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Mao, Zehui and Yan, Xinggang and Jiang, Bin and Chen, Mou (2019) Adaptive Fault-Tolerant Sliding-Mode Control for High-Speed Trains with Actuator Faults and Uncertainties. IEEE Transactions on Intelligent Transportation Systems .

### DOI

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# Adaptive Fault-Tolerant Sliding-Mode Control for High-Speed Trains with Actuator Faults and Uncertainties

Zehui Mao, Xing-Gang Yan, Bin Jiang, *Senior Member, IEEE*, Mou Chen, *Member, IEEE*

**Abstract**—In this paper, a novel adaptive fault-tolerant sliding-mode control scheme is proposed for high-speed trains, where the longitudinal dynamical model is focused, and the disturbances and actuator faults are considered. Considering the disturbances in traction force generated by the traction system, a dynamic model with actuator uncertainties modelled as input distribution matrix uncertainty is established. Then, a new sliding-mode controller with design conditions is proposed for the healthy train system, which can drive the tracking error dynamical system to a pre-designed sliding surface in finite time and maintain the sliding motion on it thereafter. In order to deal with the actuator uncertainties and unknown faults simultaneously, the adaptive technique is combined with the fault-tolerant sliding-mode control design together to guarantee that the asymptotical convergence of the tracking errors is achieved. Furthermore, the proposed adaptive fault-tolerant sliding-mode control scheme is extended to the cases of the actuator uncertainties with unknown bounds and the unparameterized actuator faults. Finally, case studies on a real train dynamic model are presented to explain the developed fault-tolerant control scheme. Simulation results show the effectiveness and feasibility of the proposed method.

**Index Terms**—Actuator faults, fault-tolerant sliding-mode control, adaptive control, actuator uncertainty, high-speed train.

## I. INTRODUCTION

Due to the increasing requirements of the reliability and safety of the modern control systems, fault detection and fault-tolerant control design have attracted more and more researchers and engineers (see [1]- [5]). High-speed trains with their high loading capacities, fast and on schedule, have been one of the most important transportation means. Similar to the other large-scale and complex control systems, faults also exist in high-speed trains, which motivates the studies of the fault detection and fault-tolerant control design for high-speed trains (see [6]- [9]).

Uncertainty, including modelling uncertainty and disturbance, widely exists in real physical systems, and thus it is essential to consider various uncertainties in control design, fault detection and fault-tolerant control design. For high-speed trains, there exist some internal and external uncertainties,

such as modelling uncertainties from the electric equipments and mechanical installations, and disturbances from the track irregularities, tunnels and slopes. It should be noted that the external disturbances can be modelled as an additional signal for the system model, while the internal uncertainties should be modelled as state or input/actuator uncertainties in the system differential dynamical equation.

It is well known that the input saturation, deadzone and hysteresis are popular problems for actuator uncertainties, see [10]- [11], for which the input signals are limited and bounded. It should be noted that the internal uncertainties cannot be considered as the external uncertainties in system modelling, since the boundedness of the system states should be ensured by the controller design, which are always used in the designed controller and cannot be assumed to be bounded, *a priori*. Actually, the complex coupling between the input distribution uncertainties and the control signal makes the control design full of challenges. Among the existing results for the controller or fault-tolerant controller design of high-speed trains, the external disturbances, which are modelled as an additional signal for the system model, are widely investigated [12]- [15]. However, the internal uncertainties, which are modelled as state or input/actuator distribution matrix uncertainties in the system differential dynamical equation, are rarely taken into considerations. Thus, the fault-tolerant control for high-speed train with actuator uncertainties is of both theoretical challenge and practical importance.

For the faulty system, the fault-tolerant control is an essential and effective technique to guarantee system stability and/or some performances (such as asymptotic tracking), in the presence of faults. Due to the unknown fault, adaptive techniques are always used to deal with this case to achieve the desired tracking performance (see [16]- [20]). As the position/speed tracking is the main task for trains to guarantee the on-time schedule, the adaptive technique is pertinent to high-speed trains with unknown faults. Moreover, the results about the adaptive fault-tolerant sliding-mode control are rare, although there are some works for the aircrafts [21], [22].

This paper is focused on the fault-tolerant control problem for the longitudinal dynamical model of high-speed trains with traction system actuator faults and uncertainties. Both the traction system actuator uncertainties and external disturbances are considered, which are modelled as the input distribution matrix uncertainties and additional disturbances in the high-speed train. For the healthy and different faulty cases, the adaptive fault-tolerant sliding-mode control schemes are proposed

This work was supported in part by the National Natural Science Foundation of China under Grant 61490703, Grant 61573180, Qing Lan Project and the Fundamental Research Funds for the Central Universities No. NE2019101. (Corresponding author: Bin Jiang.)

Z. Mao, B. Jiang and M. Chen are with College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, China, Email: zehuimao@nuaa.edu.cn (Z. Mao), binjiang@nuaa.edu.cn (B. Jiang), chenmou@nuaa.edu.cn (M. Chen).

X. Yan is with School of Engineering and Digital Arts, University of Kent, Canterbury, Kent CT2 7NT, United Kingdom, Email: x.yan@kent.ac.uk.

with the controller structure, design conditions, and adaptive laws being derived. The main contributions of this paper are summarized as follows:

- (i) Considering the traction system actuator uncertainties and external disturbances, a model with input distribution matrix uncertainty and additional disturbances is introduced to describe the dynamic properties of the high-speed trains.
- (ii) A set of conditions and the controller structure are developed for the healthy case such that the designed novel sliding-mode controller can drive the tracking error dynamical system to a pre-designed sliding surface in finite time and maintains the sliding motion on it thereafter, even in the presence of input distribution matrix uncertainty.
- (iii) For different cases (the bound of the actuator uncertainty is unknown; the actuator fault is unparameterized), the fault-tolerant sliding-mode controllers with adaptive laws are developed for the longitudinal dynamical model of high-speed trains, respectively.

The rest of this paper is organized as follows: In Section II, the longitudinal dynamical model of high-speed trains with actuator uncertainties is presented, and the actuator fault-tolerant control problem is formulated. In Section III, a sliding-mode controller with the design condition is developed for the healthy system with actuator uncertainties and external disturbances, to achieve the displacement and speed tracking. In Section IV, a new fault-tolerant sliding-mode controller with adaptive laws is proposed for the faulty system with the known bound of fault. In Section V and VI, the proposed fault-tolerant sliding-mode controller is extended to the cases of the actuator uncertainties with unknown bound and unparameterized fault, respectively. In Section VII, simulations for four cases (health and faulty cases) are presented, and the effectiveness of the fault-tolerant control scheme is verified. Finally, Section VIII concludes the paper.

## II. PROBLEM FORMULATION

For high-speed trains, the general dynamical model of longitudinal motion can be described as [6], [23], [24]

$$M(t)\ddot{x}(t) = F_t(t) - M(t)(a + bv(t) + cv^2(t)) + d(t), \quad (1)$$

where  $x(t)$  is the displacement of the train,  $M(t)$  is the mass of the train,  $F_t(t)$  is the traction force generated by the traction system, the parameters  $a$ ,  $b$  and  $c$  are resistive force coefficients of the Davis equation,  $d(t)$  models the external disturbances from weather conditions or rail conditions (ramp, tunnel, curvature, etc.).

**Remark 1:** It should be noted that the slope and curvature rails can induce additional resistances. In order to achieve a high speed for a high-speed train, the railway should be smooth, and the slope angle and the degree of curvature should be as small as possible. According to [29], under the speed 300km/h, the minimum curve radius is 4500m, and the maximum slop is 12‰. In connection with this, the train moves in a one-dimensional space, with slope and curvature resistances considered as disturbances, which are modeled as

(1). In China, a plenty of bridges are built to make the railway straight. On the other hand, the suspension system model is always used to describe the lateral and roll dynamics, which can be decoupled from the longitudinal dynamic model (1). Thus, the considering that train moves in a one-dimensional space and modelled as a rigid body, is reasonable.  $\square$

**Actuator uncertainty.** According to [6], the mass of a train can be considered as varying with respect to the stations and keeps constants between two consecutive stations. Therefore, it is reasonable to express the mass of train in the dynamics (1) as  $M(t) = \bar{M} + \Delta_M(t)$ , where  $\bar{M}$  is a constant determined by the loadings of train,  $\Delta_M(t)$  is also a constant during the two stations and only changed at the stopping stations. According to the maximum loading of a train,  $\Delta_M(t)$  is bounded and its bound can be estimated in reality.

The traction system generates the traction force, which is considered as the actuators in high-speed trains, and consists of traction motors, inverters, PWMs (pulse width modulations), rectifiers, and related mechanical drives, etc. The uncertainties widely exist in these equipments. In this paper, considering the actuator uncertainties, a dynamics model is introduced to express the traction force  $F_t(t)$  as follows:

$$F_t(t) = (1 + \Delta_f(t))F(t) + \Delta F(t), \quad (2)$$

where  $\Delta_f(t)$  and  $\Delta F(t)$  are time-varying functions to represent the uncertainties in the traction system,  $F(t)$  is the force that the motors provide. The traction force model (2) contains both additive and multiplicative uncertainties, which are used to express the most of the actuator uncertainties. Moreover, these two terms  $\Delta_f(t)$  and  $\Delta F(t)$  are bounded with their bounds obtained from the maximum traction force and mechanical installation.

