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Adaptive Constrained Constant Modulus Algorithm Based on Auxiliary Vector Filtering for Beamforming

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***Abstract*—A constrained constant modulus (CCM) algorithm with the auxiliary vector filtering (AVF) technique is introduced for robust adaptive beamforming. The proposed scheme decomposes the adaptive filter into constrained (reference vector filters) and unconstrained (auxiliary vector filters) components. The weight vector is iterated by subtracting the scaling auxiliary vector from the reference vector, which are computed according to the CCM criterion. The proposed algorithm provides an iterative exchange of information between the scalar factor and the auxiliary vector, resulting in a faster convergence and an improved steady-state performance as compared with existing techniques with large filters. The convergence properties of the proposed algorithm are analyzed. Simulation results show that the proposed beamforming algorithm outperforms existing techniques and is robust against signature mismatch problems.**

***Index Terms*—Antenna arrays, auxiliary vector, beamforming, constrained constant modulus.**

I. Introduction

Adaptive beamforming techniques are of central importance to systems equipped with antenna arrays to improve the reception of a desired signal and to suppress interference. Adaptive beamforming has found numerous applications in radar, sonar, and wireless communications [1]–[3]. In order to design adaptive beamformers, a number of adaptive filtering algorithms have been reported [4]. These algorithms usually exhibit a tradeoff between performance and computational complexity, and are based on different design criteria. Prominent design criteria are the constrained minimum variance (CMV) and the constrained constant modulus (CCM) due to their simplicity and effectiveness. The CMV criterion aims to minimize the beamformer output power while maintaining the array response on the direction of the desired signal. The CCM criterion is a positive measure [4] of the deviation of the beamformer output from a constant modulus condition subject to a constraint on the array response of the desired signal.

There are several cost-effective algorithms for the design of the adaptive beamformers. Representative examples are the stochastic gradient (SG) and recursive least squares (RLS) [5]. Although these methods provide a simple implementation of the beamformer, a major shortcoming is that they require a large number of samples to reach the steady-state when the array size is large. Besides, in dynamic scenarios, filters with many elements usually fail or provide poor performance in tracking signals embedded in interference and noise. An efficient algorithm developed to address this problem is called the multistage Wiener filter (MSWF), which was proposed with the minimum mean squared error (MMSE) criterion [7] and then extended to the CMV [8] and the CCM criteria [9]. Another well-known technique is the auxiliary vector filtering (AVF) [10] algorithm, which utilizes an iterative approach to compute the weight solution without any form of explicit input covariance matrix inversion, decomposition, or

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diagonalization. The CMV-based AVF algorithm and its application in adaptive beamforming have been reported in [11], [12], respectively.

This correspondence makes two contributions. The first contribution is the proposal of a CCM-based AVF algorithm for the design of adaptive beamformers. Compared with the CMV-based method [11], the CCM criterion exploits a constant modulus property of the transmitted signals and utilizes the deviation to provide more information for the parameter estimation of the constant modulus constellations [9]. The proposed structure decomposes the adaptive filter into constrained (reference vector filters) and unconstrained (auxiliary vector filters) components. The constrained component is initialized with the array response of the desired signal to start the iteration and ensure the constraint, and the auxiliary vector in the unconstrained component is iterated with respect to the constant modulus criterion. The weight vector is updated by means of suppressing the scaling unconstrained component from the constrained part. The main difference from the CMV-based AVF algorithm is that, in the proposed CCM-based algorithm, the auxiliary vector and the scalar factor depend on each other and are jointly calculated according to the constant modulus criterion (subject to different constraints). It provides an iterative exchange of information between the auxiliary vector and the scalar factor and also exploits the information about the constant modulus signals. This makes the beamforming algorithm more robust against uncertainties and leads to an improved performance. The second contribution is an analysis of the properties of the proposed algorithm and a comparison with previously reported techniques. Specifically, we first analyze the convergence of the auxiliary vector. Then, we establish the orthogonality of the successive auxiliary vectors. The convergence of the weight vector to the optimal weight solution is also verified. At last, we predict the trend of the mean-squared error (MSE) of the CCM criterion via the analysis and validate the result in simulations.

The rest of this correspondence is organized as follows. We outline a system model and the problem statement in Section II. The proposed scheme is introduced and the CCM-AVF algorithm is developed in Section III. The properties of the proposed algorithm are analyzed in Section IV. Simulation results are provided and discussed in Section V, and conclusions are drawn in Section VI.

II. System Model and CCM Beamformer

Let us suppose that q narrowband signals impinge on a uniform linear array (ULA) of m $(m \geq q)$ sensor elements. The sources are assumed to be in the far field with directions of arrival (DOAs) $\theta_0, \ldots, \theta_{q-1}$. The received vector $\boldsymbol{x} \in \mathbb{C}^{m\times 1}$ can be modeled as

$$\boldsymbol{x} = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{\theta} = [\theta_0, \ldots, \theta_{q-1}]^T \in \mathbb{R}^{q\times 1}$ represents the signal DOAs, $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_0), \ldots, \boldsymbol{a}(\theta_{q-1})] \in \mathbb{C}^{m\times q}$ comprises the signal steering vectors $\boldsymbol{a}(\theta_k) = [1, e^{-2\pi j(d/\lambda_c)cos\theta_k}, \ldots, e^{-2\pi j(m-1)(d/\lambda_c)cos\theta_k}]^T \in \mathbb{C}^{m\times 1}$, $(k = 0, \ldots, q-1)$, where λ_c is the wavelength and d is the inter-element distance of the ULA ($d = \lambda_c/2$ in general), $\boldsymbol{s} \in \mathbb{C}^{q\times 1}$ is the source data, $\boldsymbol{n} \in \mathbb{C}^{m\times 1}$ is assumed to be a zero-mean spatially white Gaussian process, and $(\cdot)^T$ stands for transpose. To avoid mathematical ambiguities, the steering vectors $\boldsymbol{a}(\theta_k)$ are normalized and considered to be linearly independents. The output of the beamformer is

$$y = \boldsymbol{w}^H\boldsymbol{x} \tag{2}$$

where $\boldsymbol{w} = [w_1, \ldots, w_m]^T \in \mathbb{C}^{m\times 1}$ is the complex weight vector of the beamformer, and $(\cdot)^H$ stands for Hermitian transpose.

