Adaptive Beamforming: Spatial Filter Designed Blocking Matrix

Sven Nordebo, Ingvar Claesson, and Sven Nordholm

Abstract—**Controlling the resolution in adaptive beamformers is often crucial. A simple method that works for both narrow-band and broad-band arrays is presented. This method is based on the normalized leaky LMS algorithm in conjunction with a generalized sidelobe canceller (GSC) structure, where the GSC is designed using a spatial filtering approach. In essence, the suppression of the spatial filters and the implicit noise of the leaky LMS algorithm together determine the adaptive beamformer. Analytical expressions are given for the Wiener filters and the output spectrum versus frequency and point source location. These expressions are employed in the design specification of the spatial filters and to obtain conditions for a controlled quiescent beamformer response. Simulation results are presented to illustrate the behavior of the array.**

I. INTRODUCTION

By using adaptive filters behind the array elements in delay- and sum-beamformers (see Fig. 1), the resolution can be substantially increased [1], [2] and a simultaneous cancellation of multiple jammers obtained. Such an increase in resolution and capability is often desired, but it might also lead to target cancellation caused by inaccuracies in far-field approximations or sensor and target locations.

Careful calibrations can sometimes solve the problem, but there are situations, such as with widespread sources, where this super-resolution must be constrained in some target area [3]. Typical methods are linear and quadratic constraints on the beamformer derived from eigenvector expansions [2], [4], derivatives of the signal power [5]–[7], and nulling an area of coefficients in the adaptive filters [8]. However, several of these methods are reported to be sensitive to the choice of array origin [6], [9] since the constraints imposed are origin dependent. This drawback limits their usefulness, although different methods to eliminate the origin dependence have been proposed [6], [9].

We propose a method to design the blocking structure in a GSC using a spatial filtering technique. This technique is *not* dependent on the choice of array origin since the spatial filter specification, which imposes the constraints, is origin independent. The idea is straightforward, and several filter design methods can be used to calculate the blocking structure. The major advantage using this method is the direct approach. The desired spatial resolution and the frequency interval of interest, together with the built-in noise level of the leaky LMS algorithm, directly determine which frequency and spatial/angle regions the beamformer conforms to the upper, desired branch of the GSC.

Manuscript received June 1, 1993; revised May 24, 1994. This work was supported by Nutek, the National Swedish Board for Technical Development.
S. Nordholm and I. Claesson are with the Department of Telecommunication Theory, University of Lund, Lund, Sweden.
S. Nordebo is with the Department of Signal Processing, University of Karlskrona/Ronneby, S-372 25 Ronneby, Sweden.
IEEE Log Number 9403707.

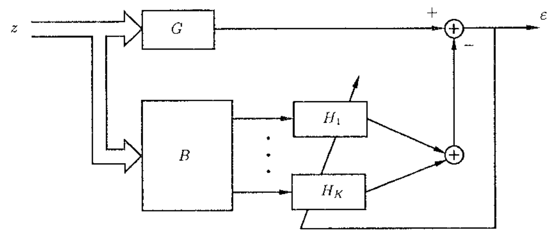

Fig. 1. A broad-band adaptive generalized sidelobe canceller. Here, G and B are fixed beamformers and $H_1 - H_K$ are adaptive filters.

II. MAIN IDEA AND GENERAL SOLUTION

The signals received by the array elements are assumed to be stationary uncorrelated point sources m_p with spectral densities $R_{m_p}(f)$. When located in the far field, the signals impinge as plane waves. The array consists of N elements taking L snapshots. Mutually uncorrelated noise is present at the array inputs with spectral density $R_n(f)$. An adaptive GSC with K adaptive filters is used (see Fig. 1), and it discriminates between the target signal and jammer signals only by their spatial locations and spectral contents. Hence, the single-input signal scenario is highly interesting, in particular with regard to the behavior of the beamformer response when this signal arrives from different points.

