Adaptive Fault-Tolerant Control of Spacecraft Attitude Dynamics with Actuator Failures

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Nomenclature

$\bar{\mathbf{A}}$	= the local vertical and local horizontal reference frame
$\bar{\mathbf{B}}$	= the body-fixed reference frame
J	= inertia matrix of spacecraft
ω	= angular velocity vector of spacecraft (rad/s)
$\bar{q}, q, q_4$	= quaternion of spacecraft
τ	= control torque input vector of spacecraft (Nm)
$\hat{\omega}$	= angular velocity vector of ideal reference model (rad/s)
$\hat{\bar{q}}, \hat{q}, \hat{q}_4$	= quaternion of ideal reference model
$\hat{\tau}$	= control input vector of ideal reference model (Nm)
$\bar{\theta}$	= adaptive parameter of angular velocity
$\bar{\delta}$	= adaptive parameter of quaternion
F_g	= the gain matrix

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g_i = actuator gain fault indicator for the ith actuator

F_d = the deviation matrix

d_i = actuator deviation fault indicator for the ith actuator

$R(\bar{q})$ = rotation matrix

I. Introduction

Spacecraft play an increasingly important role in various areas of modern society, such as telecommunication, earth observation, and space exploration. It is estimated that there have been more than 7000 spacecrafts launched all over the world. Despite rigorous testing many of these spacecraft fail on orbit due to various reasons [1], which consequently often lead to the failure of the whole mission. According to [2], over 30% of spacecraft failures occur at the subsystem level of the Attitude and Orbit Control System (AOCS). Moreover about 50% of the AOCS failures are attributed to actuator errors. The purpose of this paper is to present an actuator fault-tolerant attitude control.

In this paper, we distinguish between three types of actuator error, which are consistent with the faults that can occur in reaction wheels [3]: (i) A gain fault [4], which represents a case in which one or several actuators lose partial power but still function; (ii) A deviation fault, where an actuator delivers a constant torque in addition to the required torque; (iii) A stuck fault, which means the actuator output is stuck at a constant value of torque despite a different required torque. Previous work in the literature on fault-tolerant control focuses on just one type of fault mode, [4–6]. This paper considers a control method which could work in the presence of all of these faults.

In this paper we look at applying an adaptive control to the attitude control of a spacecraft in the presence of these actuator faults. Adaptive control refers to a control that adapts to accommodate parametric, structural, and environmental uncertainties to achieve a desired system performance, [7]. Such uncertainties often appear in aerospace actuators and automobile engines, electronic devices, and industrial processes. Payload variations or component wear and tear or even complete failure of components lead to parametric and structural uncertainties in the modelling; in addition uncertainties in the environment and the difficulty in modelling the complexity of the real system

leads us to consider a stochastic element in the modelling and a control must adapt to deal with such unknown quantities. Adaptive control has been developed in both theory and application to challenging problems of robustly controlling uncertain systems. Unlike the classical controllers, such as PID, which are conventionally based on the assumption of known system parameters, adaptive controllers do not have this strict requirement; they can adapt to parameter uncertainties by using performance error information on-line.

Conventional attitude controllers such as quaternion feedback control are tuned assuming that the system works perfectly where the parameters and constraints of the system are known [8, 9]. However, they do not take into account the re-tuning required in the event of an actuator fault. In this paper, we use an ideal reference model to identify an actuator fault where a fault is identified when the real system deviates from the ideal model. The control tracks the controlled ideal reference model to replicate it as closely as possible. Two adaptive parameters are used which increase the responsiveness of the tracking control to deviations from the ideal reference state. Moreover, it is shown that the angular velocity relative errors are more sensitive than quaternion relative errors to actuator faults. Thus the adaptive parameter shifts the emphasis to tracking the angular velocity error more aggressively than the quaternion error in the presence of a fault.

In the following section we describe the attitude kinematics and dynamics of the spacecraft and ideal reference model. Section III then addresses the problem of developing an adaptive controller in the presence of uncertainties and actuator failures. Section IV illustrates the applicability of the adaptive control through the simulation of a nano-spacecraft. Through comparing the adaptive control of this paper to a conventional proportional controller, we can see the adaptive control demonstrates an improved control performance.

II. Spacecraft System Model and Ideal Reference Model

In this paper, the dynamics of the spacecraft system can be modeled as a rigid body with negligible moving parts and no liquid propellant. In contrast to classical proportional controllers that track a reference trajectory or desired steady state, this spacecraft controller tracks the state of an idealized system under normal operating condition. This section describes the general equations

for the attitude kinematics and dynamics of the spacecraft and the ideal reference model.

The spacecraft system is considered as a rigid body. The local vertical and local horizontal (LVLH) reference frame $\bar{\mathbf{A}}$ with its origin at the centre of mass of an orbiting spacecraft has a set of unit vectors $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$, with $\vec{a}_1$ along the orbit direction, $\vec{a}_2$ perpendicular to the orbit plane, and $\vec{a}_3$ defined by the right-hand rule, towards the Earth, [9]. Define also a body-fixed reference frame $\bar{\mathbf{B}}$ with basis vectors $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$. The spacecraft attitude is then defined as the relative angle from the local-level coordinates to the body frame. Define $\omega^\times$, $\omega = [\omega_1, \omega_2, \omega_3]^T$, by the following skew-symmetric matrix:

$$\omega^\times \equiv \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{1}$$

The Euler equations of motion describing the spacecraft motion in the body-fixed reference frame can then be expressed as [9, 10]:

$$J\dot{\omega} = -\omega^\times J\omega + \tau \tag{2}$$

where $J \in \mathbb{R}^{3\times 3}$ denotes the positive definite, symmetric inertia tensor of the spacecraft; $\omega = [\omega_1, \omega_2, \omega_3]^T$ denotes the angular velocity vector of the spacecraft model with respect to the local reference frame $\bar{\mathbf{A}}$ and expressed in the body-fixed frame $\bar{\mathbf{B}}$; and $\tau = [\tau_1, \tau_2, \tau_3]^T$ denotes the control torque input vector.