The CCM beamformer is designed based on the minimization of the

$$J_{\text{cm}}(\boldsymbol{w}) = \mathbb{E}\left\{\left[|y|^2 - \nu\right]^2\right\}, \quad \text{subject to} \quad \boldsymbol{w}^H\boldsymbol{a}(\theta_0) = \gamma \tag{3}$$

where ν is set to consider the cost function as the expected deviation of the squared modulus of the beamformer output to a constant and γ is selected to ensure the convexity of the cost function [13]. The quantity θ_0 is the direction of the desired signal and $\boldsymbol{a}(\theta_0)$ is the corresponding steering vector. Since the optimization problem in (3) is a fourth-order cost function, the weight vector $\boldsymbol{w}$ obtained from (3) will be a function of the previous weight vector and the beamformer output y, which provides a positive way to exchange information between the update of the weight vector and the estimated output and thus leads to a fast convergence and tracking performance. The SG or RLS [5], [15] type algorithms can be employed to calculate the weight vector for the design of the beamformer. However, they suffer from a poor convergence and tracking performance when the dimension m is large.

III. Proposed CCM Beamformer and AVF Algorithm

In this section, we introduce an adaptive filtering structure with respect to the CCM criterion and develop a CCM-AVF algorithm for adaptive beamforming.

A. Proposed CCM-AVF Algorithm

In order to simplify the derivation, we make a transformation on the constant modulus cost function in (3), which can be written as

$$J_{\text{av}}(\boldsymbol{w}) = \mathbb{E}\left\{\left[y^*\boldsymbol{w}^H\boldsymbol{x} - \nu\right]^2\right\} = \mathbb{E}\left\{\left[\boldsymbol{w}^H\tilde{\boldsymbol{x}} - \nu\right]^2\right\} \tag{4}$$

where $\tilde{\boldsymbol{x}} = y^*\boldsymbol{x}$ can be viewed as a new received vector to the beamformer. From (4), we convert the constrained optimization problem in (3) into an unconstrained one. The constraint $\boldsymbol{w}^H\boldsymbol{a}(\theta_0) = \gamma$ is already enforced in the design by initializing $\boldsymbol{w}_0 = \gamma\boldsymbol{a}(\theta_0)/\|\boldsymbol{a}(\theta_0)\|^2$. The weight vector is iteratively computed by subtracting a scaling auxiliary vector (unconstrained component) that is orthogonal to $\boldsymbol{a}(\theta_0)$ from $\boldsymbol{w}_0$ (constrained component), which yields

$$\boldsymbol{w}_k = \boldsymbol{w}_0 - \sum_{l=1}^{k}\mu_l\boldsymbol{g}_l = \boldsymbol{w}_{k-1} - \mu_k\boldsymbol{g}_k \tag{5}$$

where $\boldsymbol{g}_k \in \mathbb{C}^{m\times 1}$ is the auxiliary vector with $\boldsymbol{g}_k^H\boldsymbol{a}(\theta_0) = 0$ and μ_k is a scalar factor to control the weight of $\boldsymbol{g}_k$. The auxiliary vector is supposed to capture the signal components in $\tilde{\boldsymbol{x}}$ that are not from the direction θ_0. The aim of (5) is to suppress the disturbance of the unconstrained component while maintaining the contribution of the signal of interest (SOI).

From (5), it is necessary to determine the auxiliary vector $\boldsymbol{g}_k$ and the scalar factor μ_k for the calculation of $\boldsymbol{w}_k$. Fixing $\boldsymbol{g}_k$, μ_k can be obtained by minimizing $\mathbb{E}\{[\boldsymbol{w}_k^H\tilde{\boldsymbol{x}} - \nu]^2\}$. Substituting the second expression of $\boldsymbol{w}_k$ in (5) into this minimization problem, computing the gradient with respect to μ_k and equating it to zero, we have

$$\mu_k = \frac{\boldsymbol{g}_k^H\tilde{\boldsymbol{R}}\boldsymbol{w}_{k-1} - \nu\boldsymbol{g}_k^H\tilde{\boldsymbol{p}}}{\boldsymbol{g}_k^H\tilde{\boldsymbol{R}}\boldsymbol{g}_k} \tag{6}$$

where $\tilde{\boldsymbol{R}} = \mathbb{E}[\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^H] \in \mathbb{C}^{m\times m}$ and $\tilde{\boldsymbol{p}} = \mathbb{E}[\tilde{\boldsymbol{x}}] \in \mathbb{C}^{m\times 1}$.

Assuming now that μ_k is known, the calculation of the auxiliary vector $\boldsymbol{g}_k$ should take the conditions $\boldsymbol{g}_k^H\boldsymbol{a}(\theta_0) = 0$ and $\boldsymbol{g}_k^H\boldsymbol{g}_k = 1$ into account. This constrained minimization problem can be transformed by the method of Lagrange multipliers into an unconstrained one, whose cost function is

