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# Adaptive Control for Structural Damage Mitigation

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Abstract - Substantial progress has been made in analyzing the integrity of composite structures when macro or nano sensors and actuators are embedded into it. The resulting structure in a dynamic environment is said to be "intelligent" if it performs a certain functional requirements related to vibrations, health, shape, etc. In health, after the damage has been detected, the subject of damage mitigation becomes important, so that in prognosis context (Farrar and Lieven, 2007), the remaining life of the structure is extended. The damage is said to be mitigated if the sensor data of the damaged structure matches with the sensor data of the healthy structure. This is done by applying an actuator loading. In this paper, Model Reference Adaptive Control (Slotine and Li, 1991) is applied for structural damage mitigation. A known finite element model resulting from the structural health monitoring and assessment techniques is adopted to determine the control parameters that mitigate the damage. An example is illustrated using a spring-mass-damper model that depicts a structural model with modal coordinates.

#### I. INTRODUCTION

n the last several years, many developments have taken place in the areas of structural health monitoring (SHM). Majority of them not only determine the presence of damage in a structure but also attempt to find the status of the structure through an accurate dynamic model that is uncertain when structural damage identification is divided into high- and low-frequency based excitation methods. Stiffness, damping and mode shape parameter changes are modeled for a relatively broad inspection zone (Yan, Yam, Cheng, and Yu, 2006., Ma and Lui, 2005., Tee, Koh, and Quek, 2005., Meng, Lin, Dong and Wei, 2006) but it applies to a specific frequency range. The bandwidth of the sensor technologies such as fiberoptic sensor (Shivakumar and Bhargava, 2005) and piezoelectric sensors (Ghasemi-Nejhad, 2005) usually limits this frequency-range, model size as well as the damage size. When size and location of the damage through SHM and assessment are known, damage prognosis (Papazian, et al., 2009) and structural health management (Xiaomo, 2010) studies assume a given sensor technology and attempt to determine the remaining life of the material. In this effort, the dynamic loading is assumed to be external and the actuator

Author <sup>α Ω β</sup>: Department of Aerospace Engineering Jain University, Jakkasandra Post, Kanakapura Taluk, Ramanagara District, Bangalore Rural 562 112, INDIA. <sup>α</sup>Professor and Corresponding Author, Email : chimpalthradi@gmail.com, <sup>Ω</sup> Visiting Professor, <sup>®</sup>Professor loading is completely ignored. Through the actuator loads, Model Reference Adaptive Control (MRAC) can be used such that the sensor data from the damaged structure can mimic the healthy structure. Although, damage prognosis study using similar sensor data for healthy and damaged structures is difficult to distinguish, the damage with such actuator loads is then said to be mitigated (Maryam and Luciana, 2010). In this paper, MRAC in state feedback format is investigated for structural damage mitigation (SDM). Some of the attributes of the MRAC are illustrated using the second order spring-mass-damper model that represents a finite element model of structural material in modal coordinate form.

MRAC has been recently investigated for Civil engineering structures (Tu, Jiang, and Stoten, 2010., Chu, Lo, and Chang, 2010), where the response of the structure in real-time is minimized under an earthquake excitations. To extend similar applications of MRAC for other aeronautical and mechanical structures, the SDM problem proposed in this paper assumes integrity of the composite material when macro (Case and Carman, 1994., Mall, 2002., Trease and Kota, 2009) or nano (Chunyu, Thostenson and Tsu-Wei, 2008) sensors and actuators are embedded into it.

The paper is organized with the problem formulation in Section II. In Section III, adaptation law is explained. In Section IV, a procedure to acquire damage and compute control parameters is briefed. In Section V, an example with spring mass-damper system is illustrated. In Section VI, conclusions are presented.

#### II. PROBLEM FORMULATION

Consider a single input healthy material in control canonical form as follows:

$$y_m^{(n)} + \overline{\alpha}_{n-1} y_m^{(n-1)} + \dots + \overline{\alpha}_0 y_m = u(t) + r(t)$$
(1)

Where r(t) is an exogenous input. The  $n^{\text{th}}$  order differentiation with respect to time variable t is denoted by  $\gamma_m^{(n)}$ . Let,

$$u(t) = -k_{n-1}y_m^{(n-1)} - \dots - k_0y_m$$
<sup>(2)</sup>

Substituting (2) in (1),

$$y_m^{(n)} + \alpha_{n-1} y_m^{(n-1)} + \dots + \alpha_0 y_m = r(t)$$
 (3)