The array response vector $\boldsymbol{d}$ from a point source to each weight is determined by the frequency $\omega = 2\pi f$ and the time delay τ to each weight. The delay τ is determined by the wave propagation velocity c, the snapshot number l, and the distance from the source to the corresponding array element n. The array response vector is, in general, also dependent on each array element characteristic which we omit in the sequel. The array response vector is thus given by

$$\boldsymbol{d} = [e^{-j\omega\tau_1} \dots e^{-j\omega\tau_{NL}}]^T \tag{1}$$

in a reflectionless and isotropic medium. The response vector is not dependent on the coordinate system since only the distance between the source and the elements enters into the expression. The filter function $G(f)$ from a point to the output of the upper beamformer is given by

$$G(f) = \boldsymbol{g}^H \boldsymbol{d} \tag{2}$$

where the vector $\boldsymbol{g}$ contains the weights of the upper beamformer.

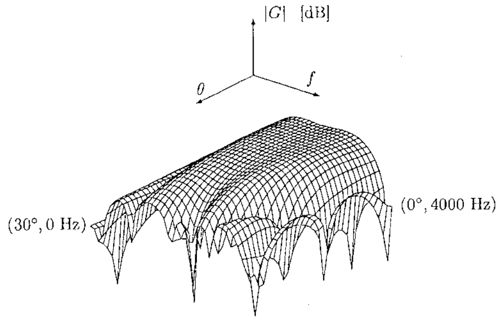

Fig. 2. Upper beamformer with quiescent response constraints imposed. Passband within f = 1600–3200 Hz and θ = 0–15°.

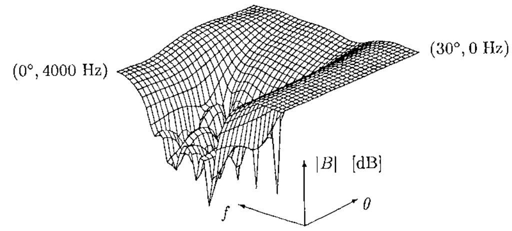

Fig. 3. Lower beamformer with no constraints imposed. Stopband within f = 1600–3200 Hz and θ = 0–15°.

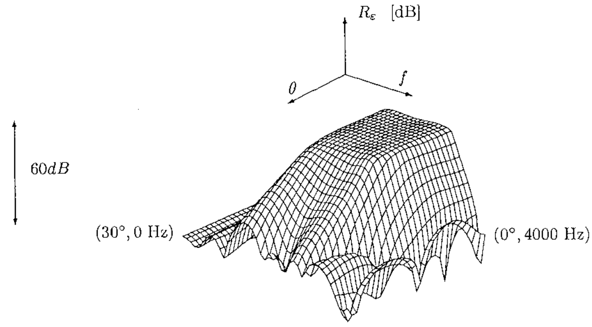

Fig. 4. Total adaptive beamformer behavior for white signal incident from different directions. The region f = 1600–3200 Hz and θ = 0–15° protected. Output spectral density plotted for varying θ = 0–30°.

Assuming that the injected noise is chosen to dominate over the sensor element noise, the Wiener solutions with a single-input signal are approximately given by (see also Appendix A)

$$R_\epsilon(f,\theta) \approx \frac{R_m(f)|G(f,\theta)|^2}{1+K(R_m(f)/R_\eta(f))|B(f,\theta)|^2} = \frac{R_m(f)|G(f,\theta)|^2}{1+K\cdot \mathrm{SNR}_x(f,\theta)} \tag{9}$$

when $\mathrm{SNR}_x(f,\theta)$ is the signal-to-noise ratio at the input of the adaptive filters. This is the key expression of the beam former. In the protected region, the adaptive beamformer is given by $|G(f,\theta)| \approx 1$ if $|B(f,\theta)|^2 \le R_\eta(f)/KR_m(f)$. Thus, in the passband region, $R_\epsilon(f,\theta) \approx R_m(f,\theta)$. In the stopband region, the signal is suppressed by the beamformer since $|G(f,\theta)|$ is small and $|B(f,\theta)| \approx 1$ implies further reduction via the canceller.

