Adaptive Consensus Control for a Class of Nonlinear Multiagent Time-Delay Systems Using Neural Networks

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***Abstract*— Because of the complicity of consensus control of nonlinear multiagent systems in state time-delay, most of previous works focused only on linear systems with input time-delay. An adaptive neural network (NN) consensus control method for a class of nonlinear multiagent systems with state time-delay is proposed in this paper. The approximation property of radial basis function neural networks (RBFNNs) is used to neutralize the uncertain nonlinear dynamics in agents. An appropriate Lyapunov–Krasovskii functional, which is obtained from the derivative of an appropriate Lyapunov function, is used to compensate the uncertainties of unknown time delays. It is proved that our proposed approach guarantees the convergence on the basis of Lyapunov stability theory. The simulation results of a nonlinear multiagent time-delay system and a multiple collaborative manipulators system show the effectiveness of the proposed consensus control algorithm.**

***Index Terms*— Consensus control, Lyapunov–Krasovskii functional, neural networks (NNs), nonlinear multiagent systems, time delay.**

I. INTRODUCTION

IN RECENT years, multiagent consensus control research has received more and more attention [1]–[5]. Their widespread applications can be discovered in many fields, such as in distributed sensor networks, cooperative control of unmanned air vehicles, flocking, and formation control, and so forth [6]–[9]. Consensus control means to design an agreeable control scheme or network control protocol based on neighbor's state information such that every agent of the multiagent system reaches a common synchronized state as time goes on.

The information exchange and interaction among all agents play a pivotal role for the consensus movements. This information is formulated by neighborhood-based graph theory. In the last decade, many significant achievements have been developed for consensus control of multiagent systems [10]–[16]. In [10], three consensus problems were explored including fixed topology in directed networks, switching topology in directed networks, and fixed topology with communication time-delay in undirected networks. In [11], a basic theoretical framework of consensus control analysis of multiagent networked systems was proposed. Both discrete and continuous update schemes were proposed in [12]. An average-consensus is achieved using a distributed algorithm in a discrete framework under switching network topologies [13]. Nevertheless, these eminent consensus control methods are limited in the linear multiagent environment. It is true that most practical multiagent systems inherent nonlinearity is more complex than linear ones. Therefore, research in nonlinear multiagent systems has more realistic significance.

With further development, time-delay problem has been included to consensus control recently. Several significant results had emerged in consensus control of multiagent linear systems, for example, [17]–[19]. However, because of the complicacy of nonlinear multiagent systems, most of the existing consensus control methods of linear multiagent systems cannot be directly applied to nonlinear multiagent systems, especially the agent's dynamic function is unknown. Because it has been proven that neural networks (NNs) have the excellent approximation ability, the use of NNs in the proof of stability in nonlinear dynamic systems has become very popular and effective [20]–[24]. With this eminent property of NNs, research on adaptive NN consensus control of nonlinear multiagent systems has gained attention [25]. Although several significant results were proposed in [26] and [27] for nonlinear multiagent systems, but time delay has not been considered. Nevertheless, time delay appears often in most control systems and it should be taken into account when building a consensus control protocol. Inclusion of time-delay component in nonlinear multiagent systems will increase the complexity of consensus control, especially when time delays are unknown. Solving state time-delay problem is popular in adaptive nonlinear tracking control and rich research results have been reported [28]–[34]. However, only a few of results are reported in consensus control of nonlinear multiagent systems [35]–[37]. In [35], a kind of intermittent consensus protocols for the second-order multiagent time-delay systems with nonlinear dynamics and fixed directed communication topology is

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introduced and reported. It is proven that the agreement control method can realize the consensus control objective if the algebraic connectivity condition is satisfied and the communication time duration is larger than their corresponding threshold values. However, in multiagent systems, all agent's nonlinear dynamics are limited to a fixed nonlinear function and the nonlinear function is required to satisfy a restrictive assumption like $\|f(x_1, x_2, t) - f(y_1, y_2, t)\| \le \sum_{i=1}^{2} \rho_i \|x_i - y_i\|$. In [36], a kind of consensus control approach of the second-order nonlinear multiagent systems with time-varying delays by pining control is proposed. It is proven that position states and velocity states of all agents can synchronize to a virtual leader. But in this paper, the time-delay terms are limited to linear forms. In [37], only the time-delay is considered in the process of information exchange.

To our best knowledge, state time-delay problem in the consensus control of nonlinear multiagent systems has not been fully studied and it is still a challenging task. Motivated by the above analysis, this paper proposes a novel NN control approach for nonlinear multiagent consensus problem with state time delay. The simulation examples prove the effectiveness of the proposed consensus control approach. The main contributions of this research are described as follows.

1) The approximation property of radial basis function neural network (RBFNN) is used to neutralize the uncertain nonlinear dynamics in agents. It is proved that the proposed approach guarantees the convergence based on Lyapunov stability theory.
2) State time-delay consensus control for a class of nonlinear multiagent systems is successfully solved by a well-designed Lyapunov–Krasovskii functional.
3) The potential singularity problem in the controller is successfully avoided by relaxing the consensus control objective to a "ball" region rather than to a single origin.

II. Problem Statement and Preliminaries

Given a class of nonlinear multiagent systems with state time-delay, every agent's dynamic is described as

$$\dot{x}_i(t) = f_i\left(x_i(t)\right) + h_i\left(x_i(t-\tau_i)\right) + u_i(t) \qquad i = 1, 2, \ldots, n \tag{1}$$

where $x_i(t) \in R^m$ is the state vector, $f_i(x_i(t))$, $h_i(x_i(t))$: $R^m \to R^m$ is unknown but continuous nonlinear vector functions and τ_i is the unknown time delay, $u_i(t) \in R^m$ is the control input vector.

In this paper, the control objective is to devise consensus controllers, $u_i(t) \in R^m$, $i = 1, \ldots, n$, for each agent in (1) such that all agents in (1) can reach to a common state finally.

Assumption 1: The unknown smooth nonlinear functions, $h_i(x_i(t))$, $i = 1, 2, \ldots, n$, satisfies the inequalities $\|h_i(x_i(t))\| \le \rho_i(x_i(t))$, $i = 1, 2, \ldots, n$, where $\rho_i(\cdot)$ is known positive smooth functions and $\|\cdot\|$ denote the two-norm.

Assumption 2: The unknown time delays $\tau_i, i = 1, \ldots, n$ are bounded and there exists a known constant $\tau_{\max}$ satisfying the conditions $\tau_i \le \tau_{\max}, i = 1, \ldots, n$.

Remark 1: Most previous research works for the nonlinear multiagent system in consensus problem do not consider any state time-delay in agent's dynamics, see [25]–[27]. However, time delay is intrinsic in most control systems, and it exists in state equations for most of the recycling processes. In addition, Assumption 2 is reasonable as it provides a boundary of unknown time delays.

Remark 2: Many practical multiagent dynamic can be depicted by nonlinear differential equations (1). For example, formation control of unmanned air vehicle, sensor networks, robotic teams, satellite clusters, and complex networks [6]–[9].

Several fundamentals are introduced in the following.

