

## **Adaptive control of MIMO electrohydraulic servosystem**

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### **ABSTRACT**

The possibility of applying model reference adaptive control to a MIMO electrohydraulic servosystem was investigated in this paper. An adaptive algorithm was developed which requires less a priori knowledge about the transfer matrix while still insuring convergence and stability of the control system. The proposed scheme is based on the estimation of the controller parameters obtained directly from input-output data. Even if the system does not satisfy the strict positive realness as in a non-minimum phase system, it is still possible to obtain asymptotically perfect model following. System simulation results are provided to illustrate the feasibility of the proposed adaptive control scheme.

### **KEYWORDS**

**Key Words:** Adaptive Control, MIMO, Electrohydraulic, Servosystem

### **INTRODUCTION**

Electrohydraulic servomechanisms are well known for their fast dynamic response, high power to inertia ratio, and control accuracy. In these systems, however, due to disturbances, variations of loads, and changing process dynamics, the system

parameters vary with the operation states and the system may contain unknown parameters. Conventional control techniques, which are very sensitive to disturbances, may lead to a degradation in performance under varying parameter conditions. Adaptive control techniques, with their ability to adjust controller parameters so as to compensate

the system parameter variations, can provide satisfactory control.

In recent years, a number of papers have dealt with SISO adaptive fluid power control systems. Hori *et al* [1], Edge and Figueredo [2], and Yun and Cho [3] investigated the application of adaptive algorithms to hydraulic servosystems with time-varying parameters and uncertain external disturbances. However, extension of adaptive algorithms for SISO systems to MIMO systems has received little attention. The transfer function properties which are easily established for SISO systems are much more complex for MIMO systems. The problem of interaction between the inputs must be properly handled. Narendra and Annaswamy [4] investigated the direct MIMO adaptive control and stability analysis of minimum phase systems. Bar-Kara and Kaufman [5] proposed a control algorithm for unknown time-varying dynamics based on the system observation. The objective of this study is to seek a control scheme which requires less a priori knowledge about the transfer matrix while still insuring convergence and stability of the adaptive algorithm. One such adaptive control algorithm

is developed in this paper based on the estimation of the controller parameters. System simulation results are provided to illustrate its feasibility.

## SYSTEM DESCRIPTION

In this paper an adaptive control technique is applied to a typical electrohydraulic servosystem which often must provide predefined outputs both on the displacement of the load and on the force of the actuator. In many processes external disturbances, often due to complex interactions between the process dynamics and the hydraulic actuator motion, usually influence the hydraulic servosystem characteristics.

A schematic diagram of the hydraulic servosystem is shown in FIG. 1. This system can be described by the following equations:

$$\begin{aligned} Q_{v1} &= c_{d1} x_{v1} \pi d \sqrt{2[p_s - p_{l1}]/\rho} \\ Q_{l1} &= A_1 \frac{dy_1}{dt} + \frac{V_{t1}}{2K_{e1}} \frac{dp_{l1}}{dt} + K_{l1} p_{l1} \\ Q_{v1} &= Q_{l1} \\ \frac{\pi}{4} D^2 p_{l1} &= M \frac{d^2 y_1}{dt^2} + K_{v1} \frac{dy_1}{dt} \\ &+ K_t y_1 + F_n(y, p_{l1}, t) \end{aligned} \quad (1)$$

