Adaptive Equalization of Multiple-Input Multiple-Output (MIMO)Channels [1]

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Abstract

The purpose of this paper is to propose and investigate a new approach to adaptive spatio-temporal equalization for MIMO (Multiple-Input Multiple-Output) channels. A system with n transmit and m ($m \geq n$) receiver antennas is assumed.

A decision Feedback equalizer is considered. A least squares solutions is first formulated, based on which a recursive solutions using Riccati recursions are proposed. The proposed solutions are tested by simulating the MIMO system. It is shown that the adaptive solutions will achieve the same performance as the optimum least squares solutions. The effect of the nondiagonal channel elements (acting as interference) on the system performance is also studied. It has been shown that in order to achieve better performance, the interference from nondiagonal channel elements needs to be minimized. This can be done by using orthogonal transmission. Moreover the proposed solutions do not require channel identification and will also enable equalizer adaptation to channel changes.

1 Introduction

Although the problem of channel equalization has been extensively studied in the literature the growth of wireless communications has presented new challenges. In particular, the introduction of space-time coding ([1],[2],[3],[4],[5]) and application of antenna arrays at both transmitter and receiver ([6],[7],[8],[9]) has encouraged new research on equalization techniques for so called Multiple-Input Multiple-Output channels.

It must be noted that although the problem of MIMO equalization has been readily studied in the literature ([10],[11],[12],[13],[14]),the adaptive methods haven't been studied extensively.

The purpose of this paper is to propose and investigate a new approach to adaptive spatio-temporal equalization for MIMO (Multiple-Input Multiple-Output) channels. We will throughout assume that the MIMO channel is a block-time-invariant frequency selective channel, and that training symbols are sent with each vector data block (due to the time varying nature of the channel) to train the receiver equalizer.

We have considered a system with n transmit and m ($m \geq n$) receiver antennas, therefore an $m \times n$ channel matrix. Each element of the resulting MIMO channel is considered to be frequency selective. The channel is also assumed to be AWGN (Additive White Gaussian Noise). The output of the m receiver antennas, after passing through the matched filter, are fed into the matrix equalizer(s) as shown in Figure(1). The Decision Feedback Equalizer (DFE) consists of an $n \times m$ feedforward matrix and an $m \times n$ feedback matrix of linear filters each with maximum L taps. The equalizer is then followed by a vector symbol detector.

In order to come up with an adaptive solution for the equalizer, we have first formulated a least squares solution. Once the least squares solution is found, it would be possible to formulate a recursive solution based on Riccati recursions. The formulated recursive solution is the adaptive equivalent of the least squares solution. It should be noted that neither of the above proposed methods requires channel identification which is necessary for other solutions such as methods based on MMSE (Minimum Mean Squared Error) criteria. Moreover the adaptive method will enable equalizer adaptation to channel changes.

The proposed solutions were tested by simulating a MIMO system with $n = 2, m = 2$. It must be noted that

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the proposed solutions are independent from the number of antennas used. A BPSK modulation scheme is used and the noise has been considered to be additive white Gaussian. To evaluate the performance of the whole system, BER (Bit Error Rate) curves versus SNR (Signal to Noise Ratio) have been presented for each case.

The paper is formatted as following. In section 2.2 we will discuss the notation used in the paper along with statement of the equalization problem. In section 3 the model of the MIMO channel is discussed. The DFE equalizer is discussed in section 4. Both the least squares formulation (subsection 4.1) and the recursive least squares (subsection 4.2) solutions are developed. Section 5 illustrates these ideas with simulations. We finally conclude in Section 6.

