Adaptive Forgetting-factor RLS-based Initialisation Per-tone Equalisation in Discrete Multitone Systems

Suchada Sitjongsataporn[1], Non-member

ABSTRACT

An adaptive forgetting-factor inverse square-root recursive least squares (AF-iQRRLS) with inverse of correlation matrix updating is presented for per-tone equalisation in discrete multitone-based systems. The proposed inverse covariance update of the square-root covariance Kalman filter is introduced to prepare for the signal flow graph (SFG). This reduced derivation of adaptive inverse square-root recursive least squares algorithm can modify via SFG. In order to reduce the computational complexity, the forgetting-factor parameter for each group called per-group forgetting-factor (PGFF) approach based on AF-iQRRLS algorithm is introduced. The forgetting-factor from the middle of each group is selected as a representative in order to find an optimal forgetting-factor parameter by using AF-iQRRLS algorithm. After convergence, it is fixed for remaining tones of whole group. Simulation results reveal that the trajectories of modified PGFF of the proposed algorithm for each individual tone can converge to their own equilibria. Moreover, the performance of the proposed algorithms are improved as compared with the existing algorithm.

Keywords: Discrete Multitone (DMT), Adaptive Algorithm, Per-Tone Equalisation (PTEQ), Adaptive Forgetting-Factor Algorithm, Adaptive Inverse Square-Root Recursive Least Squares (iQR-RLS) Algorithm, Per-Group Forgetting-Factor (PGFF) Approach, Signal Flow Graph (SFG)

1. INTRODUCTION

Discrete multitone (DMT) is a digital implementation technique widely used for high speed wired multicarrier transmission such as asymmetric digital subscriber lines (ADSLs) [1], [2] and [3]. The cyclic prefix (CP) is inserted between DMT symbols to provide subchannel independency to eliminate intersymbol interference (ISI) and intercarrier interference (ICI). The so-called shortened impulse response (SIR) which is basically the convolutional result of a (real) time-domain equaliser (TEQ) and channel impulse response (CIR) is preferably be shortened as most as possible.

By employing a TEQ, the performance of a DMT system is less sensitive to the choice of length of CP. However, TEQs have been introduced in DMT systems to alleviate the effect of ISI and ICI in case of the length of SIR or shorter than a CP [4]. The target impulse response (TIR) is a design parameter characterising the derivation of the TEQ.

In addition to TEQ, a complex one-tap frequency-domain equaliser (FEQ) is applied subsequently for each tone separately to compensate for the amplitude and phase of distortion [2]. An ultimate objective of most TEQ designs is to minimise the mean square error (MSE) between output of TEQ and TIR. This implies that TEQ and TIR are optimised in the mean square error (MSE) sense [4] and [5].

In order to improve the signal to noise ratio (SNR) and bit rate performance, a complex multitap frequency-domain equaliser structure, called *per-tone equalisation (PTEQ)* has been proposed in [6]. It can optimise the SNR for each tone separately and obtain the achievable bit rate. A technique based on transferring the (real) TEQ-operations to the frequency-domain is suggested. The sensitivity of the synchronisation delay and the size of PTEQ are reduced for the same performance.

Based on the fast convergence, the resursive least squares (RLS)-based algorithm is to provide the considerable improvement in convergence speed. Basically, the problem of RLS algorithm is the divergence when the inverse matrix loses its property of Hermitian symmetry [7]. The inverse square-root RLS (iQR-RLS) algorithm based on QR-decomposition performs especially triangularisation operation on the inverse correlation matrix [8]. In [9], a significant part of fast and cheap adaptive RLS-based computations with inverse updating can be shared among different tones leading to sufficiently low initialisation complexity. This direct initialisation is computationally intensive.

In order to reduce initialisation complexity, a per-group approach has been considered in [10] and [11]. In [10], the frequency-domain equalisation has been presented with tone grouping. Then, the per-group equalisation with the bit rate maximising for time-domain equaliser has been applied for DMT-based

Manuscript received on June 14, 2011 ; revised on August 1, 2011.

[1] The author is with the Centre of Electronic Systems Design and Signal Processing (CESdSP) Department of Electronic Engineering, Mahanakorn University of Technology 140 Cheumsamphan Road, Nong-chok, Bangkok, 10530, Thailand., Email: ssuchada@mut.ac.th

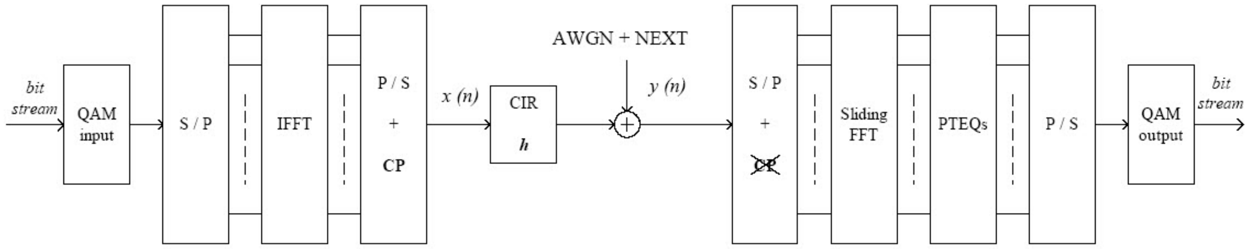

Fig.1: *Block diagram of a discrete multitone system.*

systems in [11].

