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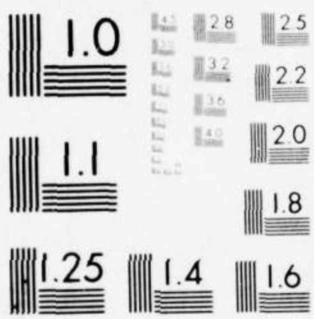

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ADAPTIVE CONTROL OF SINGLE-INPUT, SINGLE-OUTPUT LINEAR SYSTEMS*

A. Feuer and A. S. Morse

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Department of Engineering & Applied Science

Yale University

New Haven, Ct., U.S.A. 06520

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ABSTRACT

A procedure is presented for designing an adaptive control for a single-input, single-output process admitting an essentially unknown but fixed linear model, so that the resulting closed-loop system is stable with zero steady-state tracking error between the output of the process and the output of a prespecified linear reference model. The adaptive controller is a differentiator-free dynamical system forced only by the process input and output, as well as a reference input.

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A procedure is presented for designing an adaptive control for a single-input, single-output process admitting an essentially unknown but fixed linear model, so that the resulting closed-loop system is stable with zero steady-state tracking error between the output of theprocess and the output of a prespecified linear reference model. The adaptive controller is a differentiator-free dynamical system forced only by the process input and output, as well as a reference input.

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INTRODUCTION

In this paper we consider the problem of designing an adaptive control for a single-input, single-output process admitting an essentially unknown but fixed linear model, so that the tracking error between the output of the resulting controlled system and the output of a prespecified linear reference model is regulated to zero asymptotically. We assume that only the process input and output can be measured and we require the adaptive controller to be a differentiator-free dynamical system realizable with conventional analog components.

Although the development of a methodology for designing such a system is clearly one of the fundamental problems of adaptive control, surprisingly little seems to be known about the problem's possible solution. In [1] Parks put forth the idea of Liapunov redesign which can be shown to solve the problem for process models of dimension one and two. In [2] Monopoli casts the problem in a useful form (a simplification of which is used in this paper) by making the important observation that it is not really necessary to separately identify process model parameters and control feedback gains. However the arguments in [2] concerning stability contain errors and do not justify the paper's main claims; indeed there is reason to believe that the adaptive control proposed in [2] can result in an unstable system [3]. Similar errors concerning stability can be found in [4] where a form of 'implicit differentiation' is proposed to get around the problems which arise when one attempts to extend previous results to systems of dimension greater than two.

The purpose of this paper is to present what we believe to be the first solution to the aforementioned adaptive control problem which results in a stable closed-loop system in which all signals and gains are guaranteed to remain bounded. The proposed controller requires no implicit or explicit differentiation and can be realized with integrators, summers, gains and multipliers. The only assumptions made about the process are that it admits a transfer function model with left-half plane zeros and that an upper bound for the transfer function's dimension, the relative degree of the transfer function, and the sign of the transfer function 'gain' are known.

Notation

In the sequel, prime denotes transpose. If n is a positive integer, then $\underline{n} \equiv \{1,2,\ldots,n\}$ and $\underline{n}_0 \equiv \{0,1,2,\ldots,n\}$. Depending on context the letter s may be viewed as an indeterminate, a differential operator or Laplace transform variable. The zero state output of a linear system with input $f(t)$ and transfer function $\alpha(s)/\beta(s)$ is often written as $(\alpha/\beta)f(t)$. If $M(t)$ and $N(t)$ are two mxn matrices of time functions, we write $M = N$ (ε) if each element of the matrix $M - N$ is bounded in magnitude by a time function which is decaying to zero exponentially fast.

1. PARAMETERIZATION

Our basic assumption is that the relationship between the process input u and output y can be modelled by a linear system with strictly proper transfer function $g_p\alpha_p(s)/\beta_p(s)$ where g_p is a constant gain, $\alpha_p(s)$ is a stable, monic polynomial of degree m_p and $\beta_p(s)$ is a monic polynomial of degree n_p. The only data assumed known are the sign of g_p, an integer n satisfying $n \geq n_p$ and the <u>relative degree</u> $d \equiv n_p - m_p$ of the process transfer function.