The attitude kinematics of the spacecraft is parameterised using quaternions:

$$2\dot{q} = q_4\omega - \omega^\times q \tag{3}$$

$$2\dot{q}_4 = -\omega^T q \tag{4}$$

where $\bar{q} = (q, q_4) \in \mathbb{R}^3 \times \mathbb{R}$ with $q = [q_1, q_2, q_3]^T$ denotes the unit quaternion representing the attitude orientation of the ideal model in the body-fixed frame $\bar{\mathbf{B}}$ with respect to the inertial frame $\bar{\mathbf{A}}$, which are subject to the constraint

$$q^T q + q_4^2 = 1 \tag{5}$$

$\omega^\times$ is an element of the Lie algebra of the rotation group SO(3) whose Lie bracket is defined by $[X, Y] = XY - YX$. The Lie algebra $\mathfrak{so}(3)$ is isomorphic to $\mathbb{R}^3$ [11]. A rotation matrix can then be retrieved from $\bar{q}$ as:

$$R(\bar{q}) = (q_4^2 - q^T q)I_{3\times 3} + 2qq^T - 2q_4 q^\times \tag{6}$$

where $I_{3\times 3}$ is a 3×3 identity matrix.

In this paper, an idealized system under normal operating condition is set. If the adaptive control logic detects any difference between the ideal model of the system state under control and the actual state of the system, a fault is identified. The angular velocity of the ideal reference system is $\hat{\omega} = [\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3]^T$. The unit quaternion of the ideal reference system is $\hat{\bar{q}} = [\hat{q}, \hat{q}_4]$ with $\hat{q} = [\hat{q}_1, \hat{q}_2, \hat{q}_3]^T$ and the control torque of the ideal reference system is $\hat{\tau} = [\hat{\tau}_1, \hat{\tau}_2, \hat{\tau}_3]$

The dynamic and kinematics of the ideal reference model is:

$$J\dot{\hat{\omega}} = -\hat{\omega}^\times J\hat{\omega} + \hat{\tau} \tag{7}$$

$$2\dot{\hat{q}} = \hat{q}_4\hat{\omega} - \hat{\omega}^\times \hat{q} \tag{8}$$

$$2\dot{\hat{q}}_4 = -\hat{\omega}^T \hat{q} \tag{9}$$

which can be in rotation matrix form as $\dot{R}(\hat{\bar{q}}) = R(\hat{\bar{q}})\hat{\omega}^\times$

In the actual spacecraft system, the control objective is to track the ideal model with the angular velocity vector $\hat{\omega}$ and the quaternion $\hat{\bar{q}}$. The quaternion error is thus defined as

$$\begin{bmatrix} e_{q1} \\ e_{q2} \\ e_{q3} \\ e_{q4} \end{bmatrix} = \begin{bmatrix} \hat{q}_4 & \hat{q}_3 & -\hat{q}_2 & -\hat{q}_1 \\ -\hat{q}_3 & \hat{q}_4 & \hat{q}_1 & -\hat{q}_2 \\ \hat{q}_2 & -\hat{q}_1 & \hat{q}_4 & -\hat{q}_3 \\ \hat{q}_1 & \hat{q}_2 & \hat{q}_3 & \hat{q}_4 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \tag{10}$$

where $\bar{e}_q = [e_q, e_{q4}]^T$ and $e_q = [e_{q1}, e_{q2}, e_{q3}]^T$ denotes the quaternion error, which is equivalent to $R(\bar{e}_q) = R(\hat{\bar{q}})^T R(\bar{q})$ then

$$\dot{R}(\bar{e}_q) = \dot{R}(\hat{\bar{q}})^T R(\bar{q}) + R(\hat{\bar{q}})^T \dot{R}(\bar{q}) \tag{11}$$

which simplifys to

$$\dot{R}(\bar{e}_q) = R(\bar{e}_q)(\omega^\times - R(\bar{e}_q)^{-1}\hat{\omega}^\times R(\bar{e}_q)) \tag{12}$$

Defining the relative velocity as $e_\omega^\times = \omega^\times - R(\bar{e}_q)^{-1}\hat{\omega}^\times R(\bar{e}_q)$, the equation (12) comes to $\dot{R}(\bar{e}_q) = R(\bar{e}_q)e_\omega^\times$ which can be expressed in quaternion form as

$$2\dot{e_q} = e_{q4}e_\omega - e_\omega^\times e_q 2\dot{e}_{q4} = -e_\omega^T e_q \tag{13}$$

where $e_\omega = \omega - R(\bar{e}_q)^T\hat{\omega}$.

III. Adaptive Fault-Tolerant Control Logic

In this section an adaptive fault-tolerant control logic is presented which is robust to actuator faults. The actuator gain fault and deviation fault is considered. This section presents an adaptive fault-tolerant control logic that incorporates actuator faults and uncertainties.

A. Adaptive Control Logic and Stability Proof

The adaptive control logic of the actual spacecraft system is stated as

$$\tau = \omega^\times J\omega - \hat{\Omega} - \bar{\theta}e_\omega - \bar{\delta}e_{q4}e_q \tag{14}$$

where $\hat{\Omega} = Je_\omega^\times R(\bar{e}_q)^T\hat{\omega} + JR(\bar{e}_q)^T J^{-1}(\hat{\omega}^\times J\hat{\omega} - \hat{\tau})$.

This paper shows that this controller is robust to certain faults through mathematical proof and numerical simulation. However, before proceeding to address the stability properties and simulation we provide some intuition into the choice of definitions of the adaptive parameters $\bar{\theta}$ and $\bar{\delta}$.

$$\dot{\bar{\theta}} = k_\theta < e_\omega, e_\omega >_{\mathbb{R}^3} \tag{15}$$

$$\dot{\bar{\delta}} = -k_\delta < e_q, e_q >_{\mathbb{R}^3} \tag{16}$$

where $k_\theta > 0$ and $k_\delta \geq 0$. The adaptive parameters $\bar{\theta}$ and $\bar{\delta}$ are positive and are written as $\bar{\theta} = \int \dot{\bar{\theta}}\, \mathrm{d}t + \bar{\theta}(0)$ and $\bar{\delta} = \int \dot{\bar{\delta}}\, \mathrm{d}t + \bar{\delta}(0)$, where $\bar{\theta}(0) > 0$ and $\bar{\delta}(0) > 0$. The control law is initially designed with these adaptive parameters to track the ideal system.