However, the mean square error for the parameter estimates depending on the time-variations and on the forgetting-factor has been discussed in [12]. It is difficult to find an optimal forgetting-factor for different tones to provide good tracking in dealing with large model variations [13]. Therefore, the optimal forgetting-factor of RLS-based PTEQ should be adapted automatically in order to gain the bit rate improvement [14].

In this paper, we introduce how to modify the adaptive mechanism for tuning the forgetting-factor parameter based on iQR-RLS algorithm in forms of signal flow graph (SFG) that a significant part of RLS-based computations can be shared between the different tones. The advantage of using iQRRLS-based formulation is that it preserves precisely the fast convergence of standard RLS algorithm [15]. Moreover, the symmetrical LU-decomposition of inverse covariance matrix also befits to parallel implementation by means of systolic array [16] and SFG [17].

The paper is organised as follows. The system model and notation are described in Section 2. The bit rate expression is calculated in Section 3. Section 4. introduces the modified AF-iQRRLS per-tone equalisation in details and in forms of SFG. In order to reduce the computational complexity, the forgetting-factor parameter for each group so-called per-group forgetting-factor (PGFF) scheme is selected from the middle of its group based on AF-iQRRLS algorithm. The computational complexity of proposed PGFF-AFiQRRLS algorithm is given in Section 5. Simulation results and conclusion are presented in Section 6. and Section 7,. respectively.

2. SYSTEM MODEL AND NOTATION

The basic structure of the DMT transceiver is illustrated in Fig.1. The incoming bit stream is likewise reshaped to a complex-valued transmitted symbol for mapping in the quadrature amplitude modulation (QAM) and then split into N parallel paths that are instantaneously fed to the modulating the inverse fast Fourier transform (IFFT). After that, IFFT outputs are transformed into serial symbols including cyclic prefix (CP) between symbols and then fed through the channel.

The time-domain received signal is also transformed into the frequency-domain received signals without the cyclic prefix, which are fed to the sliding fast Fourier transform (FFT). Then the demodulating outputs of the sliding FFT are fed to a set of $\mathbb{T}$-tap PTEQ. The parallel of received symbols are eventually converted into serial bits in the frequency-domain. The data model and notation based on an FIR model of the transmission channel is presented in (1), where l determines the first considered sample of the k-th received DMT symbol vector $\mathbf{y}_{k,i:j} = [y_{k,i} \cdots y_{k,j}]^T$ and the subscripts $i:j$ will be omitted for conciseness. The $N \times 1$ transmitted vector $\mathbf{x}_{k,N} = [x_{k,0} \cdots x_{k,N-1}]^T$. The vector $\mathbf{n}_{k,i:j}$ is a sample vector with the additive white Gaussian noise (AWGN) and near-end crosstalk (NEXT). The vector $\bar{\mathbf{h}}$ is the channel impulse response (CIR) vector $\mathbf{h}$ in reverse order and $\bar{\mathbf{h}}$ is the CIR vector $\mathbf{h}$ in reverse order. The operators $\otimes$ and $\odot$ denote as the Kronecker product and a componentwise multiplication, respectively.

The matrix $\boldsymbol{\mathcal{F}}_N^* = \boldsymbol{\mathcal{F}}_N^H$ is the $N \times N$ (I)FFT matrix, where $\boldsymbol{\mathcal{F}}_n = [1 \quad e^{\frac{j2\pi n}{N}} \cdots \quad e^{\frac{j2\pi(N-1)}{N}}]$. The matrix $(N+\nu)\times N$ $\boldsymbol{\mathcal{P}}_\nu$ which adds the CP of length ν. The parameter Δ is a synchronisation delay and $\mathbf{I}$

$$\underbrace{\begin{bmatrix} y_{k,l+\Delta} \\ \vdots \\ y_{k,N-l+\Delta} \end{bmatrix}}_{\mathbf{y}_{k,l+\Delta:N-1+\Delta}} = \underbrace{\begin{bmatrix} \mathbf{0}_{(1)} & \overbrace{\begin{array}{|c|} [\,\bar{\mathbf{h}}^T\,]\ 0\ \cdots \\ \ddots\ \ddots \\ \cdots\ 0\ [\,\bar{\mathbf{h}}^T\,] \end{array}}^{\mathbf{H}_T} & \mathbf{0}_{(2)} \end{bmatrix}}_{\mathbf{H}} \cdot (\mathbf{I} \otimes \boldsymbol{\mathcal{P}}_\nu \boldsymbol{\mathcal{F}}_N^*) \cdot \underbrace{\begin{bmatrix} \mathbf{x}_{k-1,N} \\ \mathbf{x}_{k,N} \\ \mathbf{x}_{k+1,N} \end{bmatrix}}_{\mathbf{x}_{k-1:k+1,N}} + \underbrace{\begin{bmatrix} n_{k,l+\Delta} \\ \vdots \\ n_{k,N-l+\Delta} \end{bmatrix}}_{\mathbf{n}_{k,l+\Delta:N-1+\Delta}}, \qquad (1)$$