To motivate our selection of a reference model which the process is ultimately supposed to follow, let us recall that if Σ is any linear <u>dynamical</u> (i.e., differentiator-free) compensator with reference input r, measured input y and output u, and if $\alpha(s)/\beta(s)$ is the resulting closed-loop transfer function from r to y, then the relative degree of $\alpha(s)/\beta(s)$ cannot be less than $n_p - m_p$. Indeed, this fundamental constraint on $\alpha(s)/\beta(s)$ can be relaxed only by incorporating differentiators in Σ. Clearly any adaptive system employing a reference model not respecting this constraint {e.g. [4]} must involve some form of implicit (if not explicit) differentiation. Since we have stipulated that our adaptive controller be differentiator-free, we must require the relative degree of our reference model transfer function $\alpha(s)/\beta(s)$ to satisfy

$$\deg(\beta) - \deg(\alpha) \geq d \tag{1}$$

We further assume that $\beta(s)$ is a stable polynomial.

<u>Lemma 1</u>: <u>There exist stable, monic polynomials</u> $\gamma_0(s)$ <u>and</u> $\gamma_1(s)$ <u>of degrees one and</u> $d - 1$ <u>respectively and a stable, proper transfer function</u> $h(s)$ <u>such that</u>

$$\frac{\alpha(s)}{\beta(s)} = \frac{1}{\gamma(s)} h(s) \tag{2}$$

<u>where</u>

$$\gamma(s) = \gamma_0(s)\gamma_1(s) \tag{3}$$

The simple proof of this lemma is omitted.

To proceed, select $\pi(s)$ to be any stable monic polynomial of degree n and let $\{\mu_i(s), i \in \underline{n}\}$ be any basis for the vector space of polynomials of degree less than n {e.g., $\mu_i(s) = s^{i-1}, i \in \underline{n}\}$. Define the 2n + 1 sensitivity function vector $\theta = [\theta_0, \theta_1, \ldots, \theta_{2n}]$ by the equations

$$\left.\begin{aligned} \theta_0(t) &= h(s)\bar{r}(t) \\ \theta_i(t) &= \frac{\mu_i(s)}{\pi(s)}\, y(t) \qquad i \in \underline{n} \\ \theta_{i+n}(t) &= \frac{\mu_i(s)}{\pi(s)}\, u(t) \qquad i \in \underline{n} \end{aligned}\right\} \tag{4}$$

and write

$$e(t) \equiv y(t) - y_M(t) \tag{5}$$

for the tracking error between the process output and the output of the reference model with input $r(t)$, i.e.

$$y_M(t) = \frac{\alpha(s)}{\beta(s)}\, r(t) \tag{6}$$

<u>Proposition 1</u>: <u>There exists a constant parameter vector</u> q <u>such that</u>

$$e = \frac{g_p}{\gamma}(u + \theta q) \qquad (\varepsilon) \tag{7}$$

<u>Proof</u>: Write δ and ρ for the unique quotient and remainder of $\pi\gamma$ divided by β_p; thus

$$\pi\gamma = \delta\beta_p + \rho \tag{8}$$

where $\deg(\rho) < \deg(\beta_p) < n$; hence there exist numbers q_i such that

$$\rho/g_p = \sum_{i=1}^{n} q_i\mu_i \tag{9}$$

Since $\deg(\rho) < \deg(\beta_p)$, (8) implies that $\deg(\pi\gamma) \equiv \deg(\delta\beta_p)$ and also that δ is monic. Hence $\alpha_p\delta$ is monic and $\deg(\alpha_p\delta) = \deg(\alpha_p) - \deg(\beta_p) + \deg(\pi\gamma) = \deg(\pi)$.

Therefore, if η is defined so that

$$\eta = \alpha_p \delta - \pi \tag{10}$$

then $\deg(\eta) < \deg(\pi) = n$. Hence there exist numbers q_i such that

$$\eta = \sum_{i=1}^{n} q_{n+i} \mu_i \tag{11}$$

Using (8) and then (10) we may write

$$\begin{aligned}(\pi\gamma - \rho) y(t) &= \delta \beta_p y(t) \\ &= \delta g_p \alpha_p u(t) \\ &= g_p (\pi + \eta) u(t)\end{aligned}$$

Thus

$$y(t) = \frac{g_p}{\gamma} \left(\frac{(\rho/g_p)}{\pi} y(t) + \frac{\eta}{\pi} u(t) + u(t) \right) \qquad (\varepsilon)$$

Hence from (2),(5) and (6) we see that

$$e(t) = \frac{g_p}{\gamma} \left(u(t) + \frac{(\rho/g_p)}{\pi} y(t) + \frac{\eta}{\pi} u(t) - (h/g_p) r(t) \right) \qquad (\varepsilon)$$

Set $q_0 \equiv - 1/g_p$, and $q = \text{col.}[q_0, \ldots, q_{2n}]$.