Theorem 1 Let $e_{\omega d} = [0,0,0]^T, e_{qd} = [0,0,0,1]^T$ be the desired attitude of the system. For any tracking error (e_ω, e_q) of the spacecraft system without actuator faults (2) with the control (14). Then the tracking error $e_\omega \to e_{\omega d}$, $e_q \to e_{qd}$ as $t \to \infty$.

Proof: Define a general Lyapunov function as

$$V \equiv \frac{1}{2} < Je_\omega, e_\omega >_{\mathbb{R}^3} + \bar{\delta} < e_q, e_q >_{\mathbb{R}^3} + \frac{1}{2k_\theta}(\int \dot{\bar{\theta}}\, \mathrm{d}t)^2 \tag{17}$$

The time derivative of this Lyapunov function is given by:

$$\dot{V} =< Je_\omega, \dot{\omega} - \dot{R}(\bar{e}_q)^T\hat{\omega} - R(\bar{e}_q)^T\dot{\hat{\omega}} >_{\mathbb{R}^3} + 2\bar{\delta} < e_q, \dot{e}_q >_{\mathbb{R}^3} + \dot{\bar{\delta}} < e_q, e_q >_{\mathbb{R}^3} + \frac{1}{k_\theta}(\int \dot{\bar{\theta}}\, \mathrm{d}t)\dot{\bar{\theta}} \tag{18}$$

then substituting equation (2), (7),(15) and (16) into equation (18) gives:

$$\begin{aligned}\dot{V} = & < e_\omega, -\omega^\times J\omega + \tau + Je_\omega^\times R(\bar{e}_q)^T\hat{\omega} + JR(\bar{e}_q)^T J^{-1}(\hat{\omega}^\times J\hat{\omega} - \hat{\tau}) >_{\mathbb{R}^3} + \bar{\delta} < e_q, e_{q4}e_\omega - \\ & e_\omega^\times e_q >_{\mathbb{R}^3} - k_\delta < e_q, e_q >^2_{\mathbb{R}^3} + (\int \dot{\bar{\theta}}\, \mathrm{d}t) < e_\omega, e_\omega >_{\mathbb{R}^3}\end{aligned} \tag{19}$$

take $\hat{\Omega} = Je_\omega^\times R(\bar{e}_q)^T\hat{\omega} + JR(\bar{e}_q)^T J^{-1}(\hat{\omega}^\times J\hat{\omega} - \hat{\tau})$, and since $< e_q, -e_\omega^\times e_q >_{\mathbb{R}^3} = 0$, the equation (19) simplifies to

$$\dot{V} =< e_\omega, -\omega^\times J\omega + \tau + \hat{\Omega} + \bar{\delta}e_{q4}e_q + (\int \dot{\bar{\theta}}\, \mathrm{d}t)e_\omega >_{\mathbb{R}^3} - k_\delta < e_q, e_q >^2_{\mathbb{R}^3} \tag{20}$$

setting the control law to equation (14) gives:

$$\dot{V} = -\bar{\theta}(0) < e_\omega, e_\omega >_{\mathbb{R}^3} - k_\delta < e_q, e_q >^2_{\mathbb{R}^3} \leq 0 \tag{21}$$

Thus, the result as stated in Theorem 1 is established.

B. Actuator Fault Modes

Two main kinds of actuator fault for spacecraft are a gain fault and a deviation fault. The actuator fault changes the control torque of the spacecraft model, which is defined by:

$$\tau = F_g u + F_d \tag{22}$$

where $u = [u_1, u_2, u_3]^T$ denotes the desired control torque, $F_g \in \mathbb{R}^{3\times 3}$ is the gain fault matrix representing a gain fault and $F_d = [d_1, d_2, d_3]^T$ is the deviation fault matrix representing a deviation

fault. So for the adaptive control logic (14). The gain matrix is expressed as [4],

$$F_g = \begin{bmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \end{bmatrix} \tag{23}$$

the "gain fault indicator" $0 \leq g_i \leq 1$ can be continuous time-varying or stochastic where a stochastic element would represent uncertainty in the actuator fault. The case in which $g_i = 1$ implies that the actuator is not in a gain fault mode; $g_i = 0$ is the case in which the ith actuator is in a stuck fault mode; and $0 < g_i < 1$ corresponds to the case in which the ith actuator partially loses power (gain fault mode).

For the deviation fault matrix, d_i is the "deviation fault indicator" for the ith thruster, which is also uncertain. The case in which $d_i = 0$ implies that the ith actuator is not in deviation fault mode.

It has been shown that an increase in the gain matrix related to the angular velocity error greatly improves the control performance in the presence of an actuator fault [12]. When there is an actuator fault, the gain on the angular velocity error will increase to compensate for the increase in error. However, when the actuators are operating close to their maximum torque, the increase in the gain parameter could push the desired torque beyond the physical capability of the actuator. Therefore, to offset this increase in torque the gain related to the quaternion error could be reduced. This approach, therefore, places a greater weight on tracking the reference angular velocity relative to the reference quaternion in the presence of an actuator fault. The control presented in this paper is based on this reasoning and is also shown to guarantee asymptotic stability in the presence of a gain and stuck fault using a Lyapunov function later in the paper. This approach is informed by the observation that the angular velocity relative error is more sensitive to actuator faults than the quaternion relative error. To illustrate this point we simulate two identical spacecraft to perform a typical slew maneuver using a quaternion feedback controller of the form [9]. The first spacecraft considered (Spacecraft 1) is assumed to experience no faults and we define its angular velocity as ω_o and its quaternion as q_o. A second spacecraft (Spacecraft 2) is considered to have a gain fault described by equation (24) along with a deviation fault described by (25). The angular velocity of

Table 1 The impact of actuator fault to relative errors

$\omega(0)$	$q(0)$	$\bar{\omega}_{error}$	$\bar{q}_{error}$
$[0,0,0]$	$[0.5,0.5,0.5,0.5]$	2.2381	5.6276×10^{-6}
$[0,0,0]$	$[0,0,0,1]$	2.7211	0.0129
$[1,1,1]$	$[0.5,0.5,0.5,0.5]$	3.88005	0.0240
$[1,1,1]$	$[0,0,0,1]$	3.0393	0.0120