Where  $\alpha_{n-i} = \overline{\alpha}_{n-i} + k_{n-i}$ ,  $i = 1, 2 \cdots n$  Without loss of generality, the open loop structure with an excitation load r(t) can be studied by assuming  $k_{n-i} = 0$ ,  $i = 1, 2 \cdots n$ . Let the finite element model of a material with damage be,

$$y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_0y = \hat{u}(t)$$
(4)

The parameters  $a_{n-1}, a_{n-2}, \cdots, a_0$  are to be acquired using a structural damage assessment technique [18]. The state variables  $y, \dot{y}, \cdots y^{(n-1)}$  are measurable. The SDM problem is posed as follows: Find a control law  $u^{(t)}$  in Eq. (4) such that the sensor data  $\left\{y, \dot{y}, \cdots, y^{(n-1)}
ight\}$  mimics the sensor data of the healthy material  $\{y_m, \dot{y}_m, \cdots, y_m^{(n-1)}\}$  In the process, acquire the parameters  $\boldsymbol{a} = \{1, a_{n-1}, a_{n-2}, \cdots, a_0\}$  when applying  $u^{(t)}$  to the damaged material. MRAC is particularly attractive to address this problem. However, to separate control parameters defining the control law  $\hat{u}(t)$  from that of the system parameters **a**, a state and parameter estimation technique such as an extended Kalman filter (EKF) (Bauer and Andrisani, 1990., Speyer and Crues, 1987) is required. Currently, all the state variables are assumed to be available as the sensor data. Within this framework, the SDM problem is addressed using two design steps. First, adaptation parameters in MRAC are selected such that the damaged response to an excitation load tracks the healthy response of the material. Next, the adaptation parameters are fixed and this damaged response is utilized as sensor data to find a procedure that acquires damage and determines control parameters. Finally, the control parameters are verified to check if the damaged response is indeed tracking the healthy response.

#### III. ADAPTATION LAW

Let the Laplace variable be s. Given a Hurwitz polynomial  $s^n + \beta_{n-1}s^{n-1} + \dots + \beta_0$ , the control law  $\hat{u}(t)$  is given by (Slotine and Li, 1991),

$$\hat{u}(t) = \hat{a}_n z + \hat{a}_{n-1} y^{(n-1)} + \dots + \hat{a}_0 y = v^T(t) \hat{a}(t), \quad (5)$$

Where,  $\boldsymbol{\nu}^{T}$  refers the transpose of  $\boldsymbol{\nu}$  and

$$z(t) = y_m^{(n)} - \beta_{n-1} e^{(n-1)} - \dots - \beta_0 e,$$
  

$$e(t) = y(t) - y_m(t),$$
  

$$\hat{a} = [\hat{a}_n, \hat{a}_{n-1}, \dots, \hat{a}_1, \hat{a}_0],$$
  

$$\mathbf{v} = [z(t), y^{(n-1)}, \dots, \dot{y}, y]^T.$$

Let  $\tilde{a} = \hat{a} - a$ . Then the adaptation law is given by,

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}\boldsymbol{v}^T \widetilde{\boldsymbol{a}} ,$$

$$\boldsymbol{e} = \boldsymbol{c} \ \boldsymbol{x} ,$$
(6a)

$$\dot{\widetilde{a}} = -\Gamma v b^T P x \tag{6b}$$

Where,

$$\boldsymbol{x}^{T}(t) = \begin{bmatrix} y - y_{m}, \dot{y} - \dot{y}_{m}, \cdots y^{(n-1)} - y_{m}^{(n-1)} \end{bmatrix},$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & . \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -\beta_{0} & -\beta_{1} & -\beta_{2} & \cdots & -\beta_{n-1} \end{bmatrix}; \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix};$$

$$\boldsymbol{c} = \begin{bmatrix} 1 & 0 & \vdots & 0 & 0 \end{bmatrix}.$$

*P*, the symmetric positive definite constant matrix, is the solution matrix to the Lyapunov equation given by,

$$\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T \boldsymbol{P} = -\boldsymbol{Q}$$

**Q** is any positive definite matrix.  $\Gamma$  is a positive definite diagonal matrix whose entries refer to an adaptation mechanism with slow or fast parameter convergence depending upon the diagonal values one would like to choose. From Eq. (6), it is further observed that, upon an appropriate numerical integration scheme, the sensor data  $\{y, \dot{y}, \dots, y^{(n-1)}\}$  and the vector  $\tilde{a}$  are obtained. Clearly, the adaptation parameters in  $\Gamma$  are adjusted in such a way that,

$$y(t) \rightarrow y_m(t), \ \dot{y}(t) \rightarrow \dot{y}_m(t), \dots, y^{(n-1)} \rightarrow y_m^{(n-1)}.$$

The response  $\tilde{a}$  is coupled with the control parameters  $\hat{a}$  and system parameters a. In fact, when a is time invariant, Eq. (6b) modifies to,

