

# Adaptive Equalization of Multiple-Input Multiple-Output (MIMO) Frequency Selective Channels<sup>1</sup>

Ardavan Maleki-Tehrani, Babak Hassibi\*, John M. Cioffi

Department of Electrical Engineering, Stanford University, CA 94305, USA

\*Bell Labs, Lucent Technologies, NJ, USA

Email: ardavan@leland.Stanford.EDU

## Abstract

*The purpose of this paper is to propose and investigate a new approach to adaptive spatio-temporal equalization for MIMO (Multiple-Input Multiple-Output) channels. A system with  $n$  transmit and  $m$  ( $n \geq m$ ) receiver antennas is assumed. Adaptive MIMO linear equalizer has been considered.*

*For the considered equalizer a least squares solutions is formulated, based on which a recursive solutions using Riccati recursions are proposed. The proposed solutions are tested by simulating the MIMO system. It is shown that the adaptive solutions will achieve the same performance as the optimum least squares solutions. The effect of the nondiagonal channel elements (acting as interference) on the system performance is also studied. It has been shown that in order to achieve better performance, the interference from nondiagonal channel elements needs to be minimized. This can be done by using orthogonal transmission. Moreover the proposed solutions do not require channel identification and will also enable equalizer adaptation to channel changes.*

## 1 Introduction

Although the problem of channel equalization has been extensively studied in the literature the growth of wireless communications has presented new challenges. In particular, the introduction of space-time coding ([1],[2],[3],[4],[5]) and application of antenna arrays at both transmitter and receiver ([6],[7],[8],[9]) has encouraged new research on equalization techniques for so called Multiple-Input Multiple-Output channels.

It must be noted that although the problem of MIMO equalization has been readily studied in the literature

([10],[11],[12],[13],[14]), the adaptive methods haven't been studied extensively.

The purpose of this paper is to propose and investigate a new approach to adaptive spatio-temporal equalization for MIMO (Multiple-Input Multiple-Output) channels. We will throughout assume that the MIMO channel is a block-time-invariant frequency selective channel, and that training symbols are sent with each vector data block (due to the time varying nature of the channel) to train the receiver equalizer.

We have considered a system with  $n$  transmit and  $m$  ( $n > m$ ) receiver antennas, therefore an  $m \times n$  channel matrix. Each element of the resulting MIMO channel is considered to be frequency selective. The channel is also assumed to be AWGN (Additive White Gaussian Noise). The output of the  $m$  receiver antennas, after passing through the matched filter, are fed into the matrix equalizer(s). The linear equalizer consists of an  $n \times m$  matrix of linear filters each with maximum  $L$  taps. The equalizer is then followed by a vector symbol detector.

In order to come up with an adaptive solution for the equalizer, we have first formulated a least squares solution. Once the least squares solution is found, it would be possible to formulate a recursive solution based on Riccati recursions. The formulated recursive solution is the adaptive equivalent of the least squares solution. It should be noted that neither of the above proposed methods requires channel identification which is necessary for other solutions such as methods based on MMSE (Minimum Mean Squared Error) criteria. Moreover the adaptive method will enable equalizer adaptation to channel changes.

The proposed solutions were tested by simulating a MIMO system with  $n = 2, m = 2$ . It must be noted that the proposed solutions are independent from the num-

<sup>1</sup>Research supported in part by NSF and SK Telecom.

ber of antennas used. A BPSK modulation scheme is used and the noise has been considered to be additive white Gaussian. To evaluate the performance of the whole system, BER (Bit Error Rate) curves versus SNR (Signal to Noise Ratio) have been presented for each case. In addition we have shown the learnings curves for the adaptive algorithm.

The paper is formatted as following. In section 2.2 we will discuss the notation used in the paper along with statement of the equalization problem. In section 3 the model of the MIMO channel is discussed. The linear equalizer is discussed in section 4. Both the least squares formulation (subsection 4.1) and the recursive least squares (subsection 4.2) solutions are developed. Section 5 illustrates these ideas with simulations. We finally conclude in Section 6.