TABLE I
PROPOSED CCM-AVF ALGORITHM

For the time index $i = 1, 2, \ldots, N$.
Initialization:
$\boldsymbol{w}_0(i) = \frac{\gamma \boldsymbol{a}(\theta_0)}{\|\boldsymbol{a}(\theta_0)\|^2}$; $\mu_0(i) =$ small positive value.
$y(i) = \boldsymbol{w}_0^H(i)\boldsymbol{x}(i)$; $\tilde{\boldsymbol{x}}(i) = y^*(i)\boldsymbol{x}(i)$; $\tilde{y}(i) = \boldsymbol{w}_0^H(i)\tilde{\boldsymbol{x}}(i)$
$\hat{\tilde{\boldsymbol{R}}}(i) = \frac{1}{i}\sum_{l=1}^{i}\tilde{\boldsymbol{x}}(l)\tilde{\boldsymbol{x}}^H(l)$; $\hat{\tilde{\boldsymbol{p}}}(i) = \frac{1}{i}\sum_{l=1}^{i}\tilde{\boldsymbol{x}}(l)$
$\hat{\tilde{\boldsymbol{p}}}_{\mathrm{y}}(i) = \frac{1}{i}\sum_{l=1}^{i}\left(\nu - \tilde{y}(l)\right)^*\tilde{\boldsymbol{x}}(l)$
Iterative procedure:
For $k = 1, 2, \ldots, K$
$\boldsymbol{g}_k(i) = \mu_{k-1}^*(i)\hat{\tilde{\boldsymbol{p}}}_{\mathrm{y}}(i) - \frac{\mu_{k-1}^*(i)\boldsymbol{a}^H(\theta_0)\hat{\tilde{\boldsymbol{p}}}_{\mathrm{y}}(i)}{\|\boldsymbol{a}(\theta_0)\|^2}\boldsymbol{a}(\theta_0)$
if $\boldsymbol{g}_k(i) = \boldsymbol{0}$ **then EXIT.**
$\mu_k(i) = \frac{\boldsymbol{g}_k^H(i)\hat{\tilde{\boldsymbol{R}}}(i)\boldsymbol{w}_{k-1}(i) - \nu\boldsymbol{g}_k^H(i)\hat{\tilde{\boldsymbol{p}}}(i)}{\boldsymbol{g}_k^H(i)\hat{\tilde{\boldsymbol{R}}}(i)\boldsymbol{g}_k(i)}$
$\boldsymbol{w}_k(i) = \boldsymbol{w}_{k-1}(i) - \mu_k\boldsymbol{g}_k(i)$
Weight expression: $\boldsymbol{w}(i) = \boldsymbol{w}_{K-1}(i)$.

$$J_{\mathrm{un}}(\boldsymbol{w}_k) = \mathbb{E}\left\{\left[\boldsymbol{w}_k^H\tilde{\boldsymbol{x}} - \nu\right]^2\right\} - 2\Re\left\{\lambda_1\left[\boldsymbol{g}_k^H\boldsymbol{g}_k - 1\right] - \lambda_2\boldsymbol{g}_k^H\boldsymbol{a}(\theta_0)\right\} \tag{7}$$

where λ_1 and λ_2 are scalar Lagrange multipliers and the operator $\Re(\cdot)$ selects the real part of the argument. According to (5), computing the gradient of (7) with respect to $\boldsymbol{g}_k$, equating it to zero and solving for λ_1 and λ_2, we have

$$\boldsymbol{g}_k = \frac{\mu_k^*\tilde{\boldsymbol{p}}_{\mathrm{y}} - \frac{\mu_k^*\boldsymbol{a}^H(\theta_0)\tilde{\boldsymbol{p}}_{\mathrm{y}}}{\|\boldsymbol{a}(\theta_0)\|^2}\boldsymbol{a}(\theta_0)}{\left\|\mu_k^*\tilde{\boldsymbol{p}}_{\mathrm{y}} - \frac{\mu_k^*\boldsymbol{a}^H(\theta_0)\tilde{\boldsymbol{p}}_{\mathrm{y}}}{\|\boldsymbol{a}(\theta_0)\|^2}\boldsymbol{a}(\theta_0)\right\|} \tag{8}$$

where $\tilde{\boldsymbol{p}}_{\mathrm{y}} = \mathbb{E}[(\nu - \tilde{y})^*\tilde{\boldsymbol{x}}] \in \mathbb{C}^{m\times 1}$ and $\tilde{y} = \boldsymbol{w}^H\tilde{\boldsymbol{x}}$. Note that $\tilde{\boldsymbol{p}}_{\mathrm{y}}$ is the gradient of the CCM cost function in (4). The meaning of (8) is to subtract the components from the direction θ_0 (i.e., the projection $[\boldsymbol{I} - (\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0)/\|\boldsymbol{a}(\theta_0)\|^2)]\tilde{\boldsymbol{p}}_y$) and keep the unconstrained components in $\boldsymbol{g}_k$. The step size μ_k controls the adaptation and ensures the constraints with respect to $\boldsymbol{g}_k$. By subtracting the scaled $\boldsymbol{g}_k$ iteratively according to (5), the weight vector $\boldsymbol{w}_k$ can be updated to maintain the contribution of the SOI while attenuating the interference and noise. The proposed algorithm uses an iterative way to calculate $\boldsymbol{w}_k$ without the matrix inversion.

The expressions $\boldsymbol{w}_0$, $\boldsymbol{w}_k$, μ_k, and $\boldsymbol{g}_k$ compose the iteration of the proposed CCM-AVF algorithm. In this procedure, $\tilde{\boldsymbol{x}}$ can be viewed as a new received vector that is processed by the adaptive filter $\boldsymbol{w}_k$ (kth estimation of $\boldsymbol{w}$) to generate $\tilde{y}$, in which, $\boldsymbol{w}_k$ is determined by minimizing the MSE between the output and the desired constant modulus condition. This principle is suitable for all the iterations with $k = 1, 2, \ldots$. Generally, there exists a maximum value of k, i.e., $k_{\max} = K - 1$, that is determined by a certain rule to stop iterations and obtain the weight solution $\boldsymbol{w} = \boldsymbol{w}_{k_{\max}}$. A reasonable rule, which is adopted in the proposed algorithm, is to terminate the iteration if $\boldsymbol{g}_k = \boldsymbol{0}$ is achieved. This characteristic will be explained in the following section. Alternative rules can be found in [12]. A summary of the proposed CCM-AVF algorithm is given in Table I. It should be remarked that, for the proposed adaptive algorithm, the weight vector should be adapted following the incoming of the received vector at each time instant. Thus, the iteration procedure is performed for each time index and the time index i is included in the quantities.

B. Interpretation of the Proposed CCM-AVF Algorithm

There are several points we need to interpret in Table I. First, the initialization is very important to the proposed method. We initialize $\boldsymbol{w}_0(i)$ to estimate $\tilde{y}(i)$ and to start the iterations. Since $\mu_k(i)$ and $\boldsymbol{g}_k(i)$ depend on each other, we need a small positive value $\mu_0(i)$ to start the calculation. Under this condition, the subscript of the scalar factor for the calculation of $\boldsymbol{g}_k(i)$ should be replaced by $k-1$ instead of k, as shown in Table I.