A. Graph Theory

Let $G = (V, \varepsilon, \mathrm{A})$ denote an undirected weight graph, where $V = \{v_1, v_2, \ldots, v_n\}$ is a set of nodes, $\varepsilon \subseteq V \times V$ denotes a set of edges, and $\mathrm{A} = [a_{ij}]$ is a weighted adjacency matrix and all adjacency elements are nonnegative. Node v_i denotes the ith agent and the node indices belong to a finite index set $I = \{1, 2, \ldots, n\}$. $e_{ij} = (v_i, v_j)$ denotes an edge of G, where the node v_i is the tail of the edge and node v_j is the head of the edge and $e_{ij} = (v_i, v_j) \in \varepsilon$ if and only if there exists an information exchange between agent i and agent j. In addition, node v_i is a neighbor of node v_j if the edge $e_{ij} = (v_i, v_j)$ exists in the graph G. The value a_{ij} in adjacency matrix A associated with the edges e_{ij} denotes the communication quality between the ith agent and jth agent.

The definition of the Laplacian matrix L of graph G is described by the following formula:

$$L = B - \mathrm{A} \tag{2}$$

where $B = \mathrm{Diag}(b_1, b_2, \ldots, b_n)$ and $b_i = \sum_{j=1}^{n} a_{ij}$.

An important property of the Laplacian matrix L is that each row sum is zero, thus $\mathbf{1}_n = [1, 1, \ldots, 1]^T \in R^n$ is an eigenvector of the Laplacian matrix, L, associated with the eigenvalue $\lambda = 0$.

A network G is an undirected graph if there is a connection between two nodes v_i and v_j in G, then $a_{ij} = a_{ji} > 0$; otherwise, $a_{ij} = a_{ji} = 0$, $a_{ii} = 0$, $(i \ne j; i, j = 1, \ldots, n)$. A sequence of edges (v_{i_1}, v_{i_2}), $(v_{i_2}, v_{i_3}), \ldots, (v_{i_{k-1}}, v_{i_k})$ is a path of between node v_{i_1} and node v_{i_k}. An undirected graph, G, is called a connected graph if any two nodes, v_i, v_j satisfy $v_i, v_j \in V$ and there exists a path from v_i to v_j.

In this paper, the communication graph, G, of the nonlinear multiagent systems is an undirected and connected graph.

For an undirected connected graph, G, we have the following well-known lemma.

Lemma 1 [11]: If $G = (V, \varepsilon, \mathrm{A})$ is an undirected connected graph, then graph Laplacian matrix L is a symmetric matrix and its n real eigenvalues can be arranged in an ascending order as

$$0 = \lambda_1 < \lambda_2 \le \lambda_3 \le \cdots \le \lambda_n \le C$$

where $C = 2(\max_{1 \le i \le n} b_i)$ and λ_2 is called the algebraic connectivity and it is used to analyze the rate of consensus convergence.

Remark 3: This paper only considers fixed multiagent topology because it is the basic connected topology of different kinds of switching topologies. Its applications can be found in many fields, such as in data fusion of sensor networks, distributed computation, and formation control [38]–[40].

Lemma 2 [25], [41]: Let function $V(t) > 0$ be a continuous function defined $\forall t \geq 0$ and bounded, and $\dot{V}(t) \leq -c_1 V(t) + c_2$, where c_1 and c_2 are positive constants, then $V(t) \leq V(0) e^{-c_1 t} + c_2/c_1(1 - e^{-c_1 t})$.

B. RBFNN and Function Approximation

In this paper, RBFNN is applied to approximate the unknown nonlinear function in (1) because it has the function approximation abilities. A continuous nonlinear function $\varphi(z)$: $R^l \rightarrow R^m$ can be approximated on a compact set by the following RBFNN:

$$\varphi_{\mathrm{NN}}(W, z) = W^T S(z) \tag{3}$$

where $W \in R^{p \times m}$ is an adjustable weight matrix, p is the number of neuron, $S(z) = [s_1(z), \ldots, s_p(z)]^T$ is a basis function vector, and $s_i(z) = \exp[-(z - \mu_i)^T (z - \mu_i)/\phi_i^2]$ for $i = 1, 2, \ldots, p$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{il}]^T$ denotes the center of the receptive field, ϕ_i represents the width of the Gaussian function, and $z \in \Omega_z \subset R^l$ is the input vector.

It is well known that any nonlinear continuous function can be approximated by RBFNN to any desired accuracy over a compact set Ω. With this property, given a smooth nonlinear vector function, $\varphi(z) \in R^m$, there exists an ideal weights matrix, W^*, such that the ideal RBFNN can accurately approximate the smooth nonlinear vector function, $\varphi(z)$, on a compact set $\Omega_z \subset R^l$ as following:

$$\varphi(z) = W^{*T} S(z) + \varepsilon_z \tag{4}$$

where $W^* \in R^{p \times m}$ is the optimal weight matrix of RBF NN, p is the number of neurons, and $\varepsilon_z \in R^m$ is the approximation error and it satisfies that $\|\varepsilon_z\| \leq \varepsilon_N$. The NN approximation error denotes the minimum possible deviation between the ideal approximator $W^{*T} S(z)$ and the unknown nonlinear smooth function $\varphi(z)$. The optimal weight matrix W^* is an "artificial" quantity and is used for analytical purposes only. In general, this ideal weight matrix W^* needs to be estimated because it is unknown.

Let W^* be the value of W that minimizes $\|\varepsilon_z\|$ for all $z \in \Omega_z \subset R^l$ over a compact region

$$W^* \triangleq \arg \min_{W \in R^{p \times m}} \left\{ \sup_{z \in \Omega_z} \left\| \varphi(z) - W^T S(z) \right\| \right\}. \tag{5}$$

The NN approximation error ε_z can be decreased by increasing the number of the adjustable weights. Widespread practical application of NNs show that, if NN node number p is chosen large enough, then $\|\varepsilon_z\|$ can be reduced to arbitrary small in a compact set.

III. CONSENSUS PROTOCOL

For the multiagent system described in (1), a smooth scalar function is defined as following:

$$V_x(t) = \frac{1}{2} X^T(t)(L \otimes I_m) X(t) \tag{6}$$

where $X(t) = (x_1^T(t), x_2^T(t), \ldots, x_n^T(t))^T \in R^{nm}$.

Remark 4: In this paper, I_N denotes identity matrix of dimension $N \times N$, the symbol $\otimes$ denotes Kronecker product, that is, $D \otimes E = [d_{ij} E]$. Kronecker algebra is used to manipulate equations and governs the information between agents.

Define ith local consensus error as the following:

$$e_i(t) = \sum_{j=1}^{n} a_{ij} \left(x_i(t) - x_j(t) \right) \in R^m, \qquad i = 1, \ldots, n \tag{7}$$

where a_{ij} is the ith row and the jth column element of the adjacency matrix A.

