_i(z_i)f_i(z_i) = \begin{cases} 1, & z_i \geq \delta_i \\ 0, & |z_i| < \delta_i \\ -1, & z_i \leq \delta_i \end{cases} \tag{36}$$

To ensure the resultant functions are differentiable, we replace z_i^2 by $(|z_i| - \delta_i)^{n-i+2} f_i$ in the Lyapunov functions for $i = 1, \ldots, n$ in Section 3.1 and we also replace z_i by $(|z_i| - \delta_i)^{n-i+1} sg_i$ in the design procedure as detailed here.

- *Step* 1: We design virtual control law α_1 as

$$\alpha_1 = -\left(c_1 + \frac{1}{4}\right)(|z_1| - \delta_1)^n sg_1(z_1) - (\delta_2 + 1)sg_1(z_1) \tag{37}$$

where c_1 is a positive design parameter. We choose Lyapunov function V_1 as

$$V_1 = \frac{1}{n+1}(|z_1| - \delta_1)^{n+1} f_1. \tag{38}$$

Then, the derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= (|z_1| - \delta_1)^n f_1 sg_1(z_1)\dot{z}_1 \\ &\leq -\left(c_1 + \frac{1}{4}\right)(|z_1| - \delta_1)^{2n} f_1 \\ &\quad + (|z_1| - \delta_1)^n(|z_2| - \delta_2 - 1)f_1 \end{aligned} \tag{39}$$

where (9) and (37) have been used.

- *Step* 2: We design virtual control law α_2 as

$$\begin{aligned} \alpha_2 = -\left(c_2 + \frac{5}{4}\right)(|z_2| - \delta_2)^{n-1} sg_2(z_2) \\ + \dot{\alpha}_1 - (\delta_3 + 1)sg_2(z_2) \end{aligned} \tag{40}$$

where c_2 is positive design parameter. We design Lyapunov function V_2 as

$$V_2 = \frac{1}{n}(|z_2| - \delta_2)^n f_2 + V_1. \tag{41}$$

Then, the derivative of V_2 is

$$\begin{aligned} \dot{V}_2 \leq \sum_{i=1}^{2}(|z_i| - \delta_i)^{2(n-i+1)} f_i + M_2 \\ + (|z_2| - \delta_2)^{n-1}(|z_3| - \delta_3 - 1)f_2 \end{aligned} \tag{42}$$

where $M_2 = -(1/4)(|z_1| - \delta_1)^{2n} f_1 + (|z_1| - \delta_1)^n(|z_2| - \delta_2 - 1)f_1 - (|z_2| - \delta_2)^{2(n-1)} f_2$. Now, we show that $M_2 < 0$. It is clear that $M_2 \leq 0$ for $|z_2| < \delta_2 + 1$. For $|z_2| \geq \delta_2 + 1$

$$\begin{aligned} M_2 &\leq -\frac{1}{4}(|z_1| - \delta_1)^{2n} f_1 + \frac{1}{4}(|z_1| - \delta_1)^{2n} f_1^2 \\ &\quad + (|z_2| - \delta_2 - 1)^2 - (|z_2| - \delta_2)^{2(n-1)} \\ &< (|z_2| - \delta_2)^2 - (|z_2| - \delta_2)^{2(n-1)} \\ &= (|z_2| - \delta_2)^2(1 - (|z_2| - \delta_2)^{2(n-2)}) \leq 0. \end{aligned} \tag{43}$$

Then, (42) is written as

$$\begin{aligned} \dot{V}_2 \leq -\sum_{i=1}^{2} c_i(|z_i| - \delta_i)^{2(n-i+1)} f_i \\ + (|z_2| - \delta_2)^{n-1}(|z_3| - \delta_3 - 1)f_2. \end{aligned} \tag{44}$$