Spacecraft 2 is denoted as ω_f and its quaternion by q_f

$$g_i = 0.3 + 0.1rand(t) + 0.2sin(500t + i\pi/3) \tag{24}$$

$$d_1 = 0.00001 \tag{25}$$

We define the relative error of angular velocity as

$$\omega_{error} = \frac{\| \omega_f - \omega_o \|}{\| \omega_f \|} \tag{26}$$

and the relative error of the quaternion as

$$q_{error} = \frac{\| q_f - q_o \|}{\| q_f \|} \tag{27}$$

Table.1 shows the arithmetic mean of the relative error of angular velocity and the quaternion for a few examples of different initial conditions of the spacecraft motion. It shows clearly that the arithmetic mean value of the relative error of angular velocity $\bar{\omega}_{error}$ is much larger than the arithmetic mean value of the relative error of quaternion $\bar{q}_{error}$ in all the situations that are considered. Therefore, it is intuitive to increase the gain on the angular velocity error relative to the quaternion error due to the greater effect of a fault on the relative error of the angular velocity as shown in the control logic (14), (15), (16).

IV. Simulation Study

The following section is used to verify the effectiveness of the proposed control scheme. We take the Cubesat UKube-1 as the model used in the simulations [13], which weighs 4 kg and has

dimensions $10 \times 10 \times 10$ cm, with the moments of inertia

$$J = \begin{bmatrix} 0.0109 & 0 & 0 \\ 0 & 0.0506 & 0 \\ 0 & 0 & 0.0509 \end{bmatrix}$$

This type of nano-spacecraft does not undergo the rigorous testing of a conventional multi-tonne spacecraft and thus is more susceptible to faults. The real UKube-1 is not equipped with reaction wheels; nevertheless, this type of actuator (3-axis stabilization) is used in the simulations. In this paper we use large magnitudes for the actuator faults as it enables the demonstration of the control to be illustrated most effectively. However, using these large magnitudes for the faults means that the corresponding desired torque is out-with the current nano-spacecraft reaction wheel capability.

In this paper, a simple quaternion feedback controller [14] can be used to perform a simple slew maneuver for the ideal system:

$$\hat{\tau} = -\sigma J \hat{\omega} - k J \hat{q}_e \tag{28}$$

where $\sigma, k > 0$ are scalar constants, which can be extended to counter the unwinding problem by introducing a discontinuity. The quaternion error of the reference model $\hat{\bar{q}}_e = [\hat{q}_{1e}, \hat{q}_{2e}, \hat{q}_{3e}, \hat{q}_{4e}]^T$ is calculated from the commanded attitude quaternion $\hat{\bar{q}}_c = [\hat{q}_{1c}, \hat{q}_{2c}, \hat{q}_{3c}, \hat{q}_{4c}]^T$ and the current quaternion $\hat{\bar{q}}$ as equation (10). In this paper, the simulation is taken with a simple baseline maneuver which is planned rest-to-rest from $\hat{q}_c(0) = [0, 0, 0, 1]^T$ to $\hat{q}_c(T_f) = [0.5, 0.5, 0.5, 0.5]^T$. The initial value of the adaptive parameters in the control logic (14) can be chosen as the parameters in the ideal reference model (28) such that $\bar{\theta}(0) = \sigma$ and $\bar{\delta}(0) = k$.

A. Simulation with a Gain Fault

In general this fault can be expressed in the form $g_i = \alpha + \beta rand(t) + \epsilon sin(\gamma t + i\pi/3)$ with α as a mean value; $rand(t)$ a random number between 0 and 1, and $sin(\gamma t + i\pi/3)$ is time-varying. This general form can be used to demonstrate different types of faults that are secular, periodic and stochastic. In the simulation of this section, the random part is not considered, with the indicator g_i set as

$$g_i = 0.1 + 0.1 sin(0.1t + i\pi/3) \quad (i = 1, 2, 3) \tag{29}$$

Note that, to show the fault clearly, the value of the fault in the simulation in this section is much higher than would usually occur in the real system. The fault indicator $g_i \in [0, 1]$ implies that the actuator is operating near to perfect if g_i is near to 1. In this simulation, g_i is varied from 0 to 0.2, which means the actuator experiences a very large gain fault.