**Remark 2:** For high-speed trains, both the input saturation and deadzone exist in the actuators. Because the breaking system is working when the traction system starts, the input deadzone can be avoided, as traction force is applied to the train when the motors in the traction system work normally. Moreover, the allowed maximin speed decides the maximin traction forces and the redundances of the traction system. Then, the high-speed train cannot be operated under the input saturation. Thus, the presented traction force model (2) can mainly display the uncertainties in the high-speed train actuator.  $\square$

**Dynamic model of high-speed trains.** Let  $x_1 = x$ ,  $x_2 = \dot{x}$ ,  $m = 1/\bar{M}$ ,  $\Delta m(t) = (1 + \Delta_f(t))/M(t) - 1/\bar{M}$  and  $\bar{d}(t) = (d(t) + \Delta F(t))/M(t)$ . Due to the known bounds of  $\Delta_f(t)$ ,  $\Delta F(t)$  and  $M(t)$ , the bounds of  $\Delta m(t)$  and  $\bar{d}(t)$  can be calculated easily. The longitudinal motion dynamics (1) with (2) can be expressed as

$$\dot{x}_1(t) = x_2(t), \quad (3)$$

$$\dot{x}_2(t) = (m + \Delta m(t))F(t) - a - bx_2(t) - cx_2^2(t) + \bar{d}(t), \quad (4)$$

where  $m$ ,  $a$ ,  $b$  and  $c$  are known system parameters,  $\Delta m(t)$  and  $\bar{d}(t)$  satisfy the following conditions:

$$0 \leq \Delta m(t) \leq m_b < m, \quad |\bar{d}(t)| \leq d_b, \quad (5)$$

with  $m_b$  and  $d_b > 0$  being known constants.

**Actuator faults.** The general faults for traction system are motor faults, IGBT faults in rectifier and inverter, mechanical faults, and so on. In modelling, most of these faults can be equivalent to the effectiveness loss of the motor, and the traction force can be considered as the sum of the motor forces. The parametric fault model for one motor can be expressed as (see, e.g. [9] and [20])

$$F_i(t) = \bar{F}_i(t) = f_{i0} + \sum_{\rho=1}^{l_i} f_{i\rho} s_{i\rho}(t), \quad t \geq t_i, \quad (6)$$

for some  $i \in \{1, 2, \dots, n\}$ , where  $n$  is the number of motors. Here,  $t_i$  is the fault occurring time instant,  $i$  is the fault index,  $f_{i0}$  and  $f_{i\rho}$  are constants, which are all unknown. The basis signals  $s_{i\rho}(t)$  are known, and  $l_i$  are the number of the basis signals of the  $i$ th actuator fault.

This fault model (6) covers several practical fault conditions of the high-speed train actuators, which is shown as follows:

1) Totally fault. The motor stopping fault is a total fault. Then, Eq. (6) can be written as  $F_i(t) = \bar{F}_i(t) = f_{i0} = 0$ , with  $f_{i\rho} = 0$ , for  $\rho = 1, \dots, l_i$ .

2) Constant fault. The mechanical drives locked fault can lead the constant torque, which is a constant actuator fault. Then, Eq. (6) can be written as  $F_i(t) = \bar{F}_i(t) = f_{i0} = \text{non-zero constant}$ , with  $f_{i\rho} = 0$ , for  $\rho = i, \dots, l_i$ .

3) Periodic fault. The IGBT (Insulated Gate Bipolar Transistor) fault (from PWM) can lead the periodic fault with approximately known frequency, which could be a sine function. Then, Eq. (6) can be written as  $F_i(t) = \bar{F}_i(t) = f_{i1} \sin(\omega t)$  for some known  $\omega$ , with  $f_{i0} = 0$ ,  $f_{i1} = \text{non-zero unknown constant}$  and  $f_{i\rho} = 0$ , for  $\rho = 2, \dots, l_i$ .

In some cases, a completely parameterized fault model may be an ideal model for some time-varying actuator faults, as the knowledge of the basis functions  $f_{i\rho}(t)$  may not be available for some applications. In such cases, approximations of the basis functions  $f_{i\rho}(t)$  can be employed to achieve approximate compensation of actuator faults. Some commonly used approximation methods, such as Taylor series and neural networks, are employed to approximate the unknown actuator faults. The approximation for the actuator fault, usually will result in a bounded approximation error, the whose magnitude can be very small by proper choices of the basis functions used in approximation.

Consider that there are  $n$  motors. From (6), the input of system (3)-(4) can be rewritten as

$$F(t) = \sigma_\nu \nu(t) + \vartheta^T \zeta(t), \quad (7)$$

$$\vartheta = [\vartheta_1^T, \vartheta_2^T, \dots, \vartheta_n^T]^T,$$

$$\vartheta_i = [f_{i0}, f_{i1}, \dots, f_{il_i}]^T \in R^{l_i+1}, \quad i = 1, \dots, n, \quad (8)$$

$$\zeta(t) = [1, s_{11}(t), \dots, s_{1l_1}(t), \dots, 1, s_{n1}(t), \dots, s_{nl_n}(t)]^T, \quad (9)$$

where  $\nu(t)$  is the control input,  $\sigma_\nu$  is the number of the remaining healthy actuators,  $\vartheta$  and  $\zeta(t)$  are the actuator fault pattern parameters describing the types of faults. The vector  $\vartheta$  could change with the fault evolution, but is fixed in a time interval.

For actuator fault-tolerant control design of high-speed trains, the assumption for faults is given as: (A1) there is no more than  $\bar{n}$  ( $\bar{n} < n$ ) actuators fail, and the fault parameter  $\vartheta$  is bounded and satisfies  $\|\vartheta\|_2 \leq \vartheta_0$ , where  $\vartheta_0 > 0$  is a known constant. It implies that for  $n - \bar{n} \leq \sigma_\nu \leq n$ , the remaining healthy actuators can still achieve the desired control objective.

**Objective.** The objective of this paper is to develop an adaptive fault-tolerant sliding-mode control scheme for the high-speed trains described by (3) and (4), to guarantee the stability and asymptotic tracking properties, in the present of the actuator uncertainty  $\Delta m(t)$  and actuator faults modeled in (7)-(9).

### III. SLIDING-MODE CONTROLLER DESIGN FOR HEALTHY CASE

In this section, a controller is to be designed to make the close-loop system (3)-(4) stable and achieve the tracking performance. For high-speed trains, the Curve-To-Go is always achieved through speed tracking. Let the desired speed trajectory be  $x_d(t)$ , and the desired displacement trajectory  $y_d(t)$ . Then,  $\dot{y}_d(t) = x_d(t)$ .

**Sliding-surface design.** Denote the tracking errors  $e_1(t) = x_1(t) - y_d(t)$  and  $e_2(t) = x_2(t) - x_d(t)$ . From (3)-(4), the tracking error dynamic equation can be written as:

$$\dot{e}_1(t) = e_2(t), \quad (10)$$

$$\dot{e}_2(t) = (m + \Delta m(t))F(t) - a - bx_2(t) - cx_2^2(t) + \bar{d}(t) - \dot{x}_d(t). \quad (11)$$

For error dynamical system (10)-(11), design a sliding function:

$$\delta(e_1, e_2) = ke_1(t) + e_2(t), \quad (12)$$

where  $k > 0$  is a design parameter. The sliding surface  $\delta(t) = 0$  can be described by

$$e_2(t) = -ke_1(t). \quad (13)$$

From the structure of system (10)-(11), it is straight forward to see that system (10) dominates the sliding motion of the system (10)-(11) with respect to the sliding surface (13). From (10) and (13), the corresponding sliding mode dynamics can be described by

$$\dot{e}_1(t) = -ke_1(t), \quad (14)$$

which implies

$$e_1(t) = e^{-kt} e_1(0), \quad e_1(0) = x(0) - y_d(0). \quad (15)$$

Due to  $k > 0$ , it is clear to see that  $\lim_{t \rightarrow \infty} e_1(t) = 0$ . The analysis above shows that the sliding motion of the error dynamical system (10)-(11) associated with the sliding surface (13) is asymptotically stable. Therefore, after sliding motion occurs, it has  $\lim_{t \rightarrow \infty} (x_1(t) - y_d(t)) = 0$ , which implies that  $x_1(t)$  tracks the desired signal  $y_d(t)$  asymptotically. The objective now is to design a sliding-mode controller such that the error system (10)-(11) can be driven to the sliding surface (13) in finite time and maintains the sliding motion thereafter.



**Sliding-mode controller design.** For train dynamic system (3)-(4), consider the controller

$$F(t) = -\frac{1}{m}F_0(t) - \frac{1}{m}d_b - \frac{r(t)}{m}\text{sgn}\left(k(x_1(t) - y_d(t)) + x_2(t) - x_d(t)\right), \quad (16)$$

where

$$F_0(t) = k(x_2(t) - x_d(t)) - a - bx_2(t) - cx_2^2(t) - \dot{x}_d(t), \quad (17)$$

$r(t)$  is a nonnegative time varying gain to be designed later, and  $d_b$  satisfies (5).

Then, the following result is ready to be presented.

