Adaptive finite difference schemes based on interpolating wavelets for solving 2D Maxwell's equations

Pedro Pinho* [(1)], Margarete O. Domingues[(2)], Paulo J. S. G. Ferreira[(3)], Sônia M. Gomes[(4)], Anamaria Gomide[(5)] and José R. Pereira[(6)]
(1) Instituto Superior de Engenharia de Lisboa
(2) Instituto Nacional de Pesquisas Espaciais, São José dos Campos, SP, Brasil
(3) Signal Processing Laboratory, DET/IEETA, University of Aveiro, Portugal
(4) Universidade Estadual de Campinas, IMECC, Campinas SP, Brasil
(5) Universidade Estadual de Campinas, IC, Campinas, SP, Brasil
(6) Instituto de Telecomunicações, Universidade de Aveiro, Portugal
E-mail: ppinho@deetc.isel.ipl.pt

Introduction

Computational electromagnetic is a very active area and in the last decade was a very interesting increasing for a wide class of applications. For the numerical solution of such problems, Yee's Finite-Difference Time-Domain (FDTD) [1] scheme has become a quite popular technique. FDTD is related to the physical space representations of the fields, in which the electric and magnetic fields are described by vectors, both discretized in time and space in a grid usually uniform. The main disadvantage of this representation is therefore the possible increase of the computation time to simulate such grid. For this reason, it is important to study adaptive numerical methods that use a refined grid only in certain regions of the space, and a less refined grid in other regions where the variation of the fields is smoother. The objective is to obtain an adaptive mesh as a function of time that allows to an economy of resources and a relatively short or acceptable time of simulation. In this direction, this paper concerns with high order finite difference schemes for Maxwell's equations combined with an adaptive strategy based on interpolating techniques, the so called SPR (Sparse Point Representation) Method [2]. In this method, the mesh structure presents a heterogeneous composition: sparse in smoothness regions and dense in regions of accented variation. The method combines the simplicity and accuracy of traditional finite difference schemes with the ability of wavelet coefficients in the characterization of local regularity of functions. The application of the SPR method in staggered and non-staggered grids for the resolution of 1D Maxwell equations has been detailed by the authors in [3], which also include stability and dispersion analyses. It has been observed that high order schemes in non-staggered grids can be used with greater CFL parameters to achieve a prescribed accuracy with a certain number of cells per wavelength. Furthermore, the use of non-staggered grids facilitates the implementation of adaptive strategies. The application of the SPR method in non-staggered grids for the resolution of 2D TE Mode Maxwell equations is described by the authors in [4]. In the present paper we demonstrate some of the potential of such scheme trough the results of the numerical simulation of a 2D horn.

Wavelets tools and SPR representation

In the adaptive SPR method, the goal consists of performing the finite difference model in a more economic fashion, by taking into account local regularity information about the numerical solution.

The method has two basics parts: the representation part and the operational part. In the first part, there are wavelets tools for multiscale function representations. In such framework, functions are represented in different scale levels, and the main tools are appropriate transformations relating the information at the finest scale level to the lower ones, and vice versa. In each level, the wavelet coefficients are defined in terms of local interpolation errors at the new points of the current level of discretization. As interpolation errors, the wavelet coefficients can be used as indicators of local smoothness. Points associated with wavelet coefficients under a given threshold are removed from the grid. The resulting SPR grid is sparse in smoothness regions and dense in regions of accented variation.

In the second part spatial derivatives are discretized by traditional uniform finite differences using only the information of the SPR representation using step sizes that are proportional to the point local scale. However, since a SPR grid may be good for the function representation, it may not be good enough to be used for the representation of its derivatives. Therefore, some kind of refinement must be performed consisting in the inclusion of some of the points in order to get an extended grid more appropriated for the application of the finite difference operator. More details on the SPR method are in [2, 3, 4].

TE mode Maxwell's equations

We consider the transverse electric (TE) mode for which there are only electric field components transverse to the z-axis. We assume no variation with respect to z, which means that all partial derivatives of the fields with respect to z equal zero. The corresponding set of equations is,

$$\mu\frac{\partial H^z}{\partial t}=\frac{\partial E^x}{\partial y}-\frac{\partial E^y}{\partial x};\ \varepsilon\frac{\partial E^x}{\partial t}=\frac{\partial H^z}{\partial y}-\sigma E^x;\ \varepsilon\frac{\partial E^y}{\partial t}=-\frac{\partial H^z}{\partial x}-\sigma E^y \qquad (1)$$

Where ε is the electrical permittivity μ is the magnetic permeability and σ is the conductivity. Both fields are sampled at the same uniform grid being represented by the following equations.

$$H^{z,n}_{k,l}=H^z(k\Delta x,l\Delta y,n\Delta t);\quad E^{x,n}_{k,l}=E^x(k\Delta x,l\Delta y,n\Delta t);\quad E^{y,n}_{k,l}=E^y(k\Delta x,l\Delta y,n\Delta t) \qquad (2)$$

Based on these discrete values, using fourth order finite difference schemes for space derivatives and integrating in time using a first order Euler scheme, we obtain the following set of discrete equations.