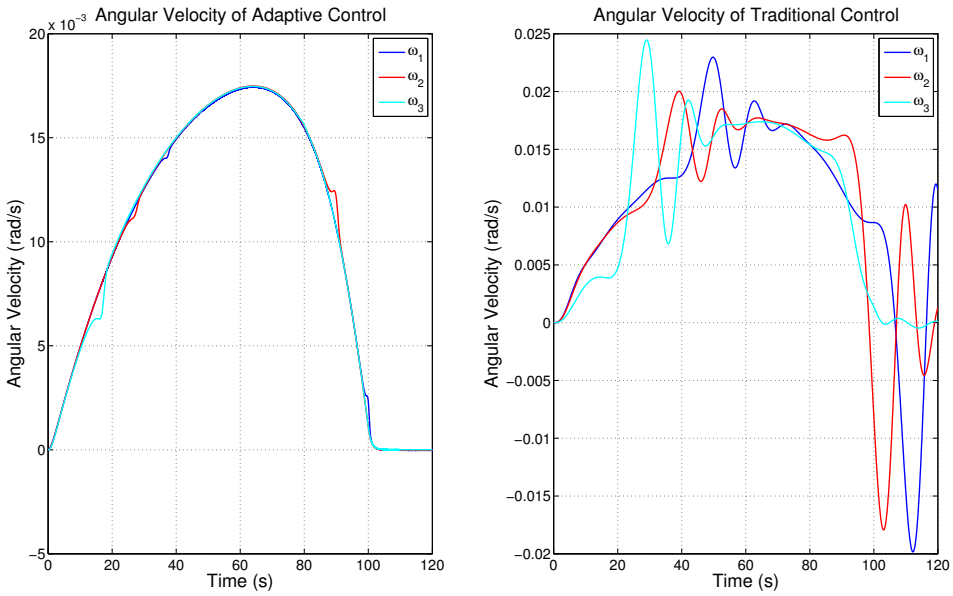

Fig. 1 Angular velocity

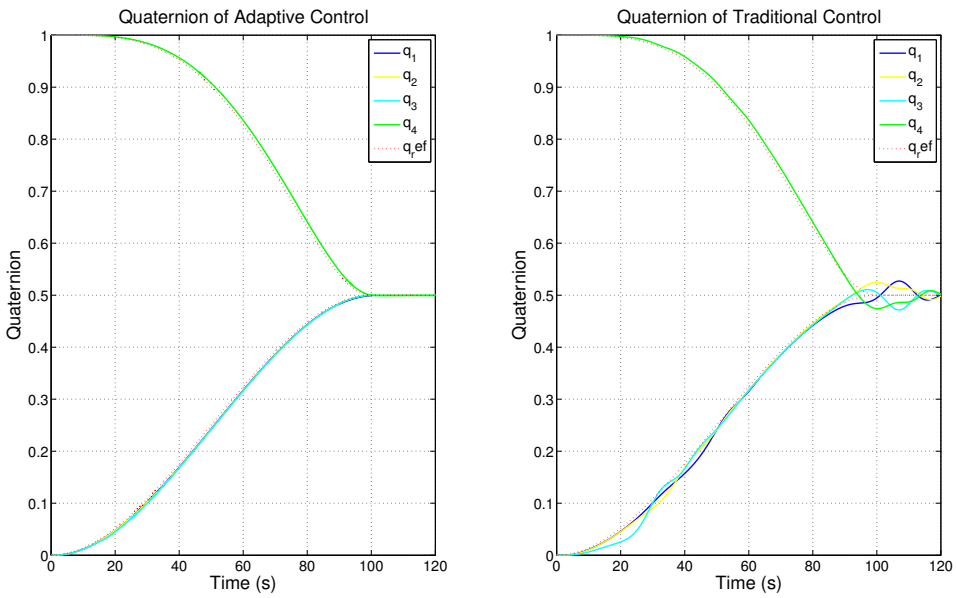

Fig. 2 Quaternion

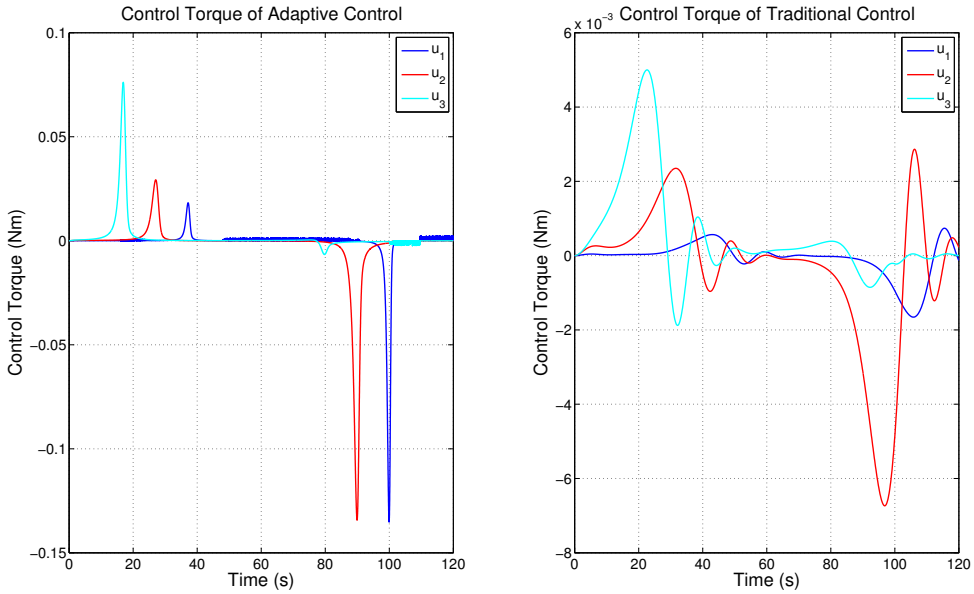

Fig. 3 Control torque

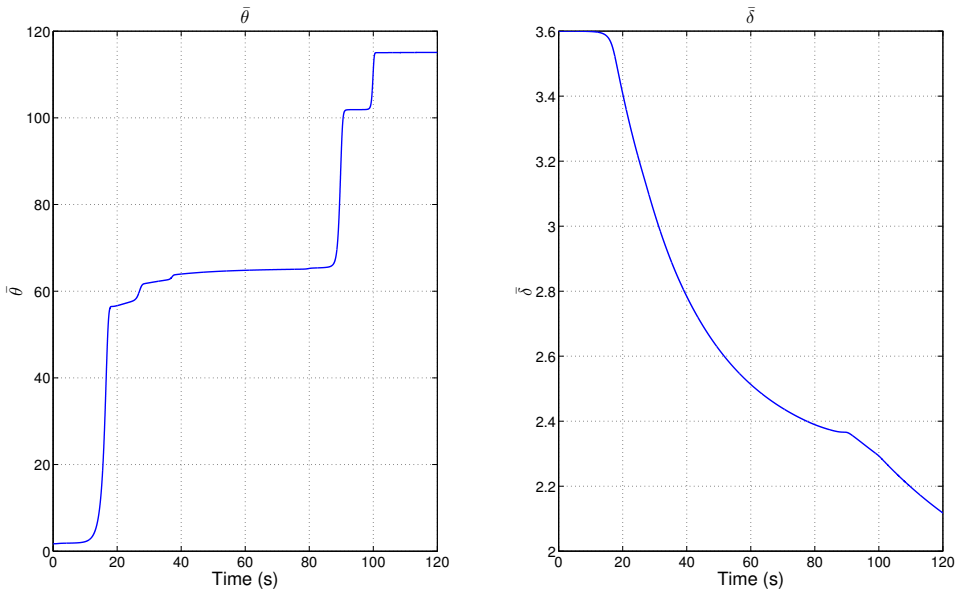

Fig. 4 Adaptive Parameters

The angular velocity of the adaptive fault-tolerant control method and the traditional quaternion feedback control method are separately shown in Fig. 1. The quaternion tracking is shown in Fig. 2. Fig. 3 shows the corresponding control torque of the adaptive and traditional control method

and Fig. 4 shows the adaptive parameters $\bar{\theta}$ and $\bar{\delta}$ against time, where $k_\theta = 10^7$ and $k_\delta = 5 \times 10^3$ is tuned experimentally for best performance. The traditional control method fails to perform the required motion whereas the adaptive control performs the maneuver.

