ADAPTIVE CONTROL OF NUCLEAR REACTORS USING A DIGITAL COMPUTER

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USING A DIGITAL COMPUTER

by

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Scope and Contents: The feasibility of adaptive control of a nuclear reactor is investigated. For practical reasons, an actual operating power plant is chosen, and a digital computer model developed for the reactor and associated control system. The effects of parameter variations on the transient response of the overall system are studied, and the advantages of using an adaptive controller established. An algorithm for the adaptation scheme is developed, and applied successfully to control the nuclear reactor.

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INTRODUCTION

Digital computers have always played a leading role in the design of nuclear reactors. The main reason for their extensive use has been the complexity of the design problems that result from the stringent requirements of good economics and safety in nuclear power plants. Since reactor control plays at least as important a role as good basic reactor design in achieving economy and reliability of operation, there is considerable incentive to utilize the unique features of the digital computer for control purposes in the operation of a power reactor.

The first time that an on-line digital computer was used to control a nuclear power plant in commercial operation occured in January 1967 at Douglas Point, Ontario. Because of the lack of experience with such a controller, the main power level regulator was still of the conventional analog form, but the duties assigned to the digital controller, namely the neutron flux tilt control, were of sufficient importance to give a good indication of the future role of digital computers for nuclear reactor control.

The performance of this first digital computer controller was sufficiently encouraging, that for the second nuclear generating station at Pickering, near Toronto, the complete reactor control is now being designed to use digital computers exclusively. Reliability of operation with the corresponding reduction of "down-time", and the possibility of more precise control, since all informations are within the one system in the same form, were the main justifications for the larger capital cost of digital computers.

At the present time, the designed system does not take real advantage of the computational facilities offered by the digital controller. Since much of the recent advances in control theory, such as optimum and adaptive control, require an on-line digital computer for their implementation, there has been considerable interest in an attempt to apply these techniques to control a nuclear reactor. The subject of this thesis has in fact originated from the control section of the Atomic Energy of Canada Ltd., and liason was maintained with interested engineers in order that practical considerations may be kept to the forefront throughout the theoretical investigation. For the same reason of practicality, an actual nuclear power plant was chosen as the subject of our study, namely the one at Douglas Point. It is a heavy water moderated system, cooled by high pressure heavy water, and uses natural uranium fuel. At 200 megawatts it is a medium size power station. While the work in this thesis follows very closely the characteristics and requirements of the Douglas Point reactor, it is also applicable to a wide range of similar reactor systems. Advantages of this choice are the availability of factual information based on extensive design and operational experience, and the possibility of treating the reactor for the purpose of power level control as a point source.

As compared to a conventional feedback controller, adaptive control for a nuclear reactor would be desirable for the following reasons.

1. The plant to be controlled is highly nonlinear. Furthermore, the operating power level may extend over eight decades, and a fixed feedback controller cannot produce transient responses with close tolerances throughout this large operating range. At best, the response will be near its desired value under a few specific conditions: when the plant parameters are close to the values assumed in the design and the power level corresponds to the one chosen for calculating the amount of compensation.

2. Several of the plant's parameters are time-varying, and some may change by as much as 100%. Good design can of course reduce the effects of these variations, but only as far as the original assumptions were correct.

3. The behaviour of a nuclear reactor when operating at about equilibrium fuel level is markedly different from the time it is first commissioned. It may take one year for the initial fuel load to burn down to the equilibrium level, and settle down to the regular refuelling cycle, in which only a portion of the burnt up fuel-rods are replaced. It is not desirable to have to keep a large engineering staff, such as associated with the design of a nuclear reactor's control system on stand-by for such a long time, just in case the system will not perform as required when steady-state operation is achieved.

The adaptive controller envisaged would change its parameters as the operating conditions and the parameters of the reactor varied, maintaining plant performance at an optimum, as defined by suitable criteria. The effects on system performance of such long term variations as fuel burn up are very similar to certain short term changes, such as altering the concentration of dissolved poison in the moderator. The control system may therefore be commissioned on the basis of initial test changes: if it can adapt to these, it should perform satisfactorily in the long run also.

The approach followed in this thesis was to first develop a digital computer model of the Douglas Point reactor and control system. The behaviour of the model was studied under a wide range of operating conditions and with the maximum likely parameter changes, in order to determine the necessity for and the improvements afforded by an adaptive control system. A number of adaptive schemes were considered for their suitability to control a nuclear reactor, and the most promising one was developed in detail.

Since most adaptive control systems in practical use take advantage of one or more special characteristics of the plant, [1], [2], [3], a good understanding of the operation of a nuclear reactor is important. In Chapter 2, the differential equations that describe the reactor are derived from a basic, physical understanding of the processes that take place inside the reactor. Chapter 3 contains a detailed explanation of that section of the present control system at Douglas Point that is considered in this thesis. In Chapter 4 the behaviour of the complete system (reactor and feedback controller) are considered, showing the results of the simulation studies. Chapter 5 is a review of optimum and adaptive control methods, and of the attempts to apply these techniques to nuclear reactor control. Chapter 6 contains the main contribution of the present work. An adaptive system that would be suitable to control a nuclear reactor of the type used at Douglas Point is developed in detail, and the performance expected from such a system is studied using our model. In Chapter 7 the conclusions drawn from this study are presented, along with recommendations for further work in the adaptive control of nuclear reactors.

CHAPTER 2.

Reactor Kinetics

[2.1] A First Order Approximation.

The fundamental energy releasing process that takes place in a nuclear reactor is the collision of a free neutron with a nucleus, that results in the splitting of the latter, and in the emission of one or more high energy neutrons. A large portion of the kinetic energy of these neutrons is converted to heat before they interact with, i.e. cause fission, of other nuclei. Since on the average more than one neutron is released per collision, a chain reaction may be sustained.

In an actual reactor not all the neutrons produced are available for fission: some are absorbed by non-fissile nuclei, others leave the reactor before they could cause fission. Accordingly, for a finite size reactor, it is customary to measure the change of the neutron population in terms of the so called multiplication factor k_{eff}, which is defined as the ratio of the number of neutrons in one generation to the number of corresponding neutrons in the immediately preceding generation. Since it is the deviation of the multiplication factor from unity that alters the operating level of the reaction, it is convenient to define reactivity as

$$\delta k = \frac{k_{eff} - 1}{k_{eff}} \qquad ....(2.1)$$

Another quantity that effects the rate at which the chain reaction proceeds is the mean time which elapses from when neutrons are produced in fission until they return again to fission or are lost to the reaction. The symbol l^* is used for the mean effective life of a neutron in an actual reactor.

Having defined these quantities, it is now possible to formulate an expression for the neutron population of a reactor. If at a given time there are n neutrons per cubic centimeter, in one generation this number will change by $n\delta k$. With an effective time of l^* between succeeding generations, the rate of change of neutrons is given by

$$\frac{dn}{dt} = n \frac{\delta k}{l^*} \qquad ....(2.2)$$

and integrating this equation gives the number of neutrons as a function of time, from an initial value of n_o, as

$$n = n_o \exp\left(\frac{\delta k}{l^*} t \right) \qquad ....(2.3)$$

For a multiplication factor greater than unity, Equation (2.3) results in an exponential rise of the neutron population. For typical values of $l^* = 7.2 \cdot 10^{-4}$ and

$\delta k = 0.003$, at the end of 2.5 seconds, the neutron population would have increased by a factor of more than 30,000. Such a rapid rise is exceedingly difficult to control. Fortunately, as will be presently discussed, such factors as delayed neutrons and the negative temperature coefficient of the fuel, decrease this rate by several orders of magnitude, making reactor control a feasible problem.

[2.2] Delayed Neutrons

In deriving Equation (2.3) it was assumed that all the neutrons were "prompt", i.e. given off instantly, and had a lifetime of l^*. Neither of these assumptions are in fact true: a small fraction of the neutrons, between one half and one percent of the total number, are emitted at discrete intervals of time after fission has taken place. For uranium 235 there are six groups of these so-called delayed neutrons, differing in concentrations and lifetimes.

The effect of delayed neutrons on the reactor kinetics may be expressed by subtracting them from the prompt ones in Equation (2.2), and adding the contributions of delayed emitters from previous generations. The resulting differential equation is of the form [4].

$$\frac{dn}{dt} = \frac{\delta k - \beta}{l^*} n + \sum_{i=1}^{6} \lambda_i C_i \qquad \text{..........(2.4)}$$

where β = fraction of total neutrons delayed
λ_i = decay constant of ith group
C_i = concentration of delayed neutrons in the ith group.

The C_i are defined by

$$\frac{dC_i}{dt} = \frac{\beta_i}{l^*} n - \lambda_i C_i \qquad \text{..........(2.5)}$$

β_i = fraction of delayed neutrons in the ith group.

Even though the delayed neutrons make up less than 1% of the total neutron population, their very long lifetimes make the average lifetime of all the neutrons much longer than would be due to prompt neutrons alone. In fact the previous rapid rise of neutron level will only take place if a reactivity greater than the delayed neutron fraction is introduced into the reactor. For smaller values, multiplication is dependent on the delayed emitters. This may be seen from Figure (2.1), where the reactor kinetic equations (2.4) and (2.5) were solved for various positive step changes of reactivity. The largest value $\delta k = \beta = 0.00487$ causes prompt criticality, since the chain reaction is sustained without the use of delayed neutrons. For smaller reactivity changes, the neutron multiplication proceeds at a much reduced rate.

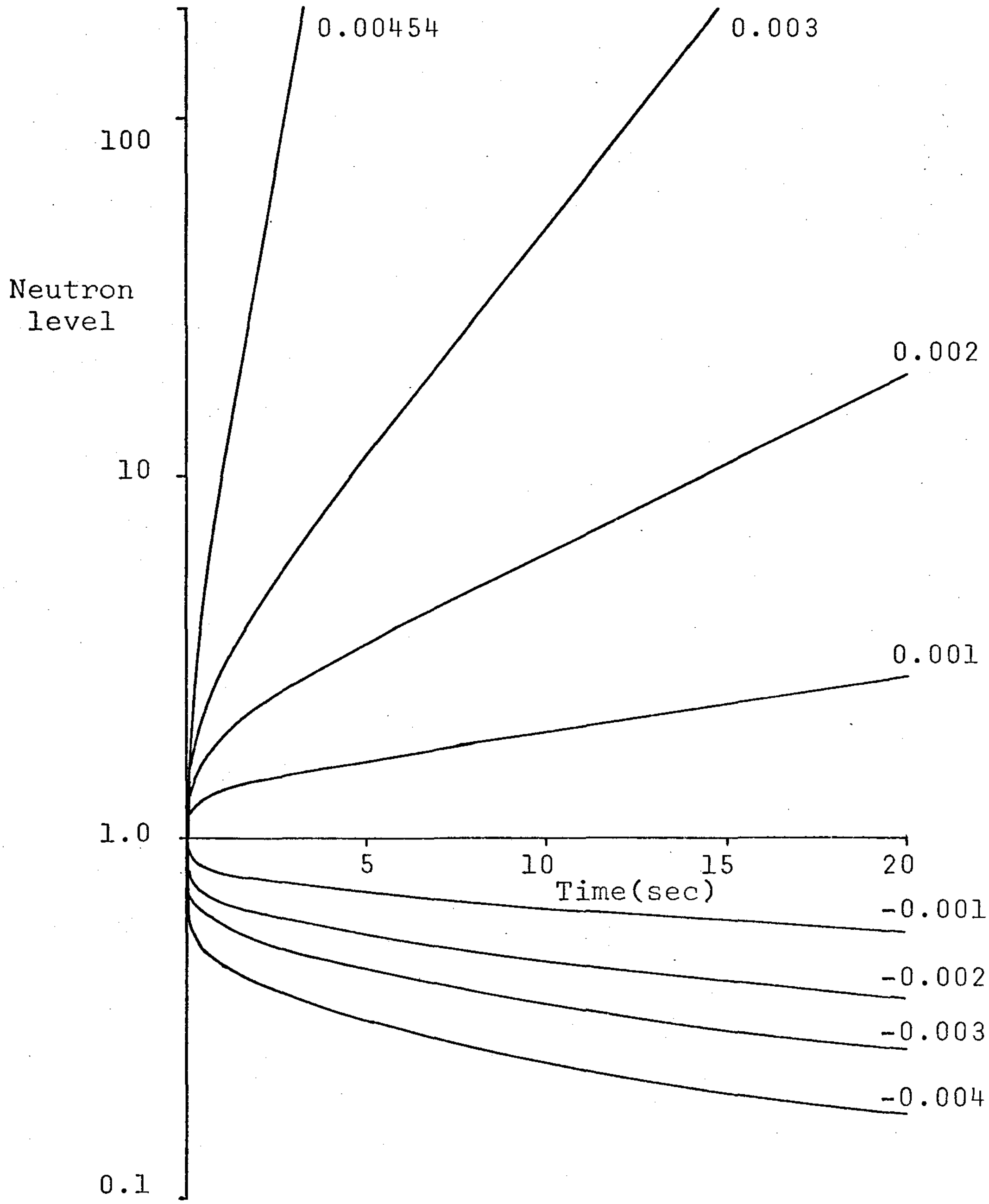


FIGURE 2.1

Solution of the simplified reactor kinetic equations for positive and negative step changes of reactivity.

The reason for the initial sudden rise of neutron level is, that at this stage the number of neutrons delayed are nearly equal to those emitted from the delayed fractions of previous populations. Following this increase, however, the number of neutrons being delayed is greater than the number entering the system, which are proportional to an earlier, lower level. Thus the rate of increase of relative neutron level gradually falls, and is ultimately determined by the lifetime of the delayed neutrons.

A similar situation arises for negative reactivity changes (Figure 2.1). After the initial fall, the neutron level cannot decrease faster than the decay of the longest living delayed emitter.

[2.3] Temperature Coefficient of Reactivity

It is apparent from Figure 2.1 that the simplified reactor kinetic equations represent a nonlinear and highly unstable system, since a positive change of reactivity produces a monotonic rise of neutron level. Such an increase cannot, of course, be maintained indefinitely in a physical system. The principal limiting factor is the negative temperature coefficient of reactivity, working through the following mechanism: an increase of reactor power causes a rise in the temperature of the moderator, decreasing its density, and hence, increasing the rate of neutron leakage.

This loss of neutrons corresponds to a negative change of reactivity, that tends to counteract the excess multiplication applied initially.

The change of reactivity associated with a change of temperature is expressed as:

$$\text{Temperature coefficient } (T_c) = \frac{d\ (\ \delta k\ )}{dT}$$

Since temperature and power level are linearly related, and it is the latter that is of final concern, the temperature coefficient is often specified in terms of the reactivity change over the operating power range. For the Douglas Point reactor, from zero power "hot" to 100% FP there is a -4.54 mk change due to the temperature coefficient. It is a negative quantity in most present day reactors, and makes the reactor self-regulating.

There is also a time-constant (τ_T) associated with the temperature coefficient, depending on the physical configurations of the reactor. In control applications, a first order lag term adequately describes the effect of temperature on reactivity. If the reactor is represented by the kinetic equations (2.4) and (2.5), the temperature coefficient of reactivity may be shown as a feedback loop around the reactor, with the externally applied reactivity change as the input (Figure 2.2).

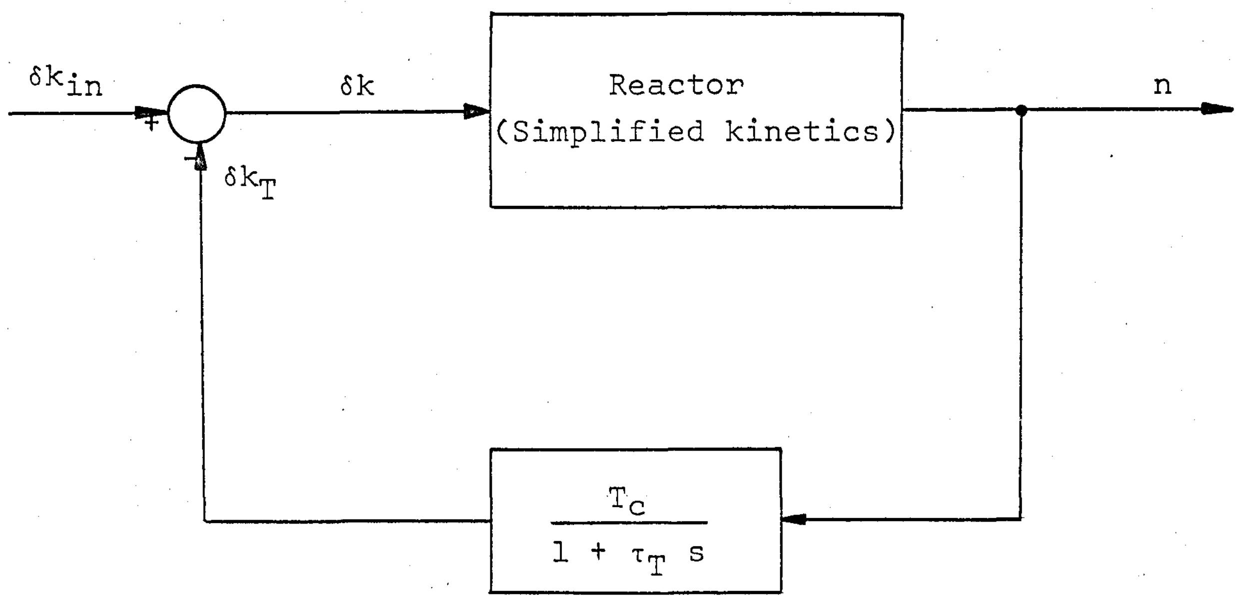


FIGURE 2.2

Feedback Representation of the Effect of Temperature on Reactivity.

The stabilizing effect of the negative temperature coefficient may be seen from Figure 2.3. For the same reactivity change ($\delta k = 0.003$), instead of the indefinite rise of neutron level when $T_c = 0$, the neutron multiplication is halted at a value dependent on the temperature coefficient. For the Douglas Point reactor $T_c = -0.00454$, and it is seen that the neutron level is not even doubled.

[2.4] Simulation of the Reactor

Apart from the temperature coefficient, many other factors alter the behaviour of a reactor from that depicted by the kinetic equations. All these effects, such as the pressure and void coefficients, poisoning, etc., were, however, considered to be negligible for the purposes of the present work.

The traditional approach to control system design has been to develop the transfer function of the plant, and use such frequency domain techniques as Nyquist or Bode plots, or Nichols charts, [5] to synthesize the necessary compensating networks that will ensure the desired response in the time-domain. With the advent of digital computers, interest has returned to the more direct method of working only in the time-domain [6]. The differential equations describing the system may be solved directly, in either closed form, or using numerical algorithms. For higher order systems, the advantages of matrix algebra have led

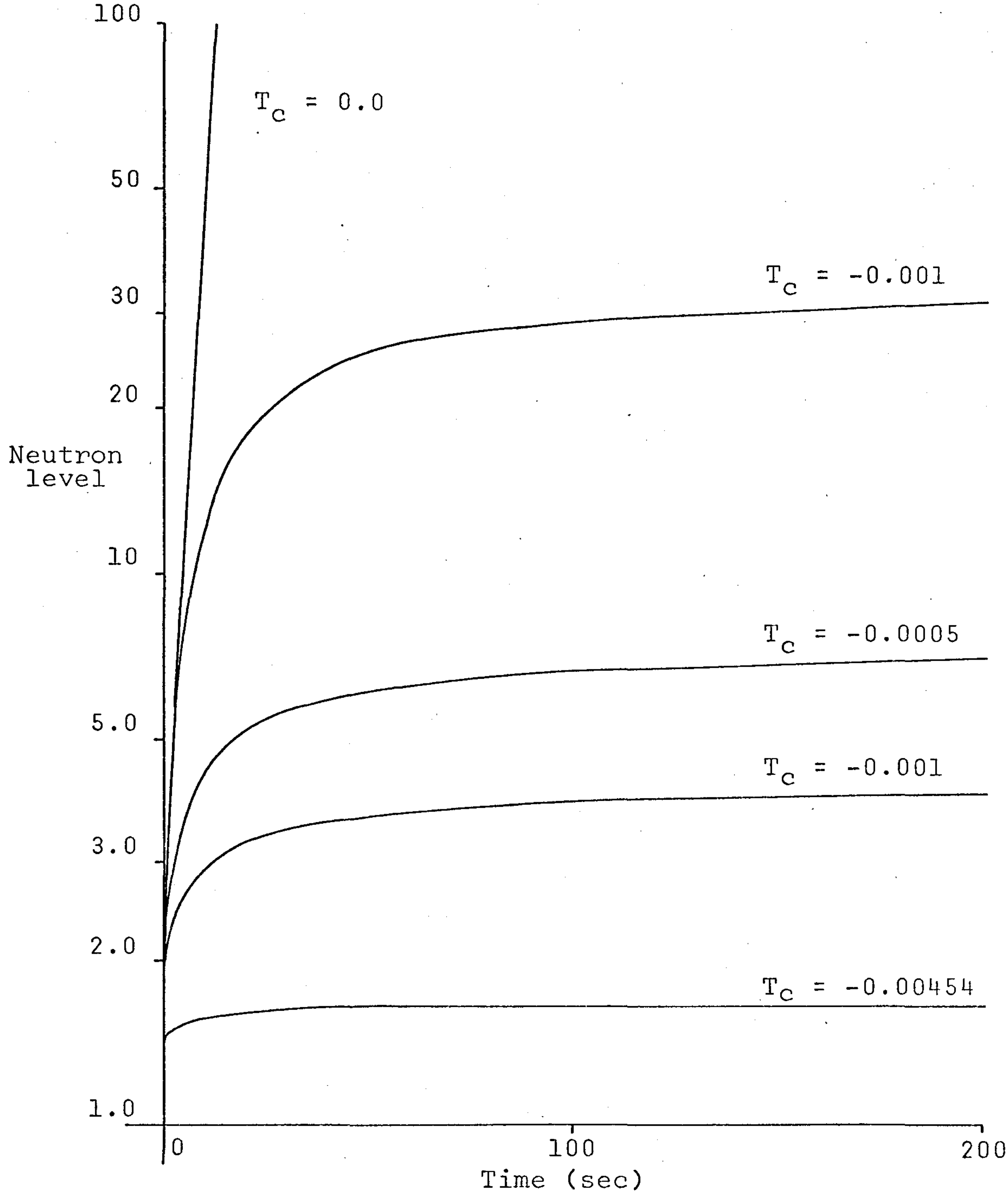


FIGURE 2.3

Effect of negative temperature coefficient (T_c), when a reactivity step of 0.003 is applied.

to the development of the state space method of analysis and synthesis [7]. Much of the classical methods of the calculus of variations have for the first time become feasible techniques to optimize control system performance.

Because of this change of emphasis from the transfer function, frequency-domain methods to time-domain techniques, and since the final interest is in the time behaviour of the system, the latter approach was adopted in this thesis.

The following equations give an adequate description of the nuclear reactor for the purpose of simulation.

$$\frac{dn}{dt} = \frac{\delta k - \beta}{l^*} n + \sum_{i=1}^{6} \lambda_i C_i \qquad \text{..........(2.4)}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{l^*} n - \lambda_i C_i \qquad \text{..........(2.5)}$$

The effect of the temperature coefficient is given by

$$\frac{d(\delta k_T)}{dt} = -\frac{\delta k_T}{\tau_T} + \frac{T_c}{\tau_T} n \qquad \text{..........(2.6)}$$

The input to the reactor is

$$\delta k = \delta k_{in} - \delta k_T$$

and the desired output is the neutron level. The values of the various coefficients for the Douglas Point reactor are given in Appendix I.

In simulating such a system, one has usually the choice of using an analog, a digital, or a hybrid computer. The reactor and the associated plant is a continuous system, represented by differential equations, and hence in a suitable form for analog simulation. The proposed control system on the other hand is digital, and will include a digital computer, so this aspect of the problem will require such a machine. A hybrid computer appears to be the natural choice, however, apart from the fact that one is not available at present, it will be shown that the reactor and all the other continuous processes may be conveniently and efficiently simulated on a digital computer.

Considering the reactor, its time response may be found by using one of the many subroutines available to solve a set of simultaneous differential equations, or by using state space techniques. However, as in most cases of interest, the above system is not only nonlinear, but also time-varying, and the required computation time becomes excessive using these methods. The state space approach, for example, would require the computation of an 8 x 8 transition matrix at every time increment.

A more efficient approach is to note that the set of equations (2.5) and (2.6) are uncoupled, and depend only on the neutron level, given by (2.4). Knowing the initial neutron level, (2.5) and (2.6) are solved for the delayed neutron concentrations and for δk_T, and these values are used to calculate the change in the number of neutrons. This iterative procedure is continued at suitable small increments of time to give the required time domain response.

The method used to solve each first order differential equation is derived in Appendix II.

The responses shown in Figures 2.1 and 2.3 were obtained using this technique. Comparison with published data indicated an accuracy of better than 1% when a time increment of 0.01 second was used. [4]

CHAPTER 3

Reactor Control

[3.1] The Basic Control Problem

Provided a given reactor has a negative temperature coefficient, we have seen, that it will be self-regulating in the sense that a thermal runaway will not occur. This property, however, is not sufficient to achieve an efficient and accurate control of the power level. An external control system is needed, that monitors the actual power level of the reactor, compares this to the demanded value, and makes the necessary adjustment of reactivity if the two differ.