Second, the expected quantities $\tilde{\boldsymbol{R}}$, $\tilde{\boldsymbol{p}}$, and $\tilde{\boldsymbol{p}}_{\mathrm{y}}$ are not available in practice. We use a sample-average approach to estimate them, namely, $\hat{\tilde{\boldsymbol{R}}}(i)$, $\hat{\tilde{\boldsymbol{p}}}(i)$, and $\hat{\tilde{\boldsymbol{p}}}_y(i)$ in Table I. To improve the estimation accuracy, these quantities can be refreshed or further regularized during the iterations. Specifically, we use $\boldsymbol{w}_k(i)$ in the iteration step instead of $\boldsymbol{w}_0(i)$ in the initialization to generate $y(i)$, and the related $\tilde{\boldsymbol{x}}(i)$ and $\tilde{y}(i)$, which are employed to update the estimates $\hat{\tilde{\boldsymbol{R}}}(i)$, $\hat{\tilde{\boldsymbol{p}}}(i)$, and $\hat{\tilde{\boldsymbol{p}}}_{\mathrm{y}}(i)$. Compared with $\boldsymbol{w}_0(i)$, $\boldsymbol{w}_k(i)$ is more efficient to estimate the desired signal. Thus, the estimates based on the current $\boldsymbol{w}_k(i)$ can be used to calculate the subsequent scalar factor and the auxiliary vector.

Third, we drop the normalization of the auxiliary vector [11], [16]. The constraint conditions ensure the orthogonality between $\boldsymbol{g}_k(i)$ and $\boldsymbol{a}(\theta_0)$. The orthogonality among the auxiliary vectors is not imposed. Actually, the successive auxiliary vectors do satisfy the orthogonality. This characteristic will be proved in the next section.

The proposed CCM-AVF algorithm employs an iterative procedure to adjust the weight vector for each time instant. It avoids any form of matrix inversion, decomposition, or diagonalization. From (6) and (8), the calculations of the scalar factor and the auxiliary vector depend on each other. It provides an iterative exchange of information between them, which are jointly computed to update the weight vector. This scheme leads to an improved convergence and tracking performance. The proposed CCM design for adaptive beamforming effectively measures the expected deviation of the beamformer output from the constant modulus condition and provides useful information for the parameter estimation.

IV. ANALYSIS OF THE PROPOSED ALGORITHM

In the following parts, we will analyze the characteristics of the proposed CCM-AVF algorithm.

A. Convergence Analysis of the Auxiliary Vector

The proposed CCM-AVF algorithm runs $k_{\max}$ iterations for each time index to update the scalar factor and the auxiliary vector. The quantities $\hat{\tilde{\boldsymbol{R}}}(i)$, $\hat{\tilde{\boldsymbol{p}}}(i)$, and $\hat{\tilde{\boldsymbol{p}}}_{\mathrm{y}}(i)$ can be assumed constant during the iteration procedure. For simplicity, we drop the time index i in the following. From (5) and (6), we have

$$\begin{aligned}\boldsymbol{g}_k^H\tilde{\boldsymbol{R}}\boldsymbol{w}_k &= \boldsymbol{g}_k^H\tilde{\boldsymbol{R}}[\boldsymbol{w}_{k-1} - \mu_k\boldsymbol{g}_k] \\ &= \boldsymbol{g}_k^H\tilde{\boldsymbol{R}}\boldsymbol{w}_{k-1} - \frac{\boldsymbol{g}_k^H\tilde{\boldsymbol{R}}\boldsymbol{w}_{k-1} - \nu\boldsymbol{g}_k^H\tilde{\boldsymbol{p}}}{\boldsymbol{g}_k^H\tilde{\boldsymbol{R}}\boldsymbol{g}_k}\boldsymbol{g}_k^H\tilde{\boldsymbol{R}}\boldsymbol{g}_k.\end{aligned} \tag{9}$$

Let us assume that the weight solution $\boldsymbol{w}_{k_{\max}} \rightarrow \boldsymbol{w}_{\mathrm{opt}}$, where $\boldsymbol{w}_{\mathrm{opt}}$ is the optimal weight solution. This assumption will be verified in the following part. In the noise free case [6], [9], [13], [14], we have $y = \boldsymbol{w}^H\boldsymbol{x} = \alpha d_0$, where α is a scaling parameter, d_0 is the transmitted data of the desired signal, and $\boldsymbol{x} = \sum_{l=0}^{q-1}\beta_l B_l d_l\boldsymbol{a}(\theta_l)$ with β_l being the fading factor with respect to the lth user, B_l being the signal amplitude, and d_l the transmitted data of the lth user, respectively. According to the definitions of $\tilde{\boldsymbol{p}}$ and $\boldsymbol{x}$, and considering the fact that d_l is an independent random variable, we obtain [5]

$$\tilde{\boldsymbol{p}} = \mathbb{E}[y^*\boldsymbol{x}] = B_0\mathbb{E}[\alpha^*\beta_0]\boldsymbol{a}(\theta_0). \tag{10}$$

According to the constraint $\boldsymbol{g}_k^H\boldsymbol{a}(\theta_0) = 0$, we have $\boldsymbol{g}_k^H\tilde{\boldsymbol{R}}\boldsymbol{w}_k = 0$. In other words, $\boldsymbol{g}_k$ and $\boldsymbol{w}_k$ are $\tilde{\boldsymbol{R}}$-orthogonal. Therefore,

$$\begin{aligned}\|\boldsymbol{w}_k + \mu_k\boldsymbol{g}_k\|_{\tilde{R}}^2 &= \|\boldsymbol{w}_k\|_{\tilde{R}}^2 + \|\mu_k\boldsymbol{g}_k\|_{\tilde{R}}^2 \\ \|\boldsymbol{w}_k\|_{\tilde{R}}^2 &= \|\boldsymbol{w}_{k-1}\|_{\tilde{R}}^2 - \|\mu_k\boldsymbol{g}_k\|_{\tilde{R}}^2.\end{aligned} \tag{11}$$

It is clear that $\{\|\boldsymbol{w}_k\|_{\tilde{R}}\}$ with $k = 1, 2, \ldots$ is a monotonically decreasing sequence of non-negative numbers. Hence, $\|\boldsymbol{w}_k\|_{\tilde{R}}$ converges. Besides, we have $\mu_k \neq 0$ since, if $\mu_k = 0$, there is no update of the weight vector $\boldsymbol{w}_k$, which is in contrast with the auxiliary vector filtering theory [11]. Thus, we have $\|\boldsymbol{g}_k\| \to 0$ or $\boldsymbol{g}_k \to \boldsymbol{0}$, as k increases. This is the termination rule we utilized in the proposed algorithm to stop the iterations. It is worth noting that $\boldsymbol{g}_k^H \tilde{\boldsymbol{p}}$ is actually not equal to but close to zero since $\tilde{\boldsymbol{p}}$ depends on y. This term provides information for the adaptation of the scalar factor.