$$H_{k,l}^{z,n+1} = H_{k,l}^{z,n} + \frac{\Delta t}{\mu \Delta y}\left(\partial^y E^{x,n}\right)_{k,l} - \frac{\Delta t}{\mu \Delta x}\left(\partial^x E^{y,n}\right)_{k,l}$$

$$E_{k,l}^{x,n+1} = \left(\frac{1-\dfrac{\sigma\Delta t}{2\varepsilon}}{1+\dfrac{\sigma\Delta t}{2\varepsilon}}\right) E_{k,l}^{x,n} + \left(\frac{\dfrac{\Delta t}{\varepsilon\Delta y}}{1+\dfrac{\sigma\Delta t}{2\varepsilon}}\right)\left(\partial^y H^{z,n+1}\right)_{k,l} \tag{4}$$

$$E_{k,l}^{y,n+1} = \left(\frac{1-\dfrac{\sigma\Delta t}{2\varepsilon}}{1+\dfrac{\sigma\Delta t}{2\varepsilon}}\right) E_{k,l}^{y,n} - \left(\frac{\dfrac{\Delta t}{\varepsilon\Delta x}}{1+\dfrac{\sigma\Delta t}{2\varepsilon}}\right)\left(\partial^x H^{z,n+1}\right)_{k,l}$$

where we introduce the finite difference operators:

$$\left(\partial^x v\right)_{k,l} = \sum_{k'} v_{k',l}\,\beta(k-k') \qquad \left(\partial^y u\right)_{k,l} = \sum_{l'} u_{k,l'}\,\beta(l-l')$$

with $\beta(1) = -2/3;\ \beta(1) = 1/12;\ \beta(-k) = -\beta(k)$

Numerical Results

Our purpose in the present section is to analyze the performance of the SPR method for the numerical solution of the TE mode system of Maxwell's equations described in the previous section. The example to be considered is a 2D horn. This structure extends to infinity in the z-direction with no change in its geometry. The model of the horn is illustrated in Fig.1. The walls of the horn are stepped.

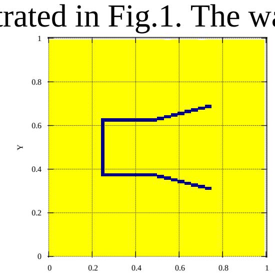
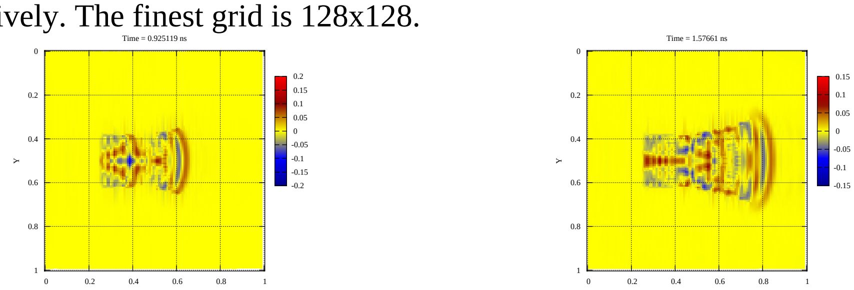


Fig. 1: Geometry of the horn

We consider that the fields at t=0 are $E^x(x,y) = E^y(x,y) = 0$ and $H^z(x,y) = e^{-2500(x-3/8)^2} e^{-2500(y-1/2)^2}$.

Fig. 2, shows the evolution with time of H^z. In the SPR representation, the thresholding limits were 10^{-2} and 10^{-4} for the electric and magnetic fields, respectively. The finest grid is 128x128.



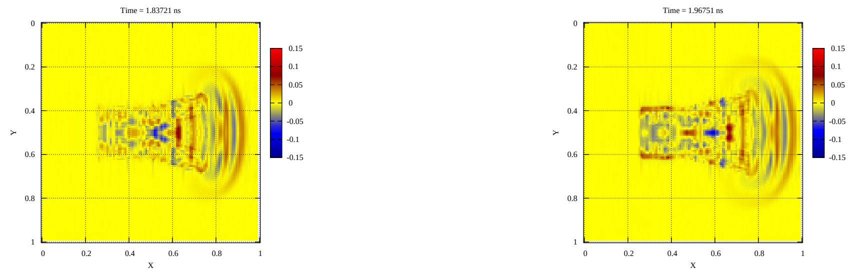


Fig. 2: Evolution with time of the solutions for H^z

Conclusions

This paper describes a 2D application of interpolating wavelets and recursive interpolation schemes with thresholding, aiming the representation of the electric and magnetic fields in nonuniform, adaptive grids. Applied to Maxwell's equations, the method leads to sparse grids that adapt in space to the local smoothness of the fields, and at the same time track the evolution of the fields over time. In general, the number of points in the grid, N_s, is below the maximum number of points, N. It is possible to control N_s, by trading off representation accuracy and data compression, and therefore speed. A numerical example is presented showing the propagation of a gaussian pulse within a 2D horn.

Acknowledgments

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