The block diagram of such an elementary reactor control system is shown in Figure 3-1. In this representation, the effect of the temperature coefficient is shown as a feedback loop around the reactor proper, that is described by the simplified kinetic equations. The effective reactivity change that will control the rate of change of neutron level, is the difference between the externally applied reactivity worth of the absorber rod and the reactivity

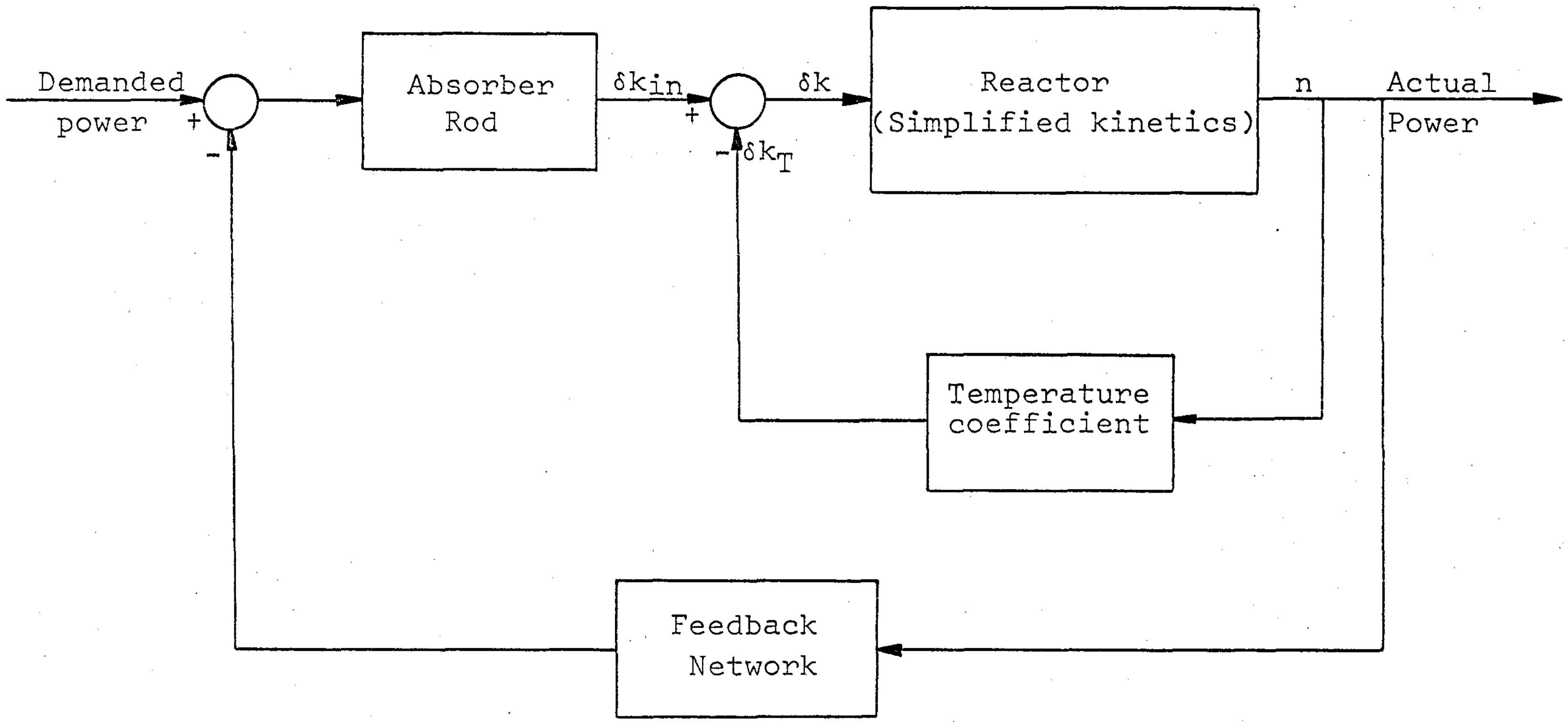


FIGURE 3.1

Block diagram representation of reactor and control system.

produced by the temperature coefficient.

Since the power output of a nuclear reactor is directly proportional to the neutron level, it is the latter that is acted upon by the control system. The neutron density may be detected by ion-chambers or inferred from the temperature rise of the coolant as it flows through the reactor. The neutron level is usually controlled by altering the amount of neutron absorbing material, typically in the form of an absorber rod, in the reactor.

It is interesting to note, that when the power level of the reactor is changed, for example increased, the absorber rod is first withdrawn, but when the power reaches its desired value, the absorber rod is reinserted to make the multiplication factor unity once again.

The steady state position of the absorber rod is therefore independent of the power level, except to the extent that the reactivity due to the temperature effect must be balanced by the absorber rod. Hence, from the steady state position of the absorber rod the value of the temperature coefficient may be inferred.

We have seen, that the neutron level of the reactor may be altered by the amount of neutron absorbing (or producing) material present in the core. At Douglas Point, the following means are available to control reactivity:

1. level of the moderator

2. poison in the moderator

3. control rods:
   (a) absorbers
   (b) boosters

In the so called "power range", namely above 15% of full power (FP), control can usually be affected by using only the absorber rods. In the simplified model considered for this thesis this is the only means of reactivity control.

[3.2] The Control System at Douglas Point.

The block diagram of the part of the control system that is considered in this work is shown in Figure 3.2. The signals processed by the feedback controller are:

1. power level of the reactor and its rate of change

2. demanded power and its rate of change.

The output is a signal that controls the reactivity by moving the absorber rod.

The feedback signals, that indicate the power level of the reactor, are derived from ion-chambers that measure the neutron concentration in the reactor, and from resistance temperature detectors (RTD's), that sense the change

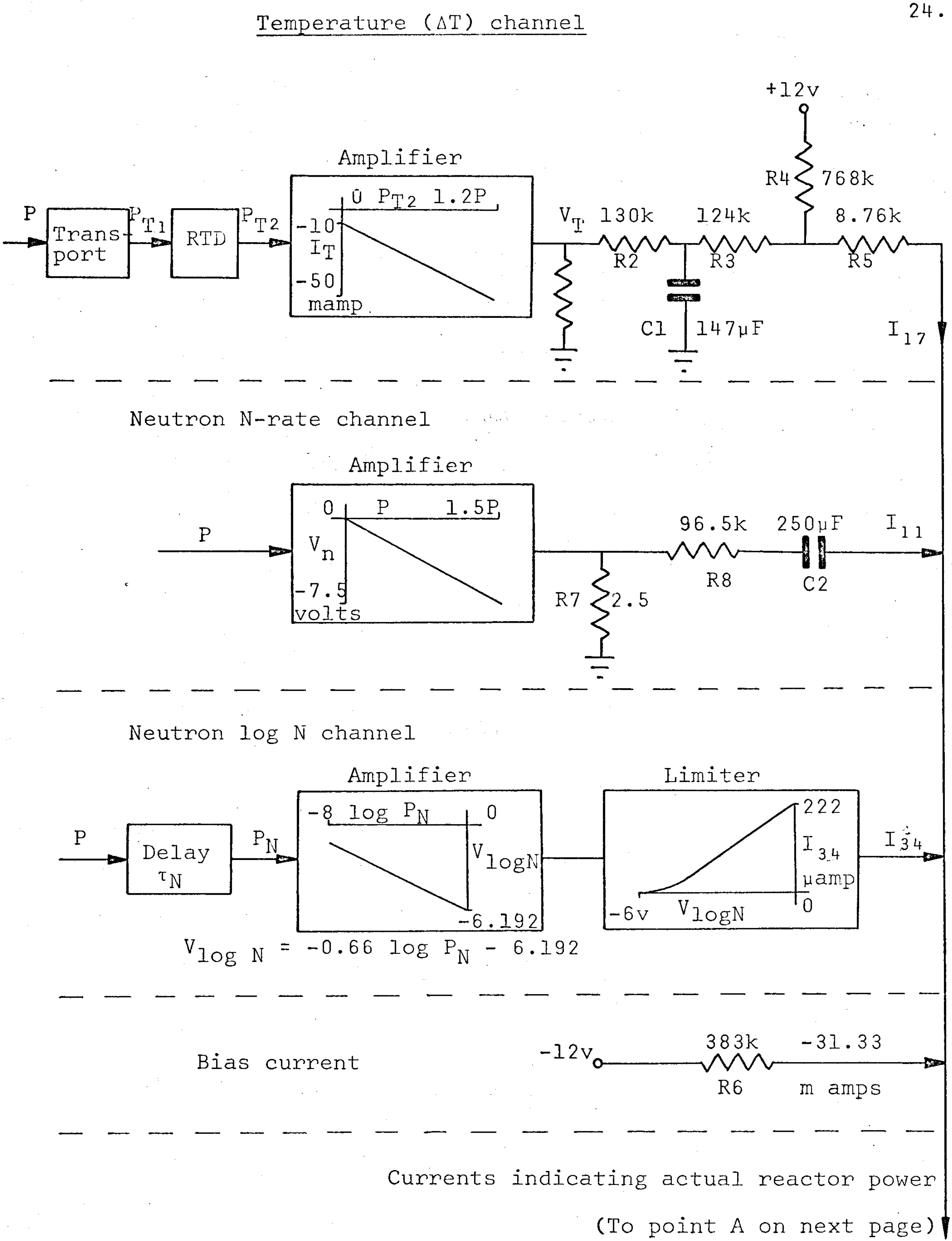


Figure 3.2a. Portion of the control system at Douglas Point

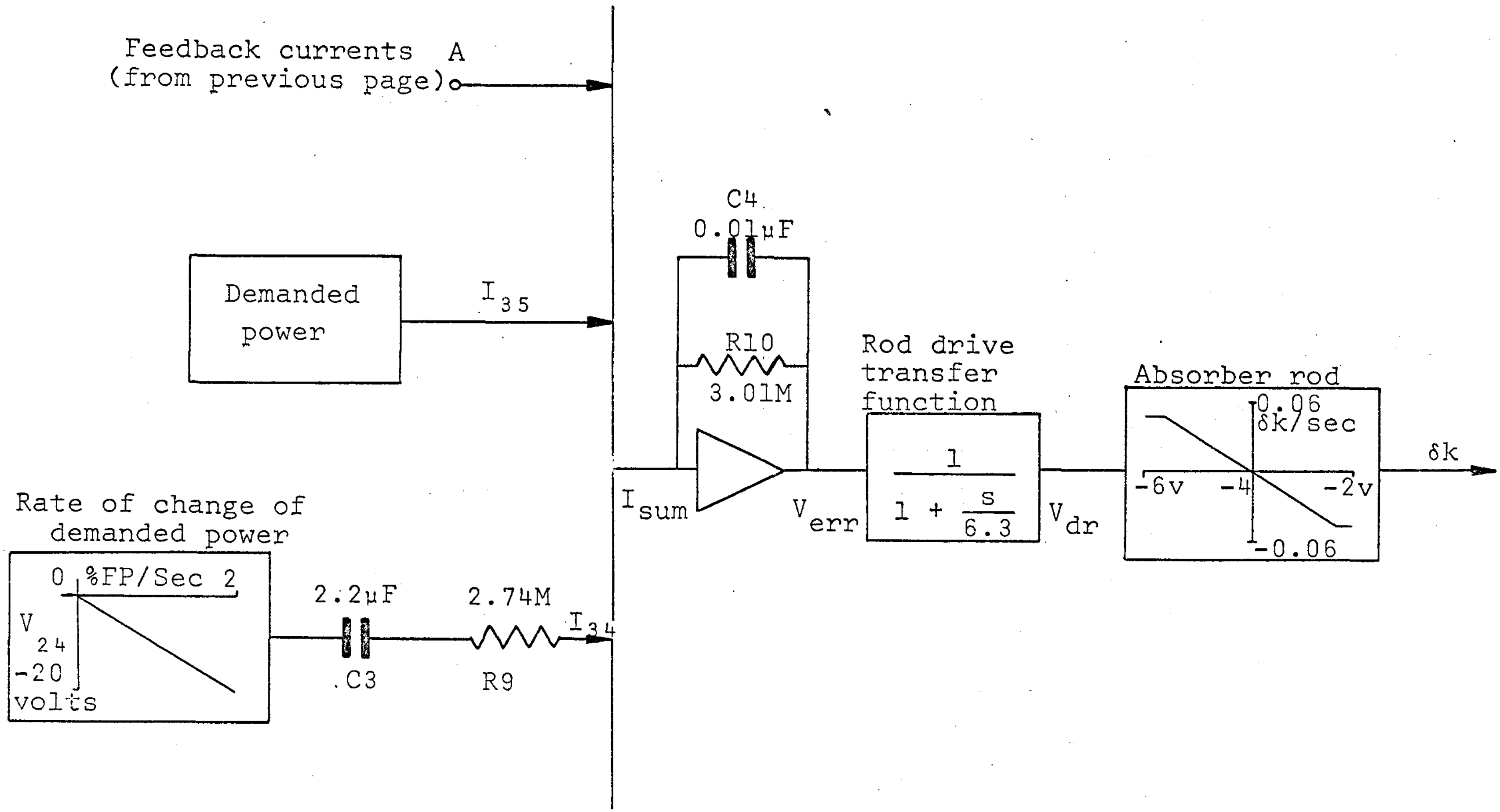


FIGURE 3.2b.

Portion of the control system at Douglas Point.

in the temperature of the primary coolant entering and leaving the reactor.

At low operating levels, viz. 10^{-8} to 10^{-3} of full power, control is based on the ion chamber output, and is proportional to the logarithm of the neutron density. The signal is processed through the channel marked "log N". Above 10^{-3} of FP the log N signal is smoothly limited and reaches zero at 0.6FP. Control is correspondingly transferred to the linear temperature channel. In the same range, the derivative of the ion chamber linear amplifier signal is also added in order to improve the transient response. This channel is marked "N-rate" in Figure 3.2.

The steady state operating characteristics of the control system are shown in Figure 3.3, displaying how the control signal arises from the combination of the log N and ΔT channel outputs. This must also be the shape of the current curve indicating demanded power. A current, I_{34}, proportional to the rate at which demanded power changes, is also added to the control signal, resulting in a larger demand signal immediately a change in power is required.

To the feedback, and demanded power signals is added a bias current, so as to produce - 4.0 volts at the output of the summing amplifier when the actual and demanded

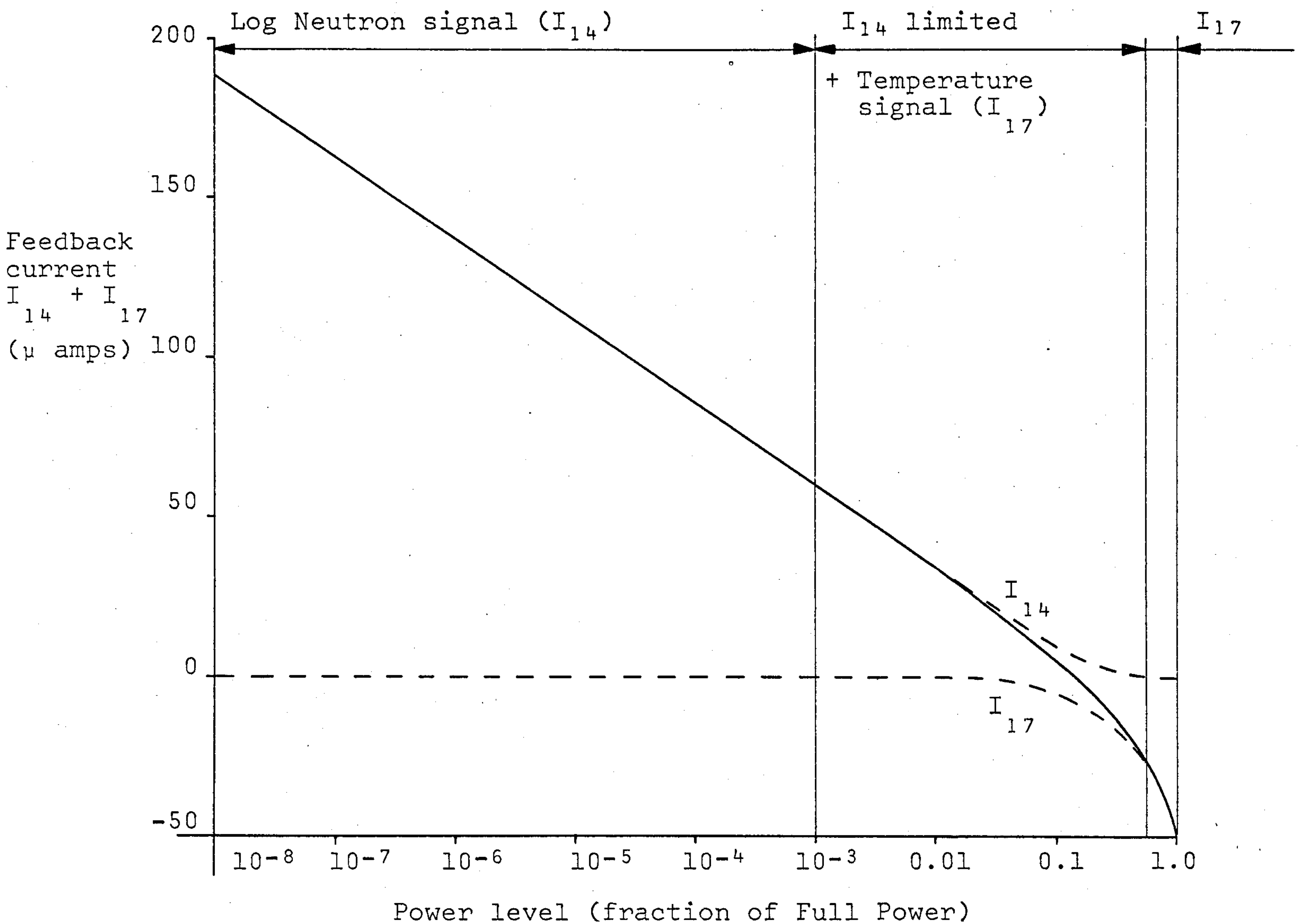


FIGURE 3.3

Steady state operating characteristic of the control system.

powers are equal in the steady state. This error voltage actuates the absorber rod, to change the reactivity in the desired direction. The transfer function of the rod drive mechanism is also included in Figure 3.2.b.

[3.3] Simulation of the Control System

The digital computer simulation of the control system shown in Figure 3.2 involves the solution of first order differential equations, the evaluation of algebraic relationships, and the logical sequencing of these operations. The method described in Appendix II was chosen to solve the differential equations. All but one of the algebraic relationships are either linear or exponential, and as such are readily calculated. According to the power level of the reactor, logical decisions guide the execution of the program along prescribed routes. (Appendix III).

In the remaining parts of this chapter each section of the control system is considered in detail, and the necessary equations for the computer simulation are derived.

[3.3.1] Log Neutron Channel

The first block (Figure 3.2) represents the delay due to the time-constant τ_N of cabel-capacitance and ion chamber amplifier input impedance. This time-constant varies over the range of 45 μ seconds at full power, to 450 seconds at 10^{-7} FP. Its effect is negligible at power operations, but tends to cause instability below 10^{-5} FP. For operating levels above 10^{-3} FP τ_N will be neglected, i.e. $P_N = P$ (P is the reactor power, normalized with respect to full power, such that P = 1.0 corresponds to 100% FP).

The signal $V_{\log N}$ is derived from the ion-chamber log N amplifier, having the characteristic

$$V_{\log N} = -\ 0.66 \log P_N - 6.192$$

The $V_{\log N}$ signal is bias shifted and limited. The characteristic of the limiter above 10^{-3} FP has been approximated by a fourth-order polynomial, using a subroutine that produces a least-square error fit to a given set of data points. Below 10^{-3} FP the I_{14} vs. $V_{\log N}$ characteristic is linear, and is given by

$$I_{14} = 222 + \frac{V_{\log N}}{0.025882} \ \mu \text{ amps.}$$

If the power level is below 10^{-3} FP, the effect of the time-constant τ_N must also be considered. In this range

$$P_N = \frac{P}{1 + \tau_N s}$$

$$\tau_N = 45 \cdot 10^{-6} P$$

The solution in difference equation form is

$$P_N\ (kT + T) = (\ 1 - T\ /\ \tau_N\ )\ P_N\ (kT) + T\ /\ \tau_N\ P$$

In the steady state, and hence initially, $P_N = P$.

[3.3.2] Temperature Channel

The block diagram of the ΔT-channel includes the time-delays denoted by "Transport" and "RTD time constant" (Figure 3.2) The transport delay arises due to the physical dimensions of the reactor: it takes a finite time for the heat generated in a fuel channel to be transported by the coolant to the measuring device, namely the RTD. The RTD's themselves have a finite response time, that must also be considered.

It is reasonable to assume that no attenuation is associated with the transport delay. A convenient method of simulating such a pure time delay on a digital computer is to store the past values of the delayed quantity (reactor power P in this case) in a linear array of dimension given by the number of samples per second multiplied by the delay τ_D in seconds. At each sampling instant, the contents of the array are shifted forward, the last member becomes the present value of P, and the first is the value P had τ_D seconds earlier.

The RTD time-constant (τ_R) may be represented by a first order lag term

$$P_{T2} = \frac{P_{T1}}{1 + \tau_R\, s}$$

The low-pass filter, consisting of R2, R3 and C1 is included to smooth out the noise on this channel, that is caused by the random arrival of coolant from different temperature zones of the reactor. The integrating action of capacitor C4 across the summing amplifier also aids in this process.

The output of the ΔT-channel, I_{17}, is related to P_{T2} through the following equations:

$$I_T = -\,10 - \frac{100}{3} P_{T2} \qquad \text{m. amps}$$

$$V_T = 400\ I_T \qquad \text{m. volts}$$

$$I_{\ } = 15 + \frac{V_T}{263.76\ (\ 1 + 9.5\ s\ )} \qquad \mu\ \text{amps.}$$

Elimination of V_T and I_T gives the desired relationship between I_{17} and P_{T2}:

$$I_{17} = -\,50.55\ \frac{P_{T2}}{1 + 9.5\ s}$$

Writing in terms of difference equations, the relationships for the simulation of the temperature channel are:

$$P_{T1}\ (kT + T) = P(kT + T - \tau_D)$$

$$P_{T_2}\ (kT + T) = (1 - T / \tau_R)\ P_{T_2}\ (kT) + T / \tau_R\ P_{T1}\ (kT)$$

$$I_{17}\ (kT + T) = (1 - T / 9.5)\ I_{17} - T\ 50.55 / 9.5\ P_{T2}\ (kT)$$

In the steady state $P_{T2} = P_{T1} = P$ and $I_{17} = -\ 50.55\ P$

Below 0.1% FP $I_{17} = 0$

[3.3.3] Neutron-rate Channel

In the power range, the rate of change of neutron level is used to compensate for the phase-delays in the ΔT-channel, giving the desired transient response. The necessary phase-lead compensation is provided by R8 and C2. Since I_{11} is only appreciable at power levels, the time-constant τN need not be considered. Hence,

$$V_N = 5.0\ P \text{ volts}$$

$$I_{11} = \frac{V_N}{4k\ \left(\frac{1 + 24.1\ s}{s}\right)} \quad \text{amps}$$

$$= \frac{-\ 1250\ s}{1 + 24.1\ s}\ P \quad \mu \text{ amps}$$

$$\text{Let } Y_N = \frac{-\ 1250\ P}{1 + 24.1\ s}$$

The necessary difference equations then are:

$$Y_N\ (kT + T) = (\ 1 - T\ /\ 24.1\ )\ Y_N\ (kT) - T\ 1250\ /\ 24.1\ P$$

and

$$I_{11}\ (kT + T) = \frac{Y_N\ (kT + T) - Y_N\ (kT)}{T}$$

In the steady state

$$Y_N = -\ 1250\ P$$

[3.3.4] Rate of Change of Power Demand

The signal proportional to the rate at which the demanded power changes has the value V_{24} = 20 volts when the power changes at ΔP = 2% FP per second. V_{24} is linearly related to ΔP, hence

$$V_{24} = 20\ \frac{\Delta P}{0.02} = 1000\ \Delta P \quad \text{volts.}$$

One half of V_{24} is used to derive I_{34} via the phase-lead network of C3 and R9:

$$I_{34} = \frac{1/2\ V_{24}}{2.74\ (\ \frac{1 + 6\ s}{6\ s})}$$

$$= \frac{3\ s\ V_{24}}{2.74\ (\ 1 + 6\ s\ )}$$

Let $$YI_{34} = \frac{3}{2.74} \cdot \frac{V_{24}}{1 + 6\ s}$$

Hence the related difference equations are:

$$YI_{34}(kT + T) = (1 - T/6)\, YI_{34}(kT) + T/5.48\, V_{24}(kT)$$

$$I_{34}(kT + T) = \frac{YI_{34}(kT + T) - YI_{34}(kT)}{T}$$

In the steady state $YI_{34} = V_{24} = 0$.

[3.3.5] Summing amplifier and Absorber rod.

The transfer impedance from the input of the summing amplifier, represented by I_{sum}, to the output V_{err}, is given by

$$\frac{V_{err}}{I_{sum}} = \frac{3.01}{1 + 0.03\, s} \quad \text{Meg. ohms.}$$

Similarly, the transfer function of the absorber rod drive is given by

$$\frac{V_{dr}}{V_{err}} = \frac{1}{1 + \frac{s}{6.3}}$$

The difference equations for the above two relationships are:

$$V_{err}(kT + T) = (1 - T/0.03)\, V_{err}(kT) + 100\, T\, I_{sum}(kT)$$

$$V_{err}(kT + T) = (1 - 6.3\, T)\, V_{dr}(kT) + 6.3\, T\, V_{err}(kT)$$

The total reactivity worth of the control rods, considered here as a single rod, is 3.0 mk. It is assumed, that in the steady state, the rod is inserted to half of its travel, and its effect is linear over the range 0.4 - 2.8 mk.

It was ascertained in the simulation studies, that the reactivity was kept within these bounds.

CHAPTER 4.

Transient Response of the Closed-loop System

In the previous two chapters the mathematical relationships that are necessary to simulate the reactor and its associated control system on a digital computer have been established. We can now "close the loop", and consider the behaviour of the overall system. In particular, we are interested in the ability of the control system to change the operating level of the reactor in a prescribed manner, or to keep it at a preset level, despite variations in the plant's parameters.

The power transients to be investigated are those encountered in the operation of the Douglas Point reactor: Demanded power may be changed at one of the following rates:

Normal: ± 0.2% FP (above 15% FP)
+ 4% actual power (below 15% FP)
Run down: - 1.0% FP

Since the system is nonlinear, the response is expected to depend on the initial power level and on the magnitude, rate and direction of the power demand change. The effects of noise on the temperature measurement, of shielding on the reading of the ion-chamber, and changes in both reactor and control system parameters will be investigated.