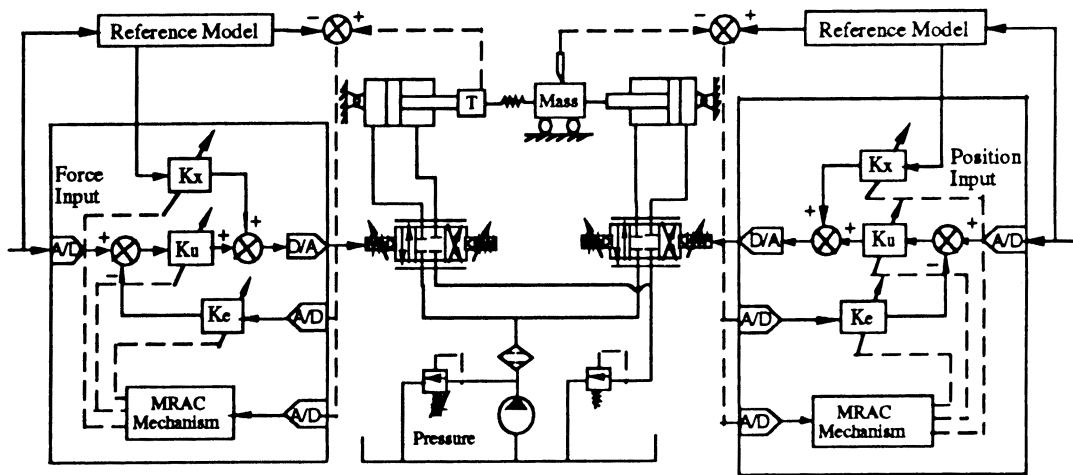


Fig. 1 Electrohydraulic Servosystem With Adaptive Control

$$\begin{aligned}
Q_{v2} &= c_{d2} x_{v2} \pi d \sqrt{2[p_s - p_{l2}]/\rho} \\
Q_{l2} &= A_2 \frac{dy_2}{dt} + \frac{V_{l2}}{2K_{e2}} \frac{dp_{l1}}{dt} + K_{l2} p_{l2} \\
Q_{v2} &= Q_{l2} \\
\frac{\pi}{4} D^2 p_{l2} &= M \frac{d^2 y_2}{dt^2} + K_{v2} \frac{dy_2}{dt} \\
&+ K_{t2} y_2 + F_n(y, p_{l2}, t)
\end{aligned} \quad (2)$$

These equations show that this highly non-linear system is time varying and contains unknown disturbances. A dynamic model of the viscous friction and oil leakage cannot be predefined. It is the purpose here to develop an adaptive control algorithm to obtain better control characteristics for this kind of system. A successful adaptation law is developed in this paper to generate the controller coefficients, and the stability of the control scheme is guaranteed.

## DISCRETE MIMO ADAPTIVE ALGORITHM

A block diagram of the proposed MRAC system is shown in FIG.2. In this approach, the proposed adaptive algorithm guarantees both the stability of the adaptive system as well as asymptotically perfect model following. In addition, if the system does not satisfy the strict positive realness condi-

tion (such as for a non-minimum phase system), perfect model following can still be obtained by means of the proposed control scheme.

## System Dynamic Model Description

The process to be controlled can be described as

$$\begin{aligned}
x_p(k+1) &= A_p x_p(k) + B_p u_p(k) \\
y_p(k) &= C_p x_p(k) + D_p u_p(k)
\end{aligned} \quad (3)$$

The plant output  $y_p(k)$  is required to follow the output  $y_m(k)$  of the asymptotically stable reference model as given by

$$\begin{aligned}
x_m(k+1) &= A_m x_m(k) + B_m u_m(k) \\
y_m(k) &= C_m x_m(k) + D_m u_m(k)
\end{aligned} \quad (4)$$

Assume a command which generates the signal

$$\begin{aligned}
v_m(k+1) &= A_v v_m(k) \\
u_m(k) &= C_v v_m(k)
\end{aligned} \quad (5)$$

where the matrices  $A_v$  and  $C_v$  are unknown.