It now follows from (9),(11) and (4), that (7) is true. □

2. CONTROL EQUATIONS

The following signals must be generated to realize the proposed adaptive controller. With γ_1 as defined by Lemma 1 and θ_i as in (4), set $\zeta = \text{col.}[\zeta_0, \zeta_1, \ldots, \zeta_{2n}]$ where

$$\zeta_i(t) = \frac{1}{\gamma_1(s)} \theta_i(t), \quad i \in \underline{2n}_0 \tag{12}$$

If $d > 1$ set $k = d - 1$ and define

$$\phi_i(t) = -\text{sign}\,(g_p) \left(\frac{s^{i-1}}{\gamma_1(s)} (\theta(t)) \right) \zeta(t), \qquad i \in \underline{k} \tag{13}$$

For $i \in \underline{k}$, let λ_i be a fixed positive number. Define signals $\omega_i(t)$, $i \in \underline{k}$ and $\psi_{i,j}(t)$, $i \in \underline{k}$, $j \in \underline{i}_0$ be the formulas

$$\left.\begin{array}{llr}
\omega_1 = \phi_1 & & \text{(a)} \\
\omega_i = \phi_i + \sum_{j=1}^{i-1} \psi_{i-1,j}\omega_j & i \in \{2,3,\ldots,k\} & \text{(b)} \\
\psi_{i,0} = 0 & i \in \underline{k} & \text{(c)} \\
\psi_{i,i} = \sum_{j=1}^{i} (\omega_j^2 + \lambda_j) & i \in \underline{k} & \text{(d)} \\
\psi_{i,j} = \dot{\psi}_{i-1,j} - \psi_{i-1,j}(\omega_j^2 + \lambda_j) + \psi_{i-1,j-1} & & \\
 & j \in \underline{i},\ \ i \in \{2,3,\ldots,k\} & \text{(e)}
\end{array}\right\} \quad (14)$$

We wish to show that each of the signals defined by (14) can be generated from y, u or r without using differentiators. To do this, we digress briefly to introduce the following terminology.

Let f(t) be a scalar-valued, piecewise-continuous function, defined for $t \geq 0$. A scalar-valued time function v(t) defined for $t \geq 0$ is said to be in class $\underline{C}^i(f)$ just in case there exists a stable transfer function h(s) of relative degree $j \geq i$ such that $v(t) = h(s)f(t)$ (ε). For example, $v \in \underline{C}^1(f)$ if and only if v is the output (mod ε) of a strictly proper stable linear system with input f.

With $i \geq 1$, let $\underline{C}^i$ denote the subring of the ring of time functions on $[0,\infty)$ (with pointwise addition and multiplication), generated by the constant functions together with the elements of $\underline{C}^i(u) \cup \underline{C}^i(y) \cup \underline{C}^{i-1}(r)$. In other words, $v \in \underline{C}^i$ just in case v can be expressed as a finite sum of finite products of time functions w (mod ε) where w is either a constant or an element of $\underline{C}^i(u) \cup \underline{C}^i(y) \cup \underline{C}^{i-1}(r)$. Clearly any signal in $\underline{C}^i$ can be generated (mod ε) using only conventional analog components.

Observe that (4) implies that

$$\theta_j \in \underline{C}^1, \quad j \in \underline{2n}_0 \tag{15}$$

This and (12) show that

$$\zeta_j \in \underline{C}^d, \quad j \in \underline{2n}_0 \tag{16}$$

Thus from (13) there follows

$$\phi_i \in \underline{C}^{(d+1-i)}, \qquad i \in \underline{k} \tag{17}$$

Proposition 2:

$$\omega_i \in \underline{C}^{(d+1-i)}, \qquad i \in \underline{k} \tag{18a}$$

$$\psi_{i,j} \in \underline{C}^{(d+1-i)}, \qquad i \in \underline{k},\ j \in \underline{i}_0 \tag{18b}$$

The proposition implies that even though the equations defining the $\psi_{i,j}$ and ω_i involve derivatives of the $\psi_{i,j}$, these signals can nevertheless be realized as sums and products of constants and outputs of stable, differentiator-free, linear systems forced by r,u and y.