**Theorem 1:** The sliding-mode control in (16) drives the error dynamical system (10)-(11) to the sliding surface (13) in finite time and maintains a sliding motion on it thereafter if  $m_b < m$  and the control gain  $r(t)$  in (16) satisfies

$$r(t) \geq \frac{m}{m - m_b} \left( \eta + \frac{m_b}{m} (|F_0(t)| + d_b) \right), \quad (18)$$

for  $\eta > 0$ .

**Proof:** From (12) and (10)-(11), the dynamic equation of sliding surface can be given by

$$\begin{aligned} \dot{\delta}(t) &= k\dot{e}_1(t) + \dot{e}_2(t) \\ &= k(x_2(t) - x_d(t)) + (m + \Delta m(t))F(t) \\ &\quad - a - bx_2(t) - cx_2^2(t) + \bar{d}(t) - \dot{x}_d(t). \end{aligned} \quad (19)$$

Substituting (16) into equation (19) yields

$$\begin{aligned} \dot{\delta}(t) &= \Delta m(t) \left( -\frac{1}{m}F_0(t) - \frac{1}{m}d_b - \frac{r(t)}{m}\text{sgn}(\delta(t)) \right) \\ &\quad - d_b + \bar{d}(t) - r(t)\text{sgn}(\delta(t)), \end{aligned} \quad (20)$$

where  $\delta(t)$  is the sliding function defined in (12).

From (20) and  $\delta(t)\text{sgn}(\delta(t)) = |\delta(t)|$ , it follows that

$$\begin{aligned} \delta(t)\dot{\delta}(t) &= \delta(t)\Delta m \left( -\frac{1}{m}F_0(t) - \frac{1}{m}d_b - \frac{r(t)}{m}\text{sgn}(\delta(t)) \right) \\ &\quad - r(t)|\delta(t)| - \delta(t)(d_b - \bar{d}(t)). \end{aligned} \quad (21)$$

From (5), (21) and  $r(t) > 0$ ,

$$\begin{aligned} \delta(t)\dot{\delta}(t) &\leq |\delta(t)|m_b \left( \frac{1}{m}|F_0(t)| + \frac{1}{m}d_b + \frac{r(t)}{m} \right) - r(t)|\delta(t)| \\ &= - \left( -\frac{m_b}{m}(|F_0(t)| + d_b + r(t)) + r(t) \right) |\delta(t)|. \end{aligned} \quad (22)$$

From (18), it has

$$\frac{m - m_b}{m}r(t) \geq \eta + \frac{m_b}{m}(|F_0(t)| + d_b). \quad (23)$$

The inequality (23) can be rewritten as

$$r(t) - \frac{m_b}{m}r(t) - \frac{m_b}{m}(|F_0(t)| + d_b) \geq \eta, \quad (24)$$

which implies that

$$r(t) - \frac{m_b}{m}(r(t) + |F_0(t)| + d_b) \geq \eta. \quad (25)$$

Substituting (25) into (21), yields

$$\delta(t)\dot{\delta}(t) \leq -\eta|\delta(t)|. \quad (26)$$

Therefore, the reachability condition holds and hence the result follows.  $\nabla$

The proposed sliding-mode controller (16) with  $F_0$  defined in (17), can drive the error dynamics (10)-(11) to the sliding surface (13) in finite time. Since the sliding motion has been asymptotically stable as analysed earlier, it follows that  $\lim_{t \rightarrow \infty} e_1(t) = 0$  and  $\lim_{t \rightarrow \infty} e_2(t) = 0$ . Thus, the proposed controller (16) can guarantee the tracking errors of health train system (3)-(4) converge to zero asymptotically.

**Remark 3:** In Theorem 1, the right hand side of the inequality (18) is a function dependent on the system state  $x_2(t)$  and desired signal  $x_d(t)$ . It is not reasonable to assume  $x_2(t)$  is bounded, *a priori*. Thus, the  $r(t)$  is designed to be a positive function dependent on the system state  $x_2(t)$  and desired trajectory  $x_d(t)$  and  $\dot{x}_d(t)$ . For different faulty cases discussed in the following sections, the controller parameter  $r(t)$  is also a function dependent on the system states, desired trajectory, basic function of fault, etc.  $\square$

**Remark 4:** The sliding-mode control has been used extensively to deal with fault-tolerant control (see, e.g. [21]-[22], [25]-[27]). However, the uncertainty existing in the input distribution matrix are rarely considered in the existing work, and specifically, the associate result for high-speed train has not been available. It should be emphasized that such a class of uncertainties is interacted with control signal and thus the traditional design method cannot be applied. This paper provides the contribution for high-speed train in this regard for the first time.  $\square$

#### IV. FAULT-TOLERANT SLIDING-MODE CONTROLLER DESIGN

In this section, a fault-tolerant controller will be designed for the train dynamic model (3)-(4) with the actuator fault described by (7). For the actuator fault model (7), the fault parameter  $\vartheta$  could be changed, and be a constant during a certain time instant. According to Assumption (A1) that the remaining healthy actuators can achieve the control performance, the fault parameter  $\vartheta$  can be assumed to be bounded.

**Faulty system.** From (3)-(4) and (7), the dynamics of the faulty system can be rewritten as:

$$\dot{x}_1(t) = x_2(t), \quad (27)$$

$$\begin{aligned} \dot{x}_2(t) &= (m + \Delta m(t))(\sigma_\nu \nu(t) + \vartheta^T \zeta(t)) - a - bx_2(t) \\ &\quad - cx_2^2(t) + \bar{d}(t), \end{aligned} \quad (28)$$

where  $\nu(t)$  is system input,  $\sigma_\nu$  is the number of the remaining health actuators and satisfies  $n - \bar{n} \leq \sigma_\nu \leq n$ ,  $\vartheta$  and  $\zeta(t)$  are defined in (8) and (9), and  $\|\vartheta\|_2 \leq \vartheta_0$  with  $\vartheta_0$  being a known constant.

With the tracking errors  $e_1(t) = x_1(t) - y_d(t)$  and  $e_2(t) = x_2(t) - \dot{x}_d(t)$ , The error dynamic equation can be written as:

$$\dot{e}_1(t) = e_2(t), \quad (29)$$

$$\begin{aligned} \dot{e}_2(t) &= (m + \Delta m(t))(\sigma_\nu \nu(t) + \vartheta^T \zeta(t)) - a - bx_2(t) \\ &\quad - cx_2^2(t) + \bar{d}(t) - \dot{x}_d(t). \end{aligned} \quad (30)$$

The objective now is to design a fault-tolerant sliding-mode controller for the error system (29)-(30), such that the close-loop signal is bounded and the tracking errors satisfy  $\lim_{t \rightarrow \infty} e_1(t) = 0$  and  $\lim_{t \rightarrow \infty} e_2(t) = 0$ , in the presence of the unknown actuator fault  $\vartheta$  and actuator uncertainty  $\Delta m(t)$ .

**Fault-tolerant sliding-mode controller design.** The modified control law is proposed to be

$$\nu(t) = -\hat{\vartheta}_\nu^T(t)\zeta(t) - \hat{\rho}_\nu(t) \left\{ \frac{1}{m}F_0(t) + \frac{1}{m}d_b + \frac{r(t)}{m} \text{sgn} \left( k(x_1(t) - y_d(t)) + x_2(t) - x_d(t) \right) \right\}, \quad (31)$$

where  $\hat{\vartheta}_\nu(t)$  and  $\hat{\rho}_\nu(t)$  are the estimates of  $\vartheta_\nu^* = \frac{\vartheta}{\sigma_\nu}$  and  $\rho_\nu^* = \frac{1}{\sigma_\nu}$ , respectively,  $F_0(t)$  is defined in (17),  $r(t)$  is a nonnegative time varying gain, and  $m$  and  $d_b$  are given in (5).

For arbitrary initial estimate  $\hat{\vartheta}_\nu(0)$  and the initial estimate  $\hat{\rho}_\nu(0) \in [\frac{1}{n}, \frac{1}{n-\bar{n}}]$ , the adaptive terms  $\hat{\vartheta}_\nu(t)$  and  $\hat{\rho}_\nu(t)$  are updated by the following adaptive laws:

$$\dot{\hat{\vartheta}}_\nu(t) = \Gamma_\vartheta \zeta(t) \delta(t), \quad (32)$$

$$\dot{\hat{\rho}}_\nu(t) = \Gamma_\rho \left( \frac{1}{m}F_0(t) + \frac{1}{m}d_b + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \delta(t) + g_\nu(t), \quad (33)$$

where the adaptive law gains  $\Gamma_\vartheta = \Gamma_\vartheta^T > 0$ ,  $\Gamma_\rho$  is a positive constant, and  $g_\nu(t)$  is given as

$$g_\nu(t) = \begin{cases} 0, & \text{if } \hat{\rho}_\nu(t) \in (\frac{1}{n}, \frac{1}{n-\bar{n}}) \text{ or} \\ & \text{if } \hat{\rho}_\nu(t) = \frac{1}{n}, \quad g(t) \geq 0 \text{ or} \\ & \text{if } \hat{\rho}_\nu(t) = \frac{1}{n-\bar{n}}, \quad g(t) \leq 0, \\ -g(t), & \text{otherwise,} \end{cases} \quad (34)$$

with  $g(t) = \Gamma_\rho \left( \frac{1}{m}F_0(t) + \frac{1}{m}d_b + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \delta(t)$ .