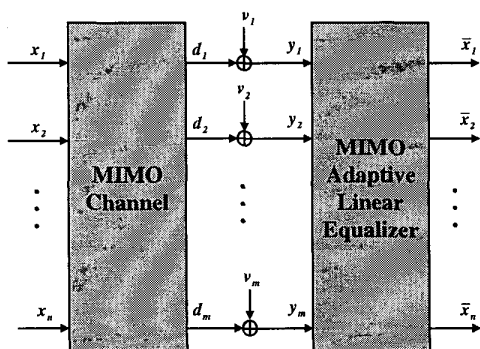


Figure 1: MIMO adaptive linear equalizer diagram

## 2 Problem Formulation

### 2.1 Notation

Standard notations are used in this paper. Bold letters denote vectors and matrices. Other notation are as follows.

$(\cdot)^*$	Hermitian
$(\cdot)'$	Transpose
$\mathbf{I}_n$	$n \times n$ Identity matrix
$\ (\cdot)\ $	2-norm of vector $(\cdot)$
$[A \ B]$	Matrices(vectors) A and B concatenated

### 2.2 Problem Statement

We are given a MIMO channel which its model is discussed in section 3. The problem is to equalize the given MIMO channel using an adaptive algorithm assuming no prior channel state information. In the following sections we will apply the linear equalizer structure to solve

the equalization problem.

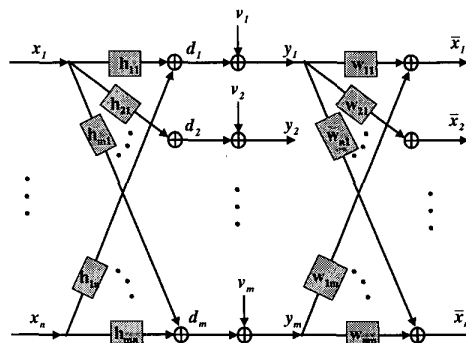


Figure 2: Structure of the MIMO adaptive linear equalizer

## 3 Channel Model

The MIMO channel is assumed to be a block-time-invariant frequency selective channel. We have considered a system with  $n$  transmit and  $m$  ( $n > m$ ) receiver antennas, therefore an  $m \times n$  channel matrix. Each element of the resulting MIMO channel is considered to be frequency selective. The channel is also assumed to be AWGN (Additive White Gaussian Noise). The following equation shows the discrete output of the  $j$ th receiving antenna:

$$y_j(k) = \sum_{i=1}^m x_i(k) * h_{j,i}(k) + v_j(k) \quad (1)$$

where  $y_j(k)$  is the discrete output of the  $j$ th receiving antenna at time  $k$ ,  $x_i(k)$  is the discrete input to the  $i$ th transmitting antenna,  $h_{j,i}(k)$  is the discrete channel impulse response from the  $i$ th transmitting antenna to the  $j$ th receiving antenna at time  $k$ , and  $v_j(k)$  is the additive white Gaussian noise at the output of the  $j$ th receiving antenna at time  $k$ . The above equation can be written in matrix form as following:

$$\mathbf{y}(k) = \mathbf{H}(k) * \mathbf{x}(k) + \mathbf{v}(k) \quad (2)$$

where  $\mathbf{y}(k) = [y_1(k) \ y_2(k) \ \dots \ y_m(k)]$ ,  $\mathbf{x}(k) = [x_1(k) \ x_2(k) \ \dots \ x_n(k)]$ ,  $\mathbf{v}(k) = [v_1(k) \ v_2(k) \ \dots \ v_m(k)]$ ,  $\mathbf{H}(k)$  is the channel matrix where  $h_{i,j}(k)$  is its  $(i, j)$ th element, and  $*$  is an element by element convolution as in a matrix product.