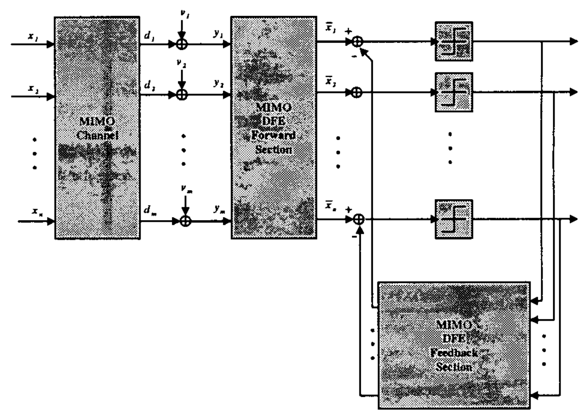

Figure 1: MIMO adaptive DFE diagram

2 Problem Formulation

2.1 Notation

Standard notations are used in this paper. Bold letters denote vectors and matrices. Other notation are as follows.

$(\cdot)^*$	Hermitian
$(\cdot)'$	Transpose
$\mathbf{I}_n$	$n \times n$ Identity matrix
$\|\|(\cdot)\|\|$	2-norm of vector $(\cdot)$
$[A \;\; B]$	Matrices(vectors) A and B concatenated

2.2 Problem Statement

We are given a MIMO channel which its model is discussed in section 3. The problem is to equalize the given MIMO channel using an adaptive algorithm assuming no prior channel state information. In the following sections we will apply the Decision Feedback Equalizer (DFE) structure to solve the equalization problem.

3 Channel Model

The MIMO channel is assumed to be a block-time-invariant frequency selective channel. We have considered a system with n transmit and m ($m \geq n$) receiver antennas, therefore an $m \times n$ channel matrix. Each element of the resulting MIMO channel is considered to be frequency selective. The channel is also assumed to be AWGN (Additive White Gaussian Noise). The following equation shows the discrete output of the jth receiving antenna:

$$y_j(k) \;\; = \;\; \sum_{i=1}^{m} x_i(k) * h_{j,i}(k) + v_j(k) \qquad (1)$$

where $y_j(k)$ is the discrete output of the jth receiving antenna at time k, $x_i(k)$ is the discrete input to the ith transmitting antenna, $h_{j,i}(k)$ is the discrete channel impulse response from the ith transmitting antenna to the jth receiving antenna at time k, and $v_j(k)$ is the additive white Gaussian noise at the output of the jth receiving antenna at time k. The above equation can be written in matrix form as following:

$$\mathbf{y}(k) \;\; = \;\; \mathbf{H}(k) * \mathbf{x}(k) + \mathbf{v}(k) \qquad (2)$$

where $\mathbf{y}(k) = [y_1(k) \;\; y_2(k) \;\; \cdots \;\; y_m(k)]$, $\mathbf{x}(k) = [x_1(k) \;\; x_2(k) \;\; \cdots \;\; x_n(k)]$, $\mathbf{v}(k) = [v_1(k) \;\; v_2(k) \;\; \cdots \;\; v_m(k)]$, $\mathbf{H}(k)$ is the channel matrix where $h_{i,j}(k)$ is its (i,j)th element, and $*$ is an element by element convolution as in a matrix product.

4 Adaptive Decision Feedback Equalization

The DFE equalizer consists of two matrices of linear filters each with maximum L taps, one for the feedforward section and one for the feedback section.

We will first formulate a least squares solution. The least squares and the recursive least squares methods will lead to the same final solution, with the recursive solution being the adaptive equivalent of the least squares solution.

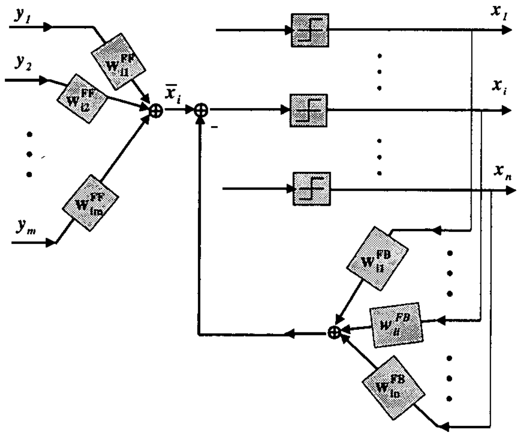

Figure 2: Structure of the MIMO adaptive linear equalizer

4.1 Least Squares solution

The least squares formulation for the DFE case is similar to the linear equalizer case [15], however two sets of terms appear, one for the feedforward and one for the feedback section. Figure(2) shows the structure of the equalizer. The ith output of the DFE equalizer can be written as:

$$\hat{x}_i(k) = \sum_{j=1}^{m} y_j(k) * w_{i,j}^{FF}(k) - \sum_{j=1}^{n} x_j(k) * w_{i,j}^{FB}(k)$$