Example: For $d = 3$ (i.e., $k = 2$) it is straightforward to verify that $\omega_1 = \phi_1$, $\psi_{1,1} = \omega_1^2 + \lambda_1$, $\psi_{2,1} = 4\phi_1\phi_2 - \psi_{1,1}(\lambda_1 + \omega_1^2)$, $\omega_2 = \phi_2 + \psi_{1,1}\omega_1$ and $\psi_{2,2} = \omega_1^2 + \lambda_1 + \omega_2^2 + \lambda_2$.

Lemma 2: If $i > 1$ and if $f(t) \in \underline{C}^i$, then $\dot{f}(t) \in \underline{C}^{(i-1)}$

The simple proof of this lemma is omitted.

Proof of Proposition 2: For $i = 1$, $\omega_1 = \phi_1$, $\psi_{1,0} = 0$ and $\psi_{1,1} = \omega_1^2 + \lambda_1$; hence from (17), $\omega_1 \in \underline{C}^d$. Since λ_1 and 0 are constants, it follows that $\psi_{1,1}$ and $\psi_{1,0}$ are elements of $\underline{C}^d$ as well. Thus (18) holds for $i = 1$.

Now suppose the proposition is true for all $i \leq j$, where $j < k$ is fixed. From (14b), (17), and the inductive hypothesis there follows $\omega_{j+1} \in \underline{C}^{(d-j)}$. This together with (14c) - (14e), Lemma 2 and the inductive hypothesis imply that $\psi_{j+1,t} \in \underline{C}^{(d-j)}$ for $t \in \{0,1,\ldots,j+1\}$. Hence by induction, the proposition is true. □

To characterize the proposed adaptive controller, let λ_0 and $a_i, i \in \underline{k}$, denote the coefficients of the polynomials defined by (3); i.e. $\gamma_0 = s + \lambda_0$, $\gamma_1 = s^k + a_k s^{k-1} + \ldots + a_2 s + a_1$. Let Q denote the lower triangular matrix.

$$Q = \begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot \\ -\psi_{11} & 1 & 0 & \cdot & \cdot \\ -\psi_{21} & -\psi_{22} & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -\psi_{k-1,1} & -\psi_{k-2,1} & & -\psi_{k-1,k-1} & 1 \end{bmatrix}_{kxk} \tag{19}$$

and let δ_i be the ith element of the row vector

$$[\delta_1,\delta_2,\ldots,\delta_k]' = [\psi_{k1},\psi_{k2},\ldots,\psi_{kk}]Q^{-1} . \tag{20}$$

Observe that since det Q = 1, the time functions δ_i are simply sums of products of the ψ_{ij}.

The remaining equations defining the proposed adaptive controller are as follows:

$$\dot{x}_0 = -\lambda_0 x_0 - \hat{g}x_1 \tag{21}$$

$$\dot{x}_i = x_{i+1} + \phi_i(x_0 + e),\ i \in \{1,2,\ldots,k-1\} \tag{22}$$

$$\dot{x}_k = (\phi_k - \omega_k)(x_0 + e) - \sum_{i=1}^{k} \delta_i x_i \tag{23}$$

$$\dot{q} = (\text{sign } g_p)(x_0 + e)\zeta \tag{24}$$

$$\dot{\hat{g}} = x_1(x_0 + e) \tag{25}$$

and finally

$$u = -\theta\hat{q} - \sum_{i=1}^{k}(\delta_i - a_i)x_i + (\phi_k - \omega_k)(x_0 + e) \tag{26}$$

The adaptive controller is thus completely described by equations (4), (12)-(14) and (19)-(26).

Remark: In the very special case when $d = 1$ (i.e., $k = 0$) the adaptive controller is described by (4),(12) and $u = -\theta\hat{q}$, where $\dot{\hat{q}}$ is now redefined to be $\dot{\hat{q}} = -(\text{sign } g_p)e\zeta$.

3. SYSTEM STABILITY

Our main result is as follows.

Theorem 1: *Let* r *be any bounded, piecewise-continuous reference signal. Then* y, u, x_i, $i \in \underline{k}_0$, $\hat{g}$, $\hat{q}$ *are bounded time functions and*

$$\lim_{t\to\infty} e(t) = 0$$

Remark: From (15)-(18),(20), and the boundedness of r,u and y it clearly follows that all the remaining time functions associated with the adaptive system (i.e., $\theta,\zeta,\phi_i,\omega_i,\psi_{i,j},\delta$) are bounded as well.