According to matrix theory and graph theory, we can easily conclude that zero is an m-multiplicity eigenvalue of matrix $(L \otimes I_m)$. The m eigenvectors of matrix $(L \otimes I_m)$ associated with the eigenvalue 0 can be given in the following form:

$$\zeta_1^T = (\kappa_1^T, \ldots, \kappa_1^T) \in R^{nm}, \ldots, \zeta_m^T = (\kappa_m^T, \ldots, \kappa_m^T) \in R^{nm}$$

where $\kappa_i \in R^m$ is a vector, in which the ith element is $1/\sqrt{n}$ and the other elements are 0. We use $\zeta_{m+1}, \zeta_{m+2}, \ldots, \zeta_{nm}$ to denote the eigenvectors of the matrix $(L \otimes I_m)$ associated with the other eigenvalues $\lambda_2, \ldots, \lambda_n$. According to matrix theory, $\zeta_1, \zeta_2, \ldots, \zeta_{nm}$ can be chosen as a set of orthogonal bases of R^{nm}. Let $M = (\zeta_1, \ldots, \zeta_{nm}) \in R^{nm \times nm}$, then $M^T M = M M^T = I_{nm}$, that is, $M^T = M^{-1}$.

With the above analysis, the following equation can be obtained:

$$\begin{aligned} V_x(t) &= \frac{1}{2} X^T(t)(L \otimes I_m) X(t) = \frac{1}{2} X^T(t) M^T \mathrm{T} M X(t) \\ &= \frac{1}{2} X^T(t) M^T \sqrt{\mathrm{T}} \sqrt{\mathrm{T}} M X(t) \\ &= \frac{1}{2} X^T(t) M^T \sqrt{\mathrm{T}} \sqrt{\hat{\mathrm{T}}} \sqrt{\hat{\mathrm{T}}^{-1}} \sqrt{\hat{\mathrm{T}}^{-1}} \sqrt{\hat{\mathrm{T}}} \sqrt{\mathrm{T}} M X(t) \\ &= \frac{1}{2} X^T(t) M^T \mathrm{T} M M^T \hat{\mathrm{T}}^{-1} M M^T \mathrm{T} M X(t) \\ &= \frac{1}{2} X^T(t)(L \otimes I_m)^T M^T \hat{\mathrm{T}}^{-1} M (L \otimes I_m) X(t) \\ &= \frac{1}{2} E^T(t) \mathrm{D} \mathrm{E}(t) \end{aligned} \tag{8}$$

where $\mathrm{T} = \mathrm{diag}\{0 I_m, \lambda_2 I_m, \ldots, \lambda_n I_m\}$, $\hat{\mathrm{T}} = \mathrm{diag}\{\lambda_2 I_m, \lambda_2 I_m, \ldots, \lambda_n I_m\}$, $E(t) = (e_1^T(t), \ldots e_n^T(t))^T \in R^{nm}$, and $D = M^T \hat{\mathrm{T}}^{-1} M$ is a positive definite matrix.

From (8), we have

$$\frac{\lambda_{\min}(D)}{2} \sum_{i=1}^{n} \|e_i(t)\|^2 \leq V_x(t) \leq \frac{\lambda_{\max}(D)}{2} \sum_{i=1}^{n} \|e_i(t)\|^2 \tag{9}$$

where $\lambda_{\min}(D)$ and $\lambda_{\max}(D)$ are the smallest eigenvalue and the largest eigenvalue of matrix D, respectively.

From the definition of Laplacian matrix, we can get the following equation:

$$\begin{aligned} V_x(t) &= \frac{1}{2} X^T(t)(L \otimes I_m) X(t) \\ &= \frac{1}{2} E^T(t) X(t) = \frac{1}{2} X^T(t) E(t) \\ &= \frac{1}{2} \sum_{i=1}^{n} e_i^T(t) x_i(t) = \frac{1}{2} \sum_{i=1}^{n} x_i^T(t) e_i(t). \end{aligned} \tag{10}$$

The time derivative of $V_x(t)$ along (1) is

$$\dot{V}_x(t) = \sum_{i=1}^{n} e_i^T(t)\left(f_i(x_i(t)) + h_i(x_i(t-\tau_i)) + u_i(t)\right). \quad (11)$$

Applying Assumption 1 and Cauchy's inequality, $\left(\sum_{i=1}^{n} x_i y_i\right)^2 \le \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right)$, to (11), we have

$$\dot{V}_x(t) \le \sum_{i=1}^{n}\Big(e_i^T(t)f_i\left(x_i(t)\right) + e_i^T(t)u_i(t) + \|e_i(t)\|\,\rho_i(x_i(t-\tau_i))\Big). \quad (12)$$

Remark 5: In (12), the consensus control design will become more difficult because the unknown function $f_i(\cdot)$ and unknown time delay τ_i are included in the inequality. Although $\rho_i(\cdot)$ is known, but because the delay term, τ_i, is unknown, the state $x_i(t-\tau_i)$ is undetermined. Since $x_i(t-\tau_i)$ relates with the consensus controller design, the control objective cannot be directly realized. In addition, because the unknown time delay τ_i and the consensus error $\|e_i(t)\|$ are entangled together, the consensus control problem becomes more complex. Therefore, we need to find a way to segregate the uncertainties, τ_i and $\|e_i(t)\|$, such that they can be dealt separately.

Applying the Young's inequalities to (12), we have the following:

$$\dot{V}_x(t) \le \sum_{i=1}^{n}\left(e_i^T(t)f_i(x_i(t)) + e_i^T(t)u_i(t) + \frac{1}{2}\|e_i(t)\|^2 + \frac{1}{2}\rho_i^2(x_i(t-\tau_i))\right). \quad (13)$$

In (13), if $\|e_i(t)\|$ and $\rho_i(\cdot)$ are separated, then the time-delay term $\rho_i^2(x_i(t-\tau_i))$ can be dealt later. To compensate the uncertainties coming from the unknown time delay $\tau_i, i = 1, 2, \ldots, n$, a Lyapunov–Krasovskii functional is designed as the following:

$$V_U(t) = \frac{1}{2}\sum_{i=1}^{n}\int_{t-\tau_i}^{t} U_i(x_i(\tau))d\tau, \qquad i = 1, \ldots, n \quad (14)$$

where $U_i(x_i(t)) = \rho_i^2(x_i(t))$.

Its time derivative is

$$V_U(t) = \frac{1}{2}\sum_{i=1}^{n}\left(\rho_i^2(x_i(t)) - \rho_i^2(x_i(t-\tau_i))\right) \qquad i = 1, \ldots, n. \quad (15)$$

It is obvious that the Lyapunov–Krasovskii functional $V_U(t)$ can use to compensate the uncertainties of the unknown time delay. The design difficulty, which comes from the unknown time delays τ_i, $i = 1, \ldots, n$, is eliminated. Because the functions, $\rho_i(x_i(t))$, $i = 1, \ldots, n$, are known, the consensus control scheme does not involve any uncertainty.

With the above analysis, if we add the term $V_U(t)$ to the right hand side of (13), then the influence of the uncertain time delay for scalar function $V_x(t)$ can be eliminated. Combining (15), the following inequality can be obtained:

$$\dot{V}(t) = \dot{V}_x(t) + \dot{V}_U(t) \le \sum_{i=1}^{n}\left(e_i^T(t)u_i(t) + e_i^T(t)f_i(x_i(t)) + \frac{1}{2}\|e_i(t)\|^2 + \frac{1}{2}\rho_i^2(x_i(t))\right) \quad (16)$$

where $V(t) = V_x(t) + V_U(t)$.