#### 4 Adaptive Linear Equalization

The linear equalizer consists of an  $n \times m$  matrix of linear filters each with maximum  $L$  taps. The equalizer is then followed by a vector symbol detector. Figure(1) shows the structure of the linear equalizer.

In order to come up with an adaptive solution for the equalizer, we first formulate a least squares solution. It is known ([15], [16],[17]) that the recursive least squares and the least squares will both converge to the same solution, where the recursive solution is the adaptive equivalent of the least squares solution.

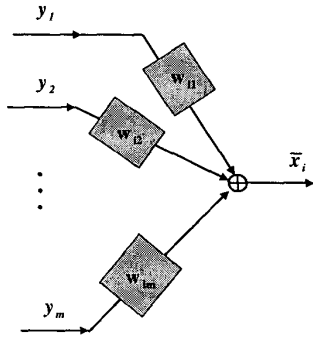


Figure 3: The  $i$ th equalizer output

##### 4.1 Least Squares solution

The least squares solution is based on formulating the MIMO equalization problem into a least squares format. The purpose is to minimize a predefined error at the output of the equalizer. Figure(1) and(2) show the block diagram and structure of the linear equalizer. The  $i$ th output of the equalizer can be written as:

$$\hat{x}_i(k) = \sum_{j=1}^m y_j(k) * w_{i,j}(k) \quad (3)$$

The error for the  $i$ th equalizer output is defined as:

$$e_i(k) = \hat{x}_i(k) - x_i(k) \quad (4)$$

The minimization problem is written for the  $i$ th equalizer output where figure(3) shows the filter elements involved. The same formulation should be repeated for each of the  $n$  outputs corresponding to the  $n$  transmitted signals. In order to formulate the least squares problem we will collect blocks of  $N$  samples from each receiving antenna output. Lets assume  $[y_i(1) \ y_i(2) \ \dots \ y_i(N)]$  depicts a block of

$N$  samples from the output of the  $i$ th antenna and  $\mathbf{x}_i = [x_i(1) \ x_i(2) \ \dots \ x_i(N)]$  depicts a block of  $N$  samples from the  $i$ th input. We will further assume that each element of the linear equalizer matrix is a linear transversal filter of maximum length  $L$ . Therefore the  $(i, j)$ th element of the equalizer matrix can be written in vector form as  $\mathbf{w}_{i,j} = [w_{i,j}(1) \ w_{i,j}(2) \ \dots \ w_{i,j}(L)]'$ . Given the above notations the minimization problem over a block of  $N$  samples can be written as:

$$\mathbf{w}_i = \text{argmin} \|\mathbf{x}_i - \mathbf{Y}\mathbf{w}_i\|^2,$$

where  $\mathbf{x}_i$  is defined as above,  $\mathbf{w}_i = [\mathbf{w}'_{i,1} \ \mathbf{w}'_{i,2} \ \dots \ \mathbf{w}'_{i,m}]'$  is a concatenated vector of the  $w_{i,j}$  vectors for  $(j = 1, \dots, m)$ , and

$$\mathbf{Y} = [\mathbf{Y}_1 \ | \ \dots \ | \ \mathbf{Y}_m]$$

where,

$$\mathbf{Y}_i = \begin{bmatrix} y_i(1) & 0 & \dots & 0 \\ y_i(2) & y_1(1) & \dots & 0 \\ \vdots & \dots & \dots & \vdots \\ y_i(L) & \dots & \dots & y_i(1) \\ \vdots & \ddots & \dots & \ddots \\ y_i(N) & \dots & \dots & y_i(N-L+1) \end{bmatrix} \quad (5)$$

Then the solution of the minimization problem is:

$$\mathbf{w}_i = (\mathbf{Y}^* \cdot \mathbf{Y})^{-1} \cdot \mathbf{Y}^* \cdot \mathbf{x}_i \quad (6)$$