is an $n \times n$ identity matrix. The matrices $\mathbf{0}_{(1)}$ and $\mathbf{0}_{(2)}$ are also the zero matrices of size $(N-l) \times (N-L+2\nu+\Delta+l)$ and $(N-l) \times (N+\nu-\Delta)$. Other parameters are as follows. The parameter n is a tone index. The n_c is of a number of the middle tone of each group index, where pg is a number of tones per group index and Ng is a number of group index. The parameter N is the (I)FFT size. The t is the index of $\mathbb{T}$, where $\mathbb{T}$ is a number of PTEQ taps. The parameter k is the DMT-symbol index of a block and K is the total size of the DMT-symbol of a block. The N_d is the range of active tones starting at tone 38 to 256 for downstream ADSL standard. The length of $s = N+\nu$ is the length of symbols adding with cyclic prefix ν. The vector $\tilde{\mathbf{x}}_{k,n}$ is a frequency-domain complex symbols vector at symbol k on tone n for $n \in N_d$.

The matrix $\tilde{\mathbf{Y}}_n$ is the *complex* demodulated output for tone n after the sliding FFT for tone n [18] as

$$\tilde{\mathbf{Y}}_n = \mathcal{F}_{\mathcal{N}} \mathbf{Y} \ , \tag{2}$$

$$\mathcal{F}_{\mathcal{N}} \cdot \mathbf{Y}(:, \sqcup + \infty) = (\mathcal{F}_{\mathcal{N}} \cdot \mathbf{Y}(:, \sqcup)) \odot \mathbf{z} + [\infty \ \cdots \ \infty]^{\mathcal{T}} \cdot (y_{k\cdot s+\nu-(t-1)} - y_{k\cdot s+s-(t-1)}), \tag{3}$$

$$\mathbf{z} = [\ \alpha^0 \alpha^1 \cdots \alpha^{N-1}\ ] \ , \tag{4}$$

$$\alpha = e^{-\jmath 2\pi (1/N)} . \tag{5}$$

$$\mathbf{Y} = \begin{bmatrix} y_{k\cdot s+\nu+1} & y_{k\cdot s+\nu} & \cdots & y_{k\cdot s+\nu-T+2} \\ y_{k\cdot s+\nu+2} & y_{k\cdot s+\nu+1} & \cdots & y_{k\cdot s+\nu-T+3} \\ \vdots & \ddots & \ddots & \vdots \\ y_{(k+1)\cdot s} & y_{(k+1)\cdot s-1} & \cdots & y_{(k+1)\cdot s-T+1} \end{bmatrix}, \quad \text{for} \quad t = 1, \cdots, \mathbb{T}-1 \tag{6}$$

where $\mathbf{Y}$ is a Toeplitz matrix. The symbol $y_{k\cdot s+\nu-(t-1)}$ is the first element of the $(t+1)^{th}$ column and $y_{k\cdot s+s-(t-1)}$ is the last element of the $(t)^{th}$ column of $\mathbf{Y}$ in (6), respectively.

The n-th sliding FFT output $\tilde{\mathbf{y}}_{k,n}[t], t \in [0, \mathbb{T}-1]$ on tone n are related by the following recursion as

$$\tilde{\mathbf{y}}_{k,n}[t] = \alpha_n \tilde{\mathbf{y}}_{k,n}[t-1] + \underbrace{(y_{k,-t} - y_{k,N-t})}_{\Delta y_{k,-t}} , \tag{7}$$

$$\begin{aligned} \Delta \mathbf{y}_k &= [(y_{k,-\mathbb{T}+1} - y_{k,-\mathbb{T}+N+1}) \cdots (y_{k,-1} - y_{k,N-1})]^T \\ &= [\Delta y_{k,-\mathbb{T}+1} \cdots \Delta y_{k,-1}]^T \ , \end{aligned} \tag{8}$$

$$\tilde{\mathbf{y}}_{k,n} = \left[\ \Delta \mathbf{y}_k^T \ \ \tilde{y}_{k,n}\ \right] \begin{bmatrix} 1 & \tilde{\alpha}_n & \cdots & \tilde{\alpha}_n^{\mathbb{T}-1} \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \tilde{\alpha}_n \\ 0 & \cdots & 0 & 1 \end{bmatrix}^T \tag{9}$$

where $t = 1, \ldots, \mathbb{T}-1$ and $\tilde{\alpha}_n = \frac{e^{-\jmath 2\pi (n-1)/N}}{\sqrt{N}}$.

The vector $\hat{\mathbf{x}}_{k,n}$ is the output vector after frequency-domain equalisation for tone $n \in N_d$ as $\hat{\mathbf{x}}_{k,n} = \mathbf{p}_n^H \otimes \tilde{\mathbf{y}}_n$, where $\mathbf{p}_n$ is the *complex* $\mathbb{T}$-tap per-tone equaliser (PTEQ) on tone n. According to the concept of PTEQ, we refer the readers to [18] for more details.

Some notation will be used throughout this paper as follows: the operator $(\cdot)^H$ and $(\cdot)^*$ denote as the Hermitian and complex conjugate operator, respectively. A tilde over the variable indicates the frequency-domain. The vectors are in bold lowercase and matrices are in bold uppercase.