The aims are to further test the suitability of the model to depict the behaviour of a reactor, to indicate the areas where adaptive control is desirable, and to provide a set of standards against which the performance of an adaptive scheme may be evaluated.

[4.1] Effect of Plant Nonlinearities

The parameters of the reactor and control system were assumed to be constant at their design values, and the power level steady. The power demand was changed at the standard rates for various lengths of time, and the response of the reactor observed.

The four responses shown in Figure 4.1 indicate the effect of altering the magnitude of the demanded power change: the larger the change the more able the system is to follow it. Above a 10% change the response did not alter significantly, and most of the remaining tests were performed at 10% and smaller variations.

Figure 4.2 shows a set of responses for a 2% power demand change at various operating levels. The effect of the initial level is most marked for small power demand

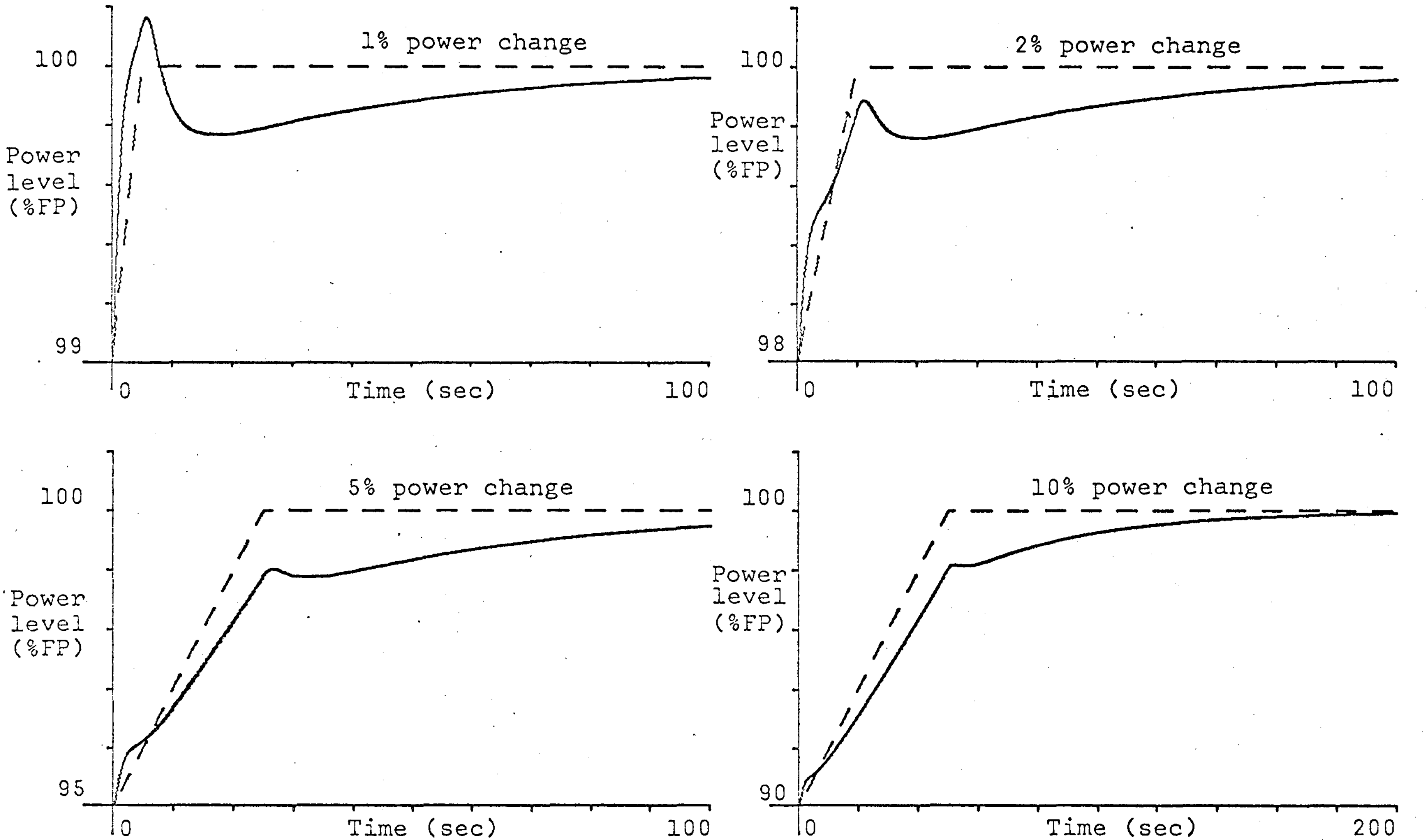


FIGURE 4.1

Effect of magnitude of power demand change.

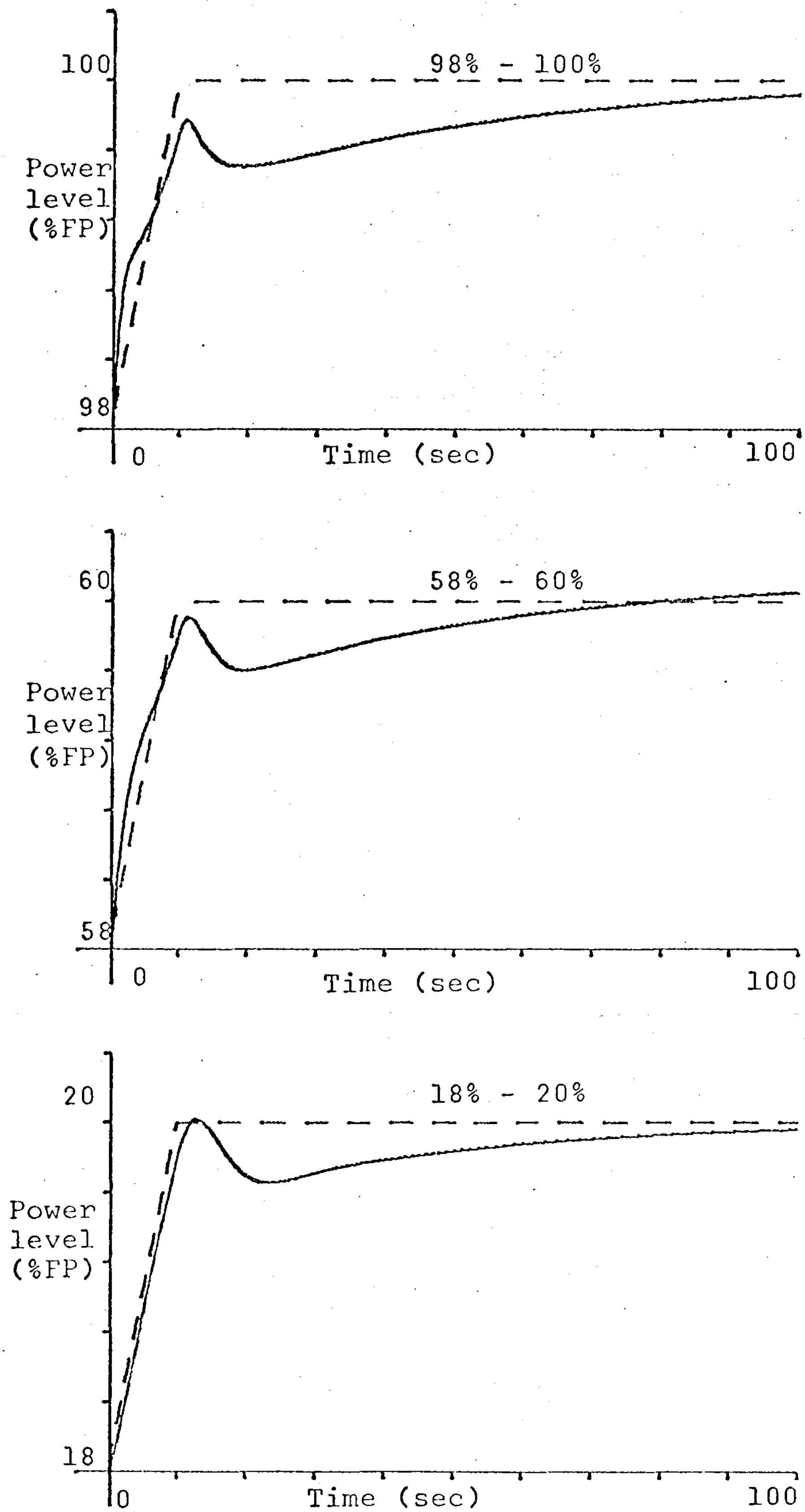


FIGURE 4.2

Effect of initial power level

changes, but even here, the differences are not very significant.

Figure 4.3 shows the response in the case of rundown. Comparison with Figure 4.1 for the same power-demand change indicates that the response deteriorates for the faster rate of change. Once again, extending the duration of the rundown resulted in a relative improvement of the response.

[4.2] Effect of Parameter Variations

4.2.1 Temperature coefficient.

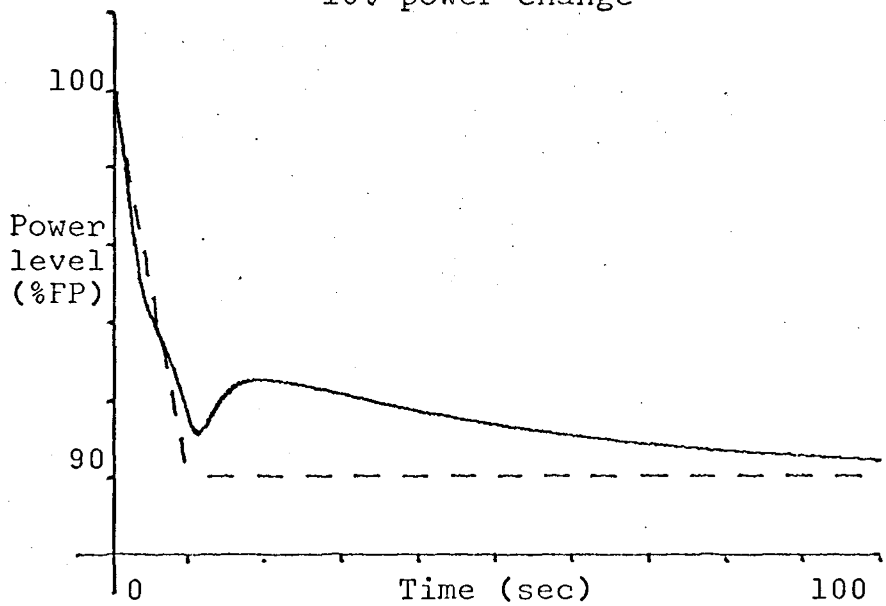
For the Douglas Point reactor the temperature coefficient has been calculated to be - 0.00454, but this value may vary over the range - 0.01 to + 0.005.

Figure 4.4 shows how the transient response changes as a function of the temperature coefficient. It is seen, that the control system can readily cope with such a wide variation in the temperature coefficient. The long time-constant of the temperature feedback effect also helps in reducing the changes in the transient response.

4.2.2. Neutron Shielding.

The ion-chamber reading of the neutron level may

-10% power change



-5% power change

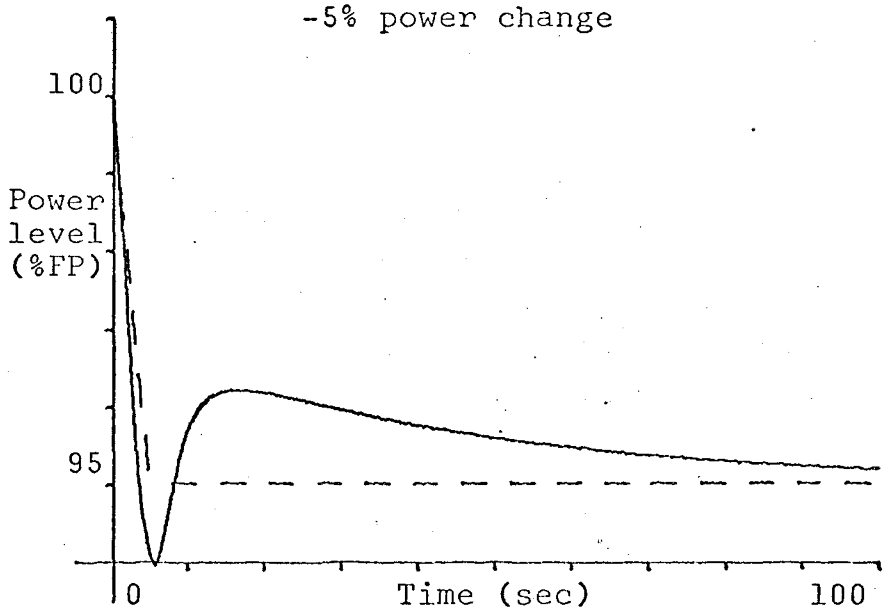


FIGURE 4.3

Response for rundown

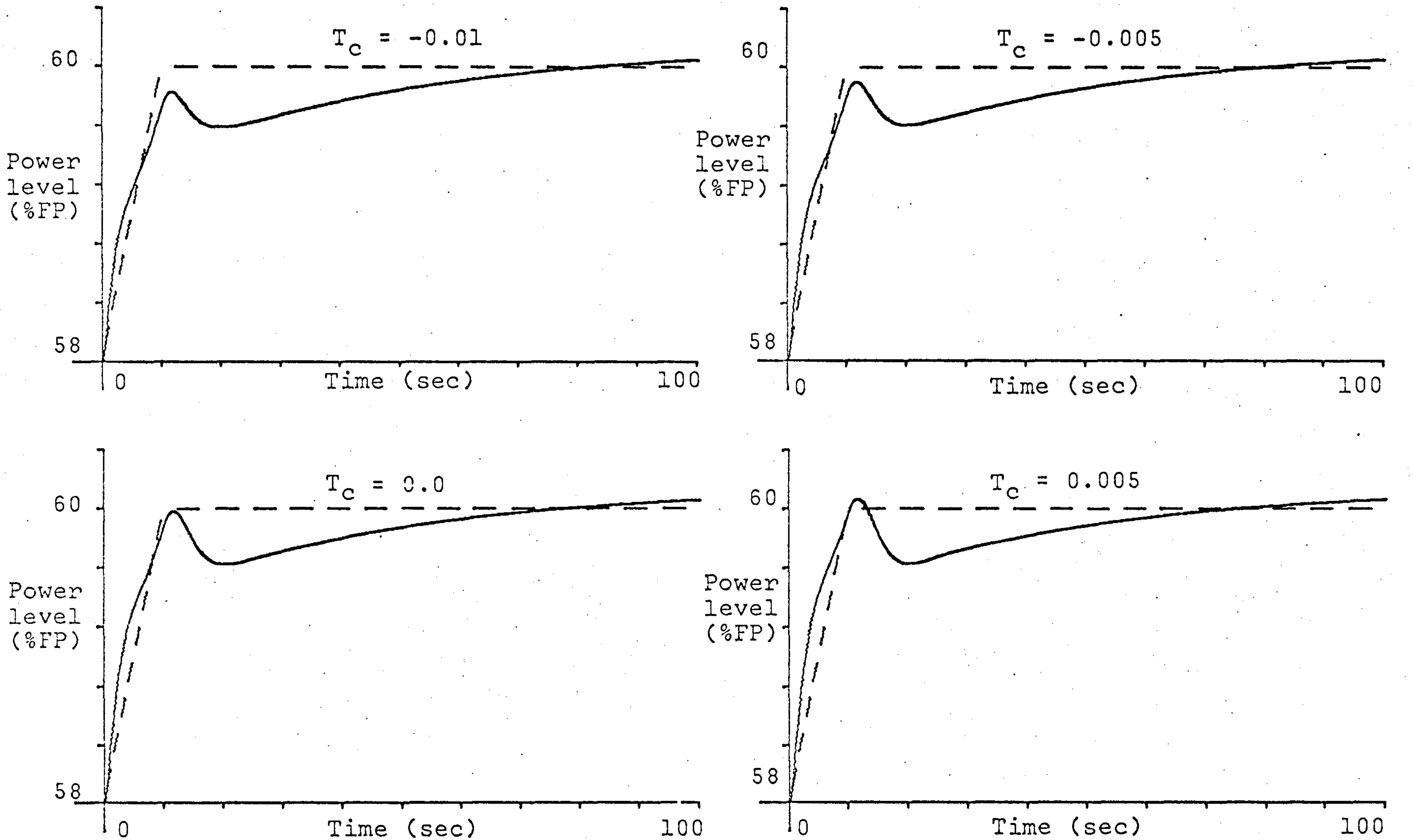


FIGURE 4.4

Effect of temperature coefficient (T_c).

be in error by as much as a factor of one half, due to the shielding effect of poison dissolved in the moderator. This poison acts as a distributed absorber rod in reducing the reactivity, but it also depresses the neutron flux as we move away from the centre of the reactor. The ion-chambers are placed in the wall of the reactor, hence the neutron density measured is very different from the mean level inside the core.

Since the ion-chamber output is used to derive two out of the three feedback signals, erroneous readings have a marked effect on the response of the reactor. The reduced signal in the log N channel results in a steady state error: the control system drives the reactor to a higher operating level than required. The transient response is also impaired, since the N-rate signal is used to improve stability.

The responses of Figure 4.5 show both of these effects. The steady state error is small near full power, because of the limiter in the log-N channel. However, for a shielding factor of 0.5, an error still results even at full power, since the ion-chamber only indicates a 50% power level, and the control system acts accordingly. At lower power levels, the steady state error becomes quite large,

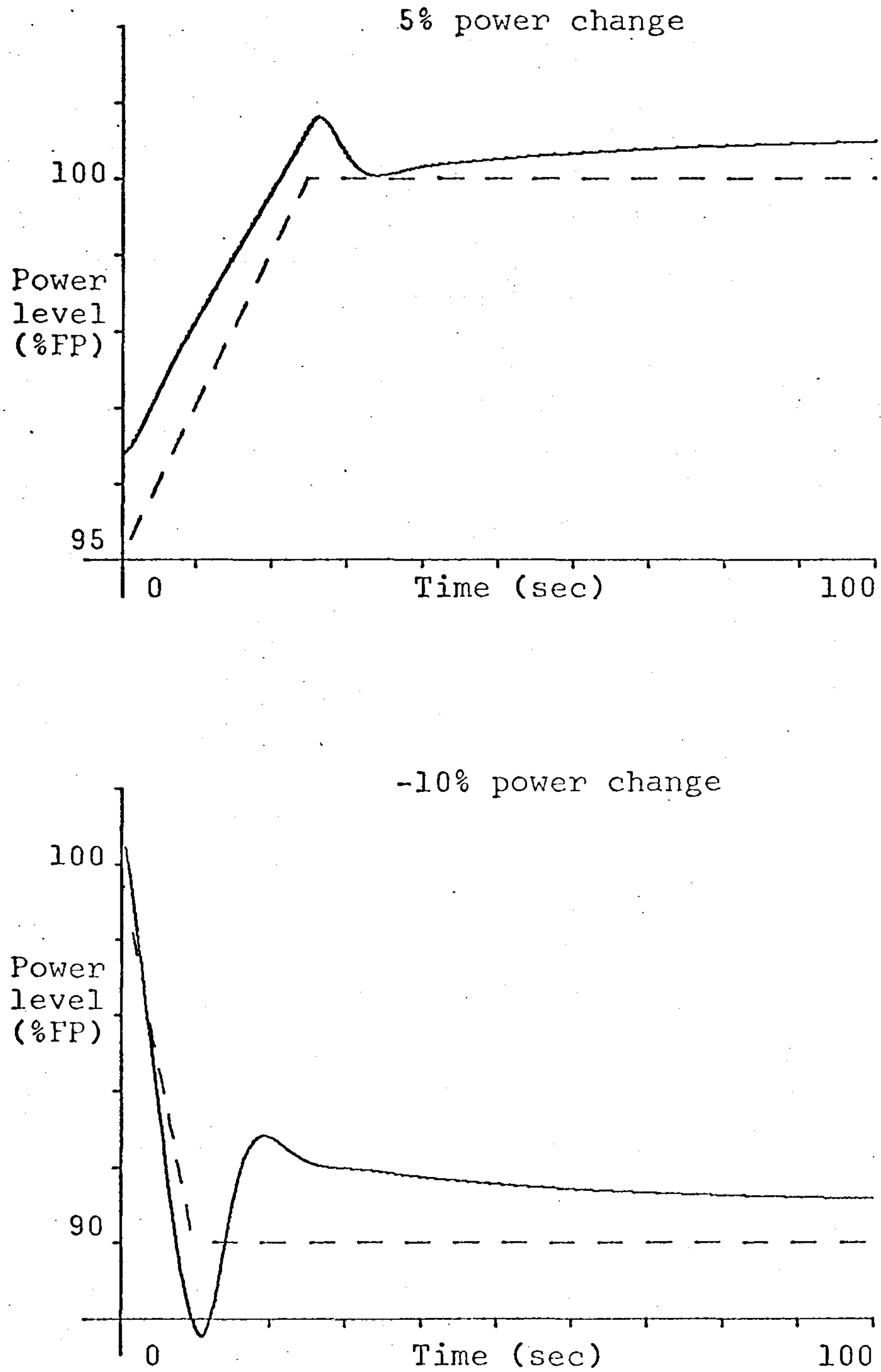


FIGURE 4.5

Effect of neutron shielding (by 0.5) near full power.

over 30% at 0.2 of full power (Figure 4.6). To obtain these responses, the power level was held constant while a step-change of - 0.5 in the shielding factor was introduced. When the reactor power achieved steady state, the demanded power was changed and the response plotted.

Since neither the level changes of the moderator nor the dissolved poison concentrations were considered for the present work, no attempts were made to study the effect of a time-varying shielding factor. It was assumed to take on a constant value in the range 0.5 to 1.0. Since the time of a power transient is very small compared with the rate of change of poison concentration, this is a reasonable assumption. No noise was introduced to the ion-chamber readings, since these are known to be relatively noise-free at the Douglas Point reactor.

[4.2.3] Transport Delay and RTD Time-constant.

The nominal value of the transport delay is 2 seconds, and it may vary by ± 25%. The RTD time-constant is 1.0 ± 50%. Despite these large deviations, neither parameter had a significant effect on the response of the system.

[4.3] Noise on the Temperature Channel

Due to the random arrival of water to the RTD's from various fuel channels of the reactor, the signal available

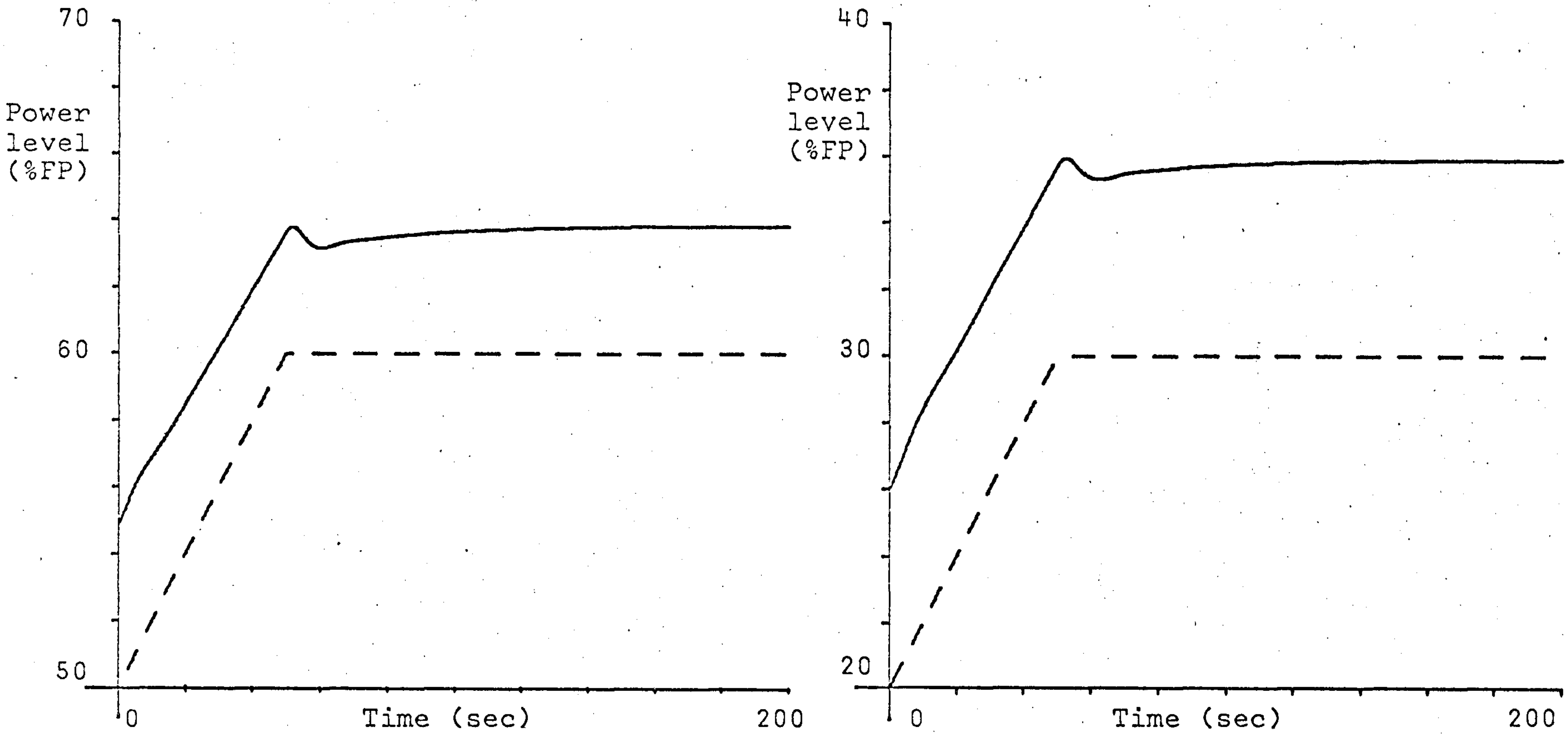


FIGURE 4.6

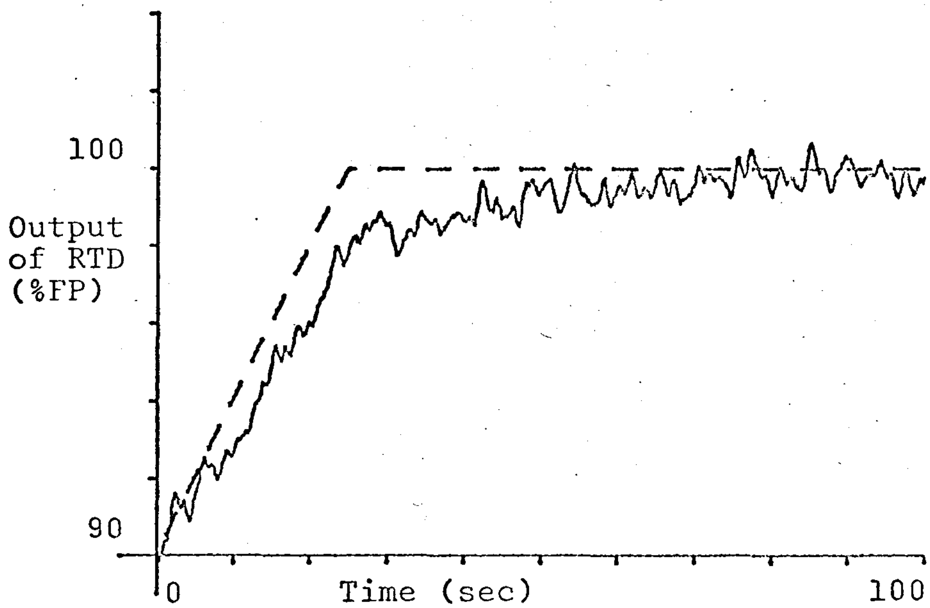
Effect of neutron shielding (by 0.5) at medium and low power levels.

to the control system from the coolant temperature change measurement has noise superimposed on it. Since only a very small sample of this signal was available, the statistical properties of the signal are not known. For the present purpose of observing the behaviour of the control system, low-frequency pseudo-random noise was generated by a computer subroutine. (Appendix IV).