Then, the following result is ready to be presented.

**Theorem 2:** The closed-loop system formed by applying the sliding-mode control in (31) and the adaptive laws given in (32)-(33) to the faulty system (27)-(28), is state bounded and its tracking errors satisfy  $\lim_{t \rightarrow \infty} e_1(t) = 0$  and  $\lim_{t \rightarrow \infty} e_2(t) = 0$ , if  $m_b < m$ , the number of the failed actuators  $\bar{n}$  in Assumption (A1) and the control gain  $r(t)$  in (31) satisfies

$$\bar{n} \leq \frac{n(m - m_b)}{m}, \quad (35)$$

$$r(t) \geq \frac{m}{m - m_b n |\hat{\rho}_\nu(t)|} \left( \eta + \frac{m_b}{m} \left( mn |\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0 \|\zeta(t)\|_2 + n |\hat{\rho}_\nu(t)| (|F_0(t)| + d_b) \right) \right), \quad (36)$$

for  $\eta > 0$ .

**Proof:** Consider the sliding function (12). From (29)-(30), by direct calculation, it follows that the time derivative of  $\delta(t)$  is given by

$$\begin{aligned} \dot{\delta}(t) = & (m + \Delta m(t))(-\sigma_\nu \hat{\vartheta}_\nu^T(t)\zeta(t) + \vartheta^T \zeta(t)) \\ & - m\sigma_\nu \hat{\rho}_\nu(t) \left( \frac{1}{m}F_0(t) + \frac{1}{m}d_b + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \\ & - \Delta m(t)\sigma_\nu \hat{\rho}_\nu(t) \left( \frac{1}{m}F_0(t) + \frac{1}{m}d_b + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \\ & + F_0(t) + \bar{d}(t) \end{aligned}$$

$$\begin{aligned} = & m\sigma_\nu \tilde{\vartheta}_\nu^T(t)\zeta(t) + \Delta m(t) \left( -\sigma_\nu \hat{\vartheta}_\nu^T(t)\zeta(t) + \vartheta^T \zeta(t) \right) \\ & + m\sigma_\nu \tilde{\rho}_\nu(t) \left( \frac{1}{m}F_0(t) + \frac{1}{m}d_b + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \\ & - \Delta m(t)\sigma_\nu \hat{\rho}_\nu(t) \left( \frac{1}{m}F_0(t) + \frac{1}{m}d_b + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \\ & - d_b + \bar{d}(t) - r(t) \text{sgn}(\delta(t)), \end{aligned} \quad (37)$$

where  $\tilde{\vartheta}_\nu(t) = \vartheta_\nu^* - \hat{\vartheta}_\nu(t)$ , and  $\tilde{\rho}_\nu = \rho_\nu^* - \hat{\rho}_\nu(t)$ .

For system (27)-(28) with adaptive laws in (32)-(33), choose a Lyapunov function candidate as

$$V = \frac{1}{2}\delta^2 + \frac{m\sigma_\nu}{2}\Gamma_\vartheta^{-1}\tilde{\vartheta}_\nu^T\tilde{\vartheta}_\nu + \frac{m\sigma_\nu}{2}\Gamma_\rho^{-1}\tilde{\rho}_\nu^2. \quad (38)$$

Let  $(T_j, T_{j+1})$ ,  $j = 0, 1, \dots, N$ , with  $T_0 = 0$ , be time intervals, and the actuators only fail at time  $T_p$ . Then, the actuator fault pattern is fixed during these time intervals, which means that  $\vartheta$  is a constant for  $t \in (T_j, T_{j+1})$ , and discontinues for  $t \in [0, \infty)$ . For  $t \in (T_j, T_{j+1})$ ,  $j = 1, \dots, N$ , take the time derivative of  $V$ :

$$\begin{aligned} \dot{V} = & \delta(t)\dot{\delta}(t) + m\sigma_\nu \Gamma_\vartheta^{-1}\tilde{\vartheta}_\nu^T(t)\dot{\tilde{\vartheta}}_\nu(t) + m\sigma_\nu \Gamma_\rho^{-1}\tilde{\rho}_\nu(t)\dot{\tilde{\rho}}_\nu(t) \\ = & \delta(t)\Delta m(t) \left( -\sigma_\nu \hat{\vartheta}_\nu^T(t)\zeta(t) + \vartheta^T \zeta(t) \right) \\ & - \delta(t)\Delta m(t)\sigma_\nu \hat{\rho}_\nu(t) \left( \frac{1}{m}F_0(t) + \frac{1}{m}d_b + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \\ & - r(t)|\delta(t)| - \delta(t)(d_b - \bar{d}(t)) + m\sigma_\nu \Gamma_\rho^{-1}\tilde{\rho}_\nu(t)g_\nu(t), \end{aligned} \quad (39)$$

where  $m\sigma_\nu \Gamma_\rho^{-1}\tilde{\rho}_\nu(t)g_\nu(t) \leq 0$ , due to  $(\rho_\nu^* - \hat{\rho}_\nu(t))g_\nu(t) \leq 0$  with  $g_\nu(t)$  defined in (34).

Consider that  $\sigma_\nu$ , the number of the remaining healthy actuators, satisfies  $\sigma_\nu \leq n$ . With the bounded condition  $\|\vartheta\|_2 \leq \vartheta_0$ , it follows from (39) and  $|\bar{d}(t)| \leq d_b$  in (5) that

$$\begin{aligned} \dot{V} \leq & |\delta(t)|m_b \left( n|\hat{\vartheta}_\nu^T(t)\zeta(t)| + \vartheta_0 \|\zeta(t)\|_2 \right. \\ & \left. + n|\hat{\rho}_\nu(t)| \left( \frac{1}{m}|F_0(t)| + \frac{1}{m}d_b + \frac{r(t)}{m} \right) \right) - r(t)|\delta(t)| \\ = & - \left( -\frac{m_b}{m} \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0 \|\zeta(t)\|_2 \right. \right. \\ & \left. \left. + n|\hat{\rho}_\nu(t)| (|F_0(t)| + d_b + r(t)) \right) + r(t) \right) |\delta(t)|. \end{aligned} \quad (40)$$

According to Assumption (A1),  $n - \bar{n} \leq \sigma_\nu \leq n$  implies  $\frac{1}{n} \leq \rho_\nu^* \leq \frac{1}{n-\bar{n}}$ , i.e.,  $\frac{1}{n} \leq \hat{\rho}_\nu(t) \leq \frac{1}{n-\bar{n}}$ . Inequality (35) can lead to  $\frac{1}{n-\bar{n}} \leq \frac{m}{m_b n}$ , so that  $\frac{1}{n} \leq \hat{\rho}_\nu(t) \leq \frac{m}{m_b n}$ , which means  $m - m_b n |\hat{\rho}_\nu(t)| > 0$ .

Using  $m - m_b n |\hat{\rho}_\nu(t)| > 0$  and (36), it has

$$\begin{aligned} \frac{m - m_b n |\hat{\rho}_\nu(t)|}{m} r(t) \geq & \eta + \frac{m_b}{m} \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| \right. \\ & \left. + m\vartheta_0 \|\zeta(t)\|_2 + n|\hat{\rho}_\nu(t)| |F_0(t)| + n|\hat{\rho}_\nu(t)| d_b \right), \end{aligned} \quad (41)$$

which implies that

$$\begin{aligned} r(t) - \frac{m_b n |\hat{\rho}_\nu(t)|}{m} r(t) - \frac{m_b}{m} \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| \right. \\ \left. + m\vartheta_0 \|\zeta(t)\|_2 + n|\hat{\rho}_\nu(t)| |F_0(t)| + n|\hat{\rho}_\nu(t)| d_b \right) \geq \eta. \end{aligned} \quad (42)$$

Due to the discontinuous parameter  $\vartheta$ ,  $V(\cdot)$  is not continuous with respect to time  $t$ . With (40) and (42), the time derivative of  $V$  for  $t \in (T_j, T_{j+1})$ ,  $j = 0, 1, \dots, N$ , becomes

$$\dot{V} \leq -\eta|\delta(t)|. \quad (43)$$

Since the number of faults occurring in the system is finite, it has

$$\dot{V} \leq -\eta|\delta(t)| \leq 0, \quad t \in (T_N, \infty). \quad (44)$$

Therefore, all the variables  $\delta(t)$ ,  $\tilde{\vartheta}_\nu = \vartheta_\nu^* - \hat{\vartheta}_\nu(t)$  and  $\tilde{\rho}_\nu = \rho_\nu^* - \hat{\rho}_\nu(t)$ , are bounded, and so are  $\hat{\vartheta}_\nu(t)$  and  $\hat{\rho}_\nu(t)$ . From (44), we have a finite energy sliding function  $\delta(t)$ :

$$\begin{aligned} & \int_0^\infty |\delta(t)| dt \\ & \leq \frac{1}{\eta} (V(\delta(0), \tilde{\vartheta}_\nu(0), \tilde{\rho}_\nu(0)) - V(\delta(\infty), \tilde{\vartheta}_\nu(\infty), \tilde{\rho}_\nu(\infty))) \\ & < \infty, \end{aligned} \quad (45)$$

which implies  $\delta(t) \in L_1$ .