A design with quiescent response constraints included is shown in Figs. 2 and 3. The lower beamformer is designed as the "inverse" of the upper beamformer. The objective in this design has been to obtain a rectangular region in frequency and direction where no signal cancellation occurs, while full adaptivity is allowed outside this region. Fig. 4 shows the result for a single target signal when the upper and lower beamformers are combined and the adaptive filters are active. Here, a target source $m(t)$ with constant spectrum was moved between 0° and 30°, and the optimum filters $\boldsymbol{H}(f,\theta)$ and the corresponding output spectrum $R_\epsilon(f,\theta)$ were calculated for each position.

IV. 1-D Spatial Filter Solution

A further restriction is now imposed on the fixed beamformers $\boldsymbol{g}$ and $\boldsymbol{b}_k$. We force their sample depth to 1, i.e., $L = 1$, and we are back to a more conventional beamformer situation. In this case, the spatial filter $B(f,\theta)$ is given by

$$B(f,\theta) = B(\Omega_1) = \sum_{m=0}^{M-1} \beta_m e^{-j\Omega_1 m} \tag{10}$$

where

$$\Omega_1 = 2\pi f d \frac{\sin\theta}{c}. \tag{11}$$

The kth spatial filter $B_k(f,\theta)$ is again obtained as

$$B_k(f,\theta) = e^{-j2\pi f(k-1)T_{sp}(\theta)} B(f,\theta). \tag{12}$$

Since we now have lost one design dimension, we propose the following procedure to obtain a desired beamformer [14].

1) Determine over which temporal frequency band $[f_l, f_u]$ the array is aimed to operate.

2) Select an angular interval $[\theta_l, \theta_u]$ over which the array is not allowed to cancel the target signal. Normally, $\theta_l = 0$ since the look direction is chosen to broadside.

3) Using the relationship in (11), a spatial filter B is designed suppressing the region $[\Omega_{1,l}, \Omega_{1,u}]$ sufficiently, where

$$\Omega_{1,l} = \frac{d\sin\theta_l}{c}\cdot 2\pi f_l$$
$$\Omega_{1,u} = \frac{d\sin\theta_u}{c}\cdot 2\pi f_u.\&(13)$$

4) Put the spatial filter coefficients in the columns of the blocking matrix $\boldsymbol{B}$, and shift them one step per column.

The filter specification above is a conventional digital high-pass filter specification if either θ_l or f_l is zero. Such a design can easily be fulfilled using equiripple filters [13]. The necessary filter stopband suppression is determined via the Wiener solution. The design parameters are, in particular, the ratio R_m/R_η and the number of adaptive filters K. With these parameters set and the filter length M chosen, the filter coefficients can effectively be found using ParksMcClellan optimum FIR filter design. Since one normally wants a perfect zero at the spatial frequency zero (no cancellation in the

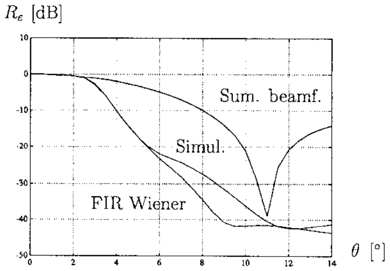

Fig. 8. Power spectrum density PSD R_ϵ and R_{y_d}: finite length Wiener solution evaluated for 3000 Hz and simulation end results (weights adapted, frozen, and response calculated). Spectral expressions evaluated for 3000 Hz. $N = 12$ elements and $d = 5$ cm, 1 spatial filter with 12 coefficients designed to protect $f = 0$–3200 Hz, and $\theta = 0$–2°. Adaptive filter with 21 coefficients.

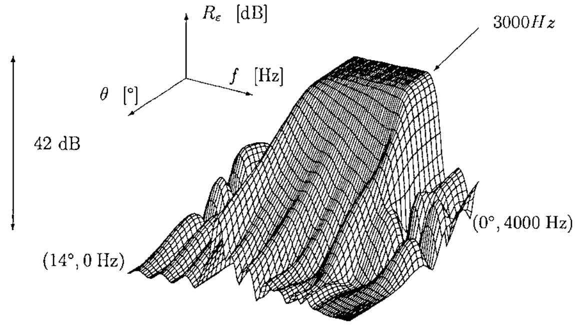

Fig. 9. R_ϵ: adapted, frozen, and calculated beamformer response for different directions. Microphone array with 12 elements and 5 cm interspacing, 1 spatial filter with 12 coefficients designed to protect $f = 0$–3200 Hz, and $\theta = 0$–2°. Adaptive filter with 21 coefficients.