The peak value of the noise was assumed to be 2% of the actual power level. This signal was superimposed on the delayed power level reading to produce the input to the RTD. The resulting waveform that is to be processed by the temperature channel is shown in Figure 4.7, along with the actual power level change of the reactor. The filtering in the control system appears adequate to reduce the effect of the noise to a reasonable level.

[4.4] Discussion of Results

The results of the simulation studies presented in this chapter show that the existing feedback control system at Douglas Point is quite adequate to control the reactor in the power range. While this fact was already known in practice under the conditions encountered during the operation of the reactor to date, it was of interest to determine if the control system could meet the specifications under extreme parameter variations. Also, the close correspondence



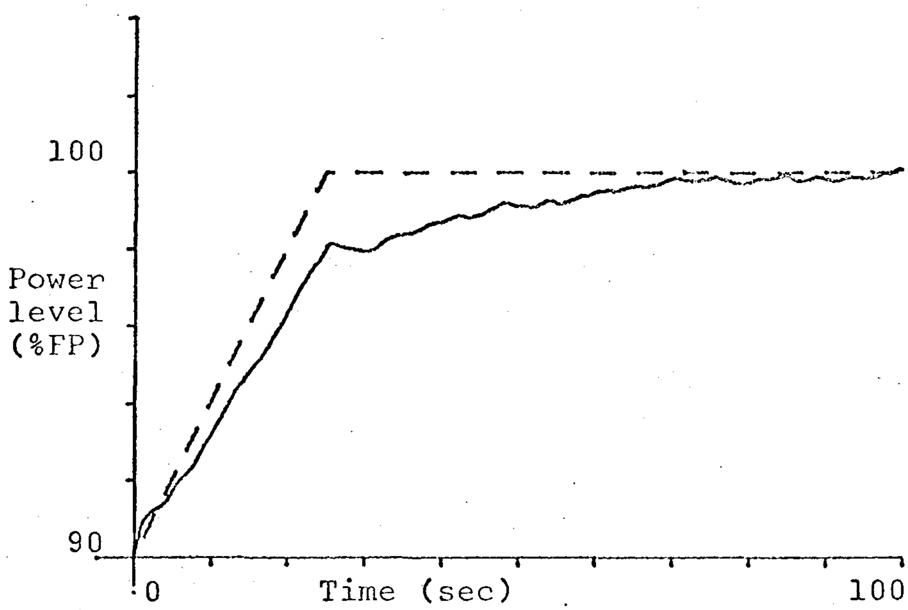


FIGURE 4.7

Effect of temperature channel noise.

of the results to those expected by the designers of the control system, confirmed that the model used in the simulation studies was a reasonable one.

The largest error is caused by neutron shielding, as much as 6% of full power at the 20% level (Figure 4.6). It is precisely for this reason that the signal derived from the ion-chambers is limited in the power range, and above 60% control is based solely on the temperature channel. In this manner, the error at full power, caused by maximum neutron shielding factor of 0.5, is reduced to 1%.

The time-lags introduced by the transport delay and RTD time-constant are effectively compensated for by the signals derived from the Neutron-rate and Demanded power rate of change circuits. The use of negative feedback reduces the effect of the changes in these time-constants to a negligible level. The low-pass filter smoothes reasonably well the noise in the temperature channel, but at the expense of introducing further time-lag into this signal.

While the overall performance of the control system was shown to be quite good by the simulation studies, it could of course be improved upon. It is realized, that such an improvement by itself is not of much practical interest, but if it is achieved by a system that is adaptive at the same time as giving a much improved performance, it should find applications.

As nuclear power plants increase in size, even a fraction of a percent represents megawatts, and thus even a slight improvement may mean a considerable saving.

CHAPTER 5.

Optimum and Adaptive Control of Nuclear Reactors

Before we consider specific control schemes for the Douglas Point reactor, a brief review of the theory of optimum and adaptive control is presented.

A survey of the applications of these theories to nuclear reactor control is considered next, and their relative advantages and disadvantages as regard to the control of large power reactors are discussed.

[5.1] Optimum Control Theory

The word "optimum" was coined from the latin optimus, meaning "best", by Leibnitz at the beginning of the eighteenth century. However, optimum control theory has only become a recognized body of work in the last two or three decades, mainly as the result of the availability of high-speed computers. The range of applications has been very wide, from maximizing the range of a rocket or the profit of a business, to minimizing the time required to change the state of a system to a new level [8].

Since the aim of optimal control theory is to determine the manner in which the best performance of a system may be achieved, its implementation requires the knowledge of the following:

1. Definition of the optimum and an accurate measure of system performance to indicate the deviation from the desired optimum. The mathematical relationship that fulfills this requirement is called the index of performance or cost function [9].

2. Accurate mathematical description of the system and all environmental factors that influence its operation.

3. The initial state and the target state of the system.

4. The class of admissible controllers.

Among the many techniques of solving optimization problems, the three fundamental ones are the calculus of variations, Pontryagin's maximum principle and Bellman's dynamic programming, based on his principle of optimality.

The calculus of variations deals with the maximization and minimization of functional expressions where entire functions must be determined [10]. The application of this technique to the design of optimum control systems generally leads to a two-point boundary-value problem. Analytical solutions for such problems are possible only in special cases, and numerical solutions must usually be resorted to. Furthermore, the variational-calculus approach is generally limited to systems that are free from inequality constraints on the control and state variables.

Pontryagin's method considers the system represented in state space form with the controller subject to saturation [11]. The cost function that is minimized is a time integral from the initial time to the final time of a function of the controlling and controlled variables. Auxiliary variables are introduced to form an adjoint system and a Hamiltonian. The optimal solution reduces to the determination of the maximum of the Hamiltonian. The technique is inherently suited to optimize systems with bounded inputs, however, obtaining a solution with nonlinear systems presents formidable difficulties due to the two-point boundary value problems inherent with this technique.

The dynamic programming approach circumvents the difficulty of having to solve a two-point boundary value problem by formulating the procedure as a multi-stage decision process instead of a set of differential equations [12]. The design of an optimal control system is viewed as a search for the optimum strategy over a multi-dimensional control space. Because of computational limitations the admissible control region is discretised, however, an exhaustive search would still not be practicable for any but trivial problems. Bellman derived a systematic procedure based on his principle of optimality, which states: an optimal policy, or optimal-control strategy, has the property that, whatever the initial state and the final decision, the remaining decision must form an optimal-control strategy with respect to the state resulting from the first decision. This principle is used in obtaining an iterative functional relationship, which is solved backward in time from the known terminal state to give the required optimum control law.

Several variations of these fundamental techniques of optimum control, as well as some other unique methods, have been described in the literature. Many of these are directed to the more efficient numerical solution of the optimum control problem, but only a very few have so far been successfully applied in practical situations.

Apart from the large amount of computer time necessary to obtain solutions to realistic problems, the fundamental requirement of optimal control theory, to have perfect knowledge of the plant, limits its application in industry.

[5.2]. Optimum Control of Nuclear Reactors.

The nature of the energy-conversion process that takes place in a nuclear reactor has been studied very extensively, it is well understood, and may be accurately represented by mathematical models. It seems therefore particularly suited for the application of optimal control theory. Furthermore, with the expensive fuel used, the growth in the size of reactors and the stringent requirements of safety, it is becoming increasingly desirable to have more accurate and predictable control of a reactor than may be possible using conventional feedback techniques. It is not surprising therefore, that a large number of papers have been published on this subject. They are reviewed here for their possible application to control a reactor of the type considered in this thesis.

Optimal control has been considered in the following six areas of nuclear reactor operation, and these are the ones that will be of likely interest for some time to come:

1. response to changes in power demand,
2. start-up,
3. shutdown in the presence of xenon poisoning,
4. regulation in the presence of random disturbances,
5. spatial flux distribution,
6. fuel management.

The first of these areas is the one of greatest interest in this thesis. It has been studied in detail by Kazuo Monta of Japan who has also conducted some trials using the derived optimum policies to control an experimental reactor. His work is described in three separate papers, references [13], [14] and [15].

Monta considers the time optimal control of nuclear reactors using Pontryagin's maximum principle. A simplified model of the reactor kinetic equations is used, that considers only one group of delayed neutrons. Since the results are to be applied to a small research reactor, temperature feedback effects are expected to be negligible. The continuous version of the maximum principle is used first, on the simplified model, in order to obtain a guide to the design of the actual discrete time system. Reactor power and reactivity are considered as state variables, with restrictions on the maximum value of both.

Following from the solution of the continuous problem, Monta uses an extension of the discrete maximum principle to obtain the time optimum control for the special case of a pulse width modulated input signal to the control rod. An important practical point is considered, namely the necessity of smooth regulation near the terminal state. Accordingly, the control law is modified in this region, and its stability determined by the second method of Liapunov.

The theory was tested on a 100 kw research reactor and the results were encouraging. In the actual implementation, it is necessary to compute the reactivity at each sample from the measured value of neutron level, and assumed delayed neutron concentrations. The inaccuracies involved with this calculation, along with the assumption of negligible temperature coefficient, are the main deficiencies of this very interesting research.

Another Japanese contribution to reactor control optimization is by Keije Miyazaki [16]. He applies Wiener's theory of least square optimization with quadratic constraint, and treats both the deterministic and stochastic cases in the form of step and ramp reference inputs, and white disturbance of reactivity. The reactor model is approximated by the single-group delayed neutron kinetics, and temperature feedback is neglected. This simplified reactor model and the optimum controller are simulated on an analog computer. Despite the severe approximations used, the rate of change of reactivity set by the optimum controller exceeded by 10^2 to 10^3 times the usual values of this constraint. It is the shortcoming of the calculus of variation approach used here that such a constraint cannot be introduced directly into the optimization process. Furthermore, the optimal solution for the case of both deterministic and random disturbances being present at the same time, remains unsolved.

A very interesting approach to the optimal reactor response problem is adopted by Duncombe and Lathbone [17]. They treat the nuclear reactor as only a part of the overall generating plant, that includes a heat exchanger, steam-turbine and condenser in the form of a secondary loop, coupled to the primary loop that includes the reactor. In such an overall view, the usual simplifications of the reactor model (single-group delayed neutron, no temperature coefficient, infinitely fast actuators, point dynamics, etc.) lead to a much smaller overall error. It is questionable, of course, to what extent an overall approach such as this jeopardises the safe operation of the reactor proper. Since the time-constants associated with the neutron multiplication are much shorter than those in the steam circuit, the associated delay may be excessive.

Pontryagin's maximum principle is used to find the optimum controller. The canonical Hamiltonian equations are converted into an initial value problem in the form of matrix Riccati equations, and these are solved by analog computer methods. Performance indices penalize core power error or other plant variables such as volume surges, together with reactivity control requirements.

The on-line implementation of the optimal controller is studied on an analog computer. The matrix Riccati equations are solved at ten times real time using the reference model for the system. Simultaneously, the state variables of the actual plant and of the reference model are compared, and the resulting error vector is used to modify the optimum feeback controller parameters produced by the Riccati equations. The results obtained, within the simplifying assumptions, are very good.

A special case of response to a power demand change is the optimum start-up of the reactor. While for large power-generating installations this is not of concern, since it should not need to take place more than perhaps once a year, it may offer significant advantages for the smaller, research type reactors. However, the real interest in optimum start-up is in connection with nuclear rocket motors. This is the subject of one of the first contributions to the field of optimum reactor control.[18].

Shen and Shaag developed a simple analog-type optimum controller that gives a constant period of start-up, followed by operation at a steady power level. The implementation requires only two amplifiers with time varying gains. The single-group delayed neutron kinetic equations are used again, and both the calculus of variations and dynamic programming give the same analytical solution. The only constraint that is included is the amount of reactivity available for control.

A more recent article on the same subject [19] makes possible the inclusion of several constraints by the use of a numerical procedure based on an iterative sequence of linear programming problems. At each step the solution is forced to satisfy all the inequality constraints and boundary conditions, thus eliminating the instability problem associated with the nuclear reactor. The assumed optimality criteria of minimum propellant consumption also leads to a minimal time solution.

The problem of optimum shutdown in the presence of xenon poisoning has received perhaps the most attention out of the attempts to apply modern control theory to nuclear reactors. It is the subject of a book by Milton Ash [20], wherein dynamic programming is used to give the desired trajectory. The simplifications in the reactor model and the coarse grid-size that had to be used in order to make the computation possible, resulted in a suboptimal solution, involving several switching times. Recent advances in the application of Pontryagin's maximum principle enabled Lewins and Babb to obtain a much improved solution to the xenon shutdown problem. [21].

The remaining areas of application that were mentioned earlier are not directly related to the present work, so will not be discussed here. The reader is referred to references [22], [23], [24], for information on these subjects.

Apart from the list of applications already given, this review of the literature on optimum control of nuclear reactors resulted in the following classifications of the optimal techniques used to date and the simplifying assumptions that were applied to various extent, to the mathematical model of the reactor.

I. <u>Methods of Optimization</u>:

1. Calculus of variations.
2. Pontryagin's maximum principle.
3. Dynamic Programming
4. Other programming techniques.

II. <u>Simplifying Assumptions</u>:

1. Prompt neutron kinetics
2. Single group delayed neutrons
3. Temperature coefficient negligible
4. Infinitely fast actuators
5. Step-inputs of reactivity
6. Point reactor dynamics
7. Constant reactor parameters.

The reason for the lack of industrial application of these optimization methods, as well as the need for an adaptive control system, are a direct consequence of the invalidity of the above assumptions.

[5.3]. Adaptive Control Systems

While in the case of optimal control we could discuss various theories, it is more realistic in the case of adaptive control to talk about proposed systems. There is, of course, a considerable amount of theory associated with a particular adaptation scheme, but the approach is typically from a practical, systems point of view. [25] [26].

The following definition of adaptive control has been adopted for the present work:

> "An adaptive control system is one that is provided with a means of continuously monitoring its own performance in relation to a given performance criterion or optimum condition and a means of modifying its own controlling parameters by closedloop action so as to approach this optimum" [27].

The advance of adaptive control systems may be viewed as a natural evolution of optimal control theory as the attempts were made to apply the latter to space problems and large scale process control. The lack of advanced precise knowledge of the operating conditions and parameter variations of these systems often precludes the direct application of optimal control theory. However, some of the techniques and performance objectives may be used to advantage, provided these can be changed or adapted as the circumstances vary.

In order to fulfill their tasks, adaptive systems must perform some or all of the following operations [28].

1. Measurement, such as input and output of plant.

2. Identification of plant parameters, signal statistics, index of performance.

3. Pattern recognition, or the comparison of present operating conditions to past records.

4. Determination of control strategy, usually to optimize some index of performance.

5. Modification of the controller parameter, or controller configuration.

Depending on the amount of importance attached to each of these five steps, various clasification schemes may be produced for adaptive systems. For the purpose of this thesis the interest is in the identification aspect, since as we have seen, optimal control may be applied to nuclear reactor control, provided we have an accurate measure of plant parameters. Accordingly, we can distinguish adaptive systems on the basis of whether or not the identification of the plant parameter takes place.

Identification schemes have been proposed based on such techniques as impulse response evaluation, quasi-linearization, pseudo-random sequences, etc. The practical problem is usually one of instrumentation, since certain parameters cannot be measured accurately with present day transducers within reasonable time.

The identification problem has been circumvented in some systems by using only the normally available measurements of input and output, and applying numerical optimum seeking methods, such as steepest descent, conjugate gradients, etc. to adapt the controller parameters [29].

For most industrial applications, it is expected that a combination of the above two methods will lead to the best engineering solution: plant-parameter variations that are readily monitored will be used to advantage, and an optimum seeking procedure based on the measured response is used to adapt for those changes that cannot economically be identified. Since the implementation of most adaptive schemes relies on the use of an on-line digital computer, analysis of the measured data should give an indication of some parameters that are not directly accessible.

[5.4]. Adaptive Control of Nuclear Reactors:

Compared to the extensive study of optimum control of nuclear reactors, it is somewhat surprising to find only a single reference to the application of adaptive techniques to reactor control. This contribution, by Corbin [30] in 1958, considers the problem in the frequency domain, and uses analog techniques of solution. The transfer function of the reactor is written in the form of a gain factor (K) times a ratio of polynomials in s, and the instantaneous values of K or of any of the pole or zero positions, are determined and corrected for in such a manner that the overall system transmittance remains invariant in a changing environment.

Corbin applied this method to a model that compensates only for variations of the gain factor. The analogue computer evaluates the actual system gain from the known input and output signals. The computed gain is compared with the desired gain, that has been, presumably, preselected for each power level, and the resultant error signal is used to vary the gain of an amplifier in the forward path, to bring the overall gain to the desired value.

CHAPTER 6

The Proposed Adaptive Control System

The development of an adaptive control system for the Douglas Point reactor was considered in two sections. One aspect of the problem was to identify, i.e. accurately determine the parameters and operating power level of the reactor, and the other to find a control law that would result in the best performance. This latter part, of course, also involved the determination of what is meant by "best response". In other words, a suitable cost function had to be selected.

[6.1] Identification.

In a practical system, such as a nuclear reactor, perfect identification can very rarely be achieved. Instead, an estimate is usually formed of the desired quantities, and this is updated as time proceeds. Hence the identification process is more realistically called parameter estimation, although the former term is commonly used in adaptive control applications. For our model of the nuclear reactor the following two parameters vary as unknown functions of time and hence need to be estimated:

(i) Temperature coefficient

(ii) Delayed-neutron concentrations.

Since none of these parameters may be measured directly, their value can only be inferred from the signals that indicate the behaviour of the overall reactor.

[6.1.1] Identification using a Reference Model

The basic effect of both the temperature coefficient and the delayed neutron concentrations is to slow down the rate of change of neutron population. However, the time constant associated with each parameter is different, ranging from 0.62 seconds to 80 seconds, the effect of the temperature coefficient being predominant. Hence, it appeared possible to estimate these parameters from the transient response of the reactor. Since it is not desirable to introduce extraneous power-level disturbances, the identification has to take place during actual power demand changes. Identification under these circumstances may be attempted by using a reference model of the reactor.

The block diagram of such a system is shown in Figure 6.1.

The portion of the system below the dotted line is a conventional feedback control arrangement. The power error actuates the controller in such a sense that the error is reduced. If now the same control signal is applied to a model of the plant, a comparison of the two outputs will

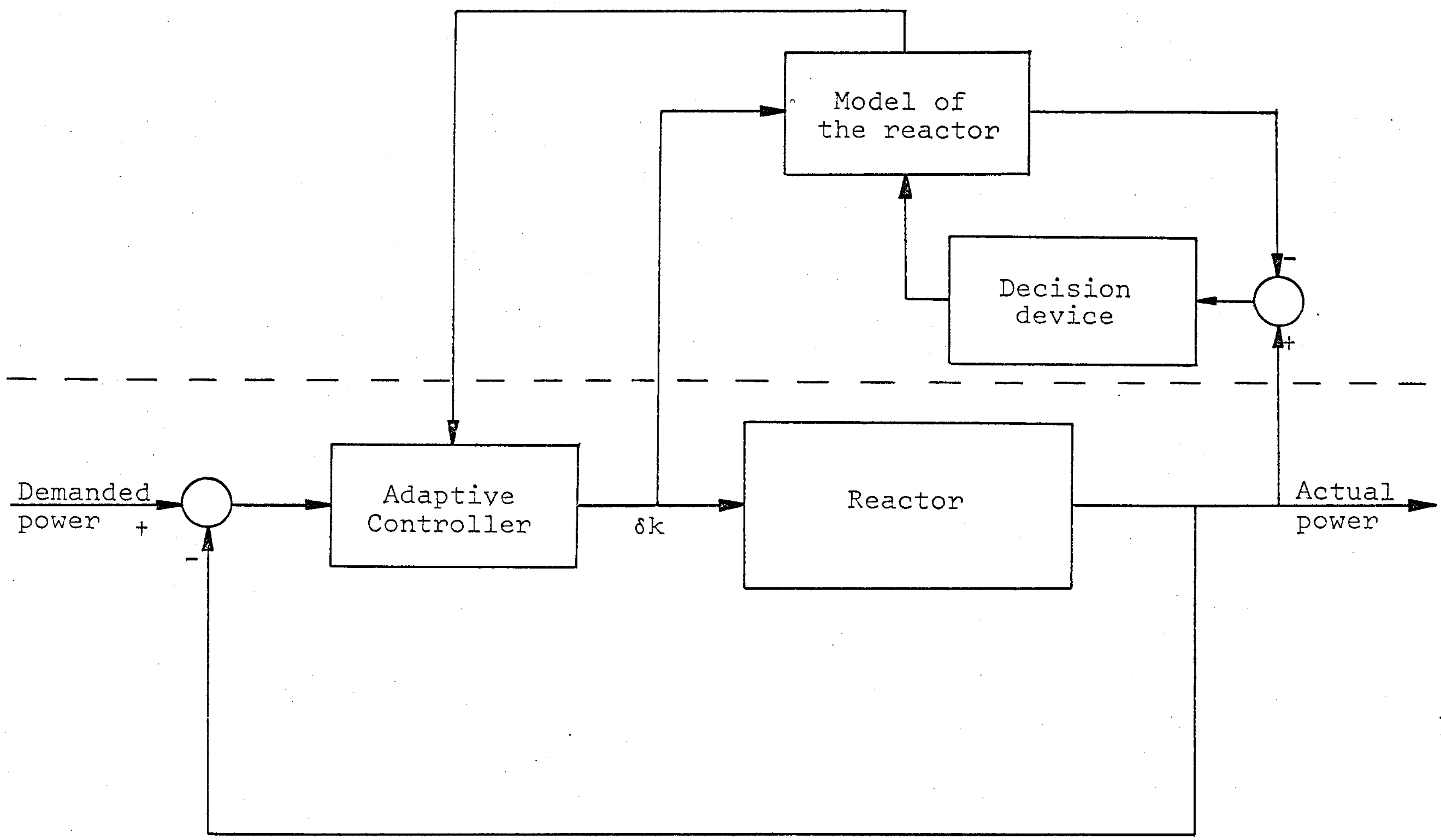


FIGURE 6.1

Identification using a reference model

indicate the nature of the difference between the assumed model and the true reactor. This information may be used to update the model to resemble more closely the actual plant. Since for the reactor the time-constants are known, and are widely different, the coefficient corresponding to each may be estimated in turn as the transient response proceeds. If changes in the parameters are observed, the adaptive controller is modified such that the response is maintained at or near its optimum value.

Results of simulation studies showed that while the method was feasible, individual parameter changes had to be detected on the basis of differences in the seventh or eighth decimal place. Since the actual reactor power cannot be measured to such a high degree of accuracy, this method of identification had to be abandoned.

[6.1.2]. Identification based on Past History.

Simulation studies of both the present control system at Douglas Point, and of the model-reference identification scheme indicated that variations in the temperature coefficient have a much greater effect on the response of the reactor than changes in the delayed neutron concentrations. It was therefore decided not to attempt identification of the changes in the delayed neutron concentration, but to use an adaptive control law that compensates for these variations without identifying them. Since small changes in the temperature coefficient produce similar results to delayed neutron concentration changes, such an adaptive controller also has the ability to compensate for small inaccuracies in the estimated value of the temperature coefficient.

In Chapter 2, the temperature coefficient was given as the change of reactivity for a given power change, i.e.

$$T_c = \frac{d\ (\delta k_T)}{dP}$$

Hence

$$\delta k_T = \int_{P_o}^{P_f} T_c\ dP = T_c\ (\ P_f - P_o\ ) \qquad \text{..........(6.1)}$$

If we assign a reactivity level $\delta k_{To} = T_c\ P_o$ to correspond to a given initial power level, we obtain

$$T_c = \frac{\delta k_T + \delta k_{To}}{P_f} \qquad \text{..........(6.2)}$$

It was assumed in the simulation studies that the initial reactivity level δk_{To} was known, and balanced by poison dissolved in the moderator to such an extent that the absorber rod was at midway of its linear range, viz. at 1.6 mk. At any given power level P_f, the amount of additional reactivity that has to be removed by the absorber rod to compensate for the effect of the temperature coefficient, gives the value of δk_T. Equation 6.2 may then be solved to give the desired quantity: the temperature coefficient T_c.