### $\dot{\hat{a}} = -\Gamma v b^T P x$ .

In this paper, an EKF algorithm is applied to separate the control parameters  $\hat{a}$  from the system parameters a appearing in  $\tilde{a}$ , where  $\tilde{a}$  can be recalled as given by  $\tilde{a} = \hat{a} - a$ . It is interesting to observe that the MRAC is sensitive to initial conditions  $\tilde{a}$  (0),  $\hat{a}$  (0) and a(0). Accordingly, the time histories of  $\hat{a}(t)$  and a(t) vary with time. However,  $\hat{u}(t)$  in Eq. (5) is guaranteed to track

$$y(t) \rightarrow y_m(t), \ \dot{y}(t) \rightarrow \dot{y}_m(t), \dots, y^{(n-1)} \rightarrow y_m^{(n-1)}$$

This attribute of MRAC is utilized to acquire damage, where an appropriate initial condition for  $\boldsymbol{a}(t)$  is selected such that  $\lim_{t\to\infty} \boldsymbol{a}(t)$  converges to a damaged model which is assumed to be known through a damage assessment techniques. This process is refereed as the *damage acquisition* for SDM. Given  $\{y, \dot{y}, \dots, y^{(n-1)}\}$  a formulation to compute by acquiring  $\boldsymbol{a}$  is presented in the next section.

## IV. COMPUTATION OF CONTROL PARAMETERS THROUGH DAMAGE ACQUISITION

Given  $\{y, \dot{y}, \dots, y^{(n-1)}\}$  obtained by integrating Eq. (6) from the previous section, EKF is applied to separate  $\hat{a}$  and a appearing in  $\tilde{a}$ . The state equations compatible to the EKF will be of the form,

$$\dot{\boldsymbol{\delta}} = \boldsymbol{F}(t, \boldsymbol{\delta}) \tag{7a}$$
$$\boldsymbol{z}_{k} = \begin{bmatrix} y_{k} \\ \dot{y}_{k} \\ \vdots \\ y_{k}^{(n-2)} \\ y_{k}^{(n-1)} \end{bmatrix}, \quad k = 1, 2 \cdots \tag{7b}$$

Where  $\delta^T = \begin{bmatrix} x^T & \hat{a}^T & a_{n-1} & \cdots & a_0 \end{bmatrix}$  is a vector with 3n+1 components,  $z_k$ ,  $k = 1, 2 \cdots$  is the measurement vector available at discrete time instants  $t_k$ ,  $k = 1, 2 \cdots$  and  $F(t, \delta)$  is a system dynamic vector presented below. Note that the sensor measurements for the EKF are the response of the damaged material computed in the previous section using the adaptation parameters specified in the matrix  $\Gamma$ .

$$\boldsymbol{F}(t,\boldsymbol{\delta}) = \begin{bmatrix} \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}\boldsymbol{v}^{T}\boldsymbol{\hat{a}} - \boldsymbol{b}\boldsymbol{v}^{T}\boldsymbol{a} \\ -\boldsymbol{\Gamma}\boldsymbol{v}\boldsymbol{b}^{T}\boldsymbol{P}\boldsymbol{x} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Eq. (7) completely specifies the required formulation to apply an EKF algorithm [19, 20]. When  $k_{n-i} = 0$ ,  $i = 1, 2 \cdots n$  Or when  $y_m(0) = \dot{y}_m(0) = \cdots = y_m^{(n-1)}(0) = 0$  the initial condition for the control parameters can be selected as  $a^{(0)}$  0. However, for *a*, one has to select the initial conditions *a* (0) to match the model of the damaged structure as stated in damage acquisition.

The computed system ( $\mathbf{a}$ ) and control ( $\hat{\mathbf{a}}$ ) parameters are used back again in Eq. (4) and the response of the system is simulated to check if it matches with the sensor data  $\{y_m, \dot{y}_m, \cdots, y_m^{(n-1)}\}$  for a for a given exogenous input r(t). If it matches, as adopted by Maryam and Luciana, 2010, the resulting structure with damage is said to be mitigated under an applied load from the actuator that is adaptive to the dynamic loads r(t) appearing in the control input  $\hat{u}(t)$ . Further, it is inferred that the MRAC adopts a certain trajectory for the system parameters *a* to compute the control parameters  $\hat{a}$  such that the response of the damaged model in Eq. (4) tracks the reference model in Eq. (3). Yet, the problem of finding a finite element model with parameters *a*, remains a fundamental problem in composite materials whenever damage is present (Reddy, 2004). It is observed that the finite element model by MRAC is given in transfer function framework. In the next section, an example is illustrated using a second order spring-mass-damper system.