The leakage factor γ is given by

$$\gamma = 1 - 2\mu\sigma_\eta^2 \tag{21}$$

where σ_η^2 is the variance of the "injected" white noise.

Signals incident from different directions will cause a wide spread in the power of the adaptive filter input signals. A power normalization of μ with an estimate of $tr\boldsymbol{R}$ is therefore vital. This facilitates stability of the algorithm, and at the same time, a reasonable excess mean-square error is obtained.

Figs. 8 and 9 show simulation results with a single source using the normalized leaky algorithm described above. After adaptation from each direction, the filter coefficients are frozen, and the spectral density of the output signal of the beamformer is calculated and compared with the finite Wiener solution. Fig. 10 shows a corresponding learning curve. Good resemblance with the theoretical curves is obtained, where the input power is significant despite the statistical fluctuations of the weights.

VI. Conclusions

In this paper, we have described a general method for controlling super-resolution in adaptive arrays using a spatial filter design for the blocking structure of a generalized sidelobe canceller. Expressions were given to explain the behavior of the adaptive array, which aids in the design. Simulations verify the expected behavior of the array.

VII. Appendix A Wiener Solutions

The infinite Wiener solution in the general case with no restrictions on geometry or causality satisfies the normal equations in the frequency domain:

$$\boldsymbol{R}_{y_d\boldsymbol{x}} = \boldsymbol{H}\boldsymbol{R}_{\boldsymbol{x}\boldsymbol{x}} \tag{22}$$

where $\boldsymbol{R}_{y_d\boldsymbol{x}}$ is a $1 \times K$ vector containing the cross-spectral densities $R_{y_dx_i}$, $\boldsymbol{H}$ is a $1 \times K$ vector of frequency functions H_i, and $\boldsymbol{R}_{\boldsymbol{x}\boldsymbol{x}}$ is a $K \times K$ matrix of cross-spectral densities $R_{x_ix_j}$. For notational convenience, the arguments t, f, and θ are omitted in this Appendix.

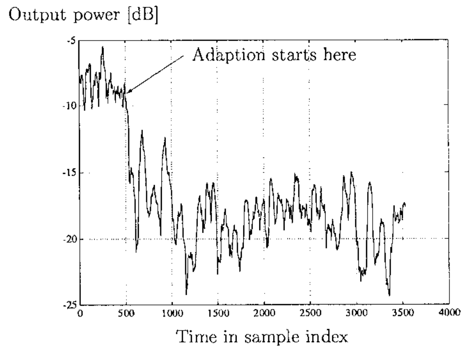

Fig. 10. Typical learning curve. Incident signal from 7°. Array with 12 elements and 5 cm interspacing, 1 spatial filter with 12 coefficients designed to protect $f = 0$–3200 Hz, and $\theta = 02°$. Adaptive filter with 21 coefficients.

The upper beamformer output signal y_d is given by

$$y_d = \boldsymbol{g}^H\boldsymbol{z}. \tag{23}$$

The array weight vector $\boldsymbol{g}$ is divided into N subvectors, one per sensor, $\boldsymbol{g}_l$ of length L so that $\boldsymbol{g} = [\boldsymbol{g}_0 \cdots \boldsymbol{g}_{N-1}]^T$. The array input vector is analogously divided, yielding $\boldsymbol{z} = [\boldsymbol{z}_0 \cdots \boldsymbol{z}_{N-1}]^T$.