##### 4.2 Modified Recursive Least Squares

The recursive least squares solution is obtained from the Kalman filter formulation of the adaptive equalization problem [15]. In this set up the problem is formulated with a state-space model. The model of interest for our equalization problem is:

$$\begin{cases} \mathbf{w}_i(\mathbf{k}+1) &= \lambda^{-1/2} \mathbf{w}_i(\mathbf{k}) \\ \hat{x}_i(k) &= \tilde{Y}(k) \mathbf{w}_i(\mathbf{k}) \end{cases} \quad (7)$$

where  $\mathbf{w}_i(k)$  is the vector of the transversal filters associated with input  $i$  at time  $k$  and is defined as above. This vector is also considered to be the state variable of the equalizer state space model.  $\tilde{Y}(k)$  is one row of the matrix  $\mathbf{Y}$  defined above at time  $k$ :

$$\tilde{Y}(k) = [\tilde{Y}_1(k) \ | \ \dots \ | \ \tilde{Y}_m(k)] \quad (8)$$

where

$$\tilde{Y}_i(k) = [y_i(k+L-1) \ \dots \ y_i(k)]$$

and  $\hat{x}_i(k)$  is the output of the equalizer corresponding to input  $i$  at time  $k$ . As in the least squares case the problem is formulated for the  $i$ th equalizer output where figure(3) shows the filter elements involved. We will not get into details of the Recursive Least Squares solution of the above state space problem and will only mention the results and refer the interested reader to [15]. The solution to the above state space problem can be formulated recursively as following:

$$\mathbf{w}_i(k+1) = \lambda^{-1/2} \left[ \mathbf{w}_i(k) + K_p(x_i(k) - \tilde{Y}(k)\mathbf{w}_i(k)) \right] \quad (9)$$

where  $x_i(k)$  is the signal sent from the  $i$ th antenna (this is the known training sequence),

$$K_p = \mathbf{P}(k)\tilde{Y}^*(k)R_e^{-1}(k)$$

$$R_e(k) = \left[ \tilde{Y}(k)\mathbf{P}(k)\tilde{Y}^*(k) + \mathbf{I}_1 \right]$$

and -

$$\mathbf{P}(k+1) = \mathbf{P}(k) - K_p\tilde{Y}(k)\mathbf{P}(k)$$

The initial states  $\mathbf{P}(0)$  and  $\mathbf{w}_i(0)$  can be arbitrarily chosen, where we have used  $\mathbf{P}(0) = \mathbf{I}_{mL}$  and  $\mathbf{w}_i(0)$  is chosen to be the zero vector. We have chosen  $\lambda = 1$ .

## 5 Simulation Results

The ideas described in previous sections were tested by simulating a MIMO system with  $n = 2, m = 2$ . It must be noted that the proposed solutions are independent from the number of antennas used. A BPSK modulation scheme has been used and the noise has been considered to be additive white Gaussian.

Each element of the channel matrix is modeled as a two ray Raleigh, which considers the impulse response to be two delta functions, which have independent fades, and have a time delay of one symbol period([18]). This is sufficient time delay to induce frequency selective fading upon the input signal. For the simulation we have considered the worst case, where we have assumed 90% correlation between the channel elements( $\rho = .9$ ).

In order to capture the effect of the nondiagonal channel interference, we have plotted the BER curves assuming gain factors of 0,0.5 and 1 for the nondiagonal channel elements(e.g.  $h_{1,2}$  and  $h_{2,1}$ ). The gain of 0 corresponds to sending completely orthogonal signals from the transmitters. The gain of 0.5 corresponds to sending nonorthogonal vectors, with crosscorrelation of .5( we may call it semi-orthogonal transmission), and

the gain of 1 corresponds to completely nonorthogonal transmission.