#### v. Example

Consider a second order spring-mass-damper system representing a structure in modal coordinates. The undamaged model is taken with the parameters 1.4  $\alpha_1 = 1.4$  And  $\alpha_0 = 1$ . The exogenous input is  $r(t) = \sin(t)$ . Let the damaged model be  $\ddot{y} + a_1\dot{y} + \dots + a_0y = \hat{u}(t)$ . Assume steady state values as  $a_1(\infty) = 1.2$  and  $a_0(\infty) = 0.8$ .

The objective in SDM is to find an initial condition for the  $\mathbf{a}^T = [1 a_1(0) a_0(0)]$  system parameters and determine the control parameters  $\hat{\mathbf{a}}^T = [\hat{a}_2, \hat{a}_1, \hat{a}_0]$  such that the control law  $\hat{u}(t)$  given by  $\hat{u}(t) = \hat{a}_2 z - \hat{a}_1 \dot{y} - \hat{a}_0$  ymitigates damage by the tracking Performance  $y(t) \rightarrow y_m(t)$  and  $\dot{y}(t) \rightarrow \dot{y}_m(t)$ , where  $y_m(t)$  and  $y_m(t)$  are the response of the undamaged material due to exogenous input r(t). Further  $\mathbf{a}(t)$  converges to  $\mathbf{a}^T(\infty) = [1 \ 1.2 \ 0.8]$ . Here z(t) is selected such that  $z(t) = \ddot{y}_m - 2\dot{e} - 2e$  with  $\beta_1 = 2$  and  $\beta_0 = 2$ . Note that the control law is adaptive to the exogenous input r(t) through the  $\ddot{y}_m$  term.

In the first design step, the tracking performance is achieved through the adaptation parameters which were found out to be  $\Gamma = diag$  (10,1, 30). Integrating Eq. (6), with an initial condition for  $\tilde{a}$  as  $\tilde{a}^{T}(0) = [-1 - 0.3 - 0.2]$  the responses of the damaged and undamaged material is shown in Fig. 1, which suggest that  $y(t) \rightarrow y_m(t)$  and  $\dot{y}(t) - \dot{y}_m(t)$ . These responses of y(t) and  $\dot{y}(t)$  are used as measurements in EKF that is used to compute control

 $(\hat{a})$  and system parameters with damage acquisition in the responses of a. In Fig. 2, the error responses of position  $y(t) - y_m(t)$  and speed  $\dot{y}(t) - \dot{y}_m(t)$  contained in  $\mathbf{x}(t)$  are compared with the Kalman filter estimates  $\hat{\mathbf{x}}(t)$ . In Fig. 3, the parameters response  $a_1(t)$  and  $a_0(t)$  are provided. We observe that the steady state values of these parameters represent the damaged state of the material. In Fig. 4, the control parameters, namely,  $\hat{a}_{2}(t), \hat{a}_{1}(t)$  and  $\hat{a}_{0}(t)$  are presented. In order to verify that these parameters indeed performs SDM, the control law with these parameters in Eq. (4) are used to get the responses  $y_s(t)$  and  $\dot{y}_s(t)$ . The error responses  $y_s(t) - y_m(t)$  and  $\dot{y}_s(t) - \dot{y}_m(t)$  are shown in Fig. 5. Clearly SDM is performed; however, the error build up in certain time intervals are due to the choice of initial condition  $\widetilde{a}$  (0) that also governs the slow and fast adaptation rates.

### VI. CONCLUSIONS

Presently, damage prognosis and structural health management schemes assume a given sensor technology and attempt to diagnose the data to predict the remaining life of the structure when an exogenous input load in a damaged structure is present. Structural damage mitigation proposed in this paper considers both sensor and actuator technologies embedded in a structure and modifies the actuator loading such that the sensor data from the damaged structure mimics the sensor data from the healthy structure. Model Reference Adaptive Control is recognized to fulfill this objective. A damaged finite element model for the material is assumed through a structural damage assessment techniques and a technique to mitigate the effects of damage in an uncertain environment is proposed. A second order spring-mass-damper model that represents a finiteelement structural model in modal coordinates is considered to illustrate the foundations of SDM using MRAC.

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Figure 2 :  $\mathbf{x}(t)$  and  $\hat{\mathbf{x}}(t)$  by EKF.







Figure 4: Control Parameters Computed by EKF a)  $\hat{a}_{2}(t)$ , b)  $\hat{a}_{1}(t)$  and c)  $\hat{a}_{0}(t)$ .



Figure 5: Simulated Errors a)  $y_s(t) - y_m(t)$  and b)  $\dot{y}_s(t) - \dot{y}_m(t)$ 

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