To implement this scheme, it is only necessary to continuously time-integrate the error signal applied to the absorber rod. Whenever estimation of the temperature coefficient is required, the value of the integral (Equation 6.1) is an estimate of δk_T, hence knowing the power level, Equation 6.1 gives the temperature coefficient. The inaccuracy of the computation stems from the fact that the actual amount of reactivity change caused by the absorber rod will not be the same as expected under ideal conditions. In the simulation studies, the time-constant associated with the absorber rod was neglected in estimating the reactivity change produced. The resultant inaccuracy in estimating the temperature coefficient will be shown to be compensated for by the adaptive controller.

[6.2]. Estimation of Power Level.

One of the vital factors in designing a control system is the availability of signals that accurately represent the quantities to be controlled. In our case, it is the power level that needs to be regulated, such that it follows closely an externally specified value. The signal that indicates the demanded power level is at the choice of the design engineer, and hence can be made as accurate as desired. On the other hand the signals that indicate the actual power level of the reactor are far from accurate. As we have seen in Chapter 4, the ion chamber reading may be in error by as much as 50%, and the signal indicating the temperature of the coolant is delayed in time, and contaminated by noise, resulting in an inaccuracy of up to ± 2% of the true power level.

Two alternate methods for estimating the power level are apparent: either the two signals are continuously combined in some manner to give a single quantity that represents the operating level, or one is processed in such a way as to give the necessary correction factor for the other. For our system the latter method was quite simply implemented, and it gave very accurate results.

The technique takes advantage of the operating characteristic of the Douglas Point power plant: being a baseload generating station, it is operated at a constant power level for long intervals of time, a number of days, as compared to the time required for changing the operating power level, a few minutes. During the steady operating level, the noise contaminated temperature channel reading may be processed to estimate its mean value, by either numerical averaging or by filtering. Once the mean value has converged sufficiently, the shielding factor of the ion-chamber reading may be computed and a subsequent power transient will be accurately controlled by the corrected reading of the ion-chamber.

Having a computer available to process the signal, the use of an optimal estimation algorithm was considered. Of the many optimal criteria, minimizing the mean square error appeared most appropriate for our problem. A linear estimator of this kind that has been popular in recent times, is known as the Kalman filter. It has been applied successfully in space research, and it was studied in detail for possible use in our control system.

[6.3] The Kalman Filter

The theoretical basis for the Kalman filter was established using the properties of matrices, and it is in this same general form that it usually appears in the literature. Since we are dealing with a scalar quantity, the operation of the filter will be described in this simplified form. In addition, as we shall see, the power level may be considered a piece-wise linear function of time, and for the present purpose, the output of an unforced system.

This output is contaminated by additive noise only. The estimation problem may therefore be represented by the block diagram shown in Figure 6.2.

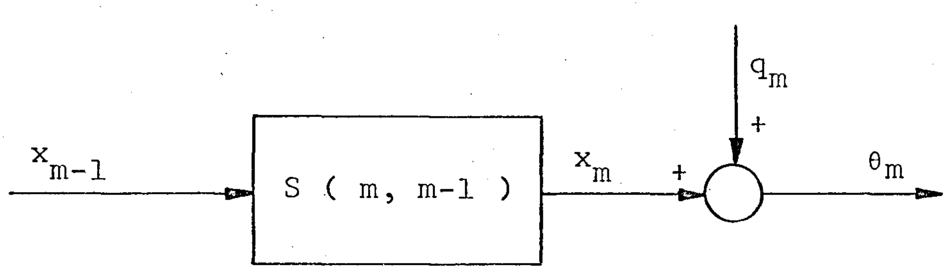


FIGURE 6.2

Problem formulation for optimal estimation process.

The quantity of interest, x, (in our case the power level as indicated by the temperature of the moderator) has a particular value at time t_{m-1}, denoted by x_{m-1}. During the next time increment it changes to x_m according to the system transition matrix S (m, m-1). The true value x_m cannot, however, be observed, only the quantity

$$\theta_m = x_m + q_m$$

where q_m is the value of the additive noise at time t_m. It is assumed that the mean value of the noise is zero, and its variance Q, is known. The problem is to determine an estimate $\hat{x}_m$ at time t_m that is a linear combination of an estimate at time t_{m-1} and the measurement data θ_m. The estimate must be optimal in the sense that the expected value of the sum of the squares of the error in the estimate is a minimum. The following description of the filter equations, based on physical reasoning, illustrates the operation of the Kalman filter.

Given the state transition matrix and the estimate $\hat{x}_{m-1}$ at t_{m-1}, it is reasonable to predict the estimate at t_m to be

$$\hat{x}_m^1 = S\ (\ m,\ m-1\ )\ \hat{x}_{m-1}$$

when no other information is available. The measurement at t_m can be used to modify this estimate. An error in the estimate is reflected by an error in the expected measurement value, i.e.

$$e_m = \theta_m - S (m, m-1) \hat{x}_{m-1}$$

According to the problem statement, the estimate is to be a linear function of the new measurements. This may be achieved if we define a quantity K_m such that the estimate $\hat{x}_m$ is given by

$$\hat{x}_m = S (m, m-1) \hat{x}_{m-1} + K_m [\theta_m - S (m, m-1) \hat{x}_{m-1}]$$

The coefficient K_m may be viewed as a gain factor, that optimally weighs together the new data θ_m and the old estimate $\hat{x}_{m-1}$ to obtain the new optimal estimate. The quantity K_m is determined so that the expectation of the error ($\hat{x}_m - x_m$) is minimized. A recursive formula for K_m can be shown to have the following form [31]:

$$K_m = \frac{C'_m}{Q + C'_m}$$

where C_m is the variance of the error, and

$$C'_m = S^2 (m, m-1) C_{m-1}$$

is the projected estimate of the variance of the estimation error, based on its value C_{m-1} at time t_{m-1}. The variance of the error in the estimate at time t_m is

$$C_m = \frac{C'_m Q}{C'_m + Q}$$

In summary, the recursive relationships of the Kalman filter in order of execution are

$$C'_m = S^2 (m, m-1) C_{m-1}$$

$$K_m = \frac{C'_m}{Q + C'_m}$$

$$\hat{x}_m = S (m, m-1) \hat{x}_{m-1} + K_m [\theta_m - S (m, m-1) \hat{x}_{m-1}]$$

$$C_m = \frac{C'_m Q}{C'_m \div Q}$$

At time t_{m-1}, C_{m-1}, Q and $\hat{x}_{m-1}$ are given, and at time t_m, θ_m is measured.

In order to apply the Kalman filter to estimate the power level of the reactor (x_m) from the noisy reading of the temperature channel (θ_m) we need to know the variance (Q) of the noise process and the transition matrix S (m, m-1). Referring again to the long periods of steady operation of the reactor, during the time control is based on the ion-chamber readings, data may be collected to compute the required variance. The transition matrix appears to be a much more difficult problem, since it is an 8 x 8 time-varying matrix, with several inexcessible components. However, with the proposed control system the transition matrix may be approximated by the piece-wise linear relationship of the demanded power to time. In other words, the transition matrix is approximated by the first two terms of the Taylor series expansion, which is accurate for each piece-wise linear segment.

[6.4]. The Adaptive Control Law.

It was indicated in section [6.1.2] that since the identification process is neither complete nor very accurate, it is necessary to use a controller that can alter its own parameters in order to keep the transient response within the desired tolerances. Such a control system, by our definition, is adaptive.

A preliminary study of controller configurations was conducted under the assumption that a perfect measurement of power-level is available. It was found that a proportional controller could not meet the dual requirements of high accuracy and good stability: as the accuracy during the initial part of the transient response improved, the overshoot became larger. The addition of error-rate control reduced the overshoot, but such a controller would need the adjustment of two parameters, and no algorithm for their updating was apparent. Since the prime concern is accuracy, an error squared controller was considered next. This method heavily penalizes any large errors in the system, while the tendency for overshoot is reduced, since a small error calls for only a very slight control effort. The performance of such a controller was studied in detail and as described in the following section, gave satisfactory results.

[6.4.1] Error-squared Controller

It was shown in Chapter 4 that the present control system at Douglas Point is able to maintain the transient response within close tolerances. For a 90-100% power change, the maximum error was 2%, while for a 98-100% change a peak deviation of only 0.2% was observed. Hence, in order to obtain appreciable improvements using an adaptive controller the aim was to reduce the power error to ± 0.02% of full power. It appeared unreasonable to assume that a large system such as a nuclear reactor could be controlled to much smaller accuracies with any certainty.

The error-squared controller was implemented as shown in Figure 6.3. For convenience, the gain constant A that multiplies the error is squared also. To study the behaviour of the system, one has to find a suitable index of performance, one that indicates when optimum operation is achieved, or the change that is required in order to drive the system towards optimum.

Since the primary aim of the control system is to keep the error to a minimum, the index of performance must include a term that evaluates the error.

In our system the interest is more in accurate overall performance, than in keeping the instantaneous value of

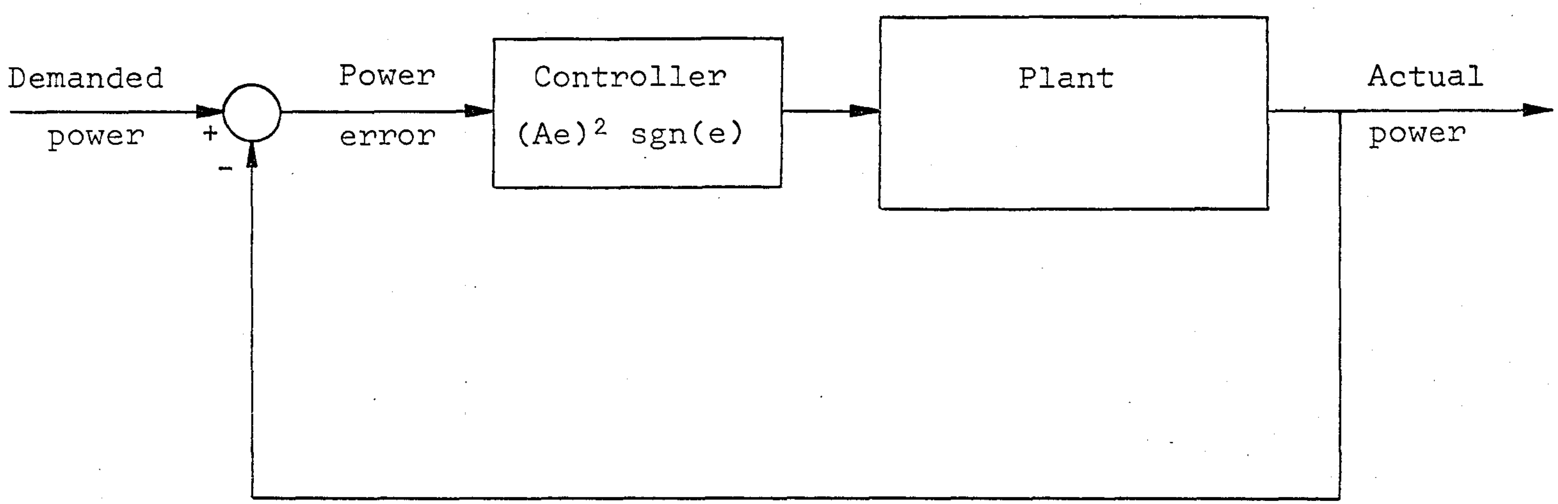


FIGURE 6.3

Error-squared controller.

the error, for example the peak overshoot, within a given tolerance. Hence a cost function or index of performance of the time-integral of the error type was considered. Typical cost functions of this form are integral of error squared, integral of time multiplied by squared error, integral of squared time multiplied by squared error, or other even functions. They each have their relative advantages and disadvantages for a particular application, since they emphasize one aspect of performance with respect to others. For our system time was not considered to be a necessary part of the cost function, since the error was to be kept at a minimum at all times. Also, since the controller itself was of the error-squared type, using a cost function containing the same term would lead to a very sensitive system. It was therefore decided to use an index of performance of the form

$$J = \int_0^T |e| \, dt$$

The integral measures the overall error during a time interval T, or gives the average value of the error if J is divided by T.

To evaluate the cost function, the initial time is assigned to the instant when a change in power is initiated, and the time period T is chosen such as to allow operation to reach steady state again. Referring to the responses obtained in Chapter 4, for a 2% power change T = 50 seconds, for a 10% power change T = 200 seconds are reasonable. To obtain a feel for the cost function, consider the limiting case when the error is at a constant 0.02%. For a 50 second interval

$$J = \int_0^{50} 0.02\, dt = 0.01$$

Since it is not desired to control the system to less than this accuracy, the optimum response for a 2% power change takes place when the resultant value of J is less than or equal to 0.01. Similarly, for a 10% power change $J \leq 0.04$.

The manner in which the controller gain A affects the response of the system was investigated by evaluating the cost functionJ under various conditions. For the 98% - 100% power change, and all parameters at their design values, the graph of J vs. A is shown in Figure 6.4. The response was observed for values of A at 100 increments, and it is seen that the desired optimum, (indicated by the broken line at J = 0.01) is achieved for $A \geq 1100$.

The disadvantage of an error-squared controller is, that since it does not penalize small errors heavily, an underdamped response with several "rings" or sign reversals of the error may be obtained. Rapid back and forth motion of the controller is undesirable in most mechanical systems. In our case we are concerned with the movement of the absorber rod, and the number of changes in the direction of its travel raises a constraint on the control system. The number of sign-reversals of the error, n, are also shown in

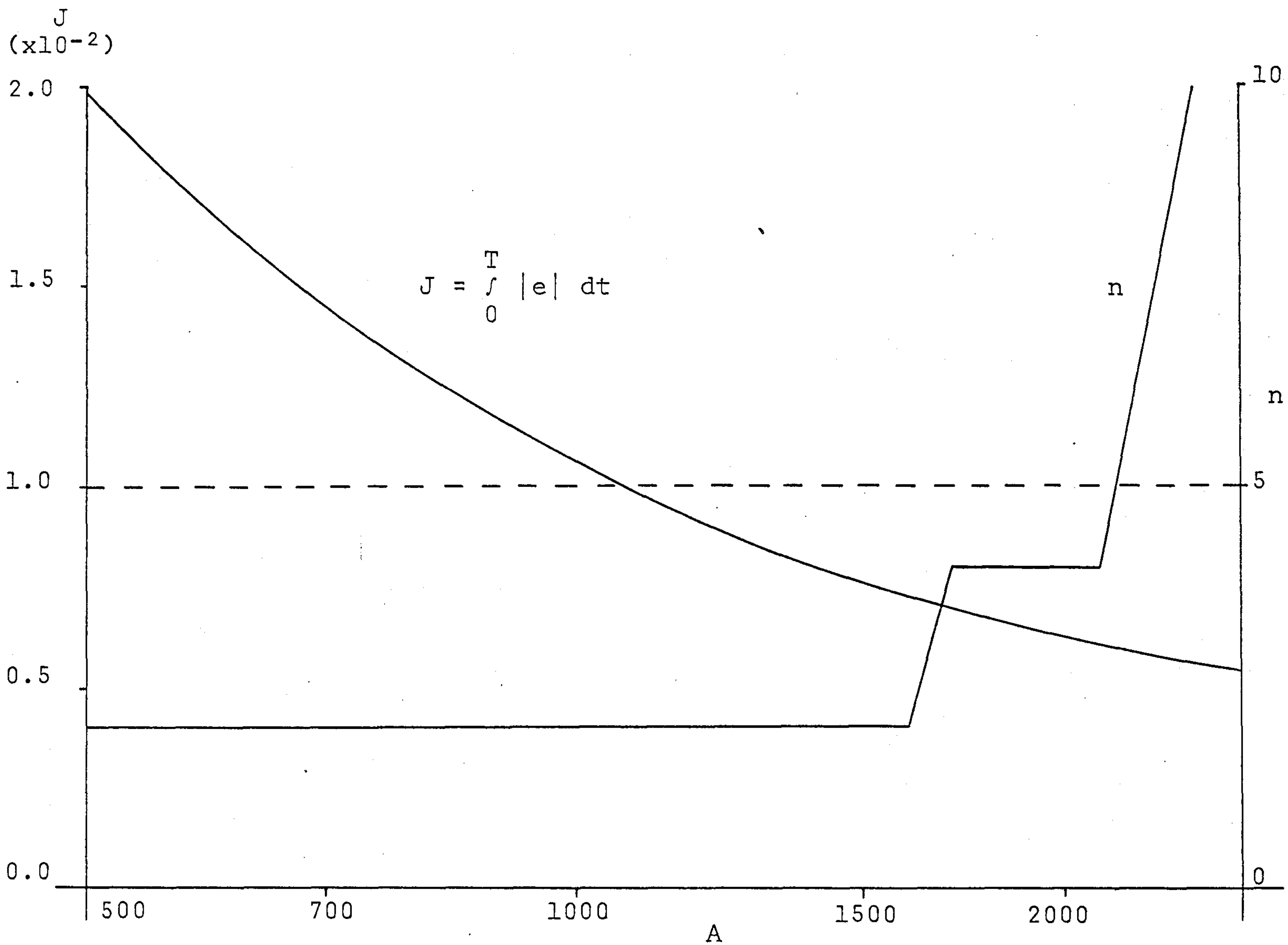


FIGURE 6.4

The cost function (J) and the constraint (n = number of sign reversals of the error) as a function of the controller gain (A).

Figure 6.4. Up to the value of A = 1600, only two sign-reversals are necessary. As the gain is further increased, however, the tendency to overcorrect the error rises, and causes a rapid increase in the number of sign reversals. The acceptable number of direction changes was set at an average of one in every 10 seconds. Hence for the 2% power change considered, five sign reversals may take place. On the scales shown in Figure 6.4 this limit corresponds to the optimum value of J. We can now state more formally our optimum control problem: it is required to find the value of the controller gain A such that

$$J = \int_0^T |e| dt \leq 0.0002T$$

subject to the constraint

$$n \leq n_o$$

where $n_o = 1$ during the first 10 seconds of the transient response and is incremented by one for each additional 10 seconds as time increases.

In optimum control theory the problem would be to find the actual minimum of J subject to the constraint. In Figure 6.4 this would result in the value A = 2100 (to the nearest 100). Alternatively, one could consider the constraint as part of the cost function, and the minimum is then found at the intersection of the two curves, and A = 1700. However, optimum control theory assumes that we have perfect knowledge of the plant, and we know in advance the desired target state. In a practical case such as ours, we have seen that neither of these requirements are met. As the parameters deviate from their assumed values and as the size of power demand change varies, the cost function and the constraint curves alter their shape and position, and the preselected value of A is no longer the optimum. It was partly for this reason, that the optimum was not selected as the minimum, but simply a value below a desirable and practically attainable level. In addition, while in Figure 6.4 there is a considerable range of A that satisfies both the cost function and the constraint, this may not be so in every case. For example, Figure 6.5 shows how the admissible values of A vary as a function of the temperature coefficient, for a power level change from 90% - 100%. It is not difficult to conceive, that if the concentration of delayed neutrons changes also, it may not be possible to satisfy the cost function requirement without violating the constraint. By selecting the minimum value of A that satisfies J the likelihood of n being exceeded is also minimized.

The range of values of A that lead to optimum responses varies not only as a function of the temperature coefficient but also with the operating power level and the magnitude of the power change. In order to cover the whole power range of operation, responses similar to those shown

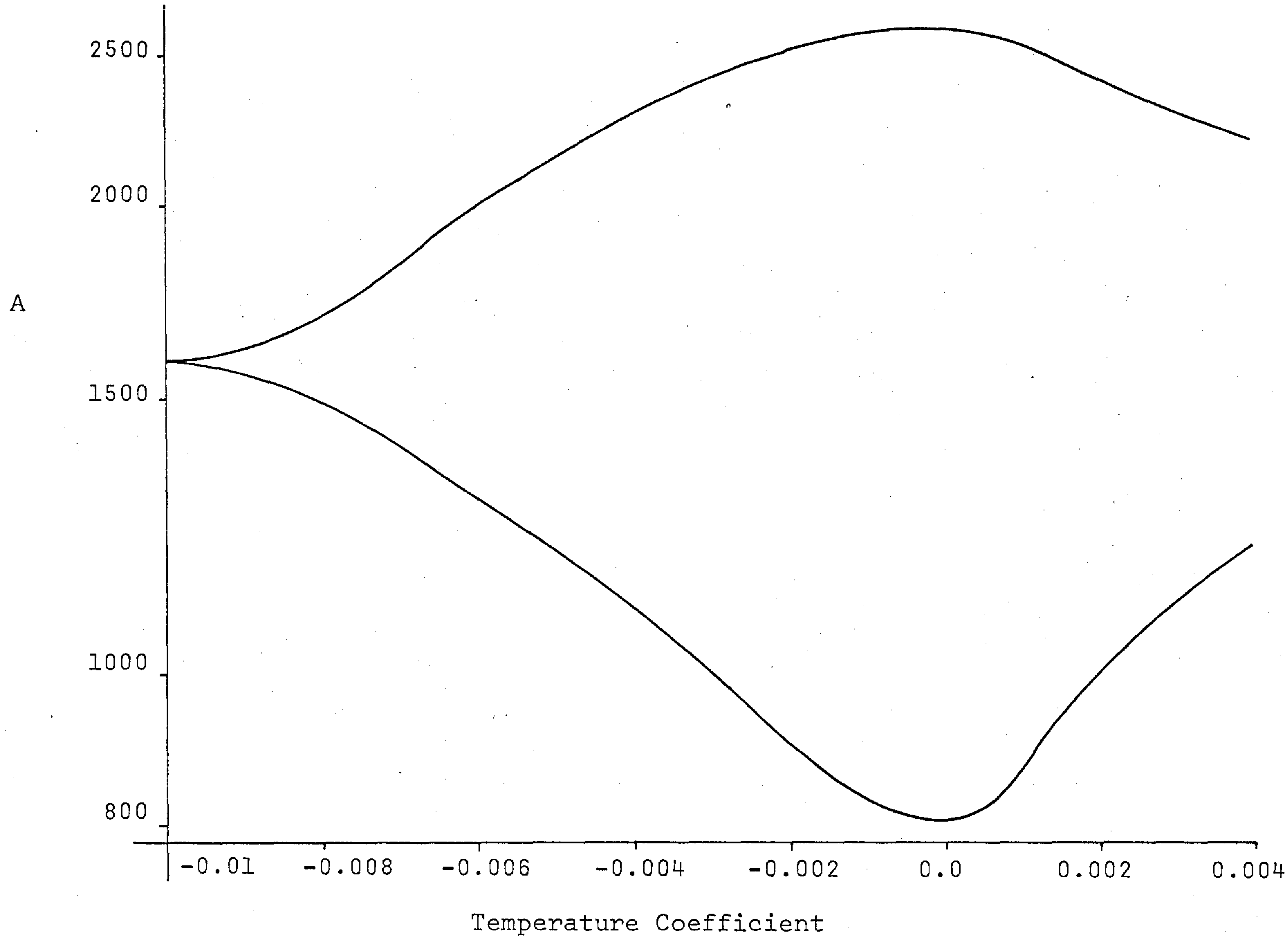


FIGURE 6.5

Admissible values of the controller gain (A) as a function of the temperature coefficient.

in Figure 6.4 were obtained for each 10% power increase, from 20% to full power, and for the temperature coefficient at intervals of 0.002. The results are tabulated in Table 6.1. It is seen that the values of A in adjacent positions vary typically by the actual increment of 100, and at the most by 200. In addition, the inaccuracy of the identification process and the lack of knowledge in advance of the size of the demanded power change, often require changing A by an amount in excess of the above differences in adjacent entries in the table. Hence the intervals used in the table are adequate.

[6.4.2] The Adaptation Algorithm

The purpose of the adaptive controller is to ensure that the controller gain A has the value that leads to the optimum response. Basically, the scheme uses the known power level and the estimated value of the temperature coefficient to select from Table 6.1 the appropriate value of A. The system then proceeds along the transient with this controller gain, and the cost function and the constraint are evaluated at regular intervals. If either measures of performance are not satisfied at any time, A is altered accordingly: increased if the error is too large, decreased if too many sign reversals have taken place.

Whenever adaptation is based on satisfying more than one requirement, the relative weighing of these will

Power level	Temperature Coefficient -0.010	-0.008	-0.006	-0.004	-0.002	0.0	0.002	0.004
20 - 30	1700	1600	1600	1600	1500	1300	1300	1500
30 - 40	1700	1500	1400	1400	1300	1200	1300	1500
40 - 50	1700	1500	1400	1300	1200	1100	1200	1400
50 - 60	1700	1500	1300	1200	1100	1000	1100	1300
60 - 70	1700	1500	1300	1100	1100	900	1100	1300
70 - 80	1700	1500	1300	1100	1000	900	1000	1300
80 - 90	1700	1500	1300	1100	1000	800	1000	1300
90 -100	1600	1500	1300	1100	900	800	1000	1200

TABLE 6.1

Optimum values of the controller gain (A) as a function of the Power level and the Temperature Coefficient.

emphasize one aspect of the response with respect to others. In our system, there are two such requirements: the cost function, measuring the average error; and the constraint, indicating the number of sign reversals of the error. By virtue of their names, we expect to give the constraint more weight. In other words, the cost function can only be reduced (by increasing A) if this does not violate the constraint. On the other hand, if the constraint is not satisfied, A will have to be reduced even if the cost function is larger than desired. In addition, the following features of the scheme add more weight to the constraint over the cost function:

1. After system identification, A is assigned its minimum value for the particular conditions.

2. The constraint n is computed every second, while the cost function only every two seconds.

3. Increases in A take place in increments of 100, but decreases in integer multiples of 100, by the amount the constraint is exceeded, viz. $n - n_o$.

4. "A" will only be increased if the average value of the error $\frac{1}{T}\int_0^T |e|\, dt$ during the

transient exceeds 0.0003, instead of the desired overall value of 0.0002.

5. The system identification is repeated every 50 seconds, resetting A each time to its expected minimum value. Hence the constraint need not be violated in order to decrease A.

[6.5] Results

The simulation program is listed in Appendix IV. In order to appreciate the results obtained from the program we need to concern ourselves only with the following aspects of its organization and timing. (The names of the various subroutines referred to are given in capital letters.)