3. BIT RATE EXPRESSION

We start from the bit rate expression ($\mathbb{BR}$) described in [19], where the actual number of bits transmitted for each tone is given by

$$\mathbb{BR} \ = \ \sum_{n=0}^{N-1} f_s \, b_n \ , \tag{10}$$

and

$$b_n \ = \ \log_2 \left(1 + \frac{SNR_n}{\Gamma_n} \right) \ \text{for} \ \ n \in N_d \ , \tag{11}$$

where f_s denotes as the sampling rate. The number b_n is the actual number of bits per transmitted symbol that is transferred on tone n and Γ_n is the SNR gap on tone n in dB.

The SNR on tone n (SNR_n) can be determined as

$$SNR_n = \frac{\text{desired signal energy}_n}{\text{energy in (received signal- desired signal)}_n} . \tag{12}$$

Therefore, the solution of SNR model can be expressed to the optimisation problem in PTEQ parameters as

$$SNR_n \ = \ \frac{\sigma^2_{n,\tilde{\mathbf{x}}_n}}{\min\limits_{\mathbf{p}_n} \|\hat{\boldsymbol{\epsilon}}_n\|^2} \ , \tag{13}$$

$$\hat{\boldsymbol{\epsilon}}_n \ = \ \tilde{\mathbf{x}}_n - \underbrace{(\mathbf{p}_n^H \otimes \tilde{\mathbf{y}}_n)}_{\hat{\mathbf{x}}_n} \ , \tag{14}$$

where $\sigma^2_{n,\tilde{\mathbf{x}}_n}$ is the variance of $\tilde{\mathbf{x}}_n$. The vector $\mathbf{p}_n$ is the complex-valued tap-weight PTEQ on tone n. The noise margin is a safety factor that accounts for unmodeled noise sources, such as nonlinearities and impulse noise [20].

4. ADAPTIVE FORGETTING-FACTOR INVERSE SQUARE-ROOT RLS (AF-IQRRLS) PER-TONE EQUALISATION

In this section, we introduce the adaptive forgetting-factor inverse QR-RLS (AF-iQRRLS) algorithm which optimises the cost function $J_{k,n}$ with the method of the exponentially weighted least squares as described in [8] for $n \in N_d$.

$$J_{k,n} = \frac{1}{2}\sum_{k=1}^{K} \lambda_n^{K-k} |\tilde{x}_{k,n} - \hat{\mathbf{p}}_{k,n}^H \tilde{\mathbf{y}}_{k,n}|^2 \ , \tag{15}$$

where λ_n denotes as the forgetting-factor at tone n. The parameter $\tilde{x}_{k,n}$ is the k^{th} transmitted DMT-symbol on tone n. The vector $\hat{\mathbf{p}}_{k,n}$ is the complex $\mathbb{T}$-tap estimated PTEQ on tone n at symbol k.

4.1 Adaptive inverse QR-RLS Algorithm

We then describe briefly the adaptive inverse QR-RLS (iQR-RLS) algorithm, which is a QR decomposition-based RLS algorithm operated on the inverse correlation matrix. Considering the step-by-step among the Kalman and RLS variables, the iQR-RLS algorithm is a fundamentally square-root covariance Kalman algorithm as given in [21], which exhibits good numerical property.

Following [8], the inverse autocorrelation $\mathbf{\Omega}_{k,n}$ may be expressed as

$$\mathbf{\Omega}_{k,n} = \lambda_n^{-1}\mathbf{\Omega}_{k-1,n} - \lambda_n^{-2}\mathbf{\Omega}_{k-1,n}\tilde{\mathbf{y}}_{k,n}\gamma_{k,n}^{-1}\tilde{\mathbf{y}}_{k,n}^H\mathbf{\Omega}_{k-1,n} \ , \tag{16}$$

where

$$\gamma_{k,n} = 1 + \lambda_n^{-1}\tilde{\mathbf{y}}_{k,n}^H\mathbf{\Omega}_{k-1,n}\tilde{\mathbf{y}}_{k,n} \ . \tag{17}$$

We then introduce the block matrix $\boldsymbol{\mathcal{M}}$, its result consists of the matrix product of right-handed on (16) using the Cholesky factorisation as

$$\begin{aligned}\boldsymbol{\mathcal{M}} &= \boldsymbol{A}\,\boldsymbol{A}^H \ , \\ &= \begin{bmatrix} 1 & \boldsymbol{\zeta}_{k,n} \\ \mathbf{0} & \lambda_n^{-\frac{1}{2}}\mathbf{\Omega}_{k-1,n}^{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} 1 & \mathbf{0}^T \\ \boldsymbol{\zeta}_{k,n}^H & \lambda_n^{-\frac{1}{2}}\mathbf{\Omega}_{k-1,n}^{\frac{H}{2}} \end{bmatrix} ,\end{aligned} \tag{18}$$

where

$$\boldsymbol{\zeta}_{k,n} = \lambda_n^{-\frac{1}{2}}\tilde{\mathbf{y}}_{k,n}^H\mathbf{\Omega}_{k-1,n}^{\frac{1}{2}} \ . \tag{19}$$

Based on the QR decomposition, we may set the prearray $\boldsymbol{A}$ to resulting postarray $\boldsymbol{B}$ transformation using the QR update procedure as

$$\begin{gathered}\boldsymbol{A}\,\Theta = \boldsymbol{B} \\ \begin{bmatrix} 1 & \lambda_n^{-\frac{1}{2}}\tilde{\mathbf{y}}_{k,n}^H\mathbf{\Omega}_{k-1,n}^{\frac{1}{2}} \\ \mathbf{0} & \lambda_n^{-\frac{1}{2}}\mathbf{\Omega}_{k-1,n}^{\frac{1}{2}} \end{bmatrix}\Theta = \begin{bmatrix} \gamma_{k,n}^{\frac{1}{2}} & \mathbf{0}^T \\ \tilde{\mathbf{k}}_{k,n}\gamma_{k,n}^{\frac{1}{2}} & \mathbf{\Omega}_{k,n}^{\frac{1}{2}} \end{bmatrix} ,\end{gathered} \tag{20}$$

where Θ is a unitary rotation.