For simplicity, we will omit the time symbols t portion inside $x_i(t)$, $e_i(t)$, $u_i(t), i = 1, \ldots, n$. From (16), $V(t) = V_x(t) + V_U(t)$ can be chosen as a Lyapunov function candidate such that the consensus controllers, $u_i(t), i = 1, 2, \ldots, n$, can be found under the assumption that system functions are known and will not be affected from the unknown time delay, τ_i.

Let $x_i \in \Omega_{x_i} \subset R^m$, Ω_{x_i} be a compact set, then $\Omega_{c_i} \subset \Omega_{x_i}$ and $\Omega_{c_i}^o$, $i = 1, \ldots, n$ as

$$\Omega_{c_i} := \{e_i \,|\, \|e_i\| < c_i\} \quad (17)$$

$$\Omega_{c_i}^o := \Omega_{x_i} - \Omega_{c_i} \quad (18)$$

where c_i is a constant that is chosen to be arbitrarily small and "−" is the complement of set Ω_{c_i}, that is, $A - B = \{x \,|\, x \in A, x \notin B\}$.

Lemma 3 [28]: Set $\Omega_{c_i}^o$ is a compact set.

The consensus controllers are designed as the following:

$$u_i(t) = \begin{cases} -k_i(t)e_i - f_i(x_i) - \frac{1}{2}e_i^{-1}\rho_i^2(x_i), & e_i \in \Omega_{c_i}^o \\ 0, & e_i \in \Omega_{c_i} \end{cases} \quad (19)$$

where $k_i(t) \ge k^* + 1/2$, $i = 1, \ldots, n$, k^* are a positive constant.

Remark 6: In this paper, using e_i^{-1} to denote $e_i/\|e_i\|^2$ and it has the property of $e_i^T e_i^{-1} = (e_i^{-1})^T e_i = 1$. Because the term $1/2e_i^{-1}\rho_i^2(x_i)$ is not well defined at $e_i = [0]_m$, where $[0]_m$ is an m-dimensional zero vector. The singularity problem in controller may take place at the point $e_i = [0]_m$, where the consensus control is reached. It is true that a multiagent system reaches the consensus; the control action is complete and should not take any power consumption. To this end, it is more practical to relax the consensus control objective to a "ball" region rather than in a single origin [43].

Next, we will prove that if system functions $f_i(\cdot)$, $i = 1, \ldots, n$ are known, the consensus controller (19) can ensure multiagent system (1) to obtain consensus state when $e_i \in \Omega_{c_i}^o$.

The Lyapunov function candidate is chosen as the following:

$$V(t) = V_x(t) + V_U(t). \quad (20)$$

According to (10) and (14), the Lyapunov function candidate, $V(t)$, can be rewritten to the following one:

$$\begin{aligned} V(t) &= V_x(t) + V_U(t) \\ &= \frac{1}{2}\sum_{i=1}^{n} e_i^T x_i + \frac{1}{2}\sum_{i=1}^{n}\int_{t-\tau_i}^{t} U_i\left(x_i(\tau)\right)d\tau. \end{aligned} \quad (21)$$

Taking the time derivative along (16), we have

$$\dot{V}(t) \le \sum_{i=1}^{n}\left(e_i^T u_i + e_i^T f_i(x_i) + \frac{1}{2}\|e_i\|^2 + \frac{1}{2}\rho_i^2(x_i)\right). \quad (22)$$

For $e_i \in \Omega_{z_i}^o$, substituting (19) into (22), yields

$$\dot{V}(t) \le -\sum_{i=1}^{n}\left(k_i(t) - \frac{1}{2}\right)\|e_i\|^2 \le -\sum_{i=1}^{n} k^* \|e_i\|^2. \quad (23)$$

According to Lyapunov stability theory, it is easy to conclude that the nonlinear multiagent system is asymptotically stable [42].

From (23), we know the nonnegative function $V(t)$ is a nonincreasing function. So we can conclude that for any $\upsilon > 0$ there exists $T > 0$ such that $\forall t > T$, $\{V(t)|V(t) \le \upsilon\}$. Because the Laplacian matrix L is a positive semidefinite matrix and the Lyapunov–Krasovskii functional $V_u(t) = 1/2\sum_{i=1}^{n}\int_{t-\tau_i}^{t} U_i(x_i(\tau))d\tau$ also is positive, so we have $V_x(t) \to 0$ when $t \to \infty$. Because the graph topology G of the nonlinear multiagent system is an undirected connected graph and $V_x(t) = 1/2E^T\mathrm{DE} \to 0$ when $t \to \infty$, we can conclude that $x_1 = x_2 = \cdots = x_n$ when $t \to \infty$, that is, the consensus behavior of the multiagent system is obtained.

In addition, because c_i is an arbitrarily small constant, it is obvious that consensus state has been obtained in the region $\|e_i\| < c_i$.

However, the proposed consensus controller (19) cannot be directly applied to the multiagent system (1) because $f_i(\cdot)$, $i = 1, \ldots, n$ is completely unknown. On the other hand, by employing the consensus controller (19), control action is only activated when $e_i \in \Omega_{c_i}^o$. Apparently, $f_i(\cdot)$ is smooth and well-defined over the compact set $\Omega_{c_i}^o$ and can be approximated by NNs to an arbitrary accuracy as

$$f_i(x_i(t)) = W_i^{*T} S_i(x_i) + \varepsilon_i(x_i) \quad (24)$$

where $W_i^* \in R^{p_i \times m}$ is the ideal weight matrix of the NN, p_i is the neuron number of the NN, $S_i(x_i) \in R^{p_i}$ are the basis function vector, and $\varepsilon_i \in R^m$ is the approximation error to satisfy $\|\varepsilon_i\| \le \varepsilon_{N_i}$.

Let $\hat{W}_i$ be the estimation of the ideal NN weight W_i^*. Construct the adaptive consensus controller and the adaptive law as the following:

$$u_i(t) = \begin{cases} -k_i(t)e_i - \hat{W}_i^T(t)S_i(x_i) - \frac{1}{2}e_i^{-1}\rho_i^2(x_i), & e_i \in \Omega_{c_i}^o \\ 0, & e_i \in \Omega_{c_i} \end{cases} \quad (25)$$

$$\dot{\hat{W}}_i(t) = \Gamma_i\left[S_i(x_i)e_i^T - \sigma_i \hat{W}_i(t)\right] \quad (26)$$

where $\Gamma_i = \Gamma_i^T \in R^{p_i \times p_i}$, $i = 1, \ldots, n$ are gain positive definite matrixes. $\sigma_i > 0$, $i = 1, \ldots, n$ are constants. The term $\sigma_i \hat{W}_i(t)$, $i = 1, \ldots, n$ are introduced to improve the robustness in the presence of the NN approximation error, ε_i, $i = 1, \ldots, n$.

The following theorem implies that the consensus control objective of multiagent system (1) can be realized by applying the proposed control laws (25).

Theorem 1: For a class of nonlinear multiagent systems described by (1), given truth in Assumptions 1 and 2, the consensus controllers provided by (25), the controller gains given as $k_i(t) = k_{i0} + k_{i1}(t)$, where $k_{i0} > 0$ is a design constant and $k_{i1}(t)$ is designed as

$$k_{i1}(t) = \frac{1}{2} + \frac{1}{\omega_i}\left[1 + \frac{\lambda_{\max}(D)}{2} + \frac{1}{\|e_i\|^2}\int_{t-\tau_{\max}}^{t} \frac{1}{2}U_i(x_i(\tau))d\tau\right] \quad (27)$$

with design constant $\omega_i > 0$, and the NN weight updated by (26), with bounded initial conditions $x_i(0)$, $\hat{W}_i(0)$, then all agents of the nonlinear multiagent system arrive at a final consensus state.