When the reference model is supplied with an input of this form, its state can be written as

$$x_m(k) = E v_m(k) + A_m \delta_0 \quad (6)$$

The matrix  $E$  satisfies the following equation:

$$\begin{aligned}
A_m E - E A_v + B_m C_v &= 0 \\
\delta_0 &= x_m(0) - E v_m(0)
\end{aligned} \quad (7)$$

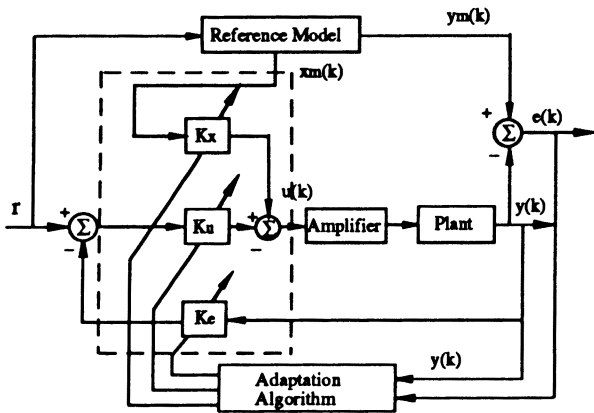


Fig. 2 MRAC System Diagram

## Discrete Adaptive Algorithm

The state error is defined as

$$e_x(k) = x_p^*(k) - x_p(k) \quad (8)$$

The output tracking error is then

$$e_y(k) = y_m(k) - y_p(k) \quad (9)$$

The adaptive algorithm generates the following plant control inputs:

$$u_p(k) = K(k+1)r(k) \quad (10)$$

where  $r^T(k) = [e_y^T(k), x_m^T(k), u_m^T(k)]$   
 $K(k) = [K_e(k), K_x(k), K_u(k)]$  (11)

and  $K(k+1) = K_v(k+1) + K_p(k+1)$   
 $K_v(k+1) = K_v(k) + e_y(k)r^T(k)T$   
 $K_p(k+1) = e_y(k)r^T(k)T$  (12)

The output error is then given by

$$\begin{aligned} e_y(k) &= C_p x_p^*(k) + D_p u_p^*(k) \\ &\quad - C_p x_p(k) + D_p u_p(k) \\ &\quad - (C_p X_{11} + D_p K_x - C_m) A_m \delta_0 \\ &= C_p e_y(k) - D_p [K(k+1) - \tilde{K}] r(k) \\ &\quad - D_p K_e e_y(k) - (C_p X_{11} + D_p K_x \\ &\quad - C_m) A_m \delta_0 \end{aligned} \quad (13)$$

and the equation of the state error is given by

$$\begin{aligned} e_x(k+1) &= x_p^*(k+1) - x_p(k+1) \\ &= A_p x_p^*(k) + B_p u_p^*(k) \\ &\quad - (A_p X_{11} - A_m X_{11} \\ &\quad + B_p \tilde{K}_x) A_m \delta_0 \\ &\quad - A_p x_p(k) - B_p u_p(k) \end{aligned} \quad (14)$$

### Stability Analysis

To prove the stability of the control system, assume a discrete quadratic Lyapunov function of the form

$$\begin{aligned} V(k) &= e_x^T(k) P e_x(k) \\ &\quad + \text{tr}[S(K_v(k) - \tilde{K})T^{-1} \\ &\quad (K_v(k) - \tilde{K})^T S^T] \end{aligned} \quad (15)$$

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= e_x^T(k+1) P e_x(k+1) \\ &\quad - e_x^T(k) P e_x(k) \\ &\quad + \text{tr}[S(K_v(k+1) - \tilde{K})T^{-1} \\ &\quad (K_v(k+1) - \tilde{K})^T S^T] \\ &\quad - \text{tr}[S(K_v(k) - \tilde{K})T^{-1} \\ &\quad (K_v(k) - \tilde{K})^T S^T] \end{aligned} \quad (16)$$