Remark: It is worth noting here that the preceding theorem says nothing about the manner in which $e(t)$ tends to zero as $t \to \infty$. A monotone $e(t)$ would be ideal but it is not difficult to see that this will not be the case except possibly if the reference signal is sufficiently rich to force the parameter errors $\hat{q}(t) - q$ and $\hat{g}(t) - g_p$ to zero as $t \to \infty$. This issue with its obvious practical implications will be examined in a future paper.

The proof of Theorem 1 depends on the following lemmas.

Lemma 3: $e = \frac{g_p}{\gamma_0}(x_1 + \zeta'(q-\hat{q}))$ (ε) (27)

Proof: Let (c,A,b) canonically realize $1/\gamma_1(s)$ with (A,b) is standard control canonical form. Then from (3),(7) and (26)

$$e = \frac{g_p}{\gamma_0} cw \qquad (\varepsilon) \tag{28}$$

where

$$\dot{w} = Aw + b(\theta(q-\hat{q}) + u_0) \tag{29}$$

and

$$u_0 \equiv -\sum_{i=1}^{k}(\delta_i - a_i)x_i + (\phi_k - \omega_k)(x_0 + e) \tag{30}$$

If we now define

$$\dot{H} = AH + b\theta \tag{31}$$

$$\dot{z} = Az + bu_0 + H\dot{\hat{q}} \tag{32}$$

and

$$\hat{w} = z + H(q-\hat{q})$$

then

$$\dot{\hat{w}} = A\hat{w} + b(\theta(q-\hat{q}) + u_0)$$

It follows from (29) and the stability of A that $cw = c\hat{w}$ (ε) and thus that

$$cw = cz + cH(q-\hat{q}) \tag{33}$$

Since c,A,b realizes $1/\gamma_1$, (31) and (12) imply that $\zeta' = cH$. From this, (28) and (33) there follows

$$e = \frac{g_p}{\gamma_0}(cz + \zeta'(q-\hat{q})) \qquad (\varepsilon) \tag{34}$$

From (13) and (31) it is straightforward to verify that

$$\begin{bmatrix} \phi_1 \\ \cdot \\ \cdot \\ \cdot \\ \phi_k \end{bmatrix} = -\text{sign }(g_p)H\zeta$$

Using this and (24) we can therefore write

$$H\dot{\hat{q}} = \begin{bmatrix} \phi_1 \\ \cdot \\ \cdot \\ \cdot \\ \phi_k \end{bmatrix} (x_0 + e)$$

By substituting this and (30) into (32) it is easy to see that the ith component of $z(t)$ must satisfy

$$\dot{z}_i = z_{i+1} + \phi_i(x_0 + e) \qquad i \in \{1,2,\ldots,k-1\}$$

$$\dot{z}_k = -\sum_{i=1}^{k} a_i(z_i - x_i) - \sum_{i=1}^{k} \delta_i x_i + (\phi_k - \omega_k)(x_0 + e)$$

From these equations and (22) and (23) it clearly follows that $(\dot{z} - \dot{x}) = A(z - x)$ where $x = \text{col. } [x_1, x_2, \ldots, x_k]$. Since A is stable we have thus shown that $z = x$ (ε) and thus that $cz = x_1$ (ε). Substitution of x_1 for cz in (34) thus yields the desired result. □

<u>Lemma 4</u>: <u>Let</u> $z_i(t)$, $i \in \underline{k}$, <u>be time functions defined by the equations</u>

$$z_1 = x_1 \tag{35}$$

$$z_i = x_i + \sum_{j=1}^{i-1} \psi_{i-1,j} z_j \qquad i \in \{2,3,\ldots,k\} \tag{36}$$

<u>Then</u>

$$\dot{z}_i = -(\lambda_i + \omega_i^2)z_i + z_{i+1} + \omega_i(x_0 + e), \quad i \in \{1,2,\ldots,k-1\}, \tag{37}$$

$$\dot{z}_k = -(\lambda_k + \omega_k^2)z_k \tag{38}$$

<u>Proof</u>: If $k = 1$, (14) implies that $\phi_k - \omega_k = 0$, and that $\psi_{kk} = \omega_1^2 + \lambda_1$; from (20), $\delta_k = \psi_{kk}$. Hence by (23) and (35) it follows that z_k satisfies (38). Thus the lemma is true if $k = 1$.