The Kalman gain vector $\tilde{\mathbf{k}}_{k,n}$ and the square-root inverse autocorrelation matrix $\mathbf{\Omega}_{k,n}^{\frac{1}{2}}$, therefore, are readily obtained from the entries in the first and second column of the postarray $\boldsymbol{B}$ in (20) by

$$\tilde{\mathbf{k}}_{k,n} = (\tilde{\mathbf{k}}_{k,n}\gamma_{k,n}^{\frac{1}{2}})\gamma_{k,n}^{-\frac{1}{2}} \ . \tag{21}$$

Note that $\mathbf{\Omega}_{k,n}^{\frac{1}{2}}$ in (20) is the upper triangular matrix. Accordingly, the inverse autocorrelation matrix $\mathbf{\Omega}_{k,n}$ may be defined with its factor as $\mathbf{\Omega}_{k,n} = \mathbf{\Omega}_{k,n}^{\frac{1}{2}}\,\mathbf{\Omega}_{k,n}^{\frac{H}{2}}$, in virtue of the product of square-root matrix and its Hermitian transpose is always a non-negative matrix as described in [21].

Therefore, the tap-weight estimated vector $\hat{\mathbf{p}}_{k,n}$ for $n \in N_d$ in the recursion form may be computed by [22]

$$\hat{\mathbf{p}}_{k,n} = \hat{\mathbf{p}}_{k-1,n} + \tilde{\mathbf{k}}_{k,n}\xi_{k,n}^* \ , \tag{22}$$

$$\xi_{k,n} = \tilde{x}_{k,n} - \hat{\mathbf{p}}_{k-1,n}^H\tilde{\mathbf{y}}_{k,n} \ , \tag{23}$$

where $\xi_{k,n}$ is the *a priori* estimation error at symbol k on tone n.

4.2 Adaptive Forgetting-factor iQR-RLS algorithm

Following [8], we demonstrate the derivation of adaptive forgetting-factor algorithm for the proposed iQR-RLS algorithm. By differentiating $J_{k,n}$ in (15) with respect to $\lambda_{k,n}$ and equating the gradient to zero, we form the stochastic approximation equation for $\lambda_{k,n}$ as

$$\lambda_{k,n} = \lambda_{k-1,n} + \alpha\,\Re\{\mathbf{\Psi}_{k,n}^H\,\tilde{\mathbf{y}}_{k,n}\,\xi_{k,n}^*\} \ , \tag{24}$$

where $\Re\{\cdot\}$ indicates as the real operator and α is the adaptation parameter for $\lambda_{k,n}$.

The derivation of $\mathbf{\Psi}_{k,n} = \frac{\partial\hat{\mathbf{p}}_{k,n}}{\partial\lambda_{k,n}}$ is defined by

$$\mathbf{\Psi}_{k,n} = (\,\mathbf{I} - \tilde{\mathbf{k}}_{k,n}\,\tilde{\mathbf{y}}_{k,n}^H\,)\,\mathbf{\Psi}_{k-1,n} + \mathbf{S}_{k-1,n}\,\tilde{\mathbf{y}}_{k,n}\,\xi_{k,n}^* \ , \tag{25}$$

and

$$\begin{aligned}\mathbf{S}_{k,n} &= \lambda_{k,n}^{-1}(\mathbf{I} - \tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{y}}_{k,n}^H)\mathbf{S}_{k-1,n}(\mathbf{I} - \tilde{\mathbf{k}}_{k,n}^H\tilde{\mathbf{y}}_{k,n}) \\ &\quad + \lambda_{k,n}^{-1}(\tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{k}}_{k,n}^H) - \lambda_{k,n}^{-1}\mathbf{\Omega}_{k,n} \ ,\end{aligned} \tag{26}$$

where $\mathbf{S}_{k,n}$ is the derivative of $\mathbf{\Omega}_{k,n}$ with respect to $\lambda_{k,n}$.

4.3 Modification of proposed AF-iQRRLS algorithm for SFG

Unfortunately, the matrix $(\mathbf{I} - \tilde{\mathbf{k}}_{k,n}^H\tilde{\mathbf{y}}_{k,n})$ of AF-iQRRLS algorithm can not be used directly on the signal flow graph (SFG). We present how to transform the matrix $(\mathbf{I} - \tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{y}}_{k,n}^H)$ for modified SFG of proposed AF-iQRRLS PTEQs which is introduced in the next section.