Let the following relations be satisfied:

$$\begin{aligned} A_p^T P B_p &= C_p^T (S^T S) - L^T W \\ A_p^T P A_p - P &= -L^T L < 0 \\ D_p^T (S^T S) + (S^T S) D_p \\ &\quad - B_p^T P B_p = W^T W > 0 \end{aligned} \quad (17)$$

where L, W are positive definite matrices. The Lyapunov function gain matrix then becomes

$$\begin{aligned} \Delta V(k) &= -[e_x^T(k) L^T \\ &\quad - r^T(k)(K(k+1) - \tilde{K})^T W^T] \\ &\quad \times [L e_x(k) - W(K(k+1) - \tilde{K})r(k)] \\ &\quad - e_y^T(S^T S) e_y(k) r^T(k)(2T + T)r(k) \\ &\quad - 2e_x(k+1) P \tilde{F} A_m \delta_0 \end{aligned} \quad (18)$$

From the above equation

$$\begin{aligned} V(k+1) &< V(k) \\ &\quad + \alpha V(k) \|A_m\|^k \\ \text{for } \alpha &< \infty \end{aligned} \quad (19)$$

Hence, it can be concluded that  $V(k)$  is bounded for all  $k$ . The quadratic form of  $V(k)$  guarantees the boundness of input gains  $K_v(k)$ , the state error  $e_x(k)$ , and the output error  $e_y(k)$ . In this case the last two terms in the Lyapunov function gain equation vanish as  $k \rightarrow \infty$ . There exists then a positive definite matrix P and a gain matrix  $\tilde{K}$  such that the conditions of eq.(17) are satisfied and all of the state variables and gains of the adaptive system are bounded and the output tracking errors vanish asymptotically.

### SYSTEM SIMULATION

Consider the system of FIG. 1 as an example of a two-input two-output electrohydraulic servosystem. This system is described by the state space equations given by eq.(3), where  $A_p =$

$$\begin{aligned}
& \begin{pmatrix} -(c_p + K_q) & 0 & -\frac{A_2 K_e}{V_2} & -\frac{A_1 K_e}{V_1} \\ 0 & -\frac{K_e K_t A_2}{K_t V_2 - K_e A_2^2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{A_1}{M} & -\frac{A_2}{M} & 0 & -\frac{K_e}{M} \end{pmatrix} \\
& B_p = \begin{pmatrix} \frac{c_{v1} K_e}{V_1} & 0 \\ 0 & \frac{c_{v2} K_e K_t}{K_t V_2 - K_e A_2^2} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \\
& C_p = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -\frac{A_2}{K_t} & 1 & 0 \end{pmatrix} \quad (20)
\end{aligned}$$

The parameters were determined to be  $A_1 = 1.6 \times 10^{-3} m^2$ ,  $A_2 = 1.0 \times 10^{-3} m^2$ ,  $c_{v0} = 8.8 \times 10^2 Nsec/m$ ,  $c_p = 2.3 \times 10^{-12} m^5/Nsec$ ,  $K_e = 3.4 \times 10^8 N/m^2$ ,  $K_t = 1.6 \times 10^{-7} N/sec^2$ ,  $K_c = 1.2 \times 10^4 N/m$ ,  $V_t = 1.7 \times 10^{-3} m^3$ ,  $M = 5.5 \times 10^2 kg$ , and  $p_s = 7.1 \times 10^6 N/m^2$ . The plant output  $y_p(k)$  is required to follow the output  $y_m(k)$  of the asymptotically stable reference model given by eq.(4), where  $A_m =$

$$\begin{aligned}
& \begin{pmatrix} -3.2 \times 10^{-2} & 0 & 0 & -5 \times 10^2 \\ 0 & -9 \times 10^{-1} & 0 & 7.8 \times 10^{-1} \\ 0 & 0 & 6.8 & 1 \\ 3 \times 10^{-4} & -1.8 \times 10^{-4} & 0 & -7.2 \times 10^2 \end{pmatrix} \\
& B_m = \begin{pmatrix} 2.3 \times 10^3 & 0 \\ 0 & 4.3 \times 10^2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \\
& C_m = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & -6 \times 10^2 & 1 & 0 \end{pmatrix} \quad (21)
\end{aligned}$$

Simulations were carried out with the reference inputs being either sinusoidal or square wave. The results are shown in FIGS. 3 to 6. From these curves it can be seen that the proposed control scheme is very effective for the electrohydraulic control system in the case of varying loads. The plant outputs follow the reference model outputs very well for the sinusoidal cases. For the square wave cases, the plant outputs experience oscillations, but still follow the reference

model satisfactorily. Thus the proposed control scheme forces the plant outputs to track those of the reference model simultaneously under changes of the plant dynamics.