B. Simulation with Deviation Fault

To illustrate the nature of deviation fault, we set

$$d_1 = \begin{cases} 0 & t \leq 50s \\ 0.01 & t \geq 50s \end{cases} \tag{30}$$

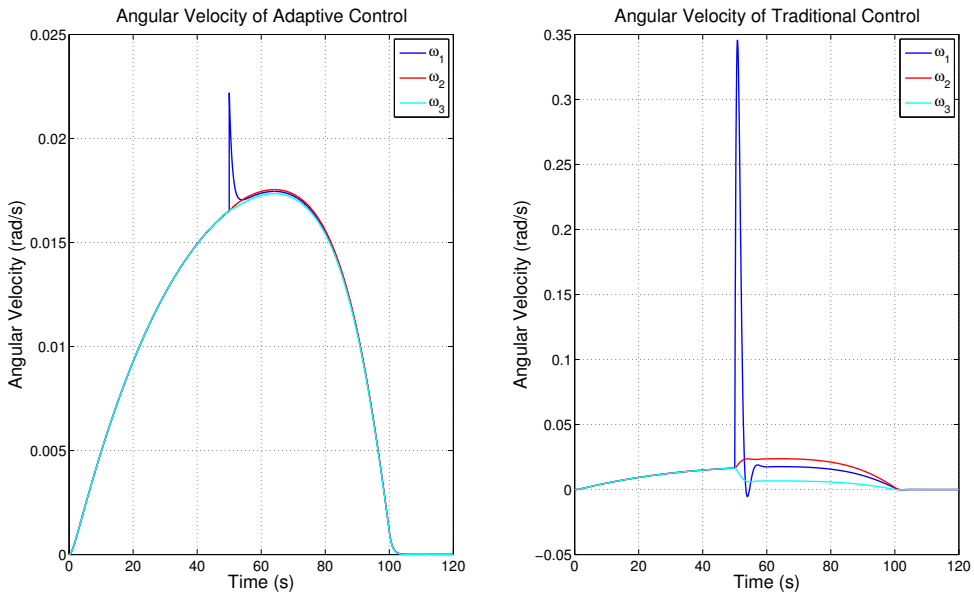

Fig. 5 Angular velocity

The angular velocity of the spacecraft using the adaptive fault-tolerant control and the traditional quaternion feedback control are separately shown in Fig. 5 and the quaternion tracking in Fig. 6. Fig. 7 illustrates the control torque of the adaptive and traditional control. Fig. 8 shows how adaptive parameter changes, where $k_\theta = 10^3$ and $k_\delta = 500$. From these figures, we can see that when the actuator deviation fault occurs at $50s$, the angular velocity on both control methods immediately deviate. In the case of the adaptive control method, the angular velocity quickly responds and moves back to the desired reference while the traditional control method completely loses control. Fig. 6 shows that the traditional control method could not control the quaternion.

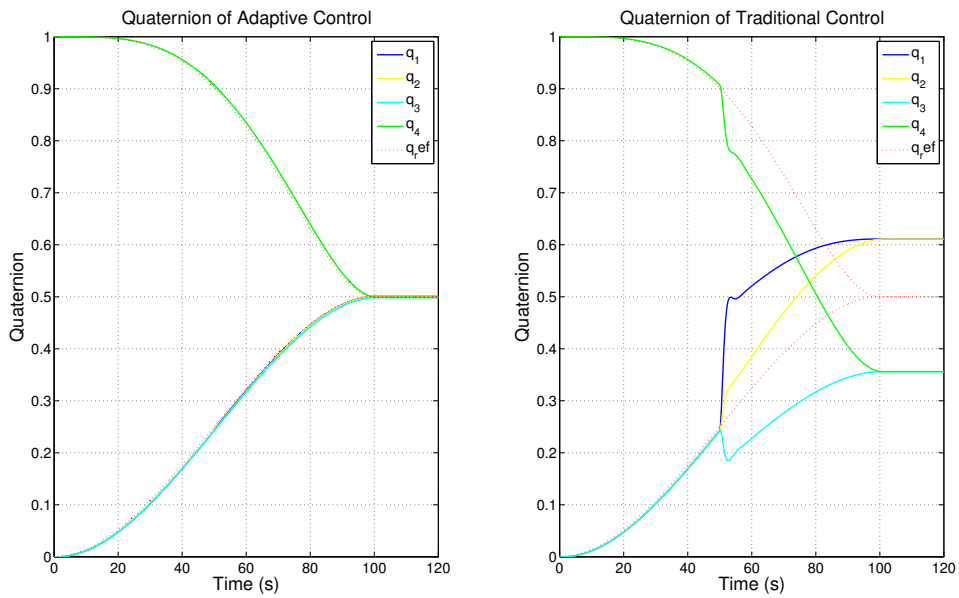

Fig. 6 Quaternion

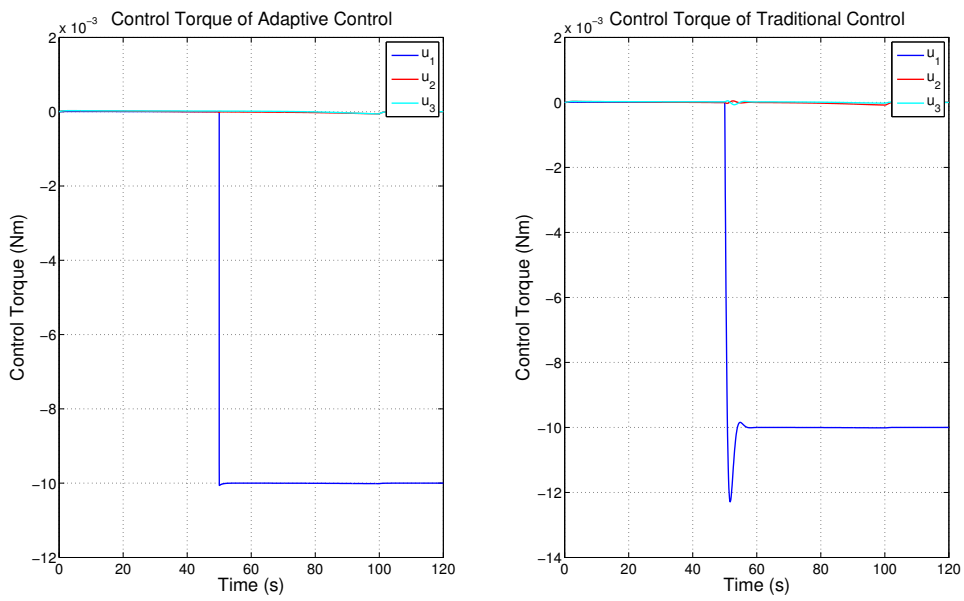

Fig. 7 Control torque

C. Simulation with Combined Actuator Faults

To show the ability of the adaptive control method in dealing with the actuator faults, both a gain fault and a deviation fault are simulated. With the gain fault as