Proof: See Appendix.

IV. Simulations

To demonstrate the effectiveness of the proposed approach, two nonlinear multiagent consensus examples are given. In both examples, each multiagent system consists of six agents. To simplify, we assumed that time delays and communication graph in these two examples are the same.

Time delays for each agent are $\tau_1 = 1.4$, $\tau_2 = 1.5$, $\tau_3 = 1.6$, $\tau_4 = 1.7$, $\tau_5 = 1.8$, $\tau_6 = 1.9$, respectively. The adjacency matrix A and the Laplacian matrix L of the two nonlinear multiagent examples are defined as follows:

$$A = \begin{pmatrix} 0.0 & 0.3 & 0.0 & 0.0 & 0.0 & 0.5 \\ 0.3 & 0.0 & 0.4 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.0 & 0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.1 & 0.0 & 0.3 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.3 & 0.0 & 0.6 \\ 0.5 & 0.0 & 0.0 & 0.0 & 0.6 & 0.0 \end{pmatrix}$$

$$L = \begin{pmatrix} 0.8 & -0.3 & 0.0 & 0.0 & 0.0 & -0.5 \\ -0.3 & 0.7 & -0.4 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.4 & 0.5 & -0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.1 & 0.4 & -0.3 & 0.0 \\ 0.0 & 0.0 & 0.0 & -0.3 & 0.9 & -0.6 \\ -0.5 & 0.0 & 0.0 & 0.0 & -0.6 & 1.1 \end{pmatrix}.$$

Example 1: The nonlinear multiagent time-delay system is given in the following:

$$\frac{d}{dt}\begin{pmatrix} x_{i1}(t) \\ x_{i2}(t) \end{pmatrix} = \begin{pmatrix} x_{i2}(t)\sin(\alpha_{i1}x_{i1}(t)) \\ x_{i1}(t)\cos(\alpha_{i2}x_{i2}^2(t)) \end{pmatrix} + u_i + \begin{pmatrix} h_{i1}(x_i(t-\tau_i)) \\ h_{i2}(x_i(t-\tau_i)) \end{pmatrix} \quad i = 1, \ldots, 6. \quad (28)$$

where $h_{i1}(x_i(t)) = \beta_{i1}x_{i1}\cos(x_{i2})$, $h_{i2}(x_i(t)) = \beta_{i2}x_{i2}$ sin (x_{i1}). α_{i1}, α_{i2}, β_{i1}, and β_{i2} are shown in Tables I and II. The initial positions of six agents are $x_1(0) = (6, 2)^T$, $x_2(0) = (3, 3\sqrt{3})^T$, $x_3(0) = (-3, 3\sqrt{3})^T$, $x_4(0) = (-6, -2)^T$, $x_5(0) = (-3, -3\sqrt{3})^T$, and $x_6(0) = (3, -3\sqrt{3})^T$, respectively.

Apparently, by choosing $\rho_i(x_i) = \sqrt{(\beta_{i1}x_{i1})^2 + (\beta_{i2}x_{i2})^2}$, Assumption 1 is satisfied and also Assumption 2 is satisfied by choosing $\tau_{\max} = 2$.

TABLE I
VALUES OF α_{i1}, α_{i2} IN THE ith AGENT'S DYNAMICS

i	1	2	3	4	5	6
α_{i1}	0.6	-0.6	7	-10	10	5
α_{i2}	0.3	0.4	-5	-11	11	10

TABLE II
VALUES OF β_{i1}, β_{i2} IN THE ith AGENT'S DYNAMICS

i	1	2	3	4	5	6
β_{i1}	0.9	3.5	-2.1	7	4.6	5.3
β_{i2}	1.2	2.6	0.6	-4.3	2.8	2.4

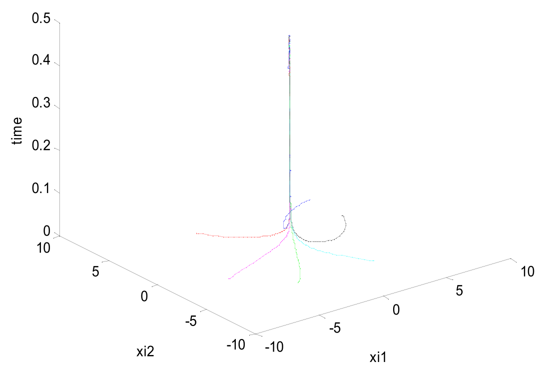

Fig. 1. Consensus performance of six agents in a 3-D view.

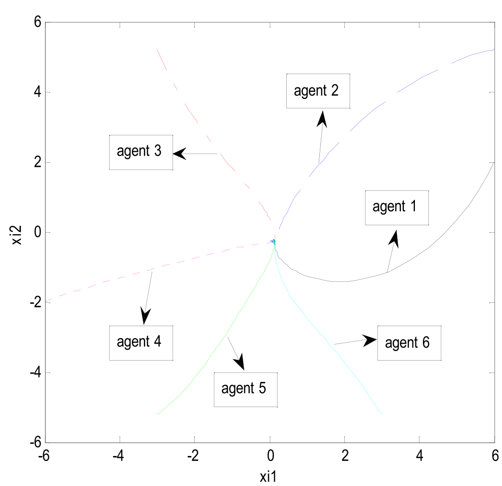

Fig. 2. Consensus performance of six agents.

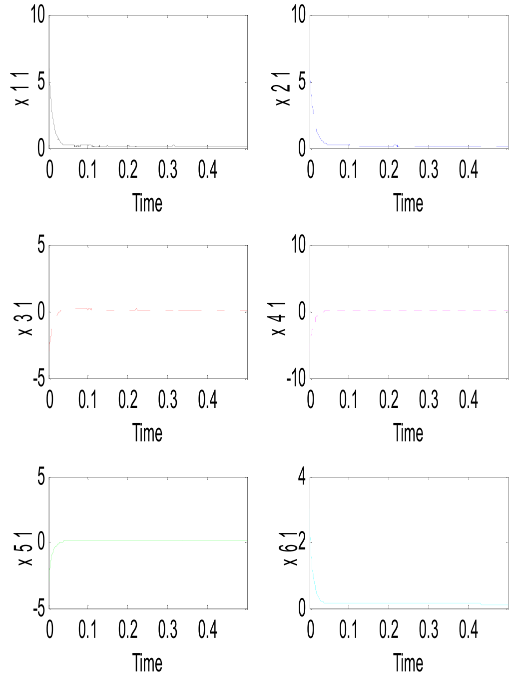

Fig. 3. Trajectory of the first coordinate of six agents.