The adaptive filter input vector $\boldsymbol{x}$ is obtained as $\boldsymbol{x} = \boldsymbol{B}^H\boldsymbol{z}+\boldsymbol{\eta}$, where $\boldsymbol{\eta}$ is a vector containing the leaky noise sources η_i. The $NL \times K$ blocking matrix $\boldsymbol{B}$ is given by

$$\boldsymbol{B} = [\boldsymbol{b}_1 \cdots \boldsymbol{b}_K] \tag{24}$$

where each column $\boldsymbol{b}_i$ is also divided into N subvectors $\boldsymbol{b}_{il}$ of length L. The input signal to the ith adaptive filter is

$$x_i = \boldsymbol{b}_i^H\boldsymbol{z} + \eta_i. \tag{25}$$

We define the vector $\boldsymbol{d}_0 = [1\ e^{-j2\pi fT} \dots e^{-j2\pi f(L-1)T}]^T$. In the upper beamformer, the source m and noise n_l are filtered by $\boldsymbol{g}^H\boldsymbol{d}$ and $\boldsymbol{g}_l^H\boldsymbol{d}_0$, respectively. In the lower beamformers, the corresponding frequency functions are $\boldsymbol{b}_i^H\boldsymbol{d}$ and $\boldsymbol{b}_{il}^H\boldsymbol{d}_0$.

VIII. Appendix B
Controlled Quiescent Response

In this Appendix, we regard all frequency functions and spectra from Appendix A as functions of the normalized frequency variable $\nu = fT$, assuming that all signals are band-limited and sampled properly with the sampling rate $1/T$. The quiescent response is the response of the beamformer with the adaptive weights frozen to the situation when the only signals present are sensor noise. A zero quiescent Wiener filter solution implies a controlled quiescent response given by (see Appendix A)

$$R_\epsilon = R_m|G|^2 + R_n \boldsymbol{g}^H \boldsymbol{D}_0 \boldsymbol{g} \approx R_m|G|^2 \tag{44}$$

where G is the response of the upper beamformer. From Appendix A, we see that the requirement for a controlled quiescent response is zero cross correlation of the sensor noise, filtered by the upper and lower beamformer, respectively. Assuming that all sensor noise sources are uncorrelated and have the same spectral density R_n, from (29), a sufficient condition for a controlled quiescent response is

$$\boldsymbol{g}^H \boldsymbol{D}_0 \boldsymbol{B} = \boldsymbol{0} \tag{45}$$

for all ν. From (26), with $G_l = \boldsymbol{g}_l^H \boldsymbol{d}_0$ and $B_{il} = \boldsymbol{b}_{il}^H \boldsymbol{d}_0$, the same condition is given by

$$\sum_{l=0}^{N-1} G_l B_{il}^* = 0 \qquad i = 1 \cdots K \tag{46}$$

for all ν. Note that G_l and B_{il} are frequency functions of the FIR filters with tap weights $g_l(n)$ and $b_{il}(n)$ corresponding to the elements in the vectors $\boldsymbol{g}_l$ and $\boldsymbol{b}_{il}$, respectively. The condition above is thus given in the time domain as

$$\sum_{l=0}^{N-1} g_l(n) \bigstar b_{il}(-n) = 0 \qquad i = 1 \cdots K \tag{47}$$

for all integers n. Here, $\bigstar$ denotes discrete-time convolution. Since the FIR filters $g_l(n)$ and $b_{il}(n)$ both are of length L, (47) implies $K(2L-1)$ linear constraints on the array weight vector $\boldsymbol{g}$, assuming that the blocking matrix $\boldsymbol{B}$ is designed unconstrained.

It is convenient to give a matrix formulation for these quiescent response constraints. Let $\boldsymbol{\Delta}_0(n)$ denote a sequence of block diagonal matrices, each with N blocks $\boldsymbol{\Delta}(n)$:

$$\boldsymbol{\Delta}_0(n) = \begin{pmatrix} \boldsymbol{\Delta}(n) & & \\ & \ddots & \\ & & \boldsymbol{\Delta}(n) \end{pmatrix} \tag{48}$$

where

$$\boldsymbol{\Delta}(n) = \begin{pmatrix} \delta(n) & \delta(n+1) & \cdots & \delta(n+L-1) \\ \delta(n-1) & \delta(n) & & \vdots \\ \vdots & & \ddots & \vdots \\ \delta(n-L+1) & \cdots & \cdots & \delta(n) \end{pmatrix} \tag{49}$$