From [8], the matrix $(\mathbf{I} - \tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{y}}_{k,n}^H)$ of $\mathbf{S}_{k,n}$ and $\mathbf{\Psi}_{k,n}$ in the AF-iQRRLS algorithm as

$$\begin{aligned}\mathbf{S}_{k,n} &= \lambda_{k,n}^{-1}(\mathbf{I} - \tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{y}}_{k,n}^H)\mathbf{S}_{k-1,n}(\mathbf{I} - \tilde{\mathbf{k}}_{k,n}^H\tilde{\mathbf{y}}_{k,n}) \\ &\quad + \lambda_{k,n}^{-1}(\tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{k}}_{k,n}^H - \mathbf{\Phi}_{k,n}^{-1}) \ ,\end{aligned} \tag{27}$$

$$\Psi_{k,n} = (\,\mathbf{I} - \tilde{\mathbf{k}}_{k,n}\,\tilde{\mathbf{y}}_{k,n}^H\,)\,\Psi_{k-1} + \mathbf{S}_{k,n}\,\tilde{\mathbf{y}}_{k,n}\,\xi_{k,n}^* \ . \tag{28}$$

Using the definition of the gain vector $\tilde{\mathbf{k}}_{k,n}$

$$\tilde{\mathbf{k}}_{k,n} = \mathbf{\Phi}_{k,n}^{-1} \, \tilde{\mathbf{y}}_{k,n} \, , \tag{29}$$

where

$$\mathbf{\Phi}_{k,n} = \lambda_{k,n}\mathbf{\Phi}_{k-1,n} + \tilde{\mathbf{y}}_{k,n}\tilde{\mathbf{y}}_{k,n}^{H} \, . \tag{30}$$

Multiplying both sides of (30) by $\mathbf{\Phi}_{k,n}^{-1}$, we get

$$\mathbf{I} - \mathbf{\Phi}_{k,n}^{-1}\tilde{\mathbf{y}}_{k,n}\tilde{\mathbf{y}}_{k,n}^{H} = \lambda_{k,n}\mathbf{\Phi}_{k,n}^{-1}\mathbf{\Phi}_{k-1,n} \, . \tag{31}$$

Substituting (29) into the matrix $(\mathbf{I} - \tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{y}}_{k,n}^{H})$ and comparing with (31), we get

$$\mathbf{I} - \tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{y}}_{k,n}^{H} = \lambda_{k,n}\mathbf{\Phi}_{k,n}^{-1}\mathbf{\Phi}_{k-1,n} \, . \tag{32}$$

In a similar fashion, the matrix $(\mathbf{I} - \tilde{\mathbf{k}}_{k,n}^{H}\tilde{\mathbf{y}}_{k,n})$ is derived by

$$\mathbf{I} - \tilde{\mathbf{k}}_{k,n}^{H}\tilde{\mathbf{y}}_{k,n} = \lambda_{k,n}\mathbf{\Phi}_{k,n}^{-H}\mathbf{\Phi}_{k-1,n}^{H} \, . \tag{33}$$

Therefore, we introduce to redefine $\mathbf{S}_{k,n}$ in (27) and $\mathbf{\Psi}_{k,n}$ in (28) using (32) and (33) which are prepared for modified SFG as

$$\mathbf{\Psi}_{k,n} = \lambda_{k,n}\mathbf{\Phi}_{k,n}^{-1}\mathbf{\Phi}_{k-1}\mathbf{\Psi}_{k-1,n} + \mathbf{S}_{k-1,n}\tilde{\mathbf{y}}_{k,n}\xi_{k,n}^{*} \, , \tag{34}$$

$$\mathbf{S}_{k,n} = \lambda_{k,n}\mathbf{\Phi}_{k,n}^{-1}\mathbf{\Phi}_{k-1,n}\mathbf{S}_{k-1,n}\mathbf{\Phi}_{k,n}^{-H}\mathbf{\Phi}_{k-1,n}^{H} + \lambda_{k,n}^{-1}(\tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{k}}_{k,n}^{H} - \mathbf{\Phi}_{k,n}^{-1}) \, . \tag{35}$$

4.4 Modified AF-iQRRLS PTEQs in DMT-based system

A recursive initialisation based on the inverse QR-RLS algorithm [23] which stores and updates the upper triangular of square-root inverse matrix $\mathbf{L}_{k,n}$ where $\mathbf{\Phi}_{k,n}^{-1} = \mathbf{L}_{k,n}\,\mathbf{L}_{k,n}^{H}$. The inverse QR-RLS algorithm preserves Hermitian symmetry of the inverse autocorrelation matrix [24], [25] in order to improve the computational efficiency and operates in the parallel implementation [9]. It is well suitable for applications.

We then introduce the modified AF-iQRRLS PTEQs that adjust adaptively their forgetting-factors and solve the following cost function for tone $n \in N_d$, where N_d is the active tones as

$$\min_{\hat{\mathbf{p}}_{k,n}} |\xi_{k,n}|^2 = \min_{\hat{\mathbf{p}}_{k,n}} |\tilde{x}_{k,n} - \hat{\mathbf{p}}_{k-1,n}^{H}\,\tilde{\mathbf{y}}_{k,n}|^2 \, . \tag{36}$$

The following pseudocode constitutes the algorithm for the proposed modified AF-iQRRLS PTEQs.

Adaptive Algorithm: modified AF-iQRRLS

For $n \in N_d$

For $k = 1, \ldots, K$

Initialise the tone independent $\mathbf{L}_0$,$\mathbf{p}_0$ and $\tilde{e}_0$.