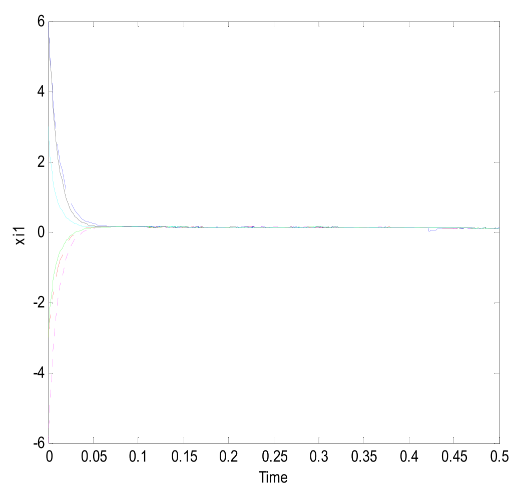

Fig. 4. Consensus performance of the first coordinate of six agents.

In this example, we choose a RBFNN that consists of 36 nodes with centers, μ_l, $l = 1, 2, \ldots, 36$, evenly spaced in the range of $[-6, 6] \times [-6, 6]$, the same widths for all, $\phi_i = 2$, and $S_i(x_i) = [s_1(x_i), \ldots, s_{36}(x_i)]^T$ with $s_l(x_i) = \exp[-(x_i - \mu_l)^T (x_i - \mu_l)/\phi_l^2]$, $l = 1, 2, \ldots, 36$.

The adaptive consensus controllers, u_i, $i = 1, \ldots, 6$, and the adaptive update laws, $\hat{W}_i(t)$, $i = 1, \ldots, 6$ can be given in the following:

$$u_i(t) = \begin{cases} -k_i e_i(t) - \hat{W}_i^T(t) S_i(x_i) - \frac{1}{2} e_i^{-1} \rho_i^2(x_i), & \text{otherwise} \\ 0, & \|e_i\| < c_i \end{cases} \tag{29}$$

$$\dot{\hat{W}}_i(t) = \Gamma_i \left[S_i(x_i) e_i^T - \sigma_i \hat{W}_i(t) \right] \tag{30}$$

where $\Gamma_i = \text{diag}\{4\}$, $\sigma_i = 2$, $c_i = 10^{-7}$, $i = 1, \ldots, 6$ and e_i is given by (7). The initial weights $\hat{W}_i(0) = [0]_{36\times 2}$, where $[0]_{36\times 2}$ denotes to a zero matrix of dimension 36×2. $k_i(t) = k_{i0} + k_{i1}(t)$, $k_{i0} = 270$, $i = 1, 2, \ldots, 6$, are constants and $k_{i1}(t)$ is chosen as $k_{i1}(t) = 1/2 + 1/\omega_i[1 + \lambda_{\max}(D)/2 + 1/\|e_i\|^2 \int_{t-\tau_{\max}}^{t} (1/2) U_i(x_i(\tau)) d\tau]$ by (27), where $\omega_i = 10$, $i = 1, \ldots, 6$.

Figs. 1–6 show the simulation results of applying consensus controller (29) to the nonlinear multiagent time-delay

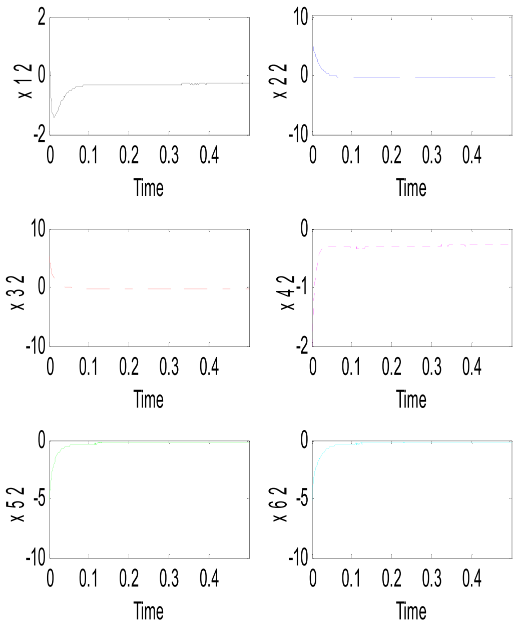

Fig. 5. Trajectory of the second coordinate of six agents.

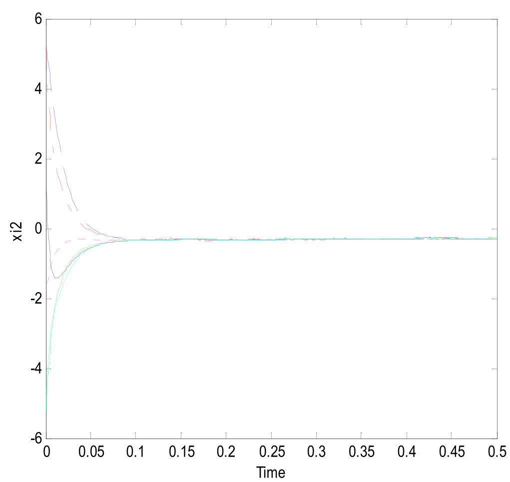

Fig. 6. Consensus performance of the second coordinate of six agents.

systems (28). From Figs. 1, 2, 4, and 6, we can see that the agreement behavior of multiagent systems (28) is achieved.

Example 2: In this example, a multiple collaborative manipulators system is used to demonstrate the effectiveness of the proposed consensus control method. This collaborative system can be considered as multiple manipulators are holding up an object or loading a workpiece. Fig. 7 shows the profile of a two-link manipulator of the multiagent systems. The multimanipulator system dynamic is described as follows:

$$\begin{aligned}\dot{q}_{i1} &= q_{i2} M_i(q_{i1})\dot{q}_{i2} + V_i(q_{i1}, q_{i2})q_{i2} + G_i(q_{i1})\\ &\quad + f_i(q_{i2}(t-\tau_i)) = \zeta_i, \qquad i = 1, \ldots, 6 \qquad (31)\end{aligned}$$

where state vector $q_{i1}, q_{i2} \in R^2$ denote the joint position, velocity of the ith manipulator, respectively, $M_i(q_{i1}) \in R^{2\times2}$ is the inertia matrix of the ith manipulator. To match the system equations described in (1), we assume $M_i(q_{i1}) = I_{2\times2}$,

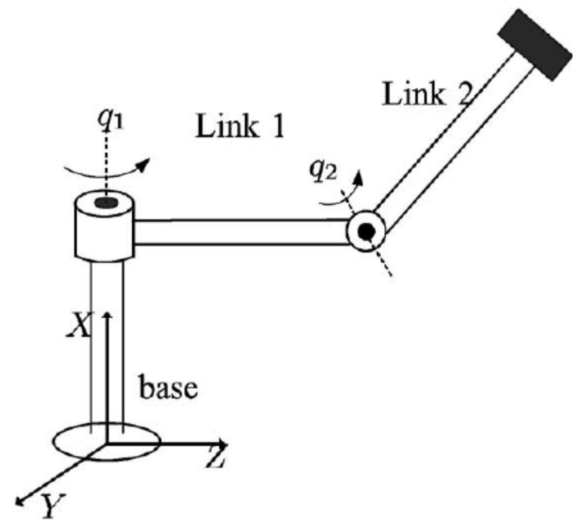

Fig. 7. Two-link revolute manipulator.