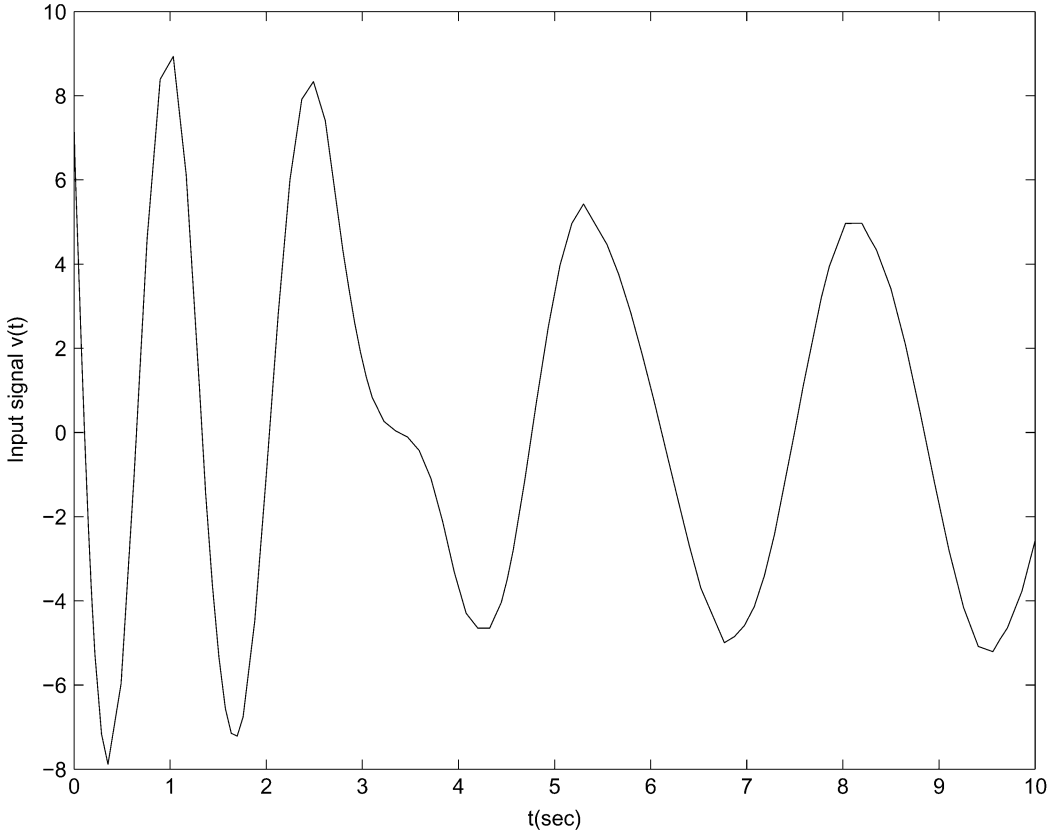

Fig. 5. Control signal $v(t)$-Scheme II.

- *Step* i ($i = 3, \ldots, n-1$): Choose

$$\alpha_i = -\left(c_i + \frac{5}{4}\right)(|z_i| - \delta_i)^{n-i+1} sg_i(z_i) + \dot{\alpha}_{i-1} - (\delta_{i+1} + 1) sg_i(z_i) \quad (45)$$

where c_i is a positive design parameter.

- *Step* n: The control law and parameter update laws are designed as follows:

$$v = \hat{e}\bar{v} \quad (46)$$

$$\bar{v} = -(c_n + 1)(|z_n| - \delta_n) sg_n(z_n) - \hat{a}^T Y - sg_n \hat{D} + y_r^{(n)} + \dot{\alpha}_{n-1} \quad (47)$$

$$\dot{\hat{e}} = -\gamma \bar{v}(|z_n| - \delta_n) f_n sg_n(z_n) \quad (48)$$

$$\dot{\hat{a}} = \Gamma Y(|z_n| - \delta_n) f_n sg_n(z_n) \quad (19)$$

$$\dot{\hat{D}} = \eta(|z_n| - \delta_n) f_n \quad (50)$$

where c_n, γ, and η are three positive design parameters, Γ is a positive–definite matrix, and $\hat{e}, \hat{a}$, and $\hat{D}$ are estimates of $e = 1/\beta$, a and D. We define a Lyapunov function as

$$V = \sum_{i=1}^{n} \frac{1}{n-i+2}(|z_i| - \delta_i)^{n-i+2} f_i + \frac{1}{2}\tilde{a}^T \Gamma^{-1} \tilde{a} + \frac{\beta}{2\gamma}\tilde{e}^2 + \frac{1}{2\eta}\tilde{D}^2. \quad (51)$$

Then, the derivative of V is given by

$$\dot{V} = \dot{V}_{n-1}(|z_n| - \delta_n)^2 f_n sg_n(z_n) \dot{z}_n + \tilde{a}^T \Gamma^{-1} \dot{\tilde{a}} + \frac{\beta}{\gamma}\tilde{e}\dot{\tilde{e}} + \frac{1}{\eta}\tilde{D}\dot{\tilde{D}}$$

$$\leq -\sum_{i=1}^{n} c_i (|z_i| - \delta_i)^{2(n-i+1)} f_i + \tilde{a}^T \Gamma^{-1}(\Gamma Y(|z_n| - \delta_n) f_n sg_n(z_n) - \dot{\hat{a}}) - \frac{\beta}{\gamma}\tilde{e}(\gamma \bar{v}(|z_n| - \delta_n) f_n sg_n(z_n) + \dot{\hat{e}}) + \frac{1}{\eta}\tilde{D}(\eta(|z_n| - \delta_n) f_n - \dot{\hat{D}}) \quad (52)$$

$$= -\sum_{i=1}^{n} c_i (|z_i| - \delta_i)^{2(n-i+1)} f_i \quad (53)$$

where (6), (37), (40), and (46)–(50) have been used.

Theorem 2: Consider the uncertain nonlinear system (1) satisfying Assumptions 1 and 2. With the application of controller (46) and the parameter update laws (48) to (50), the following statements hold.