B. Orthogonality of the Auxiliary Vectors

Now, we prove an important characteristic of the proposed CCM-AVF algorithm, namely, successive auxiliary vectors are orthogonal, i.e., $\boldsymbol{g}_k^H \boldsymbol{g}_{k+1} = 0$ with $k = 1, \ldots, k_{\max}$. We start the proof from (8), omitting the normalization [11], the auxiliary vector can be written as

$$\boldsymbol{g}_k = \mu_k^* \left[\boldsymbol{I} - \frac{\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0)}{\|\boldsymbol{a}(\theta_0)\|^2}\right] \tilde{\boldsymbol{p}}_{\mathrm{y}} = \left[\boldsymbol{I} - \frac{\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0)}{\|\boldsymbol{a}(\theta_0)\|^2}\right] \boldsymbol{g}_k \quad (12)$$

where $[\boldsymbol{I} - (\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0)/\|\boldsymbol{a}(\theta_0)\|^2)]^2 = [\boldsymbol{I} - (\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0)/\|\boldsymbol{a}(\theta_0)\|^2)]$ is considered in the expression.

From (12), we have

$$\boldsymbol{g}_k^H \boldsymbol{g}_{k+1} = \mu_{k+1}^* \boldsymbol{g}_k^H \left[\boldsymbol{I} - \frac{\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0)}{\|\boldsymbol{a}(\theta_0)\|^2}\right] \tilde{\boldsymbol{p}}_{\mathrm{y}} = \mu_{k+1}^* \boldsymbol{g}_k^H \tilde{\boldsymbol{p}}_{\mathrm{y}}. \quad (13)$$

According to the definition of $\tilde{\boldsymbol{p}}_{\mathrm{y}}$, y, and $\boldsymbol{x}$, we have

$$\begin{aligned} \tilde{\boldsymbol{p}}_{\mathrm{y}} &= \mathbb{E}\left[(\nu - \tilde{y})^* \tilde{\boldsymbol{x}}\right] = \mathbb{E}\left[\left(\nu - |y|^2\right) y^* \boldsymbol{x}\right] \\ &= \nu B_0 \mathbb{E}[\alpha^* \beta_0] \boldsymbol{a}(\theta_0) - B_0 \mathbb{E}\left[|\alpha_0|^2 \alpha^* \beta_0\right] \boldsymbol{a}(\theta_0). \end{aligned} \quad (14)$$

Since $\boldsymbol{g}_k^H \boldsymbol{a}(\theta_0) = 0$, we conclude that $\boldsymbol{g}_k^H \boldsymbol{g}_{k+1} = 0$.

C. Convergence Analysis of the Weight Vector

In order to analyze the convergence of $\boldsymbol{w}$, we need to derive the optimal weight solution of the CCM cost function in (3), which can be transformed into an unconstrained cost function

$$J_{\mathrm{ucm}}(\boldsymbol{w}) = \mathbb{E}\left\{\left[|y|^2 - \nu\right]^2\right\} + 2\Re\left\{\lambda\left[\boldsymbol{w}^H \boldsymbol{a}(\theta_0) - \gamma\right]\right\} \quad (15)$$

where λ is a scalar Lagrange multiplier. Note that γ can be adjusted to satisfy the convexity of the cost function. The related discussions can be found in [6] and [9].

Computing the gradient of (15) with respect to $\boldsymbol{w}$, equating it to a null vector and solving for λ, we obtain

$$\boldsymbol{w}_{\mathrm{opt}} = \frac{\gamma \boldsymbol{R}_{xy}^{-1} \boldsymbol{a}(\theta_0)}{\boldsymbol{a}^H(\theta_0) \boldsymbol{R}_{xy}^{-1} \boldsymbol{a}(\theta_0)} \quad (16)$$

where $\boldsymbol{R}_{xy} = \mathbb{E}[(|y|^2 - \nu)\boldsymbol{x}\boldsymbol{x}^H]$.

According to the definition of $\tilde{\boldsymbol{p}}_y$, we have

$$\tilde{\boldsymbol{p}}_y = \mathbb{E}\left[(\nu - \tilde{y})^* \tilde{\boldsymbol{x}}\right] = -\boldsymbol{R}_{xy}\boldsymbol{w}. \quad (17)$$

Considering the fact that $\boldsymbol{g}_k \to \boldsymbol{0}$ and the definition of $\boldsymbol{g}_k$ in (8), it implies

$$\left[\boldsymbol{I} - \frac{\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0)}{\|\boldsymbol{a}(\theta_0)\|^2}\right] \boldsymbol{R}_{xy}\boldsymbol{w}_k \longrightarrow \boldsymbol{0}, \quad \text{as} \quad k \to \infty \quad (18)$$

where we notice that the component of $\boldsymbol{R}_{xy}\boldsymbol{w}_k$ is orthogonal to $\boldsymbol{a}(\theta_0)$ as $k \to \infty$. Thus, to prove that $\boldsymbol{R}_{xy}\boldsymbol{w}_k$ converges, it suffices to show that the projection of $\boldsymbol{R}_{xy}\boldsymbol{w}_k$ onto $\boldsymbol{a}(\theta_0)$, i.e., $(\boldsymbol{a}^H(\theta_0)\boldsymbol{R}_{xy}\boldsymbol{w}_k/\|a(\theta_0)\|^2)a(\theta_0)$, converges. To achieve that, we multiply $\boldsymbol{R}_{xy}^{-1}$ on both sides of (18) to obtain

$$\boldsymbol{w}_k - \frac{\boldsymbol{a}^H(\theta_0)\boldsymbol{R}_{xy}\boldsymbol{w}_k}{\|\boldsymbol{a}(\theta_0)\|^2} \boldsymbol{R}_{xy}^{-1}\boldsymbol{a}(\theta_0) \longrightarrow \boldsymbol{0}, \quad \text{as} \quad k \to \infty. \quad (19)$$