Let $k > 1$ be fixed. From (22) and (35) there follows $\dot{z}_1 = x_2 + \phi_1(x_0 + e)$; thus from (36) $\dot{z}_1 = -\psi_{1,1}z_1 + z_2 + \phi_1(x_0 + e)$. Since $\psi_{1,1} = \omega_1^2 + \lambda_1$ by definition, z_1 satisfies (37). It follows that the lemma is true for $i = 1$.

Now suppose the lemma is true for $j \leq (i - 1)$, where $(i - 1) \in \{1,2,..,k-1\}$ is fixed.

From (36)

$$\dot{z}_i = \dot{x}_i + \sum_{j=1}^{i-1} (\dot{\psi}_{i-1,j}z_j + \psi_{i-1,j}\dot{z}_j)$$

Since by hypothesis z_j satisfies (37) for $j \leq i - 1$, we can write

$$\begin{aligned}\dot{z}_i &= \dot{x}_i + \sum_{j=1}^{i-1} (\dot{\psi}_{i-1,j}z_j + \psi_{i-1,j}(z_{j+1} + \omega_j(x_0 + e) - (\lambda_j + \omega_j^2)z_j)) \\ &= \dot{x}_i + \sum_{j=1}^{i-1} \psi_{i-1,j}\omega_j(x_0 + e) + \sum_{j=1}^{i-1} (\dot{\psi}_{i-1,j} + \psi_{i-1,j-1} - \psi_{i-1,j}(\lambda_j + \omega_j^2))z_j \\ &\quad + \psi_{i-1,i-1}z_i\end{aligned}$$

From this and (14) it follows that

$$\dot{z}_i = \dot{x}_i - \phi_i(x_0 + e) + \omega_i(x_0 + e) - (\omega_i^2 + \lambda_i)z_i + \sum_{j=1}^{i} \psi_{i,j}z_j \tag{39}$$

If $i < k$, (22) and (39) yield

$$\dot{z}_i = x_{i+1} + \omega_i(x_0 + e) - (\omega_i^2 + \lambda_i)z_i + \sum_{j=1}^{i} \psi_{i,j}z_j$$

Elimination of x_{i+1} using (36) shows that z_i satisfies (37).

If $i = k$, (23) and (39) yield

$$\dot{z}_k = -\sum_{j=1}^{k} \delta_j x_j - (\omega_k^2 + \lambda_k)z_k + \sum_{j=1}^{k} \psi_{k,j}z_j$$

But (35) and (36) imply $x = Qz$, with Q as defined in (19), x_i and z_i being the ith components of x and z respectively. From this and (20) it is clear that

$$-\sum_{j=1}^{k} \delta_j x_j + \sum_{j=1}^{k} \psi_{k,j} z_j = 0$$

and thus that z_k satisfies (38). By induction, the lemma is true. □

The assumed stability of the numerator polynomial α_p of the process transfer function is exploited in the following lemma.

Lemma 5: *Let $i \in \underline{k}_0$ be fixed. If y and its first i derivatives are bounded functions, then each function in $\underline{C}^{d-i}$ is bounded as well.*

Proof: Let π and σ be stable, monic polynomials of degrees $d - i$ and i respectively. Then $\alpha_p \pi\sigma$ is a stable monic, polynomial of degree n_p. If we define $\mu \equiv \beta_p - \alpha_p \sigma\pi$, then $\deg(\mu) < \deg(\alpha_p \sigma\pi)$ and

$$\begin{aligned}(\mu + \alpha_p \sigma\pi)\sigma y &= \sigma\beta_p y \\ &= \sigma g_p \alpha_p u\end{aligned}$$

Since $\alpha_p \sigma\pi$ is a stable polynomial, it follows that

$$\frac{1}{\pi} u = \frac{1}{g_p}\left(\frac{\mu}{\alpha_p \sigma\pi} \sigma y + \sigma y\right) \qquad (\varepsilon)$$

where $\mu/\alpha_p \sigma\pi$ is a strictly proper, stable transfer function. Since σ is a polynomial of degree i, it follows from the lemma's hypothesis that $\sigma y, (\mu/\alpha_p \sigma\pi)\sigma y$ and thus $(1/\pi)u$ are bounded.