- The resulting closed-loop system is globally stable.
- The tracking error approaches δ_1 asymptotically, i.e.,

$$\lim_{t \to \infty} |x(t) - y_r(t)| = \delta_1. \quad (54)$$

- The transient tracking error performance is given by

$$\| x(t) - y_r(t) \|_2 \leq \delta_1 + \frac{1}{c_1^{2n}} \left(\frac{1}{2}\tilde{a}(0)^T \Gamma^{-1} \tilde{a}(0) + \frac{\beta}{2\gamma}\tilde{e}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\right)^{1/2n} \quad (55)$$

with $z_i(0) = \delta_i, i = 1, \ldots, n$.

Proof: Based on (53), the results can be shown by following similar steps to that of Theorem 1. △△△

Note that similar remarks made in Remark 2 are also applicable here.

IV. Simulation Studies

In this section, we illustrate the aforementioned methodologies on the same example system in [1] which is described as

$$\dot{x} = a\frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + b\omega(t) \tag{56}$$

where ω represents the output of the hysteresis nonlinearity. The actual parameter values are $b = 1$ and $a = 1$. Without control, i.e., $\omega(t) = 0$, (56) is unstable as shown in [1], because $\dot{x} = (1 - e^{-x(t)})/(1 + e^{-x(t)}) > 0$ for $x > 0$, and $\dot{x} < 0$ for $x < 0$. The objective is to control the system state x to follow a desired trajectory $y_r(t) = 12.5\sin(2.3t)$ as in [1].

In the simulation of Scheme I, the robust adaptive control law (15)–(19) was used, taking $c_1 = 0.9, \gamma = \Gamma = 0.1, \eta = 0.2$. The initial values are chosen as follows: $\hat{e}(0) = 0.8/3, \hat{a}(0) = 1.5, \hat{D}(0) = 2, x(0) = 1.05$, and $v(0) = 0$ which are the same as in [1]. The simulation results presented in the Figs. 2 and 3 are system tracking error and input.

In the simulation of Scheme II by using the robust adaptive control law (46)–(50), we choose $c_1, \gamma, \eta, \Gamma$, and the initial values to be same as before and $\delta_1 = 0.1$. The simulation results presented in Figs. 4 and 5 are system tracking error and input. Clearly, all the results verify our theoretical findings and show the effectiveness of the control schemes.

V. Conclusion

This note presents two backstepping adaptive controller design schemes for a class of uncertain nonlinear single-input–single-output system preceded by unknown backlash-like hysteresis nonlinearities, where the hysteresis is modeled by a differential equation, in the presence of bounded external disturbances. In the first scheme, a sign function is involved and this can ensure perfect tracking. To avoid possible chattering caused by the sign function, we propose an alternative smooth control law and the tracking error is still ensured to approach a prescribed bound in this case. Unlike some existing control schemes, the developed backstepping controls do not require the model parameters within known intervals and the knowledge on the bound of "disturbance-like" term is not required. Besides showing global stability, we also give an explicit bound on the L_2 performance of the tracking error in terms of design parameters. Simulation results illustrates the effectiveness of our schemes.

To further improve system performance such as the tracking error, especially in the case without using sign functions, it is worthy to take the system hysteresis into account in the controller design, instead of only considering its effect like bounded disturbances. The first step of achieving this is perhaps to obtain an efficient adaptive hysteresis inverse, which is still unclear and currently under investigation.

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On Compensation of Wave Reflections in Transmission Lines and Applications to the Overvoltage Problem AC Motor Drives

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***Abstract*—In several practical applications actuators are interconnected to a controlled plant through long cables. If the actuator operates at a fast sampling rate (with respect to the propagation delay of the cable) and its impedance cannot be neglected, wave reflections will occur and the transmitted pulse will be deformed—degrading the control quality. In this note, exploiting the scattering variables representation of the transmission line, we provide a framework for the design of active compensators to reduce the wave reflection problem. The compensators, implementable with regulated current and voltage sources, can be placed either on the actuator side or the plant side, and the only required prior knowledge is the transmission line characteristic impedance and the propagation delay. An adaptive implementation that obviates the need of the lines characteristic impedance, but still requires the knowledge of the propagation delay, is also presented. We prove the existence of an *ideal* scheme that transforms the line into a pure delay transfer which, unfortunately, yields an ill-posed interconnection and therefore has to be approximated for its practical application. The proposed design method is illustrated with a benchmark ac drives example consisting of a pulsewidth modulation inverter and an induction motor.**

***Index Terms*—Impedance, infinite dimensional systems, motor control, overvoltage, pulsewidth modulation (PWM) inverter, reflection coefficient, transmission lines, wave equation.**

I. Introduction

In this note, we are interested in the problem of compensation of the wave effects that appear when a fast sampling actuator, with non-negligible impedance, is coupled to the controlled plant through long feeding cables. In this case, the connecting cables behave as a transmission line inducing a wave reflection that deforms the transmitted

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