From (19), it implies that for a given $\epsilon > 0$, there exists a $k_{\max} > 0$ such that for every $k > k_{\max}$, we have $\|\boldsymbol{w}_k - \varrho_k^* \boldsymbol{R}_{xy}^{-1}\boldsymbol{a}(\theta_0)\| < \epsilon/\|\boldsymbol{a}(\theta_0)\|^2$ with $\varrho_k = \boldsymbol{w}_k^H \boldsymbol{R}_{xy}\boldsymbol{a}(\theta_0)/\|\boldsymbol{a}(\theta_0)\|^2$. Considering the fact that $\|\boldsymbol{w}_k - \varrho_k^* \boldsymbol{R}_{xy}^{-1}\boldsymbol{a}(\theta_0)\|\|\boldsymbol{a}(\theta_0)\| \geq \|(\boldsymbol{w}_k - \varrho_k^* \boldsymbol{R}_{xy}^{-1}\boldsymbol{a}(\theta_0))^H \boldsymbol{a}(\theta_0)\|$ and the constraint $\boldsymbol{w}_k^H \boldsymbol{a}(\theta_0) = \gamma$, we have

$$\left\|\gamma - \varrho_k \boldsymbol{a}^H(\theta_0)\boldsymbol{R}_{xy}^{-1}\boldsymbol{a}(\theta_0)\right\| < \epsilon. \quad (20)$$

Thus, we find that

$$\varrho_k^* = \frac{\boldsymbol{a}^H(\theta_0)\boldsymbol{R}_{xy}\boldsymbol{w}_k}{\|\boldsymbol{a}(\theta_0)\|^2} \longrightarrow \frac{\gamma}{\boldsymbol{a}^H\theta_0 \boldsymbol{R}_{xy}^{-1}\boldsymbol{a}(\theta_0)}, \quad \text{as} \quad k \to \infty. \quad (21)$$

From (18) and (21), we conclude that $\boldsymbol{R}_{xy}\boldsymbol{w}_k$ converges. Substituting (21) into (19) and making a rearrangement, we have

$$\lim_{k\to\infty} \boldsymbol{w}_k = \frac{\gamma \boldsymbol{R}_{xy}^{-1}\boldsymbol{a}(\theta_0)}{\boldsymbol{a}^H(\theta_0)\boldsymbol{R}_{xy}^{-1}\boldsymbol{a}(\theta_0)} \quad (22)$$

which verifies the convergence of $\boldsymbol{w}_k$ to $\boldsymbol{w}_{\mathrm{opt}}$ in (16). Assuming $\boldsymbol{w} \to \boldsymbol{w}_{\mathrm{opt}}$ and $y = \boldsymbol{w}^H\boldsymbol{x} = \alpha d_0$ in the noise free case, we have $\boldsymbol{R}_{xy} \cong \mathbb{E}[(\alpha^2|d_0|^2 - \nu)\boldsymbol{x}\boldsymbol{x}^H] = \tau\boldsymbol{R}$ with $\tau = \alpha^2|d_0|^2 - \nu$ is a constant and $\boldsymbol{R} = \mathbb{E}[\boldsymbol{x}\boldsymbol{x}^H]$ [9], [13]. This approximation is also valid for sufficiently high signal-to-noise ratios or when the noise is negligible [9], [13]. Therefore, by adjusting the parameter γ, the optimal weight solution of the CCM criterion approaches the weight solution of the minimum variance distortionless-response (MVDR) filter [3], i.e., $\|\boldsymbol{w} - \boldsymbol{w}_{\mathrm{MVDR}}\| \to 0$.

D. Prediction of the Trend of MSE

In this part, we develop an expression of the MSE following the time instant i for the proposed algorithm, which can be expressed by

$$\begin{aligned} J_{\mathrm{mse}}(i) &= \mathbb{E}\left[\left|d_0(i) - \boldsymbol{w}_{\mathrm{opt}}^H \boldsymbol{x}\right|^2\right] \\ &= \epsilon_{\min} + \xi(i) - \xi_{\min} - \mathbb{E}\left[\boldsymbol{e}_w^H(i)\right]\boldsymbol{a}(\theta_0) - \boldsymbol{a}^H(\theta_0)\mathbb{E}\left[\boldsymbol{e}_w(i)\right] \end{aligned} \quad (23)$$

where $d_0(i)$ is the transmitted data of the desired user, $\epsilon_{\min} = \mathbb{E}[|d_0(i) - \boldsymbol{w}_{\mathrm{opt}}^H\boldsymbol{x}(i)|^2]$, $\xi_{\min} = \mathbb{E}[\boldsymbol{w}_{\mathrm{opt}}^H\boldsymbol{x}(i)\boldsymbol{x}^H(i)\boldsymbol{w}_{\mathrm{opt}}]$, $\xi(i) = \mathbb{E}[\boldsymbol{w}^H(i)\boldsymbol{x}(i)\boldsymbol{x}^H(i)\boldsymbol{w}(i)]$, and $\boldsymbol{e}_w(i) = \boldsymbol{w}(i) - \boldsymbol{w}_{\mathrm{opt}} = \boldsymbol{w}_{k_{\max}}(i) - \boldsymbol{w}_{\mathrm{opt}}$ is the weight error vector.

From (16), the optimal weight solution $\boldsymbol{w}_{\mathrm{opt}}$ of the CCM criterion cannot be obtained analytically since it is a function of $y(i)$, which depends on the calculated weight vector $\boldsymbol{w}(i)$ (i.e., $\boldsymbol{w}_{k_{\max}}(i)$). However, as proved in the convergence analysis of the weight vector, the optimal weight solution of the CCM criterion approaches that of the MVDR filter by adjusting γ, i.e., $\boldsymbol{w}_{\mathrm{opt}} \approx \boldsymbol{w}_{\mathrm{MVDR}}$. Thus, we use $\boldsymbol{w}_{\mathrm{MVDR}}$ instead of $\boldsymbol{w}_{\mathrm{opt}}$ for the calculation of $J_{\mathrm{mse}}(i)$ in (23).