TABLE III
VALUES OF α_{i1}, α_{i2} IN THE ith AGENT'S DYNAMICS

i	1	2	3	4	5	6
α_{i1}	10	7	-11	-8	13	12
α_{i2}	28	-15	23	17	-19	-17

TABLE IV
VALUES OF β_{i1}, β_{i2} IN THE ith AGENT'S DYNAMICS

i	1	2	3	4	5	6
β_{i1}	5	2.5	-2.1	7	6	-5.3
β_{i2}	1.1	1.6	1.6	-4.1	-8	3.4

where $I_{2\times2}$ is identity matrix of dimension 2×2

$$V_i(q_{i1}, q_{i2}) = \begin{pmatrix} V_{i11} & V_{i12} \\ V_{i21} & V_{i22} \end{pmatrix} \in R^{2\times2}$$

is the centripetal-Coriolis matrix of the ith manipulator and $V_{i11} = -m_{i2}r_{i1}r_{i2}\sin(q_{i12})q_{i22}$, $V_{i12} = -m_{i2}r_{i1}r_{i2}\sin(q_{i12})q_{i22} - m_{i2}r_{i1}r_{i2}\sin(q_{i12})q_{i21}$, $V_{i21} = m_{i2}r_{i1}r_{i2}\sin(q_{i12})q_{i22}$, $V_{i22} = 0$; $G_i = (G_{i11}, G_{i12})^T \in R^2$ is gravitational vector of the ith manipulator and $G_{i1} = (m_{i1} + m_{i2})gr_{i1}\sin(q_{i11}) + m_{i2}gr_{i2}\sin(q_{i11} + q_{i12})$, $G_{i2} = m_{i2}gd_{i2}\sin(q_{i11} + q_{i12})$; $f_i(q_{i2}(t)) = (\alpha_{i1}q_{i21} + \beta_{i1}sgn(q_{i21}), \alpha_{i2}q_{i22} + \beta_{i2}sgn(q_{i22}))^T$ is friction force vectors, α_{i1}, α_{i2}, β_{i1}, β_{i2} are shown in Tables III and IV; τ_i is time delay, $\zeta_i \in R^2$ is the ith manipulator's torque input vector. The physical parameters of each manipulator are set as $g = 9.8$ m/s^2, $d_{i1} = 1.5$ m, $d_{i2} = 1$ m, $m_{i1} = 1$ kg, and $m_{i2} = 2$ kg $(i = 1, \ldots, 6)$. The initial joint position and the initial joint velocity of six manipulators are shown in Table V.

For the multiagent consensus control, we design the consensus controller to realize the consensus control of the velocity state q_{i2}, $i = 1, \ldots, 6$. According to proposed consensus controller (25) and adaptive update law (26), the correlation parameter are chosen as $k_{i0} = 120$, $\omega_i = 6$, $\Gamma_i = \text{diag}\{0.4\}$, $\sigma_i = 0.2$, $i = 1, \ldots, 6$.

TABLE V
VALUES OF q_{i11}, q_{i12}, q_{i21} AND q_{i22} IN THE ith AGENT'S DYNAMICS

i	1	2	3	4	5	6
q_{i11}	0	0	0	0	0	0
q_{i12}	0	0	0	0	0	0
q_{i21}	4.2	2.7	-3.1	-4.5	-1.8	2.5
q_{i22}	0.5	2.8	1.9	-2.2	-3.1	-4.3

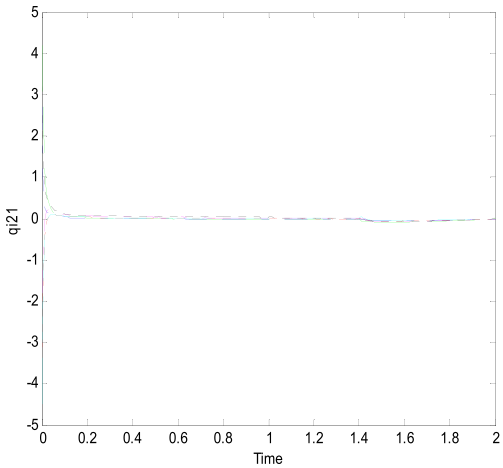

Fig. 8. Consensus performance of the first joint velocity of six manipulators.

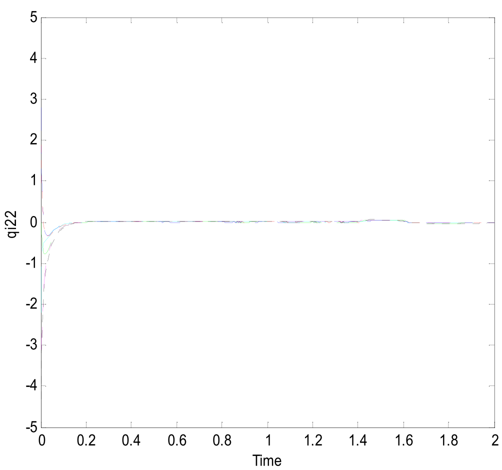

Fig. 9. Consensus performance of the second joint velocity of six manipulators.

In the example, we choose a RBFNN that consists of 25 nodes with centers, μ_l, $l = 1, 2, \ldots, 25$, evenly spaced in the range of $[-5, 5] \times [-5, 5]$, and the same widths, $\phi_i = 2$, for all.

In Figs. 8 and 9, the simulation results are obtained by applying proposed control method to the multimanipulator system (31). From Figs. 8 and 9, we can see that the velocity consensus state of six manipulators is achieved.

V. CONCLUSION

In this paper, an adaptive NN consensus control scheme is proposed for a class of nonlinear multiagent systems with state time delay. In designing the consensus controller, the uncertain nonlinearity and the time delay of agent's dynamics are compensated by employing an RBFNN that approximates the nonlinearity and by choosing an appropriate Lyapunov–Krasovskii functional, respectively. By letting the consensus error converge to a "ball" region rather than a single origin, the singularity problem in controller is avoided. Finally, stable results were obtained on the basis of Lypunov function method. Two simulations are carried out to verify the effectiveness of the proposed approach.

APPENDIX

Proof: Choosing Lyapunov function candidate as follows:

$$V(t) = V_x(t) + V_U(t) + \frac{1}{2}\sum_{i=1}^{n} \mathrm{Tr}\left(\tilde{W}_i^T(t)\Gamma_i^{-1}\tilde{W}_i(t)\right) \quad (32)$$

where $\tilde{W}_i(t) = \hat{W}_i(t) - W_i^*$.