The response of the reactor is evaluated at intervals of 0.01 second. The noise component of the temperature channel is changed every 0.1 seconds [PRNG], and the Kalman filter applied at 2 second intervals [KALMAN]. The subroutine that simulates the adaptive controller [ADAPT] is called every second.

For the first ten seconds of operation reactor control is based on the temperature channel. This time is sufficient to identify the neutron shielding factor, and thereafter the control system uses the reading of the ion chamber, corrected by the now knownfactor. Another subroutine, IDENT, keeps a continuous record of the applied reactivity changes, and gives an estimate of the value of the temperature coefficient.

When a power transient is initiated, typically after 50 seconds of steady state operation, the known power level and the latest estimate of the temperature coefficient are used to assign A the appropriate value from the array stored in memory [ADALAW]. As the transient proceeds, adaptation of A takes place, depending on the values of the cost function and the constraint. At 50 second intervals A is reset to its value contained in the array for the prevailing conditions.

The workings of the various sections of the program were tested individually, and the final results evaluated under a wide range of operating conditions. Irrespective of the initial power level, the size or direction of the power change or the value of the plant parameters, the system performance was very satisfactory. Some typical responses are shown in Figures 6.6 to 6.8 for operation near full power. In each of these cases, comparison is made with the performance of the original system under ideal conditions of no noise and unity neutron shielding, while the adapted version is subject to noise and a shielding factor of 0.5. However, since the new scheme effectively counteracts these difficulties of

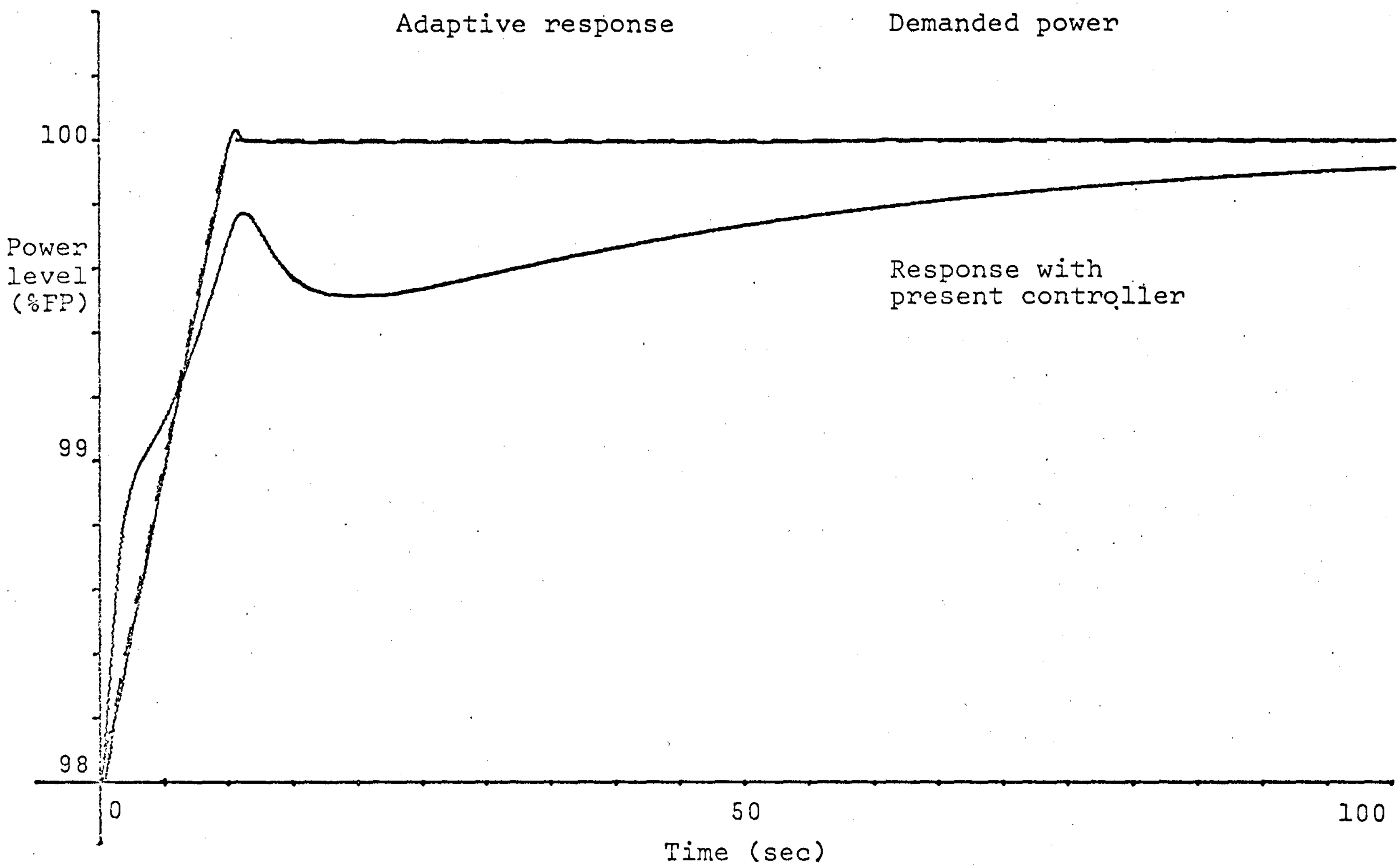


FIGURE 6.6

Response of adaptive system 98%-100%.

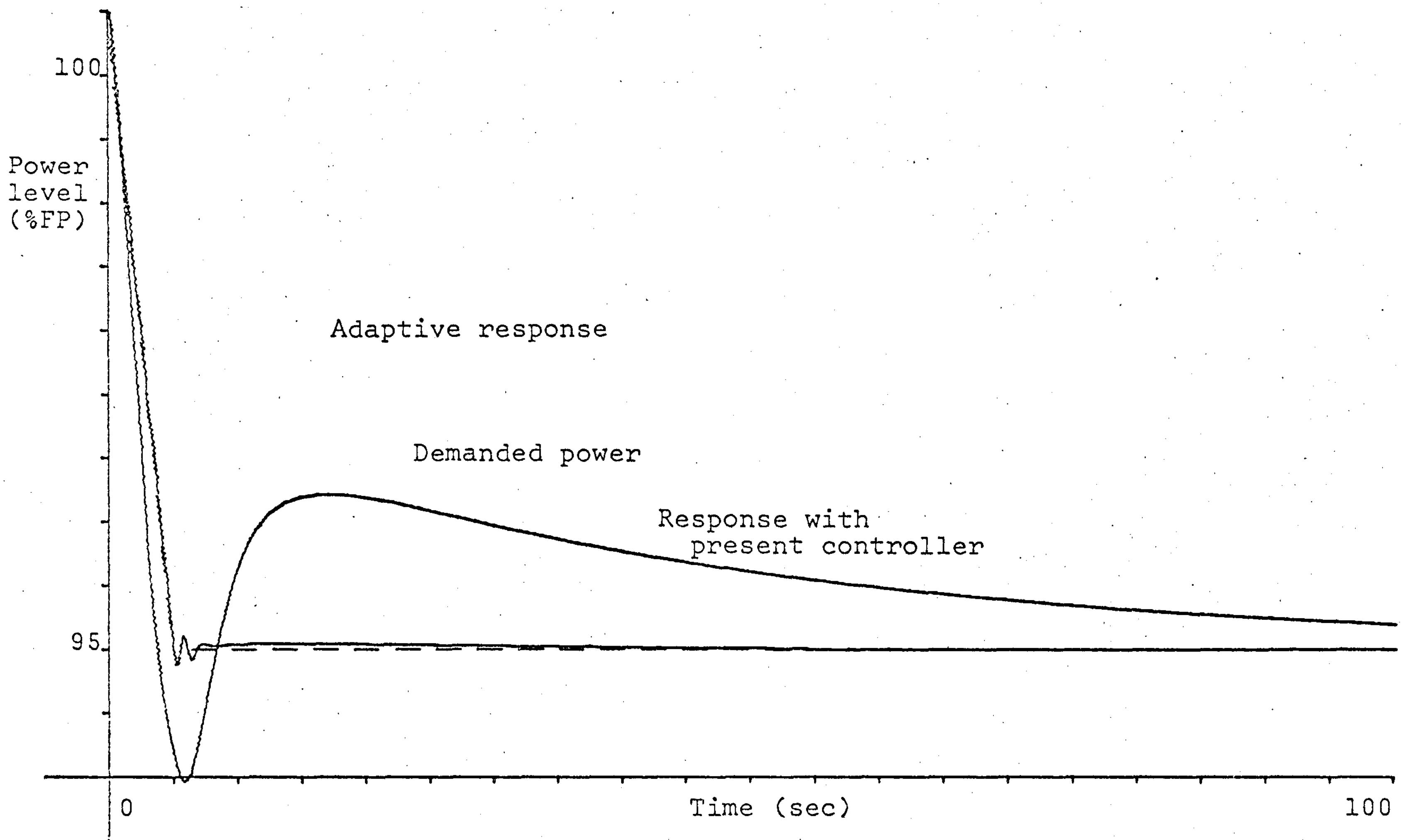


FIGURE 6.7

Response of adaptive system 100%-95%.

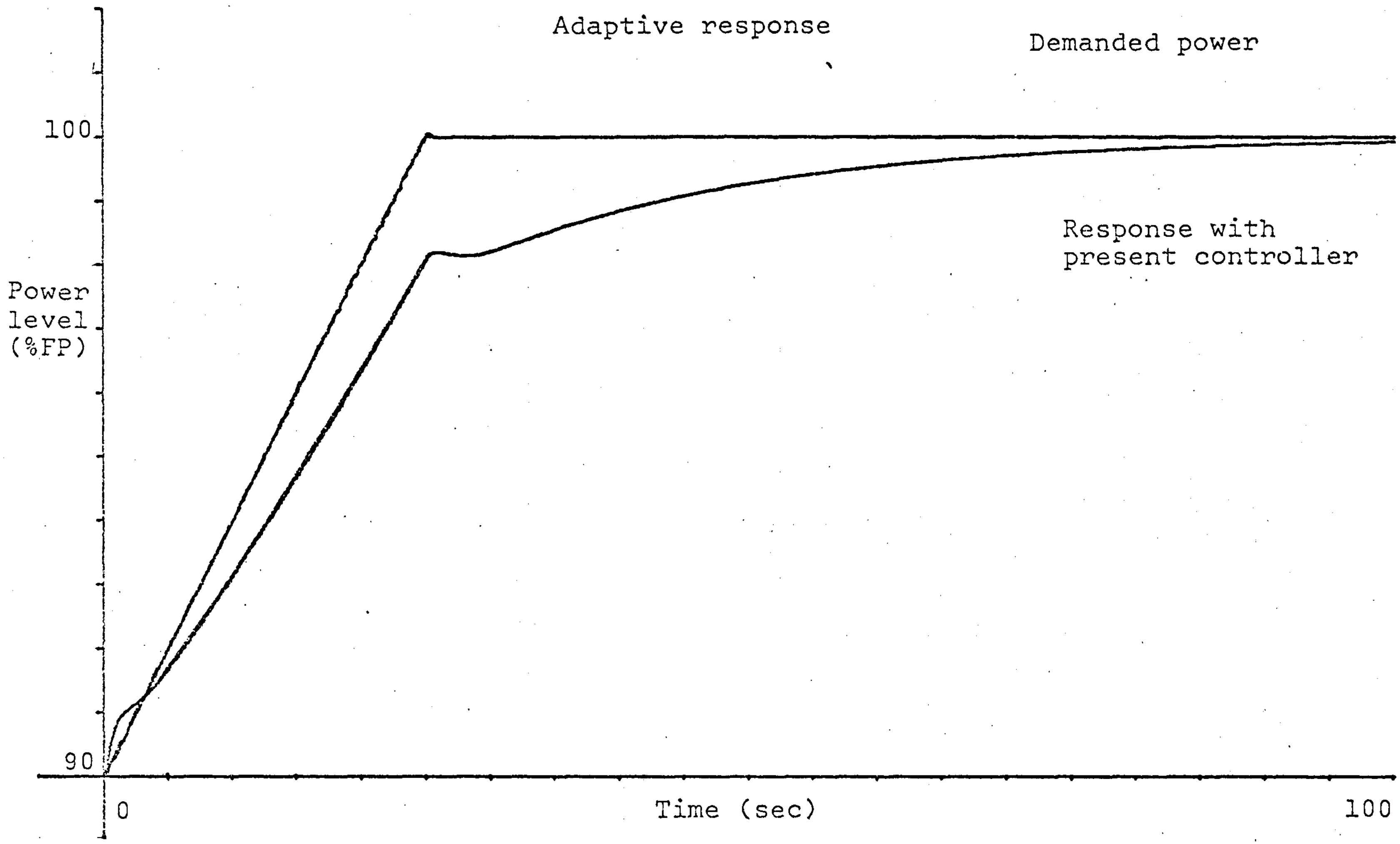


FIGURE 6.8

Response of adaptive system 90%-100%

measurements, this is the most realistic comparison. In each of these diagrams, the demanded power is indicated by the broken line, followed very closely by the response produced, when the adaptive scheme is in operation.

A considerable insight into the operation of the adaptive controller may be gained by observing the changes in the relevant control parameters: the cost function, the constraint, the gain factor and the external reactivity change that is applied to the reactor. For the three responses already discussed, these controller functions are shown in Figures 6.9 to 6.11. In each case, it is desired to reduce the cost function J below 0.3×10^{-3} without exceeding the constraint n = [1 x T / 10]* by changing the value of the gain factor A. Both the cost function and the constraint are computed from zero for each 50 second interval. The resultant control variable, the reactivity change applied by the absorber rod, is also shown.

The manner in which the gain factor is adapted to minimize the cost function is clearly indicated in Figure 6.9. For the first 12 seconds of the transient response, J is above its optimal value, hence A is increased by 100 after every 2 seconds. The value of A is 1700 when the cost function has been reduced below 0.3×10^{-3}, and since the

* The symbol [x] means "the greatest integer not greater than x ".

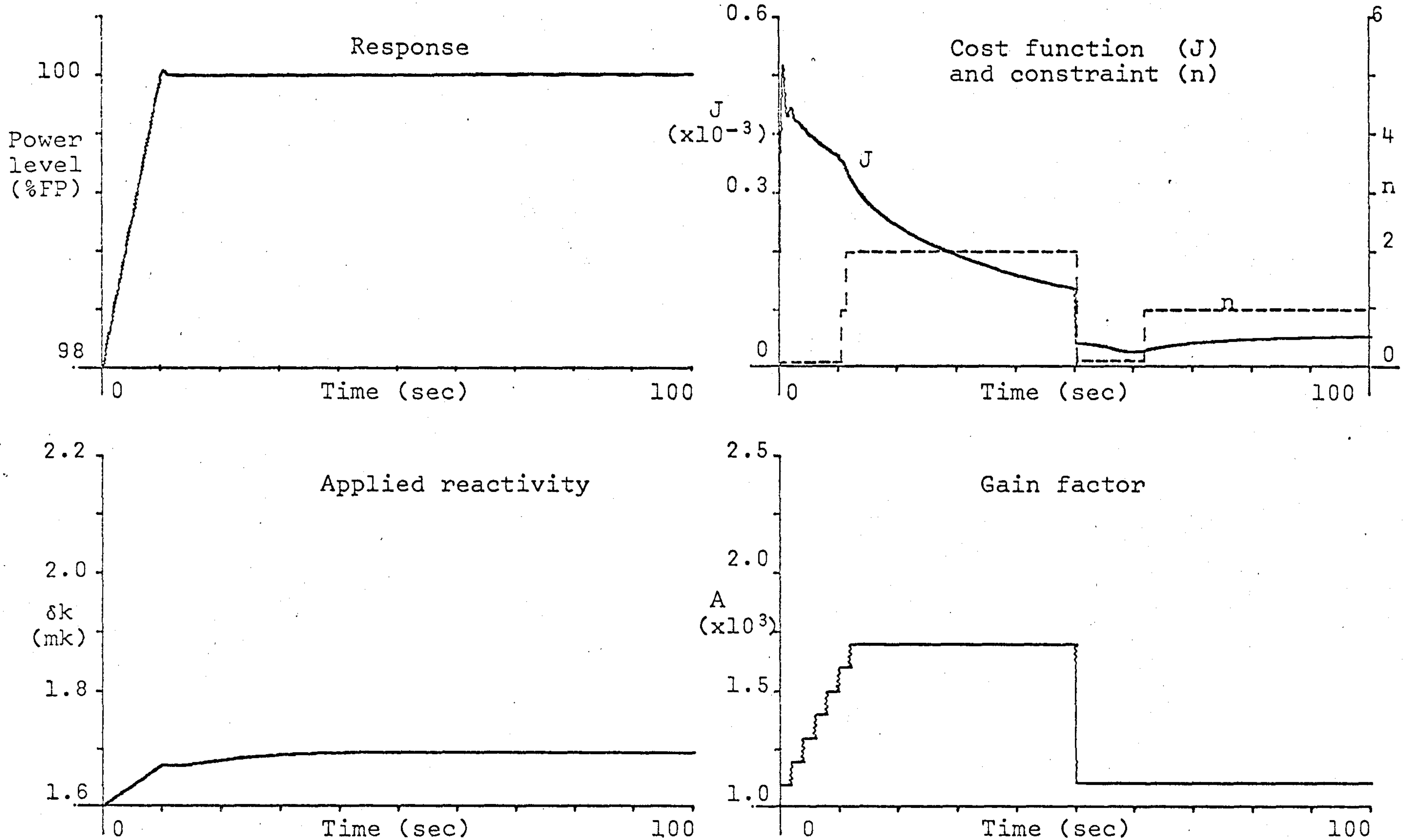


FIGURE 6.9

Response and adaptive control parameter variations

for demanded power change of 98% - 100%.

constraint is not exceeded, this value is maintained for the remaining part of the 50 second interval. It is interesting to note, that the value of A = 1700 corresponds in Figure 6.4 to the intersection of the J and n functions, and is, in a sense, the absolute minimum. Because of the time-constants present in the practical system the initial area error will always be large, and hence A will increase, and approach this absolute minimum.

The effect of the constraint on the adaptation of the gain factor is shown in Figure 6.10. For the first seconds of operation, A is increased in order to minimize J. After this time, however, several sign reversals of the error take place, and hence the gain factor is decreased. It cannot rise until the constraint is once again satisfied after the 40 second mark. Since the transient takes place in only 5 seconds, this response places the most stringent requirements onto the control system, yet it is seen to operate in a very satisfactory manner, and offer a considerable improvement over the present, unadapted response (Figure 6.7).

The controller parameter changes that govern the transient response from 90% to 100% are shown in Figure 6.11.

It was mentioned with reference to Figure 6.5 that an interesting possibility occurs when the temperature coefficient is -0.01 and the delayed neutron concentration is reduced by 20%, since in this case A cannot be reduced

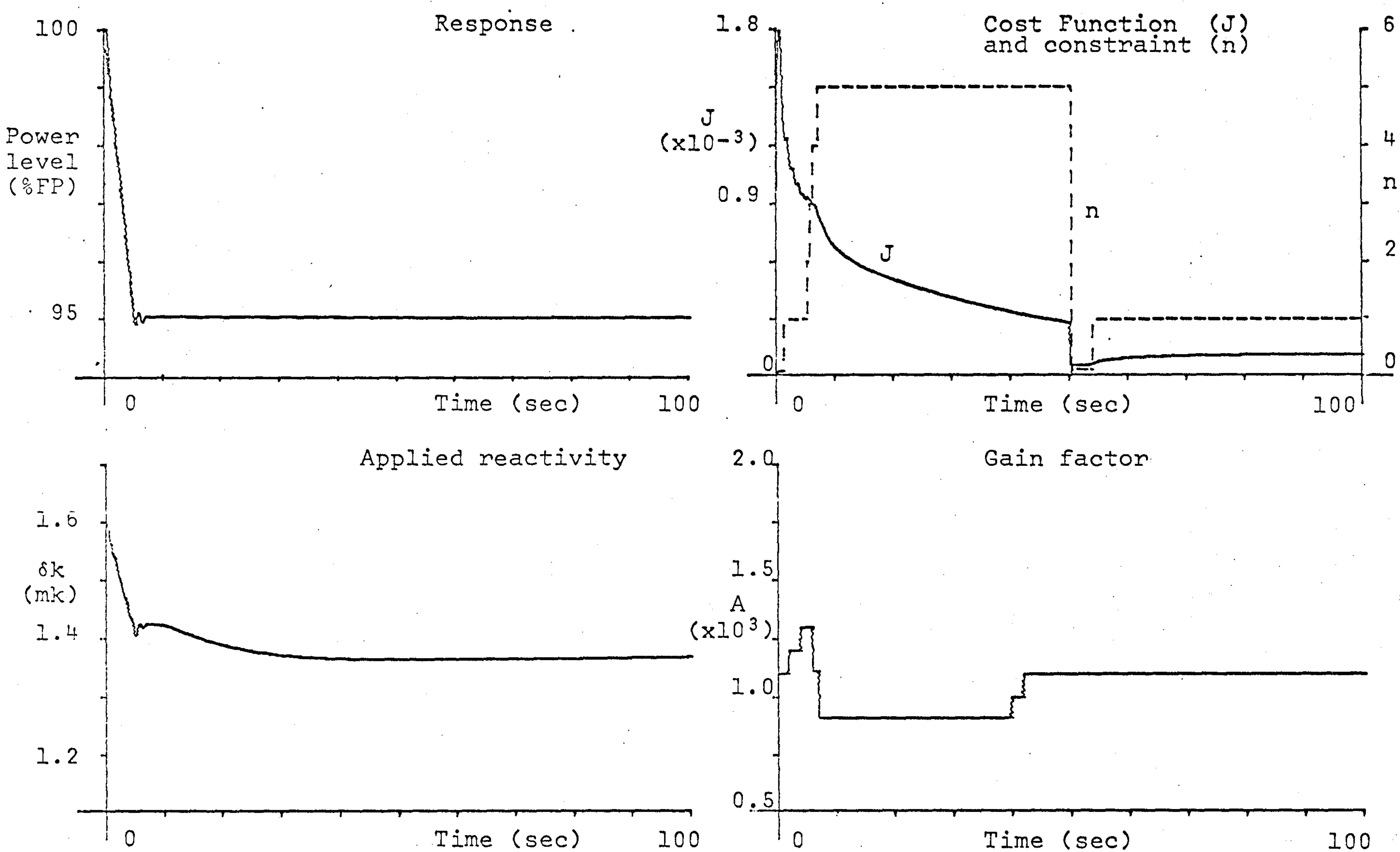


FIGURE 6.10

Response and adaptive control parameter variations for demanded power change of 100%-95%.

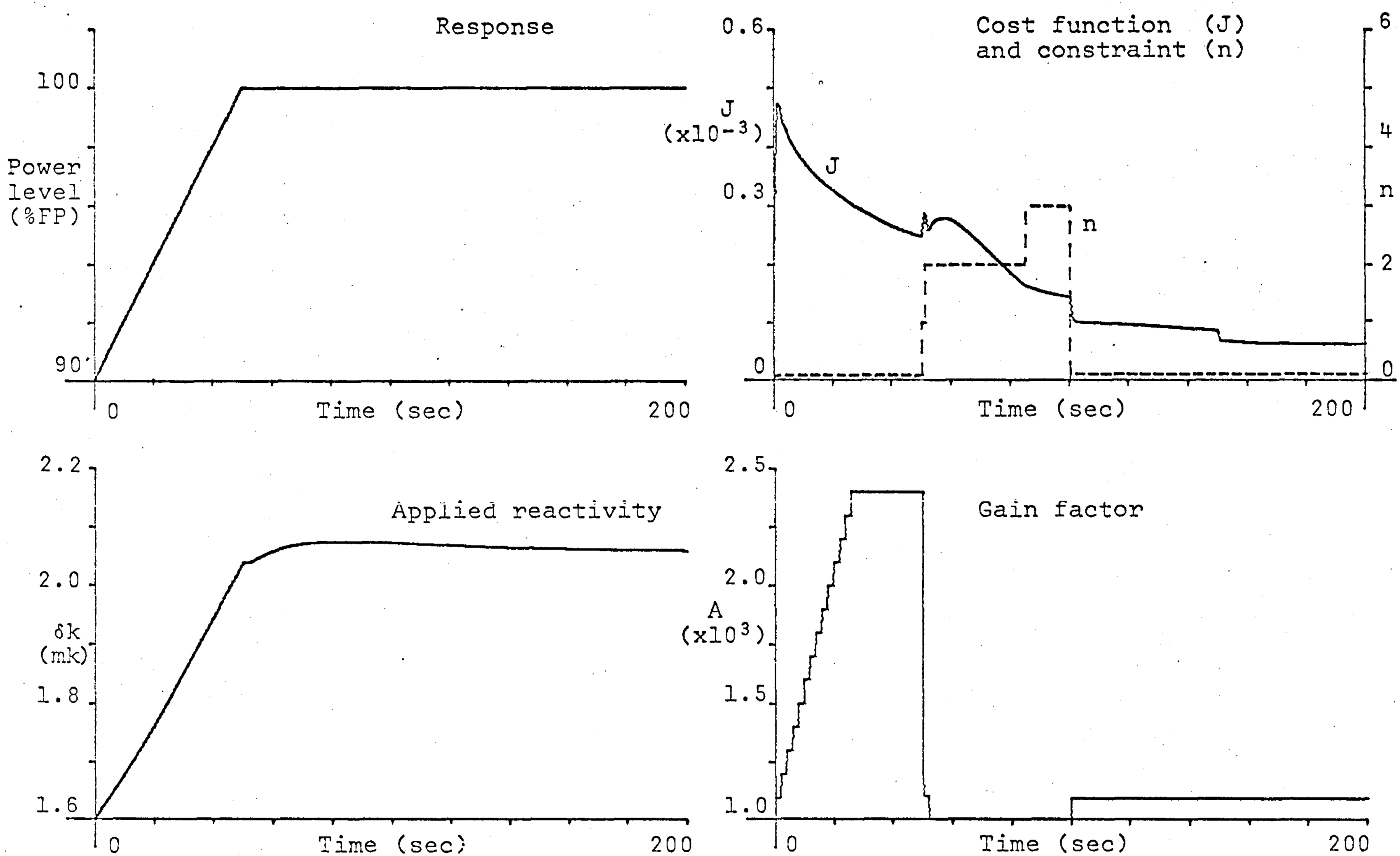


FIGURE 6.11

Response and adaptive control parameter variations

for demanded power change of 90% - 100%.

to its optimum value unless the constraint is violated.