Furthermore, from (12) and (29), it follows that

$$\begin{aligned} \dot{e}_1(t) &= -ke_1(t) + \delta(t), \\ \text{i.e., } e_1(t) &= \frac{1}{s+k}[\delta](t), \quad k > 0. \end{aligned} \quad (46)$$

Because  $k > 0$  and  $\delta(t)$  is bounded,  $e_1(t)$  and  $\dot{e}_1(t)$  are bounded, and so are  $e_2(t)$  and  $x_2(t)$ . According to [28],  $\delta(t) \in L_1$  results in  $e_1(t) \in L_1$ . Then, with the structure of the fault-tolerant controller (31), the boundedness of  $\nu(t)$  is ensured. Based on Barb  t Lemma, it has  $\lim_{t \rightarrow \infty} \delta(t) = 0$  and  $\lim_{t \rightarrow \infty} e_1(t) = 0$ . Then,  $\lim_{t \rightarrow \infty} e_2(t) = 0$ .  $\nabla$

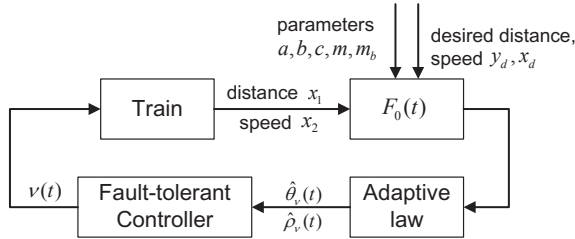


Fig. 1: The adaptive fault-tolerant sliding-mode controller

The inputs of the controller are the desired and actual distance and speed, and the parameters  $a, b, c, m, m_b, d_b$ , the signal  $\zeta(t)$  are also needed to construct the adaptive laws. Then, the output of the controller is the traction force signal  $\nu(t)$ , which requires the traction system to provide. The detail process is shown in Fig. 1.

**Discussion.** Comparing with the results for the healthy case in Section III, an additional condition about the number of the failed actuators is given as (35). From (44), we can see that the adaptive law given in (33) can make the estimate  $\hat{\rho}_\nu(t)$  belong to  $[\frac{1}{n}, \frac{1}{n-\bar{n}}]$ . Then, combining with the condition (35), the proposed fault-tolerant sliding-mode controller can make the faulty system (27)-(28) with the actuator uncertainties  $\Delta m(t)$  and fault (7) to achieve the displacement tracking.

From (44), it can be seen that the proposed adaptive fault-tolerant sliding-mode controller (31) cannot guarantee the

error dynamics (29)-(30) to be driven to the sliding surface (12), in finite time; while the controller (16) for the healthy system (3)-(4) can drive the error dynamics (10)-(11) to the sliding surface (12) in finite time. The reason is that for the fault-tolerant controller (31), the adaptive law (32)-(33) is required to deal with the unknown fault parameter  $\vartheta$ . Although the fault-tolerant controller (31) cannot drive the error dynamics (29)-(30) to the sliding surface (12) in finite time, but the tracking errors can be guaranteed to converge to zero, when  $t$  goes to infinity, while the tracking error  $e_1(t)$  exponentially converge to zero in the healthy case. The proposed fault-tolerant controller (31) can handle the actuator uncertainty  $\Delta m(t)$  and fault  $\vartheta$  simultaneously, to guarantee the closed-loop states boundedness and tracking performances.

**Remark 5:** There are many results about the fault-tolerant sliding-mode control design (see for example [21]- [22], [25]- [27]). However, the considered faults are always assumed to have known bounds or more information of faults, except the contributions mentioned in Remark 4. We would like to point out that the works of the adaptive technique combined with the sliding-mode control are rarely founded, especially for the high-speed trains. Due to the unknown system faults, we introduce the adaptive laws to estimate the faults, while the sliding-mode technique is used to deal with the input distribution matrix uncertainty and achieve the trajectory tracking. Moreover, when the bounds of the actuator uncertainties are unknown, a new adaptive law is employed to estimate the bounds, which will be shown in the next section.  $\square$

## V. CONTROLLER DESIGN FOR THE UNKNOWN BOUND $m_b$ CASE

For the healthy and faulty cases in the above sections, the bound  $m_b$  on the input distribution  $\Delta m(t)$  is known. In this section, the case that the bound  $m_b$  is unknown, will be discussed. The design procedure of the fault-tolerant sliding-mode controller for the unknown  $m_b$  is similar to that of (31), in which the unknown parameter  $m_b$  should be replaced by its estimation  $\hat{m}_b(t)$ .

For the initial estimate  $\hat{m}_b(0) \in [0, m]$ , the adaptive term  $\hat{m}_b(t)$  is updated by the following adaptive law:

$$\begin{aligned} \dot{\hat{m}}_b(t) &= \Gamma_m \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0\|\zeta(t)\|_2 \right. \\ & \quad \left. + n|\hat{\rho}_\nu(t)|(|F_0(t)| + d_b + r(t)) \right) |\delta(t)| + g_m(t) \end{aligned} \quad (47)$$

where  $\Gamma_m$  is a positive constant,  $\hat{\vartheta}_\nu(t)$  and  $\hat{\rho}_\nu(t)$  are give in (32) and (33),  $F_0(t)$  is defined in (17),  $m$  and  $d_b$  are given in (5),  $r(t)$  is a nonnegative time varying gain, and  $g_m(t)$  is given as

$$g_m(t) = \begin{cases} 0, & \text{if } \hat{m}_b(t) \in (0, m) \text{ or} \\ & \text{if } \hat{m}_b(t) = 0, \quad \bar{g}(t) \geq 0 \text{ or} \\ -\bar{g}(t), & \text{if } \hat{m}_b(t) = m, \quad \bar{g}(t) \leq 0, \\ & \text{otherwise,} \end{cases} \quad (48)$$

with  $\bar{g}(t) = \Gamma_m \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0\|\zeta(t)\|_2 + n|\hat{\rho}_\nu(t)|(|F_0(t)| + d_b + r(t)) \right) |\delta(t)|$ .

Then, the following theorem can be obtained.

**Theorem 3:** The closed-loop system formed by applying the sliding-mode control in (31) and the adaptive laws given in (32)-(33) and (47) to the faulty system (27)-(28), is state bounded and its tracking errors satisfy  $\lim_{t \rightarrow \infty} e_1(t) = 0$  and  $\lim_{t \rightarrow \infty} e_2(t) = 0$ , if the number of the failed actuators  $\bar{n}$  in Assumption (A1) and the control gain  $r(t)$  in (31) satisfy

$$\bar{n} \leq \frac{n(m - \hat{m}_b(t))}{m}, \quad (49)$$

$$r(t) \geq \frac{m}{m - \hat{m}_b(t)n|\hat{\rho}_\nu(t)|} \left( \eta + \frac{\hat{m}_b(t)}{m} \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0\|\zeta(t)\|_2 + n|\hat{\rho}_\nu(t)|(|F_0(t)| + d_b) \right) \right), \quad (50)$$

for  $\eta > 0$ .

**Proof:** The dynamic  $\delta(t)$  is the same as (37). Choose a Lyapunov function candidate modified from (38) as

$$V = \frac{1}{2}\delta^2 + \frac{m\sigma_\nu}{2}\Gamma_\vartheta^{-1}\tilde{\vartheta}_\nu^T\tilde{\vartheta}_\nu + \frac{m\sigma_\rho}{2}\Gamma_\rho^{-1}\tilde{\rho}_\nu^2 + \frac{1}{2}\Gamma_m^{-1}\tilde{m}_b^2, \quad (51)$$

where  $\tilde{\vartheta}_\nu(t) = \vartheta_\nu^* - \hat{\vartheta}_\nu(t)$ ,  $\tilde{\rho}_\nu = \rho_\nu^* - \hat{\rho}_\nu(t)$ ,  $\tilde{m}_b(t) = m_b - \hat{m}_b(t)$ ,  $\vartheta_\nu^* = \frac{\vartheta}{\sigma_\nu}$  and  $\rho_\nu^* = \frac{\rho}{\sigma_\nu}$ .

For the fault pattern fixed time intervals  $t \in (T_j, T_{j+1})$ ,  $j = 1, \dots, N$ , using  $\sigma_\nu \leq n$  and (31), take the time derivative of  $V$

$$\begin{aligned} \dot{V} &\leq |\delta(t)|m_b \left( n|\hat{\vartheta}_\nu^T(t)\zeta(t)| + \vartheta_0\|\zeta(t)\|_2 \right. \\ &\quad \left. + n|\hat{\rho}_\nu(t)| \left( \frac{1}{m}|F_0(t)| + \frac{1}{m}d_b + \frac{r(t)}{m} \right) \right) \\ &\quad - r(t)|\delta(t)| + \Gamma_m^{-1}\tilde{m}_b(t)\dot{\tilde{m}}_b(t) \\ &= \frac{\hat{m}_b(t)}{m} \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0\|\zeta(t)\|_2 \right. \\ &\quad \left. + n|\hat{\rho}_\nu(t)|(|F_0(t)| + d_b + r(t)) \right) |\delta(t)| \\ &\quad + \left( \frac{m_b}{m} - \frac{\hat{m}_b(t)}{m} \right) \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0\|\zeta(t)\|_2 \right. \\ &\quad \left. + n|\hat{\rho}_\nu(t)|(|F_0(t)| + d_b + r(t)) \right) |\delta(t)| \\ &\quad - r(t)|\delta(t)| + \Gamma_m^{-1}\tilde{m}_b(t)\dot{\tilde{m}}_b(t). \end{aligned} \quad (52)$$

From (47), (52) can be rewritten as

$$\begin{aligned} \dot{V} &= - \left( - \frac{\hat{m}_b(t)}{m} \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0\|\zeta(t)\|_2 \right. \right. \\ &\quad \left. \left. + n|\hat{\rho}_\nu(t)|(|F_0(t)| + d_b + r(t)) \right) + r(t) \right) |\delta(t)| \\ &\quad + \Gamma_m^{-1}\tilde{m}_b(t)g_m(t), \end{aligned} \quad (53)$$

where  $\Gamma_m^{-1}\tilde{m}_b(t)g_m(t) \leq 0$ , due to  $(m_b - \hat{m}_b(t))g_m(t) \leq 0$  with  $g_m(t)$  defined in (48).