This is with the assumption that the decisions at the output of the decision blocks are correct. We assume no error propagation here, however note that through usage of error correcting codes such as pre-coding techniques it is possible to eliminate this problem in a practical setup.

The error for the ith DFE equalizer output is defined as:

$$e_i(k) = \hat{x}_i(k) - x_i(k)$$

Again as the linear equalizer case only the filter elements in the feedforward and feedback sections corresponding to the ith equalizer output are considered. Figure(2) shows the diagram of the filter elements involved.

Similar to the linear equalizer case [15] we will collect blocks of N samples from output of each receiving antenna. Lets assume $[y_i(1)\ y_i(2)\ \cdots\ y_i(N)]$ depicts a block of N samples from the output of the ith antenna and $\mathbf{x_i} = [x_i(1)\ x_i(2)\ \cdots\ x_i(N)]$ depicts a block of N samples from the ith input. We will also assume that each element of the feedforward or feedback equalizer matrix is a linear transversal filter of maximum length L. Therefore the L taps of the (i,j)th element of the feedorward and feedfack matrices can be written in vector form as:

$$\begin{aligned} \mathbf{w}_{i,j}^{FF} &= [w_{i,j}^{FF}(1)\ \ w_{i,j}^{FF}(2) \cdots\ w_{i,j}^{FF}(L)]', \text{ and} \\ \mathbf{w}_{i,j}^{FB} &= [w_{i,j}^{FB}(1)\ \ w_{i,j}^{FB}(2) \cdots\ w_{i,j}^{FB}(L)]'. \end{aligned}$$

Given the above notations the minimization problem over a block of N samples can be written as:

$$\begin{aligned} \mathbf{w_i} &= \mathbf{argmin}||\mathbf{x_i} - (\mathbf{Y}\mathbf{w_i^{FF}} - \tilde{\mathbf{X}}\mathbf{w_i^{FB}})||^2 \\ &= \mathbf{argmin}||\mathbf{x_i} - ([\mathbf{Y}\ \ -\tilde{\mathbf{X}}][\begin{smallmatrix}\mathbf{w_i^{FF}} \\ \mathbf{w_i^{FB}}\end{smallmatrix}])||^2, \end{aligned}$$

where $\mathbf{w_i} = [\begin{smallmatrix}\mathbf{w_i^{FF}} \\ \mathbf{w_i^{FB}}\end{smallmatrix}]$, $\mathbf{x_i}$ is defined as above, $\mathbf{w}_i^{FF} = [(\mathbf{w}_{i,1}^{FF})'\ \ (\mathbf{w}_{i,2}^{FF})' \cdots\ \ (\mathbf{w}_{i,m}^{FF})']'$ is a concatenated vector of the $\mathbf{w}_{i,j}^{FF}$ vectors for $(j = 1, \cdots, m)$, and similarly $\mathbf{w}_i^{FB} = [(\mathbf{w}_{i,1}^{FB})'\ \ (\mathbf{w}_{i,2}^{FB})' \cdots\ \ (\mathbf{w}_{i,m}^{FB})']'$. $\mathbf{Y}$ matrix is defined as

$$\mathbf{Y} = [\ \mathbf{Y_1}\ |\ \cdots\ \ \cdots\ |\ \mathbf{Y_m}\ ]$$

where,

$$\mathbf{Y_i} = \begin{bmatrix} y_i(1) & 0 & \cdots & 0 \\ y_i(2) & y_1(1) & \cdots & 0 \\ \vdots & \cdots & & \vdots \\ y_i(L) & \cdots & \cdots & y_i(1) \\ \vdots & \ddots & & \ddots \\ y_i(N) & \cdots & \cdots & y_i(N-L+1) \end{bmatrix} \tag{3}$$

and $\tilde{\mathbf{X}}$ is defined as $\mathbf{Y}$ with $x_i(k)$ replacing $y_i(k)$ in the above definition of $\mathbf{Y}$.