$$g_i = 0.7 + 0.15rand(t) + 0.1sin(0.1t + i\pi/3) \quad (i = 1, 2, 3) \tag{31}$$

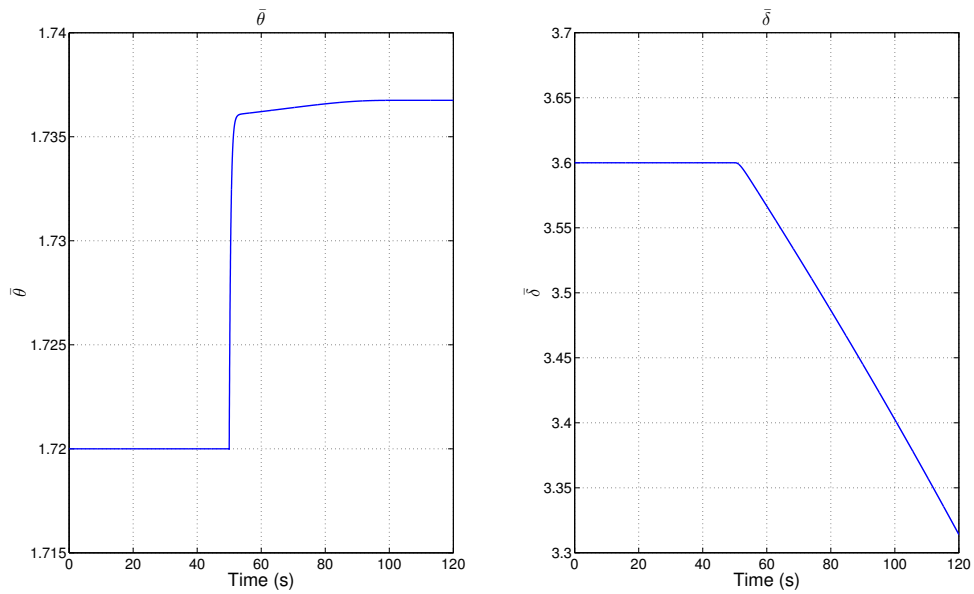

Fig. 8 Adaptive Parameters

where $rand(t)$ is a random number between 0 and 1. The deviation fault is

$$d_1 = \begin{cases} 0 & t \leq 50s \\ 0.005 & t \geq 50s \end{cases} \tag{32}$$