According to (50), it has

$$r(t) - \frac{\hat{m}_b(t)}{m} \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0\|\zeta(t)\|_2 + n|\hat{\rho}_\nu(t)|(|F_0(t)| + d_b + r(t)) \right) \geq \eta. \quad (54)$$

Further, from (53) and (54), it follows that

$$\dot{V} \leq -\eta|\delta(t)| \leq 0, \quad t \in (T_N, \infty), \quad (55)$$

which implies that all the variables  $\delta(t)$ ,  $\tilde{\vartheta}_\nu(t)$ ,  $\tilde{\rho}_\nu(t)$ , and  $\tilde{m}_b(t)$  are bounded, and  $\delta(t) \in L_1$ . From the definitions of  $\tilde{\vartheta}_\nu(t)$ ,  $\tilde{\rho}_\nu(t)$  and  $\tilde{m}_b(t)$ ,  $\hat{\vartheta}_\nu(t)$ ,  $\hat{\rho}_\nu(t)$  and  $\hat{m}_b(t)$  are bounded. Furthermore, from (12) and (29),  $e_1(t)$  and  $\dot{e}_1(t)$  are bounded, and so are  $e_2(t)$  and  $x_2(t)$ . According to [28],  $\delta(t) \in L_1$  results in  $e_1(t) \in L_1$ . Then, with the structure of the fault-tolerant controller (31), the boundedness of  $\nu(t)$  is ensured. Based on Barbalat Lemma, it has  $\lim_{t \rightarrow \infty} \delta(t) = 0$  and  $\lim_{t \rightarrow \infty} e_1(t) = 0$ . Then,  $\lim_{t \rightarrow \infty} e_2(t) = 0$ .  $\nabla$

**Discussion.** To handle the unknown bound of the input uncertainty  $\Delta m(t)$ , the adaptive law (47) is introduced to deal with the unknown parameter  $m_b$ , which is used to design the controller function  $r(t)$ . It should be noted that the adaptive law (47) is a differential equation, and contains the function  $r(t)$ . Due to  $r(t) > 0$ , we can choose  $r(t)$  as the right hand side of inequality (50) adding a positive constant  $\epsilon$ , i.e.,  $r(t) = \frac{m}{m - \hat{m}_b(t)n|\hat{\rho}_\nu(t)|} \left( \eta + \frac{\hat{m}_b(t)}{m} \left( mn|\hat{\vartheta}_\nu^T(t)\zeta(t)| + m\vartheta_0\|\zeta(t)\|_2 + n|\hat{\rho}_\nu(t)|(|F_0(t)| + d_b) \right) \right) + \epsilon$  with  $\epsilon$  being a positive constant. Then,  $r(t)$  can be substituted into (47). The fault-tolerant sliding-mode controller proposed in Theorem 3 guarantees that the corresponding closed-loop system is uniformly bounded and the tracking errors converge to zero.

**Remark 6:** For the controller design, we need the distance and speed of the train, which are available in the real high-speed train control systems. Since the distance and speed of the train are essential information, the train will stop to ensure the safety if these signals are not available under some faulty cases. Thus, there are redundant sensors to measure these signals and guarantee their accuracies. If there is one sensor failed, the remaining health sensors can provide the information that the controller needs. If there are two or more sensors failed, perhaps the train will stop, and the controller will not work. For the sensor noise, the filter method has been employed in the automatic train operating control system.  $\square$

## VI. EXTENSION TO THE UNPARAMETERIZED FAULT CASE

In this section, the fault-tolerant control problem for the unparameterized time-varying fault will be investigated. In this case, the faulty system (27)-(28) is rewritten as

$$\dot{x}_1(t) = x_2(t), \quad (56)$$

$$\begin{aligned} \dot{x}_2(t) &= (m + \Delta m(t))(\sigma_\nu \nu(t) + \vartheta(t)) - a - bx_2(t) \\ &\quad - cx_2^2(t) + \bar{d}(t), \end{aligned} \quad (57)$$

where  $\vartheta(t)$  is the bounded time-varying actuator fault, and  $|\vartheta(t)| \leq \vartheta_1$  with  $\vartheta_1$  unknown.

The fault-tolerant control law is proposed to be

$$\begin{aligned} \nu(t) &= -\hat{\vartheta}_\nu(t) - \hat{\rho}_\nu(t) \left( \frac{1}{m}F_0(t) + \frac{1}{m}d_b + \frac{1}{m}v(t) \right. \\ &\quad \left. + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right), \end{aligned} \quad (58)$$



where  $\hat{\vartheta}_\nu(t)$  and  $\hat{\rho}_\nu(t)$  are the estimates of  $\vartheta_\nu^* = \frac{\vartheta_1}{\sigma_\nu}$  and  $\rho_\nu^* = \frac{1}{\sigma_\nu}$ , respectively,  $F_0(t)$  is defined in (17),  $v(t)$  is a design signal, and  $r(t)$  is a nonnegative time varying gain.

For the initial estimate  $\hat{\rho}_\nu(0) \in [\frac{1}{n}, \frac{1}{n-\bar{n}}]$ , the design signal  $v(t)$  and the adaptive terms  $\hat{\vartheta}_\nu(t)$  and  $\hat{\rho}_\nu(t)$  are designed as

$$\dot{\hat{\vartheta}}_\nu(t) = \Gamma_\vartheta \delta(t), \quad (59)$$

$$\dot{\hat{\rho}}_\nu(t) = \Gamma_\rho \left( \frac{1}{m} F_0(t) + \frac{1}{m} d_b + \frac{1}{m} v(t) + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \delta(t) + g_\nu(t), \quad (60)$$

$$v(t) = -\mu |\delta(t)|, \quad (61)$$

where the adaptive law gains  $\Gamma_\vartheta$  and  $\Gamma_\rho$  are positive constants,  $\mu > 0$ , and  $g_\nu(t)$  is given as

$$g_\nu(t) = \begin{cases} 0, & \text{if } \hat{\rho}_\nu(t) \in (\frac{1}{n}, \frac{1}{n-\bar{n}}) \text{ or} \\ & \text{if } \hat{\rho}_\nu(t) = \frac{1}{n}, \quad g(t) \geq 0 \text{ or} \\ & \text{if } \hat{\rho}_\nu(t) = \frac{1}{n-\bar{n}}, \quad g(t) \leq 0, \\ -g(t), & \text{otherwise,} \end{cases} \quad (62)$$

with  $g(t) = \Gamma_\rho \left( \frac{1}{m} F_0(t) + \frac{1}{m} d_b + \frac{1}{m} v(t) + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \delta(t)$ .

Then, the following result is ready to be presented.

**Theorem 4:** The sliding-mode control in (58) with the adaptive laws (59)-(60) applied to the faulty system (56)-(57) guarantees that the states of the closed-loop system and the tracking errors are uniformly ultimately bounded, if the number of the failed actuators  $\bar{n}$  in Assumption (A1) and the control gain  $r(t)$  in (58) satisfies

$$\bar{n} \leq \frac{n(m - m_b)}{m}, \quad (63)$$

$$r(t) \geq \frac{m}{m - m_b n |\hat{\rho}_\nu(t)|} \left( \eta + \frac{m_b}{m} \left( mn |\hat{\vartheta}_\nu^T(t)| + n |\hat{\rho}_\nu(t)| (|v(t)| + |F_0(t)| + d_b) \right) \right), \quad (64)$$

for  $\eta > 0$ .

**Proof:** With  $\tilde{\vartheta}_\nu(t) = \vartheta_\nu^* - \hat{\vartheta}_\nu(t)$ , and  $\tilde{\rho}_\nu = \rho_\nu^* - \hat{\rho}_\nu(t)$ , the dynamic  $\delta(t)$  is expressed as

$$\begin{aligned} \dot{\delta}(t) &= m \sigma_\nu \tilde{\vartheta}_\nu(t) + m(-\vartheta_1 + \vartheta(t)) + \Delta m(t) (-\sigma_\nu \hat{\vartheta}_\nu(t) + \vartheta(t)) \\ &\quad + m \sigma_\nu \tilde{\rho}_\nu(t) \left( \frac{1}{m} F_0(t) + \frac{1}{m} d_b + \frac{1}{m} v(t) + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \\ &\quad - \Delta m(t) \sigma_\nu \hat{\rho}_\nu(t) \left( \frac{1}{m} F_0(t) + \frac{1}{m} d_b + \frac{1}{m} v(t) + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \\ &\quad - v(t) - d_b + \bar{d}(t) - r(t) \text{sgn}(\delta(t)), \end{aligned} \quad (65)$$

where  $\delta(t)$  is the sliding function defined in (12).