The solution of the above problem can be written as:

$$\mathbf{w_i} = ([\mathbf{Y}\ \ -\tilde{\mathbf{X}}]^* \cdot [\mathbf{Y}\ \ -\tilde{\mathbf{X}}])^{-1} \cdot [\mathbf{Y}\ \ -\tilde{\mathbf{X}}]^* \cdot \mathbf{x}_i$$

4.2 Modified Recursive Least Squares

For the DFE case similar to the linear equalizer case the recursive least squares solution is obtained from the Kalman filter formulation of the equalization problem [16]. The state space model is also similar:

$$\begin{cases} \mathbf{w_i(k+1)} &= \lambda^{-1/2}\mathbf{w_i(k)} \\ \hat{x}_i(k) &= [\tilde{\mathbf{y}}(k)\ \ -\tilde{\mathbf{x}}(k)]\mathbf{w_i(k)} \end{cases} \tag{4}$$

where $\mathbf{w_i(k)}$ is the vector of the transversal filters associated with input i at time k and is defined as above. It is also considered to be the state variable of the equalizer state space model. $\tilde{\mathbf{y}}(k)$ is one row of the matrix $\mathbf{Y}$ defined above at time k:

$$\tilde{\mathbf{y}}(\mathbf{k}) = [\ \tilde{\mathbf{y}}_{\mathbf{i}}(\mathbf{k})\ |\ \cdots\cdots\ |\ \tilde{\mathbf{y}}_{\mathbf{m}}(\mathbf{k})\ ] \qquad (5)$$

where

$$\tilde{\mathbf{y}}_{\mathbf{i}}(\mathbf{k}) = [\ y_i(k+L-1)\quad \cdots \quad \cdots \quad y_i(k)\ ]$$

and $\tilde{\mathbf{x}}(k)$ is one row of the matrix $\tilde{\mathbf{X}}$ at time k, defined similar to $\tilde{\mathbf{Y}}(k)$ with $x_i(k)$ replacing $y_i(k)$ in the above definition of $\tilde{\mathbf{Y}}(k)$. and $[\tilde{\mathbf{y}}\ \ -\tilde{\mathbf{x}}]$ is the concatenated vector of the above vectors. Finally $\hat{x}_i(k)$ is the ith output of the equalizer at time k. Here we will just present the Recursive Least Squares solution of the above state space problem and refer the interested reader to [16]. The solution to the above state space problem can be formulated recursively as following:

$$\mathbf{w_i}(k+1) = \lambda^{-1/2}\left[\mathbf{w_i}(k) + K_p * (x_i(k) - [\tilde{\mathbf{y}}(k)\quad -\tilde{\mathbf{x}}(k)]\mathbf{w_i}(k)\right] \qquad (6)$$

where $x_i(k)$ is the signal sent from the ith antenna (this is the known training sequence).

$$K_p = \mathbf{P}(k)[\tilde{\mathbf{y}}(\mathbf{k})\quad -\tilde{\mathbf{x}}(k)]^* R_e^{-1}(k)$$

$$R_e(k) = [[\tilde{\mathbf{y}}(k)\quad -\tilde{\mathbf{x}}(k)]\mathbf{P}(k)[\tilde{\mathbf{y}}(k)\quad -\tilde{\mathbf{x}}(k)]^* + \mathbf{I_1}]$$

and

$$\mathbf{P}(k+1) = \mathbf{P}(k) - K_p[\tilde{\mathbf{y}}(k)\quad -\tilde{\mathbf{x}}(k)]\mathbf{P}(k)$$

The initial states $\mathbf{P}(0)$ and $\mathbf{w_i}(0)$ can be arbitrarily chosen, as in the previous section, where $\mathbf{P}(0) = \mathbf{I}_{mL}$ and $\mathbf{w_i}(0)$ is chosen to be the zero vector. We have chosen $\lambda = 1$.