1. Form the matrix-vector product as:

$$\boldsymbol{a} = \tilde{\mathbf{y}}_{k,n}^{H} \cdot \mathbf{L}_{k-1,n} \, .$$

2. Determine Givens rotation [8] $\mathbf{Q}_t$ for $t = 1, \ldots, T$ as:

$$\begin{bmatrix} \mathbf{0} \\ \hline \delta \end{bmatrix} \Leftarrow \mathbf{Q}_T\mathbf{Q}_{T-1}\ldots\mathbf{Q}_1 \cdot \begin{bmatrix} \boldsymbol{a} \\ \hline 1 \end{bmatrix} \, .$$

3. Update $\mathbf{L}_{k,n}$ as:

$$\begin{bmatrix} \hat{\mathbf{L}}_{k,n} \\ \hline \delta \cdot \hat{\mathbf{k}}_{k,n} \end{bmatrix} \Leftarrow \mathbf{Q}_{T-1}\ldots\mathbf{Q}_1 \cdot \begin{bmatrix} \overbrace{\mathbf{L}_{k-1,n}(1:T-1, 1:T-1)}^{\hat{\mathbf{L}}_{k-1,n}} \\ \hline \mathbf{0} \end{bmatrix} ,$$

$$\begin{bmatrix} \tilde{\mathbf{L}}_{k,n} \\ \hline \tilde{\gamma} \cdot \tilde{\mathbf{k}}_{k,n} \end{bmatrix} \Leftarrow \mathbf{Q}_T \cdot \begin{bmatrix} \overbrace{\mathbf{L}_{k-1,n}(T, 1:T)}^{\tilde{\mathbf{L}}_{k-1,n}} \\ \hline \begin{bmatrix} \delta \cdot \hat{\mathbf{k}}_{k,n} & 0 \end{bmatrix} \end{bmatrix} ,$$

$$\begin{aligned} \mathbf{L}_{k,n} &\Leftarrow \begin{bmatrix} \hat{\mathbf{L}}_{k,n} & \tilde{\mathbf{L}}_{k,n} \end{bmatrix} , \\ \hat{\mathbf{L}}_{k,n} &\Leftarrow \lambda_{k-1,n}^{-1} \cdot \hat{\mathbf{L}}_{k,n} \, , \\ \tilde{\mathbf{L}}_{k,n} &\Leftarrow \lambda_{k-1,n}^{-1} \cdot \tilde{\mathbf{L}}_{k,n} \, . \end{aligned}$$

4. Update $\hat{\mathbf{p}}_{k,n}$ as:

$$\hat{\mathbf{p}}_{k,n} \Leftarrow \hat{\mathbf{p}}_{k-1,n} + \left(\frac{\tilde{\gamma} \cdot \tilde{\mathbf{k}}_{k,n}}{\tilde{\gamma}}\right) \cdot \xi_{k,n}^{*} \, .$$

5. Update $\lambda_{k,n}$ as:

$$\begin{aligned} \mathbf{\Phi}_{k,n}^{-1} &\Leftarrow \mathbf{L}_{k,n}\,\mathbf{L}_{k,n}^{H} \, , \\ \mathbf{\Psi}_{k,n} &\Leftarrow \lambda_{k-1,n}\,(\mathbf{\Phi}_{k,n}^{-1}\,\mathbf{\Phi}_{k-1,n})\;\mathbf{\Psi}_{k-1,n} \\ &\quad + \mathbf{S}_{k-1,n}\,(\tilde{\mathbf{y}}_{k,n}\,\xi_{k,n}^{*}), \\ \mathbf{S}_{k,n} &\Leftarrow \lambda_{k-1,n}(\mathbf{\Phi}_{k,n}^{-1}\mathbf{\Phi}_{k-1,n})\mathbf{S}_{k-1,n}(\mathbf{\Phi}_{k,n}^{-H}\mathbf{\Phi}_{k-1,n}^{H}) \\ &\quad + \lambda_{k-1,n}^{-1}(\tilde{\mathbf{k}}_{k,n}\tilde{\mathbf{k}}_{k,n}^{H} - \mathbf{\Phi}_{k,n}^{-1}) \, , \\ \lambda_{k,n} &\Leftarrow \lambda_{k-1,n} + \alpha\,\Re\{\mathbf{\Psi}_{k,n}^{H}\tilde{\mathbf{y}}_{k,n}\,\xi_{k,n}^{*}\} \, . \end{aligned}$$

end

end

By applying this algorithm to $\mathbb{T}$-tap complex-valued PTEQ vector, the modified SFG is illustrated in Fig. 2 with the building blocks described in Fig. 8. The summary of proposed AF-iQRRLS PTEQs for modified SFG is presented in Table 2. Every used tone has a $\mathbb{T}$-tap PTEQ vector which its input is the complex sliding-FFT output for that tone and $\mathbb{T} - 1$ difference terms [9].

The matrix $\mathbf{L}_{k,n}$ is stored and updated that can be used to adapt the forgetting-factor for every symbol. The update of weight-vector $\hat{\mathbf{p}}_{k,n}$ is also performed separately for each tone by means of modified adaptive forgetting-factor mechanism and inverse square-root decomposition operated on the inverse autocorrelation matrix of RLS-based algorithm.