Now let $w \in \underline{C}^{d-i}(u)$ be fixed. Hence there exists a stable transfer function $\bar{\alpha}/\bar{\beta}$ with relative degree no smaller than $d - i$ such that $w = (\bar{\alpha}/\bar{\beta})u$. The first two properties of $\bar{\alpha}/\bar{\beta}$ imply that $(\pi\bar{\alpha})/\bar{\beta}$ is a stable transfer function with nonnegative relative degree. Thus $\hat{w} \equiv (\pi\bar{\alpha}/\bar{\beta})(1/\pi)u$ is bounded and since π is stable,

$\hat{w} = \hat{w}\ (\varepsilon)$ so w is bounded as well. Therefore each function in $\underline{C}^{d-i}(\hat{u})$ is bounded. Since boundedness of r and y clearly imply boundedness of all functions in $\underline{C}^{d-i}(y) \cup \underline{C}^{d-i-1}(r)$, it follows from the definition of $\underline{C}^{d-i}$ that the lemma is true. □

To proceed it proves useful to introduce the variables

$$\left.\begin{aligned} \bar{e} &= e + x_0 \\ \bar{q} &= \hat{q} - q \\ \bar{g} &= \hat{g} - g_p \end{aligned}\right\} \tag{40}$$

Using (21), (27) and (35) we obtain

$$\dot{\bar{e}} = -\lambda_0\bar{e} - g_p\zeta'\bar{q} - \bar{g}z_1 \qquad (\varepsilon) \tag{41}$$

while from (24), (25) and (35)

$$\dot{\hat{q}} = \text{sign}(g_p)\bar{e}\zeta \tag{42}$$

$$\dot{\hat{g}} = z_1\bar{e} \tag{43}$$

Observe from (5), and (40) that y will be bounded provided each element of the set

$$\langle y \rangle \equiv \{y_M, x_o, \bar{e}\} \tag{44}$$

is. By differentiating each element in $\langle y \rangle$ and using (21),(35), (40) and (41) it is easy to see that $\dot{y}$ will be bounded provided each element in the set

$$\langle \dot{y} \rangle = \{\dot{y}_M, z_1, \zeta, \bar{q}, \bar{g}\} \cup \langle y \rangle \tag{45}$$

is. Continuing in this way it is quite straightforward to verify that for $1 < i \leq k$, the ith derivative of y, written $y^{(i)}$, will be bounded provided each element of the set

$$\langle y^{(i)} \rangle = \{y_M^{(i)}, z_i, \zeta^{(i-1)}, \omega_1^{(i-2)}, \omega_2^{(i-3)}, \ldots, \omega_{i-1}\} \cup \langle y^{(i-1)} \rangle \tag{46}$$

is. For example, by differentiating each element of the set <y>, and using (37), (42) and (43), one readily finds that the set <$\dot{y}$> defined above has the required property.

We now give a proof of Theorem 1 for the case d > 1, i.e., k > 0. The proof for the special case d = 1 involves similar (but very much simpler) arguments and will not be given.

Proof of Theorem 1: It will first be shown that $\bar{e}$, z_i, $i \in \underline{k}$, $\hat{q}$ and $\hat{g}$ are bounded time functions. For this define $\epsilon(t) \equiv \dot{\bar{e}} + \lambda_0\bar{e} + g_p\zeta'\bar{q} + z_1\bar{g}$ so that

$$\dot{\bar{e}} = -\lambda_0\bar{e} - g_p\zeta'\bar{q} - z_1\bar{g} + \epsilon(t) \tag{47}$$

and let

$$\alpha(t) = \frac{1}{2}(b_0\bar{e}^2 + \sum_{i=1}^{k} b_i z_i^2 + |g_p|b_0(\bar{q}'\bar{q}) + b_0\bar{g}^2 + (b_0^2/2\lambda_0)\int_t^\infty \epsilon^2(\tau)d\tau) \tag{48}$$

where $b_1 = 1$, $b_i = b_{i-1}/(\lambda_i\lambda_{i-1})$, $i \in \{2,3,\ldots,k\}$ and $b_0 = (1 + (\sum_{i=1}^{(k-1)} b_i)/4\lambda_0)$.