Considering the fact $\lim_{i\to\infty}\mathbb{E}[\boldsymbol{e}_w(i)] = \boldsymbol{0}$ as $\boldsymbol{w}(i) \to \boldsymbol{w}_{\mathrm{MVDR}}$, we have

$$\lim_{i\to\infty} J_{\mathrm{mse}}(i) = \epsilon_{\min} + \lim_{i\to\infty}\xi_{\mathrm{ex}}(i) \quad (24)$$

where $\xi_{\mathrm{ex}}(i) = \xi(i) - \xi_{\min}$ is the steady-state excess MSE. The quantity $\xi(i)$ can be calculated by

$$\xi(i) = \mathbb{E}\left[\boldsymbol{w}^H(i)\boldsymbol{x}(i)\boldsymbol{x}^H(i)\boldsymbol{w}(i)\right] = \mathrm{tr}\left[\boldsymbol{R}_w(i)\boldsymbol{R}\right] \quad (25)$$

where $\boldsymbol{R} = \mathbb{E}[\boldsymbol{x}(i)\boldsymbol{x}^H(i)]$ and $\boldsymbol{R}_w(i) = \mathbb{E}[\boldsymbol{w}(i)\boldsymbol{w}^H(i)] \approx \mathbb{E}[\boldsymbol{w}_{\mathrm{MVDR}}\boldsymbol{w}_{\mathrm{MVDR}}^H] + \boldsymbol{R}_{e_w}(i)$ with $\boldsymbol{R}_{e_w}(i) = \mathbb{E}[\boldsymbol{e}_w(i)\boldsymbol{e}_w^H(i)]$. In practice, $\boldsymbol{R}_{e_w}(i)$ can be estimated by the sample-average approach. Note that $J_{\mathrm{mse}}(i)$ can be regarded as a prediction of the trend of the MSE for the proposed algorithm. In the next section, we will demonstrate that $J_{\mathrm{mse}}(i)$ is able to predict the performance for the proposed CCM-AVF algorithm.

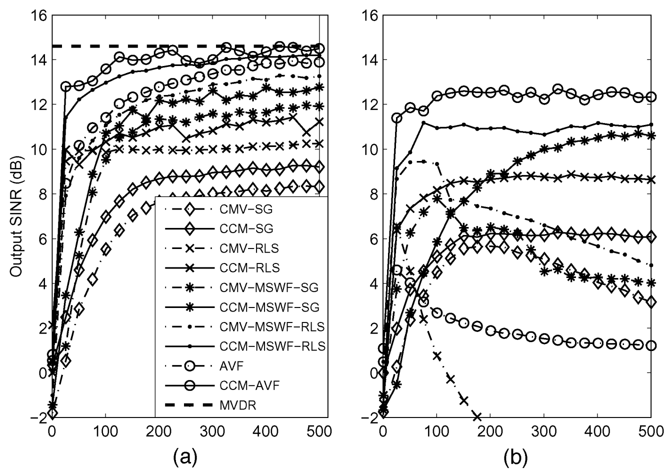

Fig. 1. Output SINR versus the number of snapshots for (a) ideal steering vector; (b) steering vector mismatch 1°.

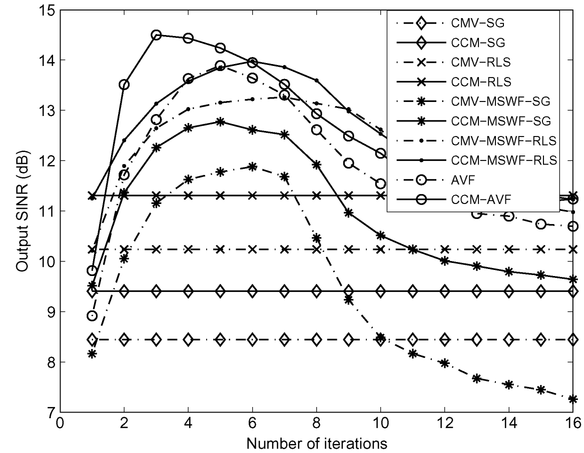

Fig. 2. Output SINR versus the number of iterations.

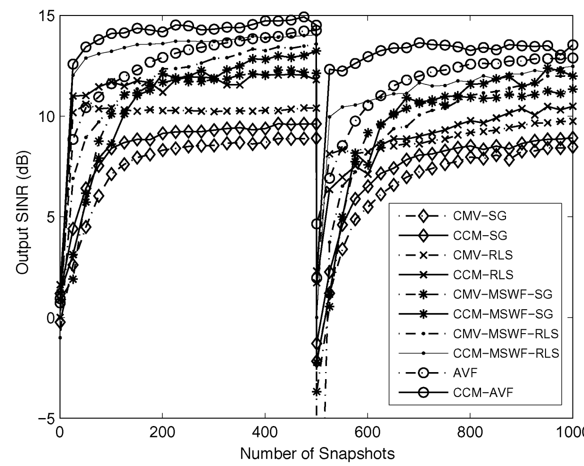

Fig. 3. Output SINR versus the number of snapshots in a scenario where additional interferers suddenly enter the system.

V. Simulations

In this section, we evaluate the performance of the proposed algorithm and verify the analysis in Section IV. Simulations are performed for a ULA containing $m = 40$ sensor elements with half-wavelength interelement spacing. We compare the proposed algorithm (CCM-AVF) with the SG [5], RLS [6], MSWF [9], and AVF [11] methods. With respect to each method, we consider the CMV and the CCM criteria for beamforming. All the results are averaged over 1000 simulation runs. In all experiments, BPSK sources' powers (desired user and interferers) are $\sigma_S^2 = \sigma_I^2 = 1$ and the input $\mathrm{SNR} = 0$ dB with spatially and temporally white Gaussian noise. We set $\gamma = \nu = 1$ for the proposed algorithm.