5 Simulation Results

The ideas described in previous sections were tested by simulating a MIMO system with $n = 2, m = 2$. It must be noted that the proposed solutions are independent from the number of antennas used. A BPSK modulation scheme has been used and the noise has been considered to be additive white Gaussian.

Each element of the channel matrix is modeled as a two ray Raleigh, which considers the impulse response to be two delta functions, which have independent fades, and have a time delay of one symbol period([17]). This is sufficient time delay to induce frequency selective fading upon the input signal. For the simulation we have considered the worst case, where we have assumed 90% correlation between the channel elements($\rho = .9$).

In order to capture the effect of the nondiagonal channel interference, we have plotted the BER curves assuming gain factors of 0,0.5 and 1 for the nondiagonal channel elements(e.g. $h_{1,2}$ and $h_{2,1}$). The gain of 0 corresponds to sending completety orthogonal signals from the transmitters. The gain of 0.5 corresponds to sending nonorthogonal vectors, with crosscorrelation of .5(we may call it semi-orthogonal transmission), and the gain of 1 corresponds to completely nonorthogonal transmission.

We will discuss the orthogonal transmission concept in a seperate paper, however we just mention here that in order to achieve better performance, the interference from nondiagonal channel elements needs to be minimized (as shown in the following), and one way to achieve this goal, is to use orthogonal transmission. In the MIMO system case as opposed to a multi-user case we have control over the transmitter, therefore it is possible to use orthogonal vectors as trasmitted signals. This will cause the nondiagonal channel elements to become smaller, and eliminates the possibility of having singularities in the channel matrix. One exmple of orthogonal transmission is using orthogonal spreading codes for each antenna, and another example is usage of orthogonal constellations.

In Figure(3), we show the BER vs. SNR curves for the DFE equalizer. The BER curves are for the output of the first antenna (though it can be arbitrarily chosen to be the second antenna). For each element of the equalizer matrices the number of taps, L, is chosen to be 10. The figure shows the BER curves for the least squares solution with $N = 100$ and $N = 500$, and also the BER curve when using the RLS algorithm (the least squares curves for lower gains are identical and were not drawn). As seen from the plots the curves shift to the right as we increase the gain of the nondiagonal channel since the interference from the other antenna acts as a constant Gaussian noise (the transmitted signals are white Gaussian). The figure also shows that when the power of the interfering channels is equal to ($Gain = 1$) the main channels (e.g. $h_{1,1}$ and $h_{2,2}$), then the algorithms performs very poorly (e.g. singular channel matrix).

6 Conclusion

In this paper we proposed and investigated a new approach to adaptive spatio-temporal equalization for MIMO (Multiple-Input Multiple-Output) channel equalization. A decision Feedback equalizer was considered. A least squares solutions was first formulated, based on which a recursive solutions using Riccatirecursions was proposed.

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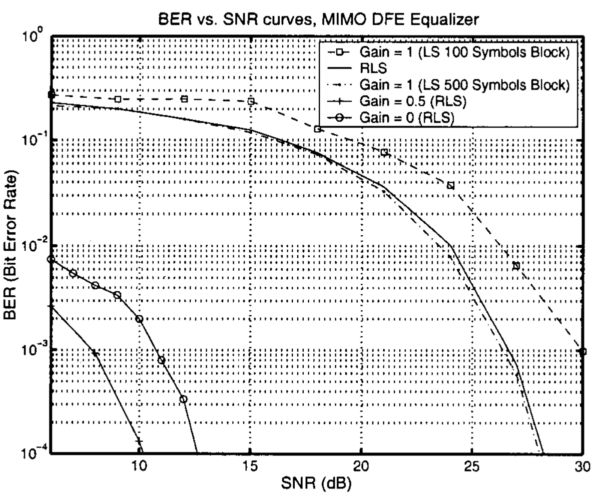

Figure 3: SNR vs. BER curves

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