and $\delta(n) = 1$ for $n = 0$ and zero otherwise. The quiescent response condition according to (47) is now given by

$$\boldsymbol{g}^H \boldsymbol{\Delta}_0(n) \boldsymbol{b}_i = 0 \qquad i = 1 \cdots K \tag{50}$$

for all integers n, or

$$\boldsymbol{B}^H \boldsymbol{\Delta}_0^H(n) \boldsymbol{g} = \boldsymbol{0} \tag{51}$$

for all integers n. Note that the left side of (45) is the discrete-time Fourier transform of the left side of (51) transposed. Since $\boldsymbol{\Delta}_0(n)$ is zero unless $|n| \leq L-1$, (51) implies K linear constraints on $\boldsymbol{g}$ for every n such that $|n| \leq L-1$. These constraints can be collected in a constraint matrix $\boldsymbol{C}$ of dimension $r \times NL$ where $r \leq K(2L-1)$. Thus, any linear dependent constraints are assumed to be removed so that $\boldsymbol{C}$ is a full rank matrix. The controlled quiescent response condition is compactly expressed as

$$\boldsymbol{C}\boldsymbol{g} = \boldsymbol{0}. \tag{52}$$

References

[1] B. Widrow and S. D. Stearns, *Adaptive Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1985.

[2] H. Cox, R. M. Zeskind, and M. M. Owen, "Robust adaptive beamforming," *IEEE Trans. Acoust., Speech, Signal Processing*, Oct. 1987.

[3] I. Claesson, S. Nordholm, P. Eriksson, and B. Bengtsson, "A multi-DSP implementation of a broadband adaptive beamformer for use in a hands-free mobile radio telephone," *IEEE Trans. Veh. Technol.*, Feb. 1991.

[4] M. H. Er and A. Cantoni, "A unified approach to the design of robust narrow-band antenna array processors," *IEEE Trans. Antennas Propagat.*, Jan. 1990.

[5] ——, "Derivative constraints for broad-band element space antenna array processors," *IEEE Trans. Acoust., Speech, Signal Processing*, Dec. 1983.

[6] ——, "A new set of constraints for derivative-constrained broad-band beam-formers," *IE Australia and IREE Australia*, vol. 11, June 1991.

[7] K. M. Buckley and L. J. Griffiths, "An adaptive generalized sidelobe canceller with derivative constraints," *IEEE Trans. Antennas Propagat.*, Mar. 1986.

[8] B. Widrow and J. M. McCool, "A comparison of adaptive algorithms based on the methods of steepest descent and random search," *IEEE Trans. Antennas Propagat.*, Sept. 1976.

[9] Y. H. Leung, A. Cantoni, and M. H. Er, "Quadratically constrained array processor-dependence on the coordinate system and a new formulation," in *Proc. ISSPA92*, Gold Coast, Australia, Aug. 1992.

[10] L. J. Griffiths and K. M. Buckley, "Quiescent pattern control in linearly constrained adaptive arrays," *IEEE Trans. Acoust., Speech, Signal Processing*, July 1987.

[11] W. Lu and A. Antoniou, *Two-Dimensional Digital Filters*. New York: Marcel Dekker, 1992.

[12] P. P. Vaidyanathan and T. Q. Nguyen, "Eigenfilters: A new approach to least-squares FIR filter design and applications including Nyquist filters," *IEEE Trans. Circuits Syst.*, Jan. 1987.

[13] T. W. Parks and C. S. Burrus, *Digital Filter Design*. New York: Wiley, 1987.

[14] S. Nordholm, "Broadband adaptive beamforming with applications in acoustic signal processing," Ph.D. dissertation, Lund Univ., LUTEDX/TETT-91/1010+190P.

Sven Nordebo was born in Sweden in 1963. He received the M.S. degree from the Royal Institute of Technology, Stockholm, Sweden, in 1989.

Since 1990 he has been working toward the Ph.D. degree in signal processing at Luleå University of Technology. His interests are in signal processing, antenna arrays, and optimum digital filters.