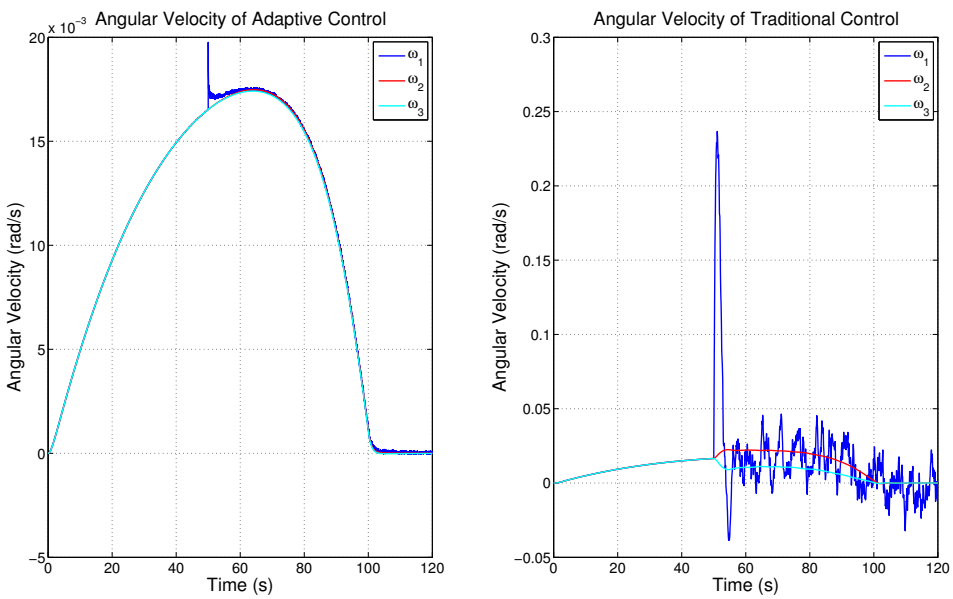

Fig. 9 Angular velocity

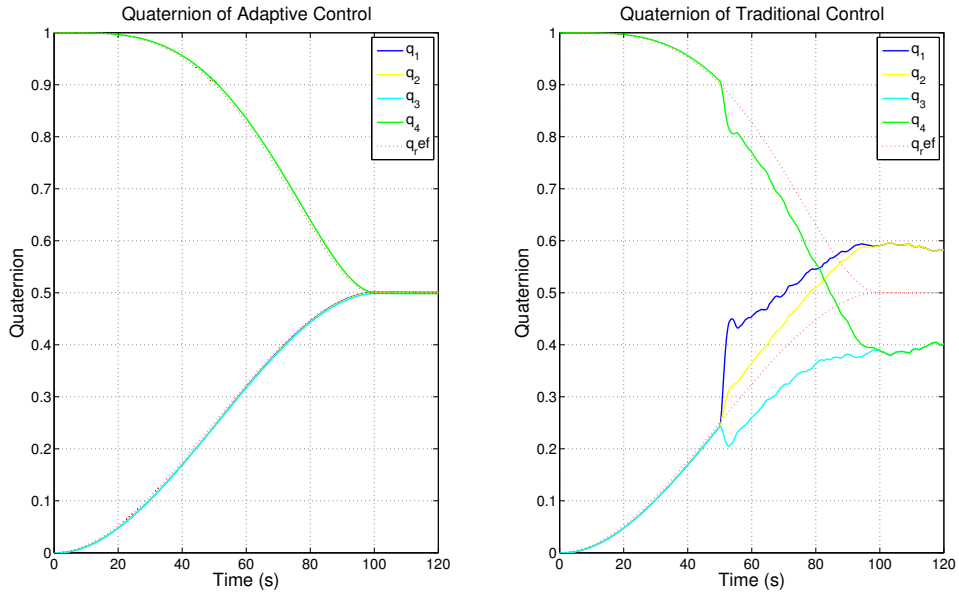

Fig. 10 Quaternion

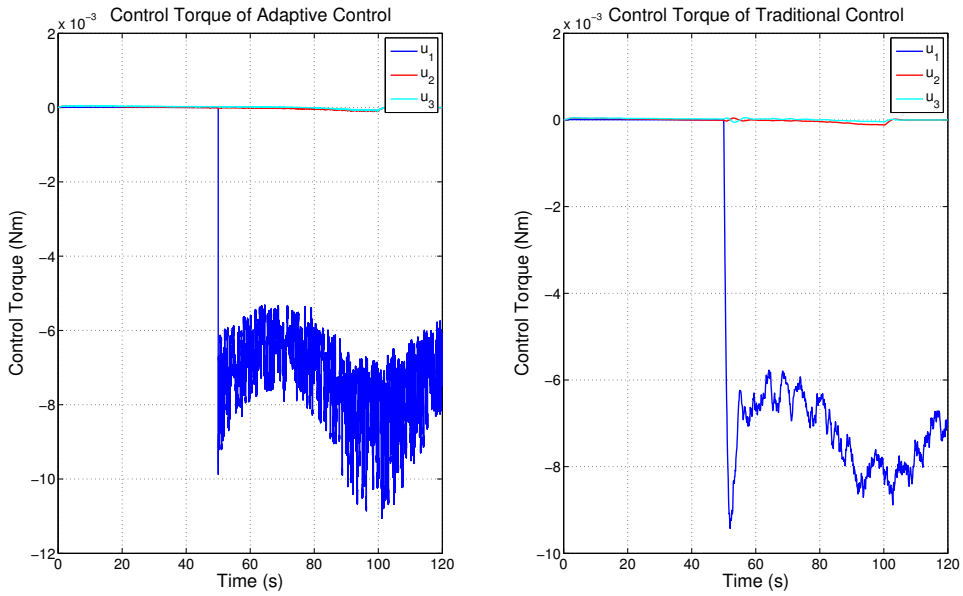

Fig. 11 Control torque

The angular velocity of the adaptive fault-tolerant control method and the traditional quaternion feedback control method are shown in Fig. 9. The quaternions are shown in Fig. 10. Fig. 11 illustrates the control torque of the adaptive control method and the traditional control method, respectively. Fig. 12 shows the variation of the two adaptive parameters $\bar{\theta}$ and $\bar{\delta}$, in this simulation

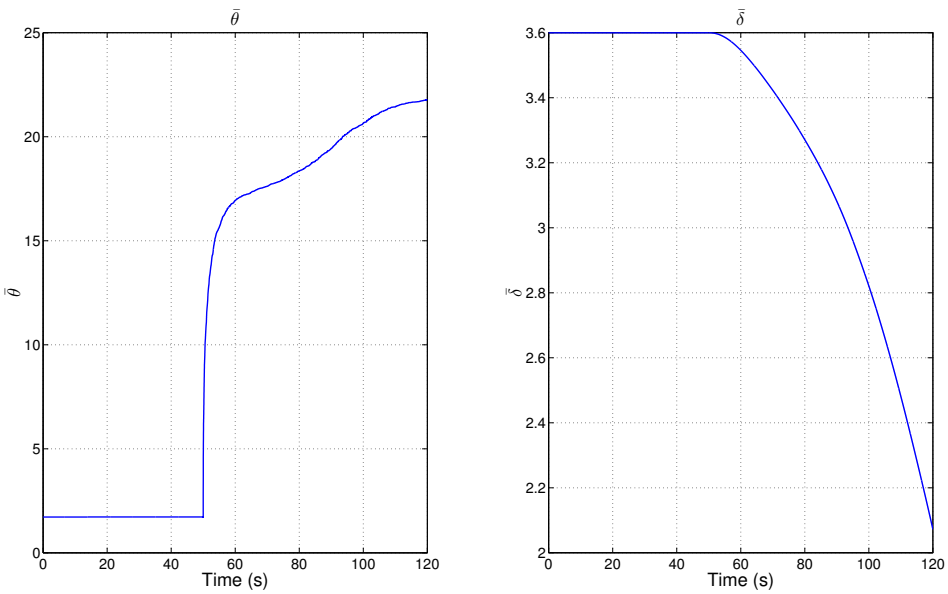

Fig. 12 Adaptive Parameters

$k_\theta = 10^7$ and $k_\delta = 3 \times 10^3$. From these figures, compared with the traditional quaternion control law, the adaptive fault-tolerant control law demonstrates a good control performance relative to the conventional controller (28) and reaches the desired state.

V. Conclusion

Two kinds of spacecraft actuator failures were considered: a gain fault, and a deviation fault. An adaptive fault-tolerant control method is proposed for the spacecraft experiencing these actuator failures. The fault-tolerant control in this paper relies on an ideal reference model to identify when a fault occurs. The control tracks the ideal reference model to replicate it as closely as possible. This control employs adaptive parameters to improve the responsiveness of the angular velocity error and the quaternion error due to actuator faults. Moreover, the angular velocity error magnitude is more sensitive than the quaternion error to an actuator fault. This sensitivity can be exploited in the control design by more aggressively tracking the angular velocity of the ideal system relative to the quaternion error. This is achieved by introducing time-dependent parameters that weight the components of the feedback control respectively. In the case of a gain fault (and stuck fault) the stability is proved by a Lyapunov function. This adaptive control has been shown to significantly

improve the performance over a conventional control in the presence of these actuator faults.

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