We have discussed the orthogonal transmission concept in a separate paper [19]. However we just mention here that in order to achieve better performance, the interference from nondiagonal channel elements needs to be minimized (as shown in the following), and one way to achieve this goal, is to use orthogonal transmission. In the MIMO system case as opposed to a multi-user case we have control over the transmitter, therefore it is possible to use orthogonal vectors as transmitted signals. This will cause the nondiagonal channel elements to become smaller, and eliminates the possibility of having singularities in the channel matrix. One example of orthogonal transmission is using orthogonal spreading codes for each antenna, and another example is usage of orthogonal constellations.

In Figure(4), we show the BER vs. SNR curves for the matrix linear equalizer. The BER curves are for the output of the first antenna (though it can be arbitrarily chosen to be the second antenna). For each element of the linear equalizer matrix the number of taps,  $L$ , is chosen to be 10. The figure shows the BER curves for the least squares solution with  $N = 100$  and  $N = 500$ , and also the BER curve when using the RLS algorithm (the least squares curves for lower gains are identical and were not drawn). As seen from the plots the curves shift to the right as we increase the gain of the nondiagonal channel since the interference from the other antenna acts as a constant Gaussian noise (the transmitted signals are white Gaussian). The figure also shows that when the power of the interfering channels is equal to ( $Gain = 1$ ) the main channels (e.g.  $h_{1,1}$  and  $h_{2,2}$ ), then the algorithms performs very poorly (e.g. singular channel matrix).

In figure (5) we show the learning curves of the RLS algorithm for the linear equalizer case, for different nondiagonal channel gains (each curve is obtained from averaging 500 runs) . As seen the algorithm converges very fast, however the steady state Mean Squared Error (MSE) is higher when the nondiagonal channel elements have a higher gain. This is due to higher interference level from the nondiagonal channel elements. We would like to note that we have included the MIMO MMSE equalizer performance results in [19].

## 6 Conclusion

In this paper we proposed and investigated a new approach to adaptive spatio-temporal equalization for MIMO (Multiple-Input Multiple-Output) channel equal-

ization. MIMO Adaptive linear equalizer has been considered.

The proposed solutions were tested by simulating the MIMO system. It was shown that the adaptive solutions will achieve the same performance as the optimum least squares solutions. Furthermore, the effect of the nondiagonal channel elements (acting as interference) on the system performance was studied. It has been shown that in order to achieve better performance, the interference from nondiagonal channel elements needs to be minimized. This can be done by using orthogonal transmission. We must also mention that the proposed solutions do not require channel identification and will also enable equalizer adaptation to channel changes.