Choose a Lyapunov function candidate as

$$V = \frac{1}{2} \delta^2 + \frac{m \sigma_\nu}{2} \Gamma_\vartheta^{-1} \tilde{\vartheta}_\nu^2 + \frac{m \sigma_\nu}{2} \Gamma_\rho^{-1} \tilde{\rho}_\nu^2. \quad (66)$$

Then, from (65), (59) and (60), the time derivative of  $V$  is given by

$$\begin{aligned} \dot{V} &= \delta(t) m (-\vartheta_1 + \vartheta(t)) + \delta(t) \Delta m(t) (-\sigma_\nu \hat{\vartheta}_\nu(t) + \vartheta(t)) \\ &\quad - \delta(t) \Delta m(t) \sigma_\nu \hat{\rho}_\nu(t) \left( \frac{1}{m} F_0(t) + \frac{1}{m} d_b + \frac{1}{m} v(t) + \frac{r(t)}{m} \text{sgn}(\delta(t)) \right) \\ &\quad - \delta(t) v(t) - r(t) |\delta(t)| \\ &\quad - \delta(t) (d_b - \bar{d}(t)) + m \sigma_\nu \Gamma_\rho^{-1} \tilde{\rho}_\nu(t) g_\nu(t). \end{aligned} \quad (67)$$

Using (59)-(61), it follows that

$$\dot{V} \leq -\eta_1 |\delta(t)| + m_b \vartheta_1 |\delta(t)| - \mu |\delta(t)|^2. \quad (68)$$

where  $\eta_1 = \eta + m(\vartheta_1 - \vartheta(t))$ . Because  $|\vartheta(t)| \leq \vartheta_1$  and  $m > 0$ , it has  $m(\vartheta_1 - \vartheta(t)) \geq 0$ , which means  $(\eta + m(\vartheta_1 - \vartheta(t))) > 0$ .

The term  $m_b \vartheta_1 |\delta(t)| - \mu |\delta(t)|^2$  attains a maximum value  $\frac{m_b^2 \vartheta_1^2}{4\mu}$  at  $|\delta(t)| = \frac{m_b \vartheta_1}{2\mu}$  for  $t \in [0, +\infty)$ , i.e.,

$$\dot{V} \leq -\eta_1 |\delta(t)| + \frac{m_b^2 \vartheta_1^2}{4\mu}. \quad (69)$$

Therefore, for any initial state  $\delta(0)$ , the solution of the closed-loop system is uniformly ultimately bounded, i.e.,  $\delta(t)$ ,  $\hat{\vartheta}_\nu$  and  $\hat{\rho}_\nu$  are uniformly ultimately bounded. From (46), it has  $e_1(t)$  is uniformly ultimately bounded. Then, so is  $e_2(t)$ .  $\nabla$

**Discussion.** To handle the unparameterized fault  $\vartheta(t)$  with unknown bound  $\vartheta_1$ , the nonlinear damping is applied. With the sliding-mode technique, the proposed fault-tolerant controller (58) can make the closed-loop system is uniformly bounded, in the presence of actuator uncertainty  $\Delta m(t)$  and unknown actuator fault  $\vartheta$ . Compared with the tracking performances of controller (16) for healthy case and the fault-tolerant controller (31) for the case that the bound of fault parameter is known, although the performance of the controller (58) is degraded, the tracking errors can be smaller enough by choosing appropriate controller parameters.

For high-speed trains, the topic of the reliability and safety has attracted many researchers and engineers. Until now, there are still some problems in the existing fault diagnosis and fault-tolerant control scheme for high-speed trains. For example, the actuator uncertainties that are modelled as the input distribution matrix uncertainty, are not taken into account in the controller and fault-tolerant controller design. This paper considers uncertainty in input matrix in high-speed train using adaptive and sliding-mode techniques, for the first time. This is one of the main contributions in the area. Moreover, in the future, the high-speed trains will have higher speeds, which require advanced control to achieve the high-accuracy speed and position tracking to guarantee the safety of the trains. Considering that the sliding-mode technique is widely used in industrial systems, we propose the adaptive fault-tolerant sliding-mode controller.

**Remark 7:** The controllers proposed in Theorems 1-4, show the traction force that the traction system provides to the train for achieving the desired trajectory tracking, in the presence of the actuator uncertainties and faults. From Theorems 1-4, it can be seen that the information (desired and actual speed, distance and some parameters of the train) that the controller

design needs, are all available in practice. Then, the proposed control methods can be implemented in the automatic train operating control system. Moreover, perhaps there are some delays in the information transfer. Due to the robustness of the sliding-mode controller, the proposed adaptive fault-tolerant sliding-mode controller can handle some small delays.  $\square$

**Remark 8:** In train control, there are mainly two types of models used in the literatures, namely, the single mass point model and the cascade mass point model. The proposed method can be extended to the cascade mass point model with inputs acting on every car. Under the assumption that the speeds of all cars are synchronous, the cascade mass point model can be considered as a single mass point model. The high-speed trains require advanced control to achieve the high-accuracy speed and position tracking to guarantee the safety of the trains. For this case, the disturbances, uncertainties and faults should be considered and dealt with. So, it is meaningful to study the uncertainty existing in the input distribution matrix, which is rarely investigated in both train and car vehicles.  $\square$

## VII. SIMULATION STUDY

To demonstrate the effectiveness of the proposed adaptive fault-tolerant sliding-mode controllers, the simulation studies on a high-speed train will be presented. The considered train contains 8 vehicles (4 locomotives and 4 carriages), which means there are 16 motors in the considered system, and the simulation parameters are from a CRH type train in [29].

**Simulation conditions.** The parameters of the train are chosen as  $\bar{M} = 380$  (ton),  $a = 8.63 \times 10^{-3}$  (kN),  $b = 7.295 \times 10^{-6}$  (kN s/m),  $c = 1.12 \times 10^{-6}$  (kN s<sup>2</sup>/m<sup>2</sup>),  $\Delta_M(t) = 20$  (ton),  $\Delta_f(t) = 1 - e^{-0.05t}$ , and  $\Delta F(t) = 10 \sin(0.03t)$  (kN).

*Case 1 (Healthy mode):* The disturbance is set as

$$d(t) = \begin{cases} 0, & t \in [0, 200]; \\ 200(1 - e^{-10t}), & t \in [200, 500]; \\ 100 \sin(0.03t), & t \in [500, 2000]. \end{cases} \quad (70)$$

which can represent that the train runs in a straight track during  $0 \leq t < 200$ . During  $200 \leq t < 500$ , as the train accelerates, the aerodynamic force increases, then the train enters a tunnel. From  $t = 500$ , the train travels in a slope track, and has some time-varying disturbances, such as winds, meeting another train, etc. We choose the parameter of sliding surface as  $k = 8$  and the initial sates as  $x(0) = [0.55 \ 0]^T$ .

*Case 2 (Known fault bound mode):* For the train with 4 locomotives, there are 16 motors with same type, i.e.,  $n = 16$ . Because most faults can be considered as the effectiveness loss of the traction force, the parameters of the fault expression in (7) are chosen as  $\sigma_\nu = 15$  and

- (i) for  $t \in [400, 600]$ ,  $\xi = 2 \times 10^5$ ,  $\varpi(t) = 1$ ;
- (ii) for  $t \in [600, 800]$ ,  $\xi = 2 \times 10^5$ ,  $\varpi(t) = 1 + \sin(0.05t - 30)$ ;
- (iii) for  $t \in [800, 2000]$ ,  $\xi = 0$ ,  $\varpi(t) = 1$ ;

which means at the beginning, a motor has a constant fault and after some time, the fault becomes a time-varying fault. The failed motor completely stops working, at last. Further, it has  $\vartheta_0 = 4 \times 10^5$ .

The initial parameter estimates are 80% of their ideal values, and the initial conditions are chosen as  $x(0) = [0.5 \ 0]^T$ . The gains of the adaptive laws in (32)-(33) are chosen as 0.2, and the parameter of sliding surface as  $k = 8$ .

*Case 3 (Unknown fault bound mode):* The fault is the same as Case 2, while  $m_b$  is unknown. The initial parameter estimates are 90% of their nominal values, and the initial conditions are chosen as  $x(0) = [0.1 \ 0]^T$ . The gains of the adaptive laws in (32)-(33) are chosen as 0.2, and the parameters as  $k = 12$  and  $\mu = 2$ .

*Case 4 (Unparameterized fault mode):* In this case, an unparameterized time-varying fault is considered, whose corresponding parameters are chosen as  $\sigma_\nu = 15$  and

$$\vartheta(t) = 2 \times 10^5 \sin(0.01t - 30), \text{ for } t \geq 600, \quad (71)$$

which implies a fault occurs in a motor after 600s.  $\vartheta(t)$  is unknown.