When $e_i \in \Omega_{c_i}^o$, according to (22), the time derivative of $V(t)$ is

$$\dot{V}(t) \le \sum_{i=1}^{n}\left(e_i^T u_i + e_i^T f_i(x_i) + \frac{1}{2}\|e_i\|^2 + \frac{1}{2}\rho_i^2(x_i)\right) + \sum_{i=1}^{n} \mathrm{Tr}\left(\tilde{W}_i^T(t)\Gamma_i^{-1}\dot{\hat{W}}_i(t)\right). \quad (33)$$

Substituting (24)–(26) into (33), we have

$$\begin{aligned}\dot{V}(t) \le& \sum_{i=1}^{n}\Big(-k_i(t)\|e_i\|^2 - e_i^T\hat{W}_i^T(t)S_i(x_i) \\ &+ e_i^T W_i^{*T}S_i(x_i) + e_i^T\varepsilon_i(x_i) + \frac{1}{2}\|e_i\|^2\Big) \\ &+ \sum_{i=1}^{n}\mathrm{Tr}\left(\tilde{W}_i^T(t)\left[S_i(x_i)e_i^T - \sigma_i\hat{W}_i(t)\right]\right) \\ \le& \sum_{i=1}^{n}\Big(-\left(k_i(t) - \frac{1}{2}\right)\|e_i\|^2 - e_i^T\tilde{W}_i^T(t)S_i(x_i) \\ &+ e_i^T\varepsilon_i(x_i) + \mathrm{Tr}\left(\tilde{W}_i^T(t)S_i(x_i)e_i^T\right) \\ &- \sigma_i\mathrm{Tr}\left(\tilde{W}_i^T(t)\hat{W}_i(t)\right)\Big). \end{aligned} \quad (34)$$

According to trace operator property in matrix algebra below

$$a^T b = \mathrm{Tr}\left(ab^T\right) = \mathrm{Tr}\left(ba^T\right) \qquad \forall a, b \in R^n \quad (35)$$

the following inequality can be satisfied:

$$\dot{V}(t) \le \sum_{i=1}^{n}\Big(-\left(k_i(t) - \frac{1}{2}\right)\|e_i\|^2 + e_i^T\varepsilon_i(x_i) - \sigma_i\mathrm{Tr}(\tilde{W}_i^T(t)\hat{W}_i(t))\Big). \quad (36)$$

Given $k_i(t) = k_{i0} + k_{i1}(t)$, (36) becomes

$$\dot{V}(t) \le \sum_{i=1}^{n}\Big(-\left(k_{i1}(t) - \frac{1}{2}\right)\|e_i\|^2 - k_{i0}\|e_i\|^2 + e_i^T\varepsilon_i(x_i) - \sigma_i\mathrm{Tr}\left(\tilde{W}_i^T(t)\hat{W}_i(t)\right)\Big). \quad (37)$$

According to the Cauchy's inequality and Young's inequality, the following inequalities can be obtained:

$$-k_{i0}\,\|e_i\|^2 + e_i^T\varepsilon_i(x_i) \le \frac{\|\varepsilon_i(x_i)\|^2}{4k_{i0}} \le \frac{\varepsilon_{N_i}^2}{4k_{i0}}, \qquad i=1,\ldots,n. \tag{38}$$

With $\mathrm{Tr}(\tilde{W}_i^T(t)\hat{W}_i(t)) = 1/2(\mathrm{Tr}(\tilde{W}_i^T(t)\tilde{W}_i(t)) + \mathrm{Tr}(\hat{W}_i^T(t)\hat{W}_i(t)) - \mathrm{Tr}(W_i^{*T}W_i^*))$, $i=1,\ldots,n$, the following inequalities are obtained:

$$\begin{aligned}-\sigma_i\mathrm{Tr}\left(\tilde{W}_i^T(t)\hat{W}(t)_i\right) \le{}& -\frac{1}{2}\sigma_i\left(\mathrm{Tr}\left(\tilde{W}_i^T(t)\tilde{W}_i(t)\right)\right.\\ &\left.+\mathrm{Tr}\left(W_i^{*T}W_i^*\right)\right)\quad i=1,\ldots,n.\end{aligned} \tag{39}$$

Substituting (27), (38), and (39) into (37), we have

$$\begin{aligned}\dot{V}(t) \le \sum_{i=1}^{n}\Bigg(&-\frac{1}{\omega_i}\left[1+\frac{\lambda_{\max}(D)}{2}\right]\|e_i\|^2 - \frac{1}{2\omega_i}\int_{t-\tau_{\max}}^{t}U_i(x_i(\tau))d\tau\\ &-\frac{1}{2}\sigma_i\mathrm{Tr}(\tilde{W}_i^T(t)\tilde{W}_i(t)) + \Delta_i\Bigg)\end{aligned} \tag{40}$$

where $\Delta_i = (1/2)\sigma_i\mathrm{Tr}(W_i^{*T}W_i^*) + \varepsilon_{N_i}^2/4k_{i0}$, $i=1,\ldots,n$

With the condition of Assumption 2 that $\tau_i \le \tau_{\max}$, the inequalities $1/2\int_{t-\tau_i}^{t}U_i(x_i(\tau))d\tau \le 1/2\int_{t-\tau_{\max}}^{t}U_i(x_i(\tau))d\tau$, $i=1,\ldots,n$ are hold.

From (9) and (14), we have

$$\begin{aligned}\dot{V}(t) \le{}& -\frac{1}{\omega}V_x(t) - \frac{1}{\omega}V_U(t)\\ &-\frac{1}{2}\sum_{i=1}^{n}\left(\sigma_i\mathrm{Tr}\left(\tilde{W}_i^T(t)\tilde{W}_i(t)\right)\right) + \sum_{i=1}^{n}\Delta_i\end{aligned} \tag{41}$$

where $\omega = \max\{\omega_1,\omega_2,\ldots,\omega_n\}$.

Furthermore, we have the following inequality:

$$\begin{aligned}\dot{V}(t) \le{}& -\frac{1}{\omega}V_x(t) - \frac{1}{\omega}V_U(t)\\ &-\frac{1}{2}\sum_{i=1}^{n}\left[\frac{\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}\mathrm{Tr}\left(\tilde{W}_i^T(t)\Gamma_i^{-1}\tilde{W}_i(t)\right)\right] + \sum_{i=1}^{n}\Delta_i\\ \le{}& -KV(t) + \Delta\end{aligned} \tag{42}$$

where $\Delta = \sum_{i=1}^{n}\Delta_i$ and the positive constant K is defined by

$$\sigma = \min_{1\le i\le n}\left\{\frac{\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}\right\},\quad K = \min\left\{\frac{1}{\omega},\sigma\right\}. \tag{43}$$

For $e_i \in \Omega_{c_i}^o$, according to Lemma 2 and (42), the following fact can be obtained:

$$0 < V(t) \le \rho + (V(0)-\rho)e^{-Kt} \tag{44}$$

where $\rho = \Delta/K$.

From (9), (20) and (44), we have

$$\begin{aligned}\frac{\lambda_{\min}(D)}{2}\sum_{i=1}^{n}\|e_i(t)\|^2 \le{}& V_x \le \rho + (V(0)-\rho)e^{-Kt}\\ &\le \rho + V(0)e^{-Kt}.\end{aligned} \tag{45}$$

Further, we have

$$\sum_{i=1}^{n}\|e_i(t)\|^2 \le \frac{2}{\lambda_{\min}(D)}\rho + \frac{2}{\lambda_{\min}(D)}V(0)e^{-Kt} \tag{46}$$

which implies that given $\upsilon > \sqrt{2\rho/\lambda_{\min}(D)}$, there exists $T>0$ such that for all $t > T$, then

$$\|e_i(t)\| < \upsilon,\quad i=1,\ldots,n \tag{47}$$

where υ can be a small constant which depends on the NN approximation error ε_i and controller parameters ω_i, σ_i, and Γ_i. It is easily know that the decrease in the control gain ω_i and increase the adaptive gain Γ_i and NN node number p_i will result in a better consensus performance.

In addition, it is obvious that the consensus state has been obtained when $e_i \in \Omega_{c_i}$. $\square$

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