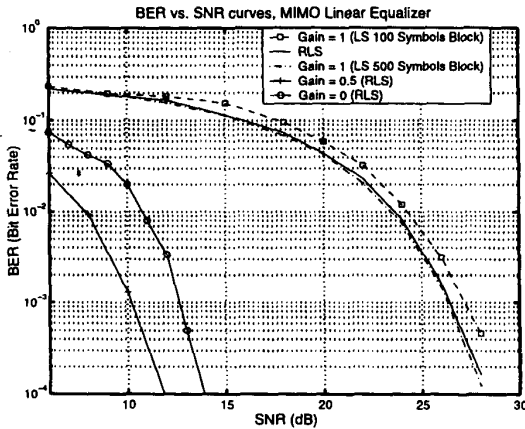


Figure 4: SNR vs. BER curves

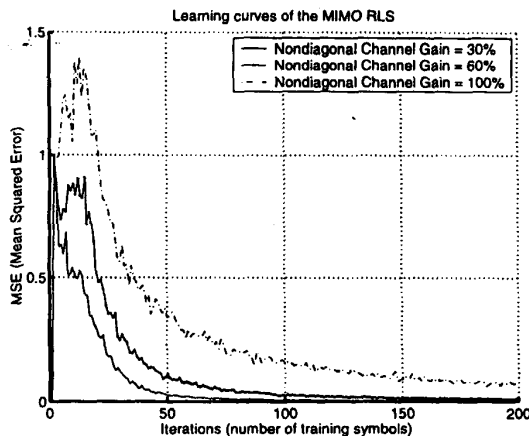


Figure 5: Learning curves for the MIMO RLS

## References

- [1] G. Foschini, "Layered Space-time Architecture for Wireless Communication in a fading environments using multi-element antennas," *Bell Labs Technical Journal*, vol. 1, pp. 41-59, Autumn 1996.
- [2] V. Tarokh and N. Seshadri and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE TRANSACTIONS ON INFORMATION THEORY*, vol. 44, pp. 744-65, 1998.
- [3] B. Ng, J. Chen, and A. Paulraj, "Space Time Processing for Fast Fading channels with Co-channel Interference," *IEEE 46th Vehicular Technology Conference*, vol. 3, pp. 1491-5, 1996.
- [4] R. Negi, A. M. Tehrani, and J. Cioffi, "Adaptive antennas for space-time coding over block-time invariant multi-path fading channels," *IEEE 49th Vehicular Technology Conference 1999*, vol. 1, pp. 70-74, 1999.
- [5] A. M. Tehrani, R. Negi, and J. Cioffi, "Space-time coding and transmission optimization for wireless channels," *Conference Record of the Thirty-Second Asilomar Conference on Signals, Systems and Computers, 1998*, vol. 2, pp. 1798-1802, 1998.
- [6] S. Roy, J. Yang, and P. Kumar, "Joint Transmitter/Receiver Optimization for Multiuser Communication," in *Cyclostationarity in Communications and Signal Processing* (W. A. Gardner, ed.), IEEE Press, 1994.
- [7] G. Wornell, "Signal Processing Techniques for Efficient Use of Transmit Diversity in Wireless Communications," *1996 IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 2, pp. 1057-60, 1996.
- [8] G. Raleigh and J. Cioffi, "Spatio-temporal Coding for Wireless Communications," *IEEE GLOBECOM 96*, vol. 3, p-p. 1809-14, 1996.
- [9] A. M. Tehrani, A. Hassibi, J. Cioffi, and S. Boyd, "An implementation of discrete multi-tone over slowly time-varying multiple-input/multiple-output channels," *IEEE GLOBECOM 1998*, vol. 5, pp. 2806-2811, 1998.
- [10] A. Duel-Hallen, "Equalizers for Multiple Input Multiple Output Channels and PAM Systems with Cyclostationary Input Sequences," *IEEE Journal on Selected Areas in Communications*, vol. 10, pp. 630-39, April 1992.
- [11] J. Saltz, "Digital Transmission Over Cross-Coupled Linear Channels," *AT & T Technical Journal*, vol. 64, pp. 1147-1159, July-August 1985.
- [12] Y. Kim and S. Shamsunder, "Multichannel algorithms for Simultaneous Equalization and Interference Suppression," *Wireless Personal Communications*, vol. 8, pp. 219-37, 1998.
- [13] Y. Li and K. J. R. Liu, "Adaptive Blind source Separation and Equalization for Multiple-Input/Multiple-Output Systems," *IEEE Transactions on Information Theory*, vol. 44, pp. 2864-2876, 1998.
- [14] L. Vandendorpe and O. V. de Wiel, "MIMO DFE Equalization for Multitone DS/SS Systems over Multipath Channels," *IEEE Journal on Selected Areas in Communications*, vol. 14, p-p. 502-511, 1996.
- [15] A. Sayed and T. Kailath, "A state-space approach to adaptive RLS filtering," *IEEE Signal Processing Magazine*, vol. 11, pp. 18-60, July 1994.
- [16] B. Widrow and S. Stearns, *Adaptive Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1985.
- [17] S. Haykin, *Adaptive Filter Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1996.
- [18] W. C. Jakes, *Microwave Mobile Communications*. IEEE Press, 1994.
- [19] A. M. Tehrani and J. Cioffi, "Adaptive Equalization of MIMO channels," *Under preparation for IEEE ICC 2000, New Orleans, June 2000*.