This was, therefore, considered to be a good test for the adaptive controller. The results are shown in Figure 6.12, indicating the expected interplay of the contradictory requirements. Initially both J and n are satisfied, so the gain factor remains constant. After 10 seconds the cost function exceeds 0.3×10^{-3}, hence A is increased and the rise of J is halted. However, this results in exceeding the constraint, so A has to be reduced. The cost function continues to rise until at the 50 second mark the constraint is reset to zero. A similar pattern is repeated for the next 50 second interval, after which steady state operation has been reached.

The manner in which the relative weighing of the cost function and the constraint is realized has a strong bearing on the behaviour of the adaptive controller. For example, instead of resetting the gain factor A to its nominal value every 50 seconds, it could be regulated entirely by the interaction of the cost function and the constraint. This would tend to give A its maximum possible value, hence reducing the cost function, but would result in the more frequent violation of the constraint than is the case for the basic scheme.

The effect of resetting A at regular intervals is

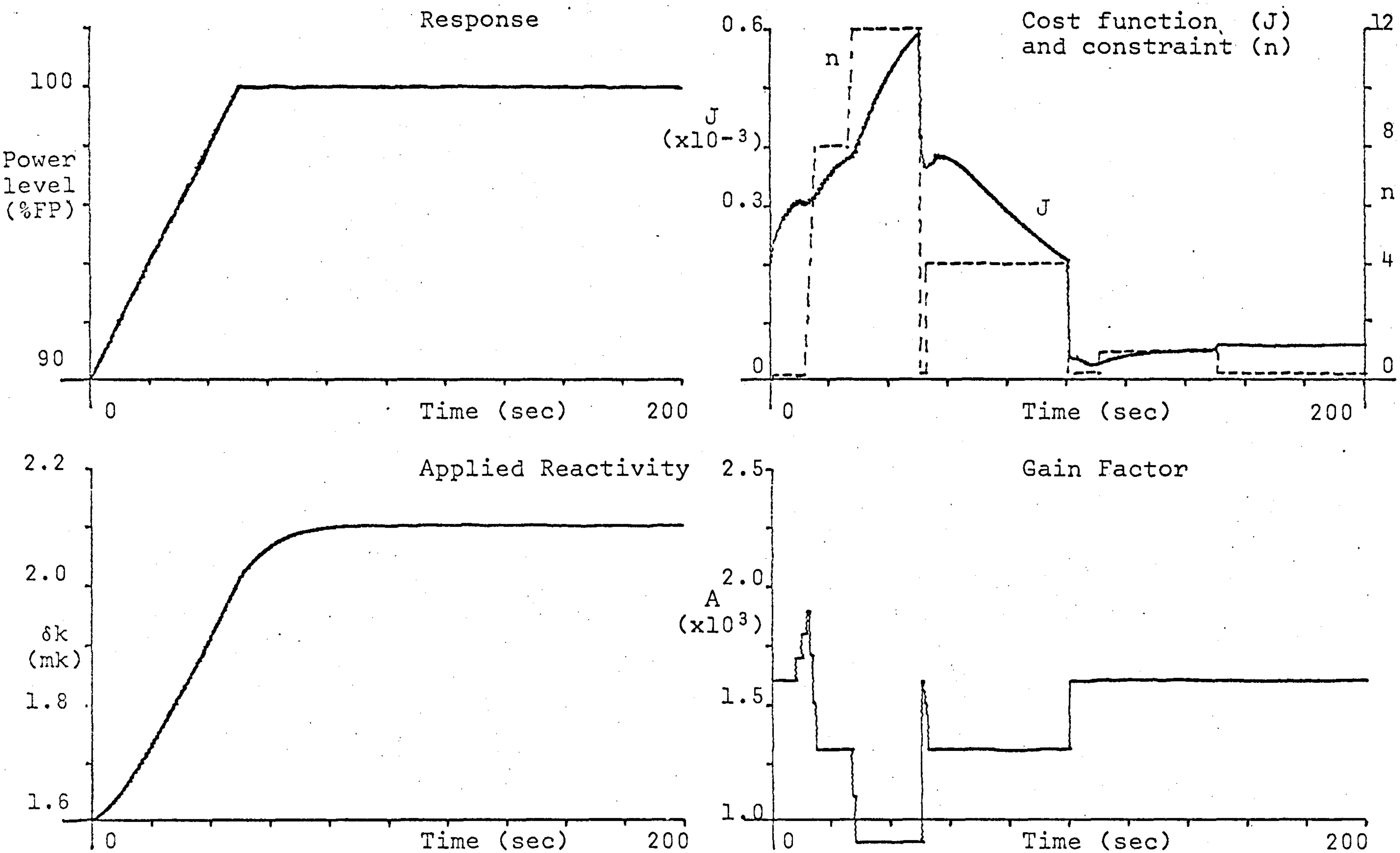


FIGURE 6.12

Response and adaptive control parameter variations for demanded power change of 90%-100%. $T_c = -0.01$, $\beta_1 = 0.8\ \beta$

particularly important during large power level changes, so the two methods are compared for a 20% - 100% change in demanded power. Figure 6.13 shows the behaviour of the originally proposed method. The gain factor is clearly shown to be reset every 50 seconds to its nominal, lower value, to be increased by the least amount necessary to bring J below 0.3×10^{-3}. The sensitivity of the gain factor to changes in the cost function is well illustrated by these diagrams. In addition, the gain factor, after the initial rise, is maintained near its nominal value, and the number of sign reversals are kept to a minimum. The basic scheme may therefore be regarded as a rather conservative one, which is in line with the importance placed on safety in nuclear reactor operations.

In comparison, Figure 6.14 shows the response of the system when the gain factor is a function of J and n only. As expected, the cost function is reduced rapidly, and to a lower value than in the previous case, but only at the expense of many more error sign reversals and a higher average value of the gain factor.

One additional aspect of the adaptive control algorithm was investigated, namely the effect of increasing the time interval at which the reactor power is sampled and changes in the reactivity are applied. A ten-fold increase from 0.01 second to 0.1 second resulted in less than

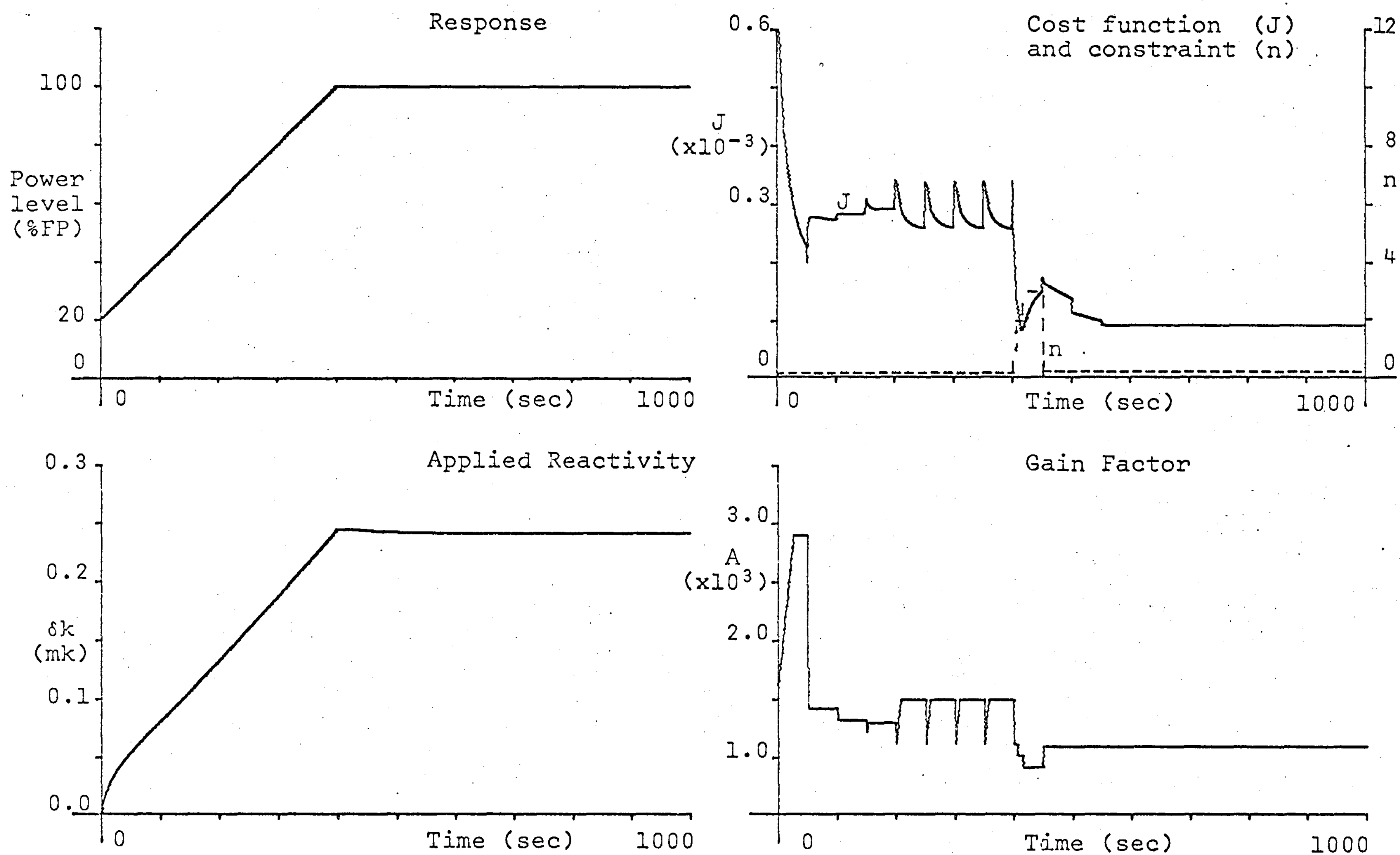


FIGURE 6.13

Response and adaptive control parameter variations for demanded power change of 20%-100% $T_c = -0.003$.

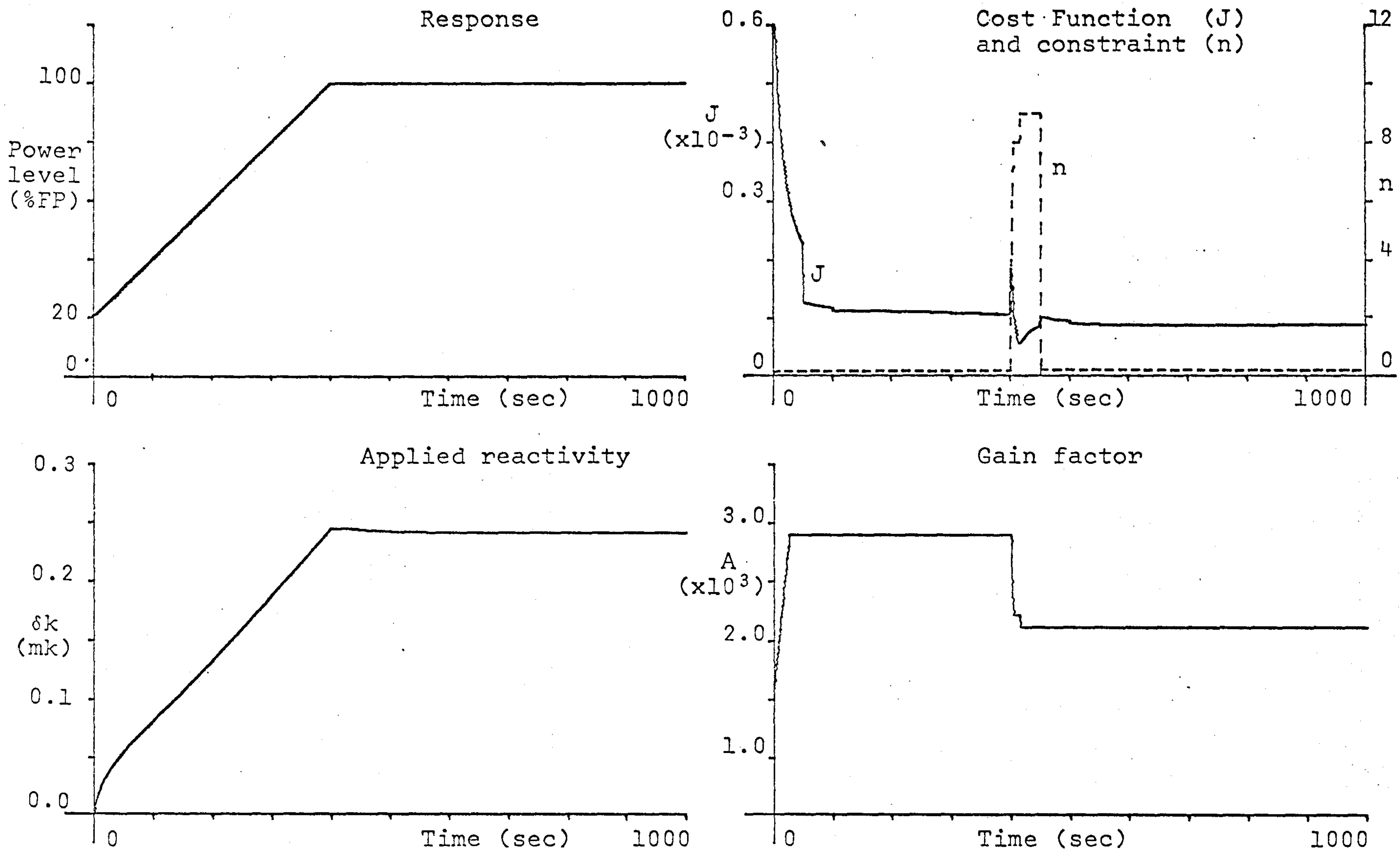


FIGURE 6.14

Response and adaptive control parameter variations for demanded power change of

20%-100% $T_c = -0.003$

Gain factor (A) a function of J and n only.

doubling the peak power error, so the sampling frequency could be reduced in most applications. The basic clock rate used for the simulation studies was also confirmed to be adequate: reducing it to 0.001 second caused less than 1% change in the peak power deviaticn.

CHAPTER 7.

Conclusions and Recommendations for Further Work.

The feasibility of adaptive control of nuclear reactors by a digital computer has been demonstrated in this thesis. In order to keep practical considerations to the forefront, an actual nuclear power plant was considered. A digital computer model of the reactor and the associated analog control system was developed, and the response of this system to power demand changes, in the presence of maximum likely parameter variations, was studied. The nature of the model chosen permitted investigations only in the power range, extending from 20% to 100% of full power. It was found, that the feedback controller in existing use was quite suitable to maintain the system response within a few percent of the desired power level, despite large parameter variations.

In order to illustrate the substantial improvements possible by using adaptive control, the power level was to be kept to less than 0.1% of full power. A controller of the error squared type was found capable of meeting the above specification, provided the associated gain factor was set to a value appropriate to the operating power level and the estimated value of the temperature coefficient, and then adapted during the transient response to compensate for inaccurate parameter identification.

The problem of estimating the power level of the reactor was solved by filtering the noisy reading of the temperature channel, and hence determining the neutron shielding factor, while the reactor is in steady state operation. A transient response may thereafter be controlled using the corrected reading of the ion-chamber.

Since the digital computer will be controlling the reactor "on line", it is essential that the computations associated with the adaptive algorithm be performed in less than a millisecond. The proposed scheme requires only the evaluation of the incremental change of the error, a few logical decisions, and the possible change of the gain factor by a constant amount. In addition, fast memory access is required for the 8 x 8 array that contains the nominal values of the gain factor. The scheme could therefore be very readily implemented on a digital computer, or on a simple special purpose digital machine containing only storage and logic elements.

The adaptive scheme presented in this thesis was developed for the particular reactor we considered. However, the approach used may be applied to the adaptive control of any other type of system, provided a mathematical model for it exists, and the ranges of parameter changes are known. The basic philosophy is that optimum trajectories may be found for the model, and stored in the computer, but since in practice perfect plant identification is rarely possible, the supposedly optimum controller will need to be adapted in order to provide the desired response. The scheme may also be extended to include a learning loop: the originally calculated optimum gain factor may be modified based on evaluation of the actual performance. However, because of the addition of another feedback loop, stability problems may arise.

An important practical feature of the adaptation algorithm is, that the desired performance is achieved by the least possible gain factor, and hence the least control effort. This results in minimizing the amount of absorber material placed into the core, hence increasing its lifetime.

The main purpose of using adaptive control for nuclear reactors is to compensate for the time-varying nature of the plant. However, even on a short-time basis, the improved accuracy of control should result in a sizeable increase in efficiency. For several hundred megawatt generating stations, a saving of even a fraction of a percent represents an amount of considerable commercial significance.

Apart from the already mentioned possibility of including a learning loop in the system, two more major areas of work became apparent during the course of this thesis. One is a natural extension, based on considering a more complete model of the reactor and the rest of the power plant, the other aims at a new appraisal of the philosophy of nuclear reactor control.

The model considered in this thesis did not take into account the effect of the moderator on reactivity, thus preventing a study of the response at low power levels. Since the reactor may be critical over a power range of eight decades, adaptive control may be used to advantage to maintain the desired performance. In addition, the change of poison concentration in the moderator and its effect on the neutron shielding factor should be investigated.

Xenon poisoning and spatial flux disturbances will also be more important for larger size reactors. Since the purpose of a power plant is the generation of electric energy, the reactor should be considered as an integral part of the generating plant, and optimum reactor control should take into consideration the overall efficiency of operation.

It was noted several times in this thesis, that the present, conventional feedback control system is quite satisfactory in following power demand changes. Hence, as long as the specifications for reactor control systems are as liberal as at the present, there is little room for improvement in the performance. The approach for future work should therefore be to establish new criteria of optimality, subject only to the physical limitations of the system and safety requirements

The efficient operation of a large scale power plant, such as the present day nuclear reactors, necessitates the use of digital computers for control purposes. These machines, in turn, offer facilities far beyond the most sophisticated control systems of the past. In addition, the recent developments of optimization theory and adaptive control techniques make possible the effective use of the computer to control the reactor in the most efficient manner. The gap, however, between modern control theory and its application to systems in actual use is quite wide at present, but its bridging offers a challenging and rewarding field of endeavour.

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APPENDICES

APPENDIX I

Parameters of Douglas Point Reactor

l^* = mean effective lifetime of a neutron

$= 7.216 \cdot 10^{-4}$ second

β = fraction of total neutrons delayed

$= 4.867 \cdot 10^{-3}$

β_i = fraction of neutrons delayed in the ith group

λ_i = the decay constant of the ith delayed neutron group

Delay Group	β_i (%)	λ_i (sec^{-1})
2	0.05667	1.61
3	0.16067	0.457
4	0.14200	0.154
5	0.11067	0.0315
6	0.01667	0.0125

APPENDIX II

Discrete-time approximate solution of first-order differential equations.

Consider the following d.e.

$$\dot{x} = A\,x + B\,u \qquad (1)$$

By definition of a derivative, the LHS of (1) may be written as

$$\dot{x}\,(t) = \lim_{\Delta t \to 0} \frac{x\,(t + \Delta t) - x\,(t)}{\Delta t} \qquad (2)$$

For a sufficiently small interval $T = \Delta t$ the function $x\,(t)$ may be regarded as constant, hence we can equate the RHS's of equations (1) and (2)

$$\frac{x\,(t + T) - x\,(t)}{T} = A\,x\,(t) + B\,u\,(t)$$

Rearranging, we obtain

$$x\,(t + T) = T\,A\,x\,(t) + T\,B\,u\,(t) + x\,(t)$$

Letting $t = k\,T$

$$x\,[\,(k + 1)\,T\,] = (\,1 + T\,A\,)\,x\,(kT) + T\,B\,u\,(kT)$$

APPENDIX III

SIMULATION PROGRAM FOR THE DOUGLAS POINT REACTOR

AND THE EXISTING CONTROL SYSTEM

```
C  CD(I)     CONCENTRATION OF DELAYED NEUTRONS IN GROUP I
C  BD(I)     FRACTION OF DELAYED NEUTRONS IN GROUP I
C  DL(I)     DECAY CONSTANT OF DELAYED NEUTRONS IN GROUP I
C  APT1      ARRAY STORING DELAYED POWER VALUES OF TEMPERATURE CHANNEL
C  TCO       TEMPERATURE COEFFICIENT
C  SILD      SHILDING FACTOR (NEUTRON)
C  TORD      TRANSPORT DELAY
C  TORR      RTD TIME-CONSTANT
C  PDEMO     DEMANDED POWER AT T=0
C  DELP      RATE OF CHANGE OF DEMANDED POWER  FP/SEC
C  TD        TIME DURATION OF POWER DEMAND CHANGE
C  T         TIME INCREMENT
C IDATA      =0 IF LAST SET OF DATA,  =1 OTHERWISE
C  NCL       NUMBER OF ITERATIONS BEFORE PRINT-OUT OCCURS
C  NCP       NUMBER OF ITERATIONS BEFORE PUNCHED OUTPUT OCCURS
C  BETAF     FRACTIONAL DEVIATION FROM ASSUMED VALUE OF BETA
C  PDEM      DEMANDED POWER
C  P         ACTUAL REACTOR POWER
C  PINC      DEMANDED POWER INCREASE
C  TT        TOTAL TIME
C  DKT       REACTIVITY CHANGE DUE TO TEMPERATURE COEFFICIENT
C  DK        REACTIVITY DUE TO ABSORBER ROD
C  DKA       ACTUAL VALUE OF REACTIVITY
C  ANEUT     NEUTRON LEVEL
C
C
      DIMENSION APT1(200), AOUTP(2000)
      DIMENSION CD(6), BD(6), DL(6)
C
    1 FORMAT ( E8.1, F7.4, F6.2, F6.3, I2, I6, I4, I3 )
    2 FORMAT ( 1X, F7.2,     8E15.5, I4)
    3 FORMAT ( 1H1, 4X, 1HT, 10X, 4HPDEM, 13X, 1HP, 12X, 3HI17, 12X,
     1  3HI11, 12X, 3HI14, 12X, 3HI35, 12X, 4H I34, 11X, 3HDKR,/)
    4 FORMAT ( 8F10.1 )
    6 FORMAT ( ////8H PDEMO =, E8.1, /8H DELP  =, F7.4, /5H TD =, F6.2,/
     1 5H T  =, F6.3,/5H ITD=, I6, /5H NCL=, I4, /8H IDATA =, I2)
    7 FORMAT ( /// 8X, 2F5.2, / 5X, F8.5, F5.2)
    8 FORMAT ( I10, 7F10.5 )
    9 FORMAT ( 10F8.6 )
```

```
C
      DATA BD / 0.0, 0.0005667, 0.0016066, 0.00142, 0.0011067,0.0001666/
      DATA DL / 0.0, 1.61, 0.457, 0.154, 0.0315, 0.0125/
      DATA TCO, SILD /-0.00454, 1.0/
      DATA TORD, TORR/ 2.0, 1.00/
      BETAF = 1.0
      BETAFO = 1.0
   10 READ ( 5, 1 ) PDEMO, DELP, TD, T, IDATA, ITD, NCL, NCP
      READ ( 5, 4 ) TCO, SILD, BETAF
      IF ( BETAF .EQ. BETAFO ) GO TO 13
      DO 99 ID = 2, 6
      BD ( ID ) = BETAF * BD(ID) / BETAFO
   99 CONTINUE
      BETAFO = BETAF
   13 CONTINUE
      PDEM = PDEMO
      P = PDEM
      NCNT = 0
      NCNP = 0
      IR = 0
      PINC = DELP * TD
      TT = 0.0
      CI14 = 0.0
      CI35 = 0.0
C
C  INITIALIZE CONTROL SYSTEM
C
C
C
      YI34 = 0.0
      YIO34 = YI34
      CI34 = 0.0
C
      IPT = TORD / T  + 0.1
      DO 200 I = 1, IPT
      APT1(I) = P
  200 CONTINUE
      PT2 = P
      CI17 = - 50.55 * P
C
      PN = P * SILD
      YN = - 1250.0 * PN
      YNO = YN
      CI11 = 0.0
C
      VERRI = -4.0
      VDRIVE = - 4.0
      WRITE ( 6, 3 )
      WRITE ( 6, 2 ) TT,PDEM,P,CI17, CI11, CI14, CI35, CI34
C
```

```
C  INITIALIZE REACTOR
C
      BETA = 0.0048666 * BETAF
      SL = 0.0007216
      DKT =-TCO * P
      DK = 0.0
      DK = 0.0016
      DKO = DKT - DK
      ANEUT = P
      SUM = 0.0
      DO 11 ID = 2, 6
      CD(ID) = BD(ID) * ANEUT / SL / DL(ID)
   11 SUM = SUM + DL(ID) * CD(ID)
C
C  THE CONTROL SYSTEM
C  ******************
C
C  TO GENERATE POWER DEMAND
C
      DO 25 IT = 1, ITD
      TT = IT
      TT = TT * T
      IF ( TT .GT. TD ) GO TO 35
      CI35 = 0.0
      IF ( PDEM .GT. 0.150 .OR. DELP .LT. 0.0) GO TO 23
      PDEM = PDEMO * EXP( 0.04 *TT)
      VDP = -0.66 * ALOG10(PDEM) - 6.192
      IF ( PDEM .LE. 0.001) GO TO 30
      IF ( PDEM .LE. 0.010) GO TO 31
      GO TO 32
   31 CI35 = 40000.0 / 3.0 / 263.76 * PDEM
   30 CI35 = CI35 - ( 222.0 + VDP / 0.025882)
      GO TO 26
   23 PDEM = PDEM + DELP * T
   32 IF ( PDEM .GT. 0.6 ) GO TO 33
      VDP = -0.66 * ALOG10(PDEM) - 6.192
      X = VDP
      CI35 = -(2625.64 + 1895.99*X + 546.119*X**2 + 72.4341*X**3
     1  + 3.65433*X**4 )
   33 CI35 = CI35 + 40000.0 / 3.0 / 263.76 * PDEM
C
C  RATE OF CHANGE OF POWER DEMAND
C
   26 V24 = 1000.0 * DELP
      DEPO = 0.04 * PDEM
      IF ( P .LT. 0.15 .AND. DELP .GT. 0.0 ) V24 = 1000.0 * DEPO
   35 YI34 = ( 1.0 - T / 6.0 ) * YI34 + T /5.48 * V24
      CI34 = ( YI34 - YIO34 ) / T
      IF ( ABS( CI34) .LT. 0.0001) CI34 = 0.0
      YIO34 = YI34
```

```
C   POWER INPUT FROM REACTOR
C
C   TEMPERATURE CHANNEL
C
   34 IF ( P .LT. 0.001 ) GO TO 27
      DO 201 I = 2, IPT
      APT1(I) = APT1(I-1)
  201 CONTINUE
      APT1(1) = P
      PT1 = APT1(IPT)
      PT2 = PT2 + T * ( PT1 - PT2 ) / TORR
      CI17 = ( 1.0 - T / 9.5 ) * CI17 - T * 50.55 / 9.5 * PT2
      GO TO 28
   27 CI17 = 0.0
C
C   NEUTRON RATE CIRCUIT
C
   28 TORN = 0.000045 / P
      PN = P * SILD
      YN = ( 1.0 - T / 24.1 ) * YN - T *1250.0/ 24.1 * PN
      CI11 = ( YN - YNO ) / T
      YNO = YN
C
C   LOG NEUTRON CIRCUIT
C
      PLOGN = PN
      VLOGN = -0.66 * ALOG10(PLOGN) - 6.192
      IF ( PLOGN .GT. 0.010) GO TO 20
      CI14 = 222.0 + VLOGN / 0.025882
      GO TO 22
   20 IF ( PLOGN .LT. 0.6) GO TO 21
      CI14 = 0.0
      GO TO 22
   21 X = VLOGN
      CI14 =    2625.64 + 1895.99*X + 546.119*X**2 + 72.4341*X**3
     1 + 3.65433*X**4
      IF ( CI14 .LT. 0.0 ) CI14 = 0.0
C
C   ERROR VOLTAGE
C
   22 VERR = -3.01 * ( CI17 + CI11 + CI14 + CI35 + 4.0 / 3.01 + CI34 )
      VERRI = ( 1.0 - T / 0.03 ) * VERRI + T / 0.03 * VERR
      IF ( ABS(VERRI+ 4.0 ) .LT. 0.001) VERRI= -4.0
C
C   ABSORBER ROD
C
      VDRIVE = ( 1.0 - 6.3 * T ) * VDRIVE + 6.3 * T * VERRI
      DKR = -0.00003 * (VDRIVE+ 4.0 )
      IF (VDRIVE.LE. -6.0 ) DKR = 0.00006
      IF (VDRIVE.GE. -2.0 ) DKR =-0.00006
C
```

```
C   THE REACTOR
C   ***********
C
C   COMPUTE NEUTRON LEVEL
C
      DKT = ( 1.0 - 0.08* T ) * DKT - T * 0.08*TCO * P
      DK = DK  + T * DKR
      IF ( DK .GT. 0.003 ) DK = 0.003
      IF ( DK .LT. 0.0   ) DK = 0.0
      DKA = DK - DKT + DKO
      EN = T * (DKA- BETA) / SL
      ANEUT = ( EN + 1.0 ) * ANEUT + T * SUM
      P = ANEUT
      IF ( P .GT. 1000.0 ) GO TO 39
C
C   COMPUTE DELAYED NEUTRONS
C
      SUM = 0.0
      DO 14 ID = 2, 6
      BINL = BD(ID) * ANEUT / SL
      TLI =T *DL(ID)
      CD(ID) = ( 1.0 - TLI ) * CD(ID) + T * BINL
   14 SUM = SUM + DL(ID) * CD(ID)
C
C   WRITE AND PUNCH RESULTS
C
      NCNT = NCNT + 1
      IF ( NCNT .LT. NCL) GO TO 37
      NCNT = 0
      IPPC= 10000.0 * ( PDEM - P )
      IDK = 10000.0 * DK
      WRITE ( 6, 2 ) TT,PDEM,P,CI17, CI11, CI14, CI35, CI34, DKR, IDK
   37 NCNP = NCNP + 1
      IF ( NCNP .LT. NCP ) GO TO 25
      NCNP = 0
      IR = IR + 1
      AOUTP(IR) = P
   25 CONTINUE
      WRITE ( 6, 6 )PDEMO, DELP, TD, T, ITD, NCL, IDATA
   39 WRITE ( 6, 7 ) TORD, TORR, TCC, SILD
      WRITE ( 6, 7 ) BETAF
      TNCL = NCP
      T = T * TNCL
      WRITE ( 7, 8 ) IR, PDEMO, DELP, PINC, TD, T
      WRITE ( 7, 9 ) TCO, SILD, BETAF
      WRITE ( 7, 9 ) (AOUTP(I), I = 1, IR )
   29 IF ( IDATA .NE. 0 ) GO TO 10
      STOP
      END
```