Observe that the integral in (48) is finite since by (41), $\epsilon(t)$ must approach zero exponentially fast. From (37), (38), (42), (43) and (47), it follows by direct verification that

$$\begin{aligned}\dot{\alpha} = &-\lambda_0(\bar{e} - (b_0/2\lambda_0)\epsilon)^2 - (b_1\lambda_1/2)z_1^2 - b_k(\lambda_k/2 + \omega_k^2)z_k^2 \\ &- \sum_{i=1}^{k-1} b_i((\omega_i z_i - \bar{e}/2)^2 + (\lambda_i/2)(z_i - z_{i+1}/\lambda_i)^2)\end{aligned} \tag{49}$$

Since $\alpha(t) \geq 0$ and $\dot{\alpha}(t) \leq 0$, it follows that $\alpha(t) \leq \alpha(0) < \infty$. Thus $\alpha(t)$ is bounded. From (48) it is now clear that $\bar{e}$, z_i, $i \in \underline{k}$, $\bar{q}$ and $\bar{g}$ are bounded; it follows from (40) that $\hat{q}$ and $\hat{g}$ are bounded as well.

Boundedness of $\hat{g}$ and z_1 together with (35) and (21), imply that x_0 is bounded. Since y_M is clearly bounded, it follows from (44) that y is also. Thus by Lemma 5, all functions in $\underline{C}^d$ are bounded. This and (16) imply that ζ is bounded. Since

y_M is clearly bounded, it follows from (45) that y is also. This and Lemma 5 show that $\underline{C}^1$ is bounded for the case $d = 2$.

To reach the same conclusion for $d > 2$ (i.e., $k > 1$), suppose that for fixed $i \in \{2,\dots,k\}$, all elements of the set $<y^{(i-1)}>$ are bounded; this, of course, implies that $y,\dots,y^{(i-1)}$ are bounded as well. Since $y_M^{(j)}$ is bounded for all $j \leq k$, $y_M^{(i)}$ is bounded as is z_i as was shown previously. Now for any integer $j \in \{1,2,\dots,i-1\}$, we have by Proposition 2 that $\omega_j \in \underline{C}^{(d+1-j)}$. Hence by Lemma 2 $\omega_j^{(i-(j+1))} \in \underline{C}^{(d+1-(i-1))}$ which, by definition, is a subset of $\underline{C}^{(d-(i-1))}$; since by Lemma 5 all functions in $\underline{C}^{(d-(i-1))}$ are bounded, it follows that $\omega_j^{(i-(j+1))}$ is bounded for $j \in \{1,2,\dots,i-1\}$. Thus from (46), all elements of the set $<y^{(i)}>$ are bounded. By induction, it now follows that all elements of $<y^{(k)}>$ are bounded implying that $y,\dots,y^{(k)}$ are bounded. Since $k = d - 1$, it now follows from Lemma 5, that $\underline{C}^1$ contains only bounded functions.

The boundedness of the functions in $\underline{C}^1$ together with (15)-(18) imply that θ,ζ, all ω_i and all $\psi_{i,j}$ are bounded as well. This and Lemma 4 shows that x_i, $i \in \underline{k}$ are bounded. It now follows from (26) that the first assertion of Theorem 1 is true.

To prove that $e(t) \to 0$, we note from (49), and the boundedness of all functions, that $\ddot{\alpha}(t)$ is bounded. This together with the boundedness of $\dot{\alpha}$ and the fact that the integral $\int_0^\infty \dot{\alpha}(t)dt$ converges, allows us to claim that $\lim_{t\to\infty} \dot{\alpha}(t) = 0$. From this and (49) it follows that as $t \to \infty$, all z_i and $\tilde{e}$ approach zero. Since $x_1 = z_1$ and $\hat{g}$ is bounded, $\hat{g}x_1$ is bounded and approaches zero as $t \to \infty$. From this, (21), and the fact that $\lambda_0 > 0$ it follows that $x_0 \to 0$ as $t \to \infty$. Therefore by (40), $\lim_{t\to\infty} e(t) = 0$, as claimed. □

CONCLUDING REMARKS

In this paper we have shown that it is possible to construct an adaptive control for a linear process model which results in a stable closed-loop system with zero steady-state output tracking error. While the proposed controller is admittedly complex, to our knowledge it is the only differentiator-free dynamical adaptive control proposed thus far which has been shown to produce stable closed-loop operation. The existence of such a control actually runs contrary to our own earlier expectations [5], and possibly to those of others [6]. Indeed, it would be interesting to draw connections between the modelling assumptions in this paper and the results of [6].

Since Theorem 1 is true, independent of the stability of the open-loop process model, the results presented here are potentially applicable to the problem of identifying process models not assumed to be open-loop stable. Be the application identification or control, it is of course important to insure that the reference signal can be selected so as to yield zero steady-state system parameter errors. This matter will be considered in a future paper.

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