In Fig. 1, we evaluate the output signal-to-interference-plus-noise ratio (SINR) performance of the proposed algorithm. It includes two experiments. There are $q = 10$ users, including one desired user in the system. The scalar factor is $\mu_0(i) = 0.01$. In Fig. 1(a), we assume that the exact DOA of the SOI is known at the receiver. The CCM-based algorithms achieve better convergence and steady-state performance than those of the CMV-based. The proposed CCM-AVF algorithm enjoys faster convergence than the existing methods and the steady-state performance, which is close to that of the MVDR filter. It should be remarked that the existing methods should be able to converge to the optimum MVDR filter. However, this will require a large number of snapshots. The proposed algorithm can converge to the optimum MVDR filter performance with a reduced data record. In Fig. 1(b), we assume that the DOA of the SOI estimated by the receiver to be 1° away from the actual direction. Under this mismatch scenario, the SINR performance of all the algorithms degrades. Since the CCM-based methods exploit the constant modulus property of the transmitted signal and measure the deviation for the parameter estimation, they are more robust to the mismatch than the CMV-based ones. The proposed algorithm converges fast and reaches a superior performance to other previously reported techniques.

In Fig. 2, we check the impact of the selection of the iteration number to the performance of the existing and proposed methods. We keep the same scenario as that in Fig. 1(a) and set the number of snapshots $N = 500$. From this experiment, we find that the most adequate iteration number for the proposed CCM-AVF algorithm is $k_{\max} = 3$, which is comparatively lower than other AVF and MSWF algorithms, but reach the superior performance. We also checked that this value is rather insensitive to the number of users in the system, to the number of sensor elements, and work efficiently for the studied scenarios. The SINR values of the algorithms should converge to the same steady-state level as the number of iterations increases. However, the convergence depends on the number of snapshots, namely, it requires a large number of snapshots to converge if the number of iterations is large. In this experiment, the number of snapshots is fixed at 500, which is set since the proposed algorithm reaches the steady-state with a small number of iterations. It illustrates the advantage of the proposed algorithm over existing methods under the condition where the number of snapshots is small.

In the next experiment, we evaluate the performance of the proposed and analyzed algorithms in a non-stationary scenario, namely, when the number of users changes. In Fig. 3, the scenario starts with $q = 8$ users including one desired user. From the first stage (first 500 snapshots), the convergence and steady-state performance of the proposed CCM-AVF algorithm is superior to other existing methods. Four more users enter the system at the time instant $i = 500$. This change makes the output SINR reduce suddenly and degrades the performance of all the methods. The proposed CCM-AVF algorithm rapidly tracks the change and converges to a higher value of SINR at steady-state.

Fig. 4 is carried out under the same scenario as that in Fig. 1(a) and verifies the analysis about the prediction of the trend of the MSE. We increase the number of snapshots to $N = 1000$ and compare the MSE result obtained via simulations in Table I with that obtained by the prediction in (23). Note that we use $\boldsymbol{w}_{\mathrm{MVDR}}$ [3] instead of $\boldsymbol{w}_{\mathrm{opt}}$ for the

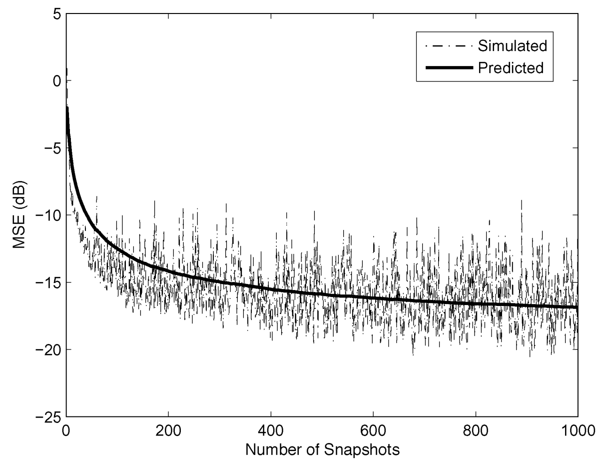

Fig. 4. MSE predicted versus simulated performance.

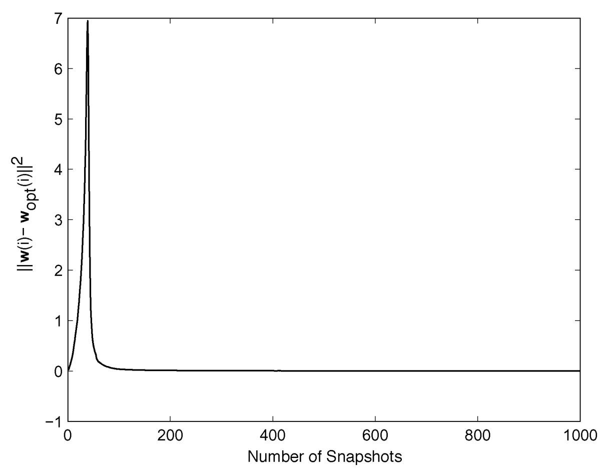

Fig. 5. Square estimation error between the weight solution and the optimal weight vector.

calculation of $J_{\rm mse}(i)$. It indicates that the curve obtained with the prediction agrees with that obtained via the simulation, verifying the validity of our analysis.

Finally, we show the convergence of the weigh vector for the proposed algorithm to that of the MVDR filter. In order to exhibit the convergence of the iteration procedure for each time instant, we use the sample-average approach to estimate $\boldsymbol{R}$ for the computation of $\boldsymbol{w}_{\rm MVDR}(i)$. Note that $\boldsymbol{w}_{\rm MVDR}(i)$ is calculated following the incoming of the received vector, which is different from $\boldsymbol{w}_{\rm MVDR}$ that is independent of i. The scenario is the same as that in Fig. 4. We evaluate the square estimation error $\|\boldsymbol{w}(i) - \boldsymbol{w}_{\rm MVDR}(i)\|^2$ against the number of snapshots. It exhibits that the square estimation error always keeps a relative low level which is close to zero except the first $i \leq 40$ since the number of snapshots are not enough $(i \leq m)$ to provide accurate estimate of $\boldsymbol{R}$. It should be remarked that in addition to convergence to the MVDR, the proposed CCM-AVF algorithm is robust against signature mismatches. The existing CMV based algorithms do not have this feature.

VI. Concluding Remarks

We proposed a CCM-AVF algorithm for robust adaptive beamforming. It exploits the constant modulus property of the transmitted signals to measure the expected deviation of the beamformer output from the constant modulus condition. The proposed CCM-AVF algorithm provides an iterative exchange of information between the scalar factor and the auxiliary vector to update the weight vector for improving the performance. The convergence and related characteristics of the proposed algorithm were analyzed. Simulations were performed to show the improved performance of the proposed method over existing techniques and to verify the analysis.

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