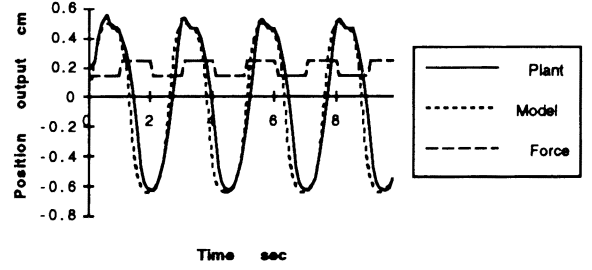


Fig.3 Simulation of Position Tracking

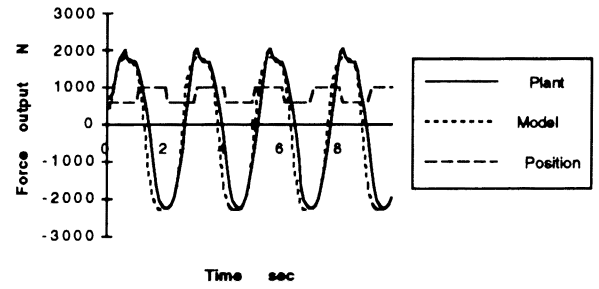


Fig. 4 Simulation of Force Tracking

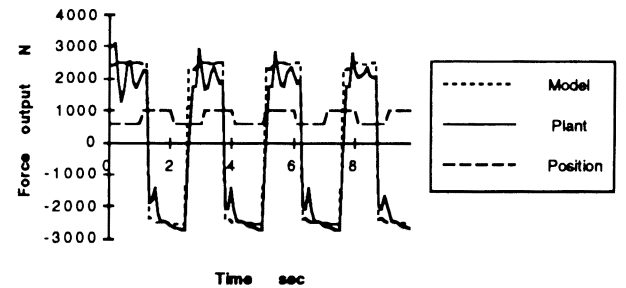


Fig. 5 Simulation of Position Tracking

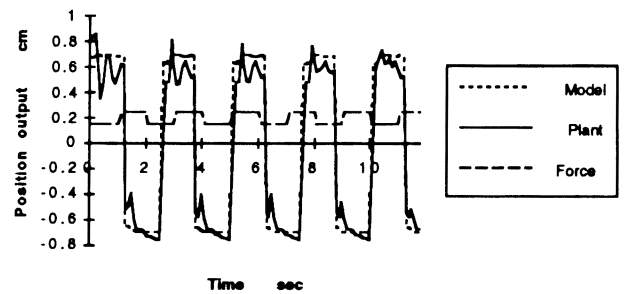


Fig.6 Simulation of Force Tracking

## CONCLUDING REMARKS

As a result of this investigation, the following insight has been gained into some prospective adaptive algorithms for a MIMO electrohydraulic servosystem:

- 1) Adaptive control techniques can improve the system performance through the introduction of adjustable parameters of the controller so as to compensate for varying process dynamics due to changes of the external loads and disturbances.
- 2) The proposed adaptive algorithm generates the control signals directly from the system input-output information which is easy to obtain in most electrohydraulic systems.
- 3) By using the reference model state, this approach requires less knowledge about the process dynamics.
- 4) The system dynamics are inherently highly nonlinear. The linearization will, in some cases, cause a considerable error in the description of system dynamics. A nonlinear algorithm should be introduced to improve the control system performance. Further investigation is recommended in this area.

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