APPENDIX IV

SIMULATION PROGRAM FOR THE DOUGLAS POINT REACTOR

USING AN ADAPTIVE CONTROLLER

```
C   ATCOOL   ARRAY CONTAINING DELAYED VALUES OF REACTOR POWER AS INDICATED
C            BY COOLANT TEMPERATURE
C   TSTDY    PERIOD OF INITIAL STEADY OPERATION
C   TRANS    TIME AT WHICH POWER TRANSIENT IS INITIATED
C   TFINAL   DURATION OF SIMULATION STUDY
C   PTEMP    POWER READING OF TEMPERATURE CHANNEL
C   PTEST    ESTIMATE OF POWER READING OF THE TEMPERATURE CHANNEL
C   PNEST    ESTIMATE OF POWER READING OF NEUTRON CHANNEL
C   PP       REACTOR POWER AT PREVIOUS TIME INCREMENT
C   EPR      OUTPUT OF ERROR SQUARED CONTROLLER
C   PEMAX    MAXIMUM POWER ERROR
C   SLDEST   ESTIMATE OF SHILDING FACTOR
C   TCOEST   ESTIMATE OF TEMPERATURE COEFFICIENT
C
      EXTERNAL PRNG, KALMAN, SHIELD, IDENT, ADALAW, ADAPT
C
      DIMENSION CD(6), BD(6), DL(6)
      DIMENSION ATCOOL(200)
      DIMENSION APOWER ( 1000 )
      DIMENSION AA(1000), AE(1000), ADK(1000), NA(1000)
C
    1 FORMAT ( 8F10.1)
    2 FORMAT ( 1X, F7.2,     8E15.5, I3, F4.1)
    3 FORMAT ( 1H1, 4X, 1HT, 10X, 4HPDEM, 13X, 1HP, 10X, 5HPNEST,10X,
     1 5HPTEST,12X, 3H DK, 12X, 3HEPR, 12X, 4HAREA, 11X,4HVERR,/)
    4 FORMAT ( 3F20.6, I10 )
    5 FORMAT ( 1H1, F20.6, // )
    6 FORMAT ( / 50X, E20.6)
    7 FORMAT ( /// 8X, 2F5.2, / 5X, F8.5, F5.2)
    8 FORMAT ( / 22H MAXIMUM POWER ERROR =, F9.6,
     1   10H AT TIME =, F7.3, 8H SECONDS, /
     2   35H NUMBER OF ABSORBER ROD REVERSALS =, I3 )
    9 FORMAT ( 8I10)
  100 FORMAT ( 10F8.6 )
  101 FORMAT ( I10, 7F10.5 )
  102 FORMAT ( 25I3 )
C
      DATA DL / 0.0, 1.61, 0.457, 0.154, 0.0315, 0.0125/
      DATA BD / 0.0, 0.0005667, 0.0016066, 0.00142, 0.0011067,0.0001666/
      BETAFO = 1.0
```

```
      TCO = - 0.00454
      TCO =-0.003
   10 READ ( 5, 1 ) PDEMO, DELPO, T, TSTDY, TRANS, TFINAL, SILD, BETAF
      READ ( 5, 9 ) NCL, NCP, IDATA
C
      IF ( BETAF .EQ. BETAFO ) GO TO 13
      DO 99 ID = 2, 6
      BD ( ID ) = BETAF * BD(ID) / BETAFO
   99 CONTINUE
      BETAFO = BETAF
   13 WRITE ( 6, 3 )
      DELP = DELPO
      PDEM = PDEMO
      P = PDEM
      PTEMP = PDEM
      PNEST = PDEM
      PTEST = PDEM
      PEST = PDEM
      PP = P
C
      IPT = 2.0 / T + 0.1
      DO 40 I = 1, IPT
      ATCOOL(I) = P
   40 CONTINUE
C
      A = 500.0
      VDRIVE = - 4.0
      PEMAX = 0.0
      NCNT = NCL - 1
      NCNP = 0
      PSIGN =  1.0
      NREV = 0
      NDREV = 0
      AREA = 0.0
      DAREA = 0.0
      COUNT = 1.0
      NSEC = 0
      KTIME = 0
      MTIME = 1.0 / T + 0.1
      TT = -T
      TTT = 0.0
      TD = TRANS - TSTDY
      PINC = DELP * TD
      TSP1 = TSTDY + 1.0
      TSP3 = TSTDY + 3.0 * TD
      TSEC = 0.0
      IRUN = 0
      TORR = 1.0
      SLDEST = 1.0
      TCOEST = -0.00454
      IR = 0
      ACC = 0.00005
```

```
C
C   INITIALIZE REACTOR
C
      BETA = 0.0048666 * BETAF
      SL = 0.0007216
      DKT =-TCO * P
      DK = 0.0016
      IF ( TD .GT. 100.0 ) DK = 0.0
      IF ( DELPO .LT. 0.0 .AND. TD .GT. 20.0 ) DK = 0.003
      DKO = DKT - DK — !! Reactor not critical in steady state ?!
      ANEUT = P
      SUM = 0.0
      DO 11 ID = 2, 6
      CD(ID) = BD(ID) * ANEUT / SL / DL(ID)
   11 SUM = SUM + DL(ID) * CD(ID)
C
C   TO GENERATE POWER DEMAND
C
   20 TT = TT + T
      TTR = 0.0
      IF ( TT .LT. TSTDY .OR. TT .GE. TRANS ) GO TO 21
      IF ( PDEM .GE. 0.150 .OR. DELP .LT. 0.0) PDEM = PDEM + DELP * T
      IF ( PDEM .LT. 0.15 ) PDEM = PDEMO * EXP( 0.04 * TT )
      TTR = 1.0
C
   21 IF ( TT .LT. TSTDY ) GO TO 22
      ISEC = ( TT - TSTDY ) / 50.0
      TSEC = ISEC
      KTIME = KTIME + 1
      TTT = TTT + T
C
C   ERROR VOLTAGE
C
   22 PERROR = PDEM - PNEST
      IF ( TT .LE.10.0 ) PERROR = PDEM - PTEST
      IF ( TT .LE. TSP1 ) GO TO 15
      IF ( SIGN ( 1.0, PERROR ) .EQ. PSIGN ) GOTO 12
      IF ( ABS(PERROR) .LT.    ACC    ) GO TO 12
      PSIGN = - PSIGN
      NREV = NREV + 1
      NDREV = NDREV + 1
      IF ( NREV .GT. 50 .OR.  A .LT.  100.0 ) GO TO 30
   12 IF ( TT .LT. TSTDY + 5.0 ) GO TO 15
      IF ( ABS ( PERROR ) .LE. ABS ( PEMAX ) ) GO TO 15
      PEMAX = PERROR
      TEMAX = TT
   15 EPR = SIGN (( A * ( PERROR    )) ** 2, ( PERROR    ))
      IF ( ABS ( PERROR ) .LT.    ACC   ) EPR = 0.0
      VERR = - 4.0 - EPR
      PP = P
```

```
C
C   ABSORBER ROD
C
        VDRIVE = ( 1.0 - 6.3 * T ) * VDRIVE + 6.3 * T * VERR
        DKR = -0.00003 * (VDRIVE+ 4.0 )
        IF (VDRIVE.LE. -6.0 ) DKR = 0.00006
        IF (VDRIVE.GE. -2.0 ) DKR =-0.00006
C
C   THE REACTOR
C
C   COMPUTE NEUTRON LEVEL
C
        DK = DK  + T * DKR
        IF ( DK .GT. 0.003 ) DK = 0.003
        IF ( DK .LT. 0.0   ) DK = 0.0
        DKT = ( 1.0 - 0.08* T ) * DKT - T * 0.08*TCO * P
        DKA = DK - DKT + DKO
        EN = T * (DKA- BETA) / SL
        ANEUT = ( EN + 1.0 ) * ANEUT + T * SUM
        P = ANEUT
        IF ( P .GT. 1000.0 ) GO TO 30
C
C   COMPUTE DELAYED NEUTRONS
C
        SUM = 0.0
        DO 14 ID = 2, 6
        BINL = BD(ID) * ANEUT / SL
        TLI =T *DL(ID)
        CD(ID) = ( 1.0 - TLI ) * CD(ID) + T * BINL
    14  SUM = SUM + DL(ID) * CD(ID)
C
C   TEMPERATURE CHANNEL
C
        DO 41 I = 2, IPT
        ATCOOL(I) = ATCOOL(I-1)
    41  CONTINUE
        ATCOOL(1) = P
C
        CALL PRNG ( IRUN, TNOISE, PDEMO )
C
        PCOOL = ATCOOL(IPT) + TNOISE
C
C   LOW PASS ACTION OF RTD
C
        PTEMP = ( 1.0 - T / TORR ) * PTEMP + T / TORR * PCOOL
        CALL KALMAN ( IRUN, TTR, PDEM,  DELP, PTEMP, PTEST,EPR,T,Q,C,K)
C
C   NEUTRON CHANNEL
C
C
        CALL SHIELD ( P, PTEST, PNEST, SILD, SLDEST, EPR, TT )
C
        CALL IDENT ( IRUN, DKO, DK, DKR, T, PNEST, TCOEST)
C
```

```
C
C   ADAPTIVE CONTROLLER
C
C
      IF ( ABS ( TT - TSTDY - TSEC *50.0) .LE. T )
     1  CALL ADALAW ( TCOEST, PDEM, IRUN, A, DAREA, NDREV, TTT, NSEC,
     2  KREV, NREVP)
C
      IF ( KTIME .EQ. MTIME ) CALL ADAPT
     1  ( KTIME, NSEC,NDREV, KREV, NREVP, TTT, DAREA, A, EPR)
C
C
      IF ( A .LT. 500.0 ) A = 500.0
      IF ( A .GT. 5000.0 ) A = 5000.0
C
C   COMPUTE AREA ERROR
C
      IF ( TT .LT. TSTDY ) GO TO 31
      XHR = ABS( P - PDEM )
      XHL = ABS ( PP - PDEM + DELP * T )
      AREA = AREA + ( XHR + XHL ) / 2.0 * T
      DAREA=DAREA + ( XHR + XHL ) / 2.0 * T
C
C   WRITE AND PUNCH RESULTS
C
   31 NCNT = NCNT + 1
      IRUN = 1
      IF ( NCNT .LT. NCL) GO TO 27
      NCNT = 0
      PPC = 100.0 * (PP - PDEM )
      AK = 0.001 * A
      WRITE ( 6, 2 )TT,PDEM, P,PNEST,PTEST,DK,EPR,AREA,SLDEST,NREV,AK
   27 IF ( TT .LT. TSTDY ) GO TO 25
      NCNP = NCNP + 1
      IF ( NCNP .LT. NCP ) GO TO 25
      NCNP = 0
      IR = IR + 1
      APOWER ( IR ) = P
      AA ( IR ) = A/ 1000.0
      TTT = TTT + T
      AE ( IR ) = DAREA /TTT* 1000.0
      TTT = TTT - T
      ADK( IR ) = DK * 1000.0
      NA ( IR ) = NDREV
   25 IF ( TT .LT. TFINAL ) GO TO 20
   30 WRITE ( 6, 6 ) PDEMO, DELPO, T, SILD, SLDEST,A,TCO,TCOEST, Q, C, K
      WRITE (6, 8 ) PEMAX, TEMAX, NREV
      TNCP = NCP
      T = T * TNCP
```

```
      WRITE ( 7, 101 ) IR, PDEMO, DELP, PINC, TD, T
      WRITE ( 7, 100 ) TCO, SILD, BETAF
      WRITE ( 7, 100 ) ( APOWER(I), I = 1, IR )
      WRITE ( 7, 100 ) TCO, SILD, BETAF
      WRITE ( 7, 100 ) ( AE ( I ) , I = 1, IR )
      WRITE ( 7, 102 ) ( NA ( I ) , I = 1, IR )
      WRITE ( 7, 100 ) ( AA ( I ) , I = 1, IR )
      WRITE ( 7, 100 ) TCO, SILD, BETAF
      WRITE ( 7, 100 ) ( ADK( I ) , I = 1, IR )
   29 IF ( IDATA .NE. 0 ) GO TO 10
      STOP
      END
```

```
      SUBROUTINE PRNG (IRUN, TNOISE, PDEMO )
C
C   PSEUDO RANDOM NOISE GENERATOR
C
C   TNOISE  POWER READING OF TEMPERATURE CHANNEL INCLUDING ADDITIVE NOISE
C   N       ARRAY REPRESENTING SHIFT-REGISTER
C
      DIMENSION N(16)
C
C
C   SET FLIP-FLOPS TO 1
C
      DATA N / -1, -1, 1, -1, -1, -1, 1, 1, -1, 1, 1, -1, 1, -1,1,1/
      DATA KOUNT/ 9 /
C
C   FORM EXCLUSIVE OR
C
   10 KOUNT = KOUNT + 1
      IF ( KOUNT .LT. 10 ) RETURN
      KOUNT = 0
      IP = - N(1) * N(3) * N(12) * N(16)
C
C   SHIFT FORWARD
C
      DO 11 IN = 1, 15
      I = 17 - IN
   11 N(I) = N(I-1)
      N(1) = IP
C
C   SUM OUTPUT OF EACH STAGE
C
      NSUM = 0
      DO 12 I = 1, 16
      NSUM = NSUM + N(I)
   12 CONTINUE
      SUM = NSUM
      SUM = SUM / 16.0
      IPDEM = 10.0 * PDEMO + 0.1
      SCALE = IPDEM
      TNOISE = SUM * SCALE / 100.0
      RETURN
      END
```

```
      SUBROUTINE KALMAN ( IRUN, TTR, PDEM,  DELP, PTEMP, X, EPR,T,Q,C,K)
C
C   THE KALMAN FILTER
C
C   TTR      =1 DURING POWER DEMAND TRANSIENT  =0 OTHERWISE
C   X        ESTIMATE OF DESIRED QUANTITY (PTEST)
C   Q        VARIANCE OF NOISE
C   C        VARIANCE OF THE ERROR
C   K        KALMAN GAIN
      REAL K, NAVQ
C
      IF ( IRUN .NE. 0 ) GO TO 10
C
C   INITIALISE FILTER
C
      X = PDEM
      IPDEM = 10.0 * PDEM + 0.1
      SCALE = IPDEM
      SCALEO= SCALE/100.0
      Q = 0.5 * 0.06 * SCALEO** 2
      QO = Q
      C = 5.928E-09
      K = 9.879E-06
      KOUNT = 199
      NAVQ = 1000.0
C
C   KALMAN FILTER
C   X = PTEST
C
   10 KOUNT = KOUNT + 1
      Q1 = ( PTEMP - X ) ** 2
      NAVQ = NAVQ + 1.0
      Q = ( Q * ( NAVQ - 1.0 ) + Q1 ) / NAVQ
      IF ( KOUNT .LT.200 ) RETURN
      KOUNT = 0
      IF ( TTR .EQ. 0.0 ) GO TO 11
      IPDEM = 10.0 * PDEM + 0.1
      SCALE = IPDEM
      SCALE = SCALE/100.0
      Q = QO/ SCALEO**2 * SCALE**2
   11 CONTINUE
      X = X + DELP * T * TTR *200.0
      X = X + K * ( PTEMP - X )
      C = C + ( DELP * T ) ** 2 * TTR
      C = C * Q / ( C + Q )
      IF ( ABS ( C ) .LT. 1.0E-12 ) C = SIGN ( 1.0E-12, C )
      K = C / ( Q + C )
      RETURN
      END
```

```
      SUBROUTINE SHIELD ( P, PTEST, PNEST, SILD, SLDEST, EPR, TT )
C
C   EVALUATION OF NEUTRON SHILDING FACTOR
C
C
C   SILD    NEUTRON SHILDING FACTOR
C   SLDEST  ESTIMATED VALUE OF SILD
C
      REAL KOUNT
      DATA KOUNT, SLDE/ 1.0, 1.0 /
      PNEUT = P * SILD
      IF ( TT .GT. 50.0 )  GO TO 10
      AK = 100.0
      IF ( TT .LE.  10.0 ) GO TO 12
      AK = 1000.0
   12 KOUNT = KOUNT + 1.0
      SLD = PNEUT / PTEST
      SLDE = ( SLDE * ( KOUNT -1.0) + SLD ) / KOUNT
      IF ( KOUNT .LT. AK   ) GO TO 10
      SLDEST = SLDE
   11 KOUNT = 1
   10 PNEST = PNEUT / SLDEST
      RETURN
      END
```

```
      SUBROUTINE IDENT ( IRUN, DKO, DK, DKR, T, PEST,  TCOEST )
C
C  IDENTIFICATION OR ESTIMATION OF THE TEMPERATURE COEFFICIENT
C
C
C  DKEST    ESTIMATE OF REACTIVITY CHANGE APPLIED BY THE ABSORBER ROD
C  TCOEST    ESTIMATE OF TEMPERATURE COEFFICIENT
C
      IF ( IRUN .NE. 0 ) GO TO 10
      DKEST = DK
   10 DKEST = DKEST + T * DKR
      IF ( DKEST .GT. 0.003 ) DKEST = 0.003
      IF ( DKEST .LT. 0.0   ) DKEST = 0.0
      TCOEST = - ( DKO + DKEST ) / PEST
      RETURN
      END
```

```
      SUBROUTINE ADALAW ( TCOEST, PDEM, IRUN, A,DAREA,NDREV,TTT,NSEC,
     1                    KREV, NREVP)
C
C  DETERMINATION OF OPTIMAL VALUE OF THE CONTROLLER GAIN FACTOR
C
C
C  AA      ARRAY CONTAINING OPTIMAL VALUES OF GAIN FACTOR (A) AS A
C          FUNCTION OF THE POWER LEVEL AND THE TEMPERATURE COEFFICIENT
      DIMENSION AA ( 8, 8 )
C
      DATA  AA /
     1 1700.0, 1700.0, 1700.0, 1700.0, 1700.0, 1700.0, 1700.0, 1600.0,
     2 1600.0, 1500.0, 1500.0, 1500.0, 1500.0, 1500.0, 1500.0, 1500.0,
     3 1600.0, 1400.0, 1400.0, 1300.0, 1300.0, 1300.0, 1300.0, 1300.0,
     4 1600.0, 1400.0, 1300.0, 1200.0, 1100.0, 1100.0, 1100.0, 1100.0,
     5 1500.0, 1300.0, 1200.0, 1100.0, 1100.0, 1000.0, 1000.0,  900.0,
     6 1300.0, 1200.0, 1100.0, 1000.0,  900.0,  900.0,  800.0,  800.0,
     7 1300.0, 1300.0, 1200.0, 1100.0, 1100.0, 1000.0, 1000.0, 1000.0,
     8 1500.0, 1500.0, 1400.0, 1300.0, 1300.0, 1300.0, 1300.0, 1200.0/
C
   10 JTCO = ( TCOEST * 1000.0 + 12.0 ) / 2.0 + 0.5
C**   IF ( TTT .GT. 10.0 ) GO TO 11
      IF ( JTCO .LT. 1 ) JTCO = 1
      IF ( JTCO .GT. 8 ) JTCO = 8
      IPDEM = PDEM * 10.0 - 0.5
      IF (IPDEM .LT. 1 )IPDEM = 1
      IF (IPDEM .GT. 8 )IPDEM = 8
      A = AA ( IPDEM, JTCO )
   11 CONTINUE
      DAREA = 0.0
      NDREV = 0
      NREVP = 0
      KREV = 0
      TTT = 0.0
      NSEC = 0
      RETURN
      END
```

```
      SUBROUTINE ADAPT2 ( KTIME, NSEC, NREV, KREV, NREVP, TT,AREA,A,EPR)
C
C   THE ADAPTIVE CONTROLLER
C
C   ADAPTATION OF THE CONTROLLER GAIN FACTOR BASED ON THE EVALUATION OF
C           THE COST FUNCTION AND THE CONSTRAINT
C
C
C   AREA    AREA ERROR GIVING VALUE OF THE COST FUNCTION
C   NREV    NUMBER OF SIGN-REVARSALS OF THE ERROR GIVING VALUE OF THE
C           CONSTRAINT
C
      KTIME = 0
      NSEC = NSEC + 1
      NREVM = 1 + NSEC / 10
      IF ( EPR .EQ. 0.0 ) GO TO 11
      IF ( 2 * ( NSEC / 2 ) .NE. NSEC ) GO TO 11
      IF ( ABS ( EPR ) .GT.10.0 ) GO TO 11
      IF ( AREA .GT. 0.0003 * TT .AND. NREV .LE. NREVM ) A = A + 100.00
   11 IF ( NREV .GT. NREVM ) GO TO 10
      NREVP = 0
      KREV = 0
      RETURN
   10 IF ( KREV .EQ. 0 ) AN = NREV - NREVM
      IF ( KREV .EQ. 1 ) AN = NREV  - NREVP
      A = A - AN * 100.0
      KREV = 1
      NREVP = NREV
      RETURN
      END
```