The initial parameter estimates are 90% of their nominal values, and the initial conditions are set as  $x(0) = [0.05 \ 0]^T$ . The gains of the adaptive laws in (59)-(60) are chosen as 0.2, and the parameters as  $k = 12$  and  $\mu = 3$ .

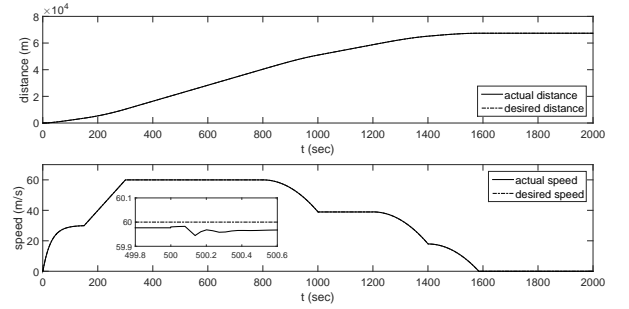


Fig. 2: Distances and speed trajectories for the healthy system

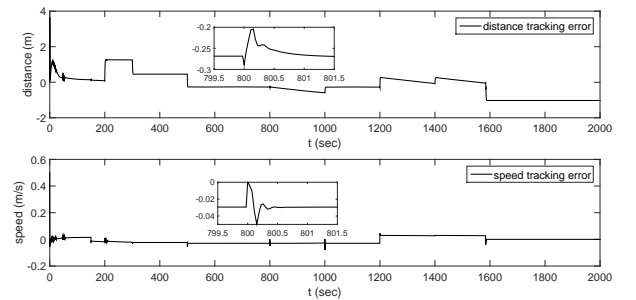


Fig. 3: The tracking errors for the healthy system

**Simulation results.** The simulate results for the four cases above are shown in Figs. 1-5, respectively. The operating conditions including acceleration, reacceleration, constant speed, deceleration, constant speed, redeceleration, and slowing down until fully stop, are considered [8]. The total running time is 2000 seconds (about 34 minutes), which can describe a train running from a station to another one.

The simulation results of the healthy system including the plant distance (solid) and desired distance (dashed) are shown in Fig. 2, and the distance and speed tracking errors for cases 1-4 are shown in Figs. 3-6.

In fig. 3, there are some transit responses occurring at the instants that the accelerations are abruptly changed. Since the acceleration information is used to design the controller, it is better to choose a smooth acceleration curve to avoid the transit responses.

Figs. 4-6 demonstrate that, while the actuator fault occurs at the 400th or 600th second, the proposed adaptive fault-tolerant sliding-mode controller can regulate the train speed and displacement states close to the desired trajectories after some transit responses, so that the tracking performances and the system stability is achieved, during the train operation.

It is visible from the simulation results that a little bit chattering occurs in Figs. 3-6. This is caused by the discontinuous controllers (16), (31) and (58), due to the sign function, which results in a discontinuous right hand side in the dynamical equations. The chattering has been reduced by using boundary layer method in which the discontinuous sign function is approximated by the continuous saturation function proposed in [30], [31] and [32]. To show the sliding-mode properties, the chattering is not completely removed and the small accepted amplitude of the chattering is retained.

These simulation results show that the proposed adaptive fault-tolerant sliding-mode controller can achieve the close-loop stability and asymptotic tracking properties of the train even in the presence of unknown actuator uncertainties and faults.

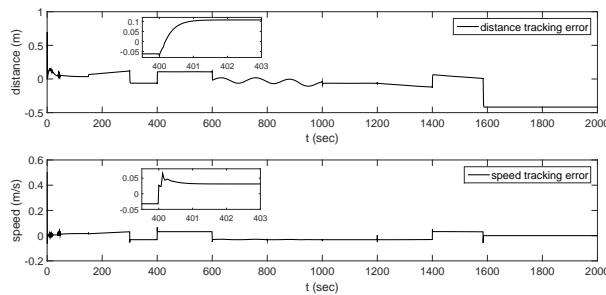


Fig. 4: The tracking errors for the parameterized fault

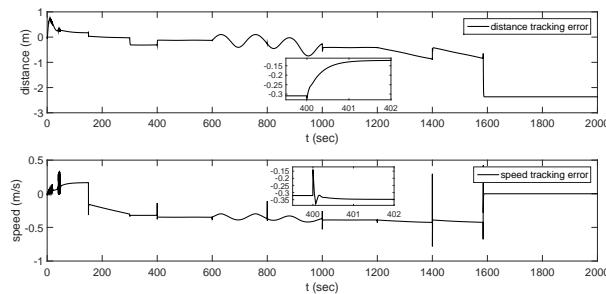


Fig. 5: The tracking errors for the unknown uncertainty bound

### VIII. CONCLUSIONS

In this paper, a new adaptive fault-tolerant sliding-mode control scheme is proposed for high-speed train with unknown actuator uncertainties and faults. For the healthy train system,

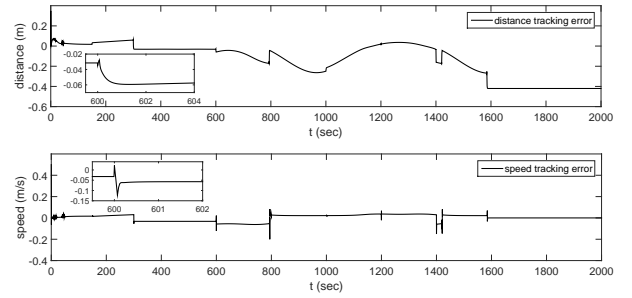


Fig. 6: The tracking errors for the unparameterized fault

a sliding-mode controller is designed to guarantee that the tracking error dynamics can asymptotically converge to zero. The cases that the bound of the actuator fault parameter is known, the bound of the actuator uncertainty is unknown, and the fault is modelled as an unparameterized time-varying function, have been considered as well. Combining with the adaptive technique, an adaptive fault-tolerant sliding-mode control scheme is proposed to handel the actuator uncertainties and faults, simultaneously. The simulation examples on a realistic train dynamic model are given to demonstrate the effectiveness of the proposed methods.

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**Zehui Mao** received her Ph.D. degree in Control Theory and Control Engineering from Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2009. She is now an associate professor at the College of Automation Engineering in Nanjing University of Aeronautics and Astronautics, China. She has been a visiting scholar in University of Virginia from October 2015 to November 2016. She worked in the areas of fault diagnosis, with particular interests in nonlinear control systems, sampled-data systems and networked control systems. Her current

research interests include fault diagnosis and fault-tolerant control of systems with disturbance and incipient faults, and high-speed train and spacecraft flight control applications.



**Xing-Gang Yan** received the B.Sc. degree from Shaanxi Normal University, in 1985, the M.Sc. degree from Qufu Normal University in 1991, and the Ph.D. degree of engineering from Northeastern University, China in 1997. Currently, he is Senior Lecturer of Control Engineering at the University of Kent, United Kingdom. He worked as a Research Fellow/Research Associate in the Northwestern Polytechnical University, University of Hong Kong, China, Nanyang Technological University, Singapore and the University of Leicester, United Kingdom. He was a Lecturer in Qingdao University, China from 1991 to 1994. He is a Chartered Engineer. He is the Editor-In-Chief of the International Journal of Engineering Research and Science & Technology. He serves as a member of the Editorial Board of several engineering journals and a member of TPC for several conferences. His research interests include sliding mode control, decentralised control, fault detection and isolation and time delay system with applications. He has published three books, five invited book chapters and over 170 referred papers in these research areas.



**Bin Jiang** received the Ph.D. degree in Automatic Control from Northeastern University, Shenyang, China, in 1995. He had ever been postdoctoral fellow, research fellow, invited professor and visiting professor in Singapore, France, USA and Canada, respectively. Now he is a Chair Professor of Cheung Kong Scholar Program in Ministry of Education and Vice President of Nanjing University of Aeronautics and Astronautics, China. He serves as Associate Editor or Editorial Board Member for a number of journals such as IEEE Trans. On Control Systems Technology; Int. J. Of Control, Automation and Systems; Neurocomputing; Journal of Astronautics; Control and Decision, Systems Engineering and Electronics Technologies, etc. He is a senior member of IEEE, Chair of Control Systems Chapter in IEEE Nanjing Section, a member of IFAC Technical Committee on Fault Detection, Supervision, and Safety of Technical Processes. His research interests include fault diagnosis and fault tolerant control and their applications to helicopters, satellites and high-speed trains.

He has been the principle investigator on several projects of National Natural Science Foundation of China. He is the author of 8 books and over 200 referred international journal papers and conference papers. He won Second Class Prize of National Natural Science Award of China in 2018.



**Mou Chen** (M'10) received his B.S. degree in material science and engineering at Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 1998, the M.S. degree and the Ph.D. degree in Automatic control engineering at Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2004. He is currently a professor in College of Automation Engineering at Nanjing University of Aeronautics and Astronautics, China.

He was an Academic Visitor at the Department of Aeronautical and Automotive Engineering, Loughborough University, UK, from November 2007 to February 2008. From June 2008 to September 2009, he was a research fellow in the Department of Electrical and Computer Engineering, the National University of Singapore. He was a senior Academic Visitor at the School of Electrical and Electronic Engineering, The University of Adelaide, AUS, from May 2014 to November 2014. His research interests include nonlinear system control, intelligent control, and flight control.