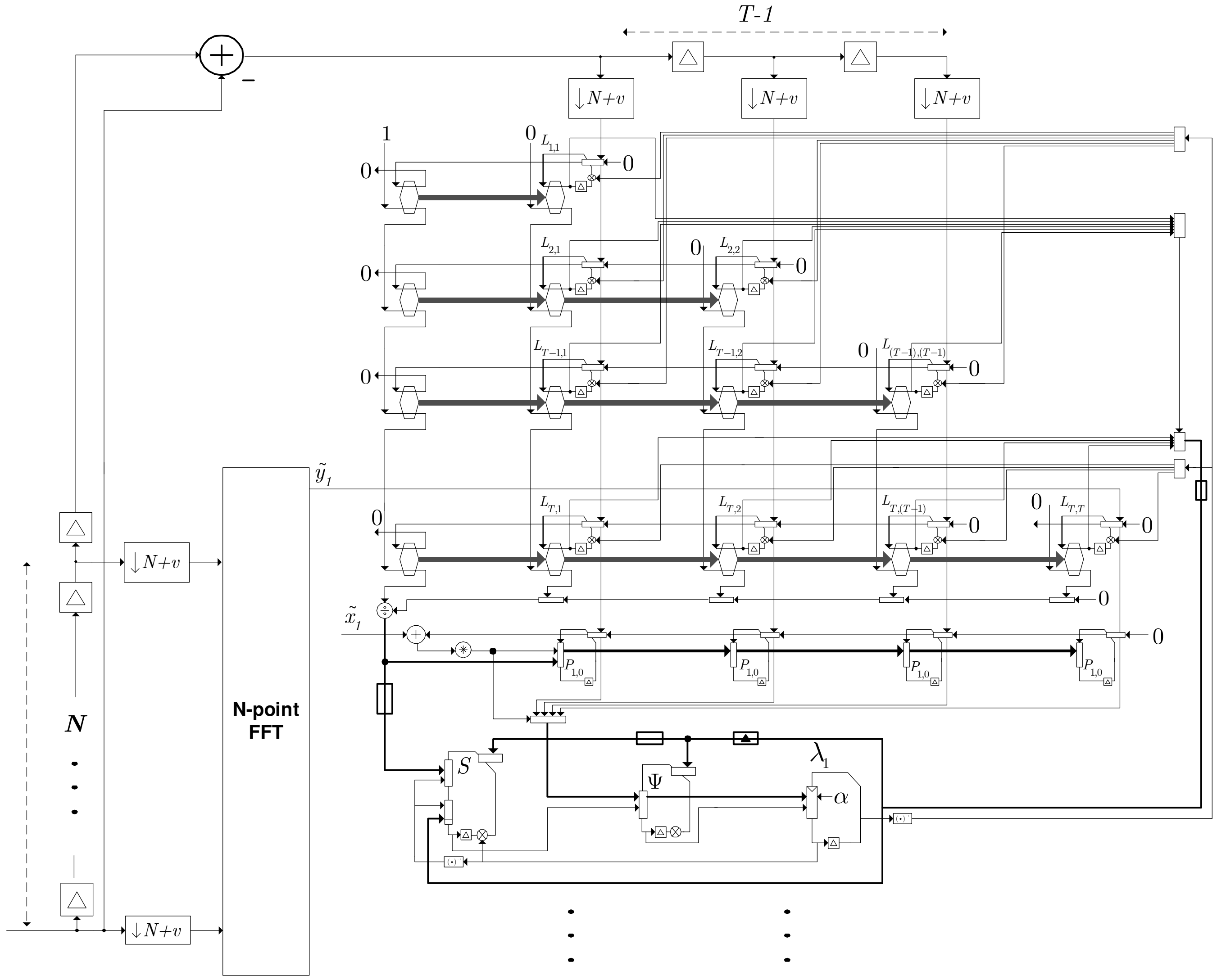

Fig.2: *Block diagram of the modified SFG of the proposed AF-iQRRLS PTEQs.*

4.5 Per-group forgetting-factor AF-iQRRLS algorithm

In order to reduce initialisation complexity, we then apply to search adaptively forgetting-factor parameter per group by using the modified AF-iQRRLS algorithm. Thus, a per-group forgetting-factor (PGFF) AF-iQRRLS algorithm is presented for PTEQs. The idea is that the centre tone of each group is computed to find an optimal forgetting-factor, then this PTEQs of each group is used this forgetting-factor for this whole group, which is the extension of proposed AF-iQRRLS algorithm.

We apply to combine tones to find a forgetting-factor per group, i.e. λ_{c_1} for group n_1. Thus, the real-valued λ_{c_1} of group n_1 is computed using the modified AF-iQRRLS algorithm with the considered DMT-symbols of the middle tones c_1 of this group. After convergence, we use this forgetting-factor λ_{c_1} on middle tone c_1 to attain the overall complex-valued PTEQs for each tone with the adaptive iQR-RLS algorithm for group n_1. Block diagram of proposed AF-iQRRLS PTEQ using per-group forgetting-factor approach is introduced in Fig. 4.

Following [6], the 11-combining tones for each group is presented to compute for $\boldsymbol{\lambda}_{c_n}$ with the method of proposed AF-iQRRLS algorithm is defined as

$$\lambda_{k,c_n} = \lambda_{k-1,c_n} + \alpha \, \Re\{\boldsymbol{\Psi}^H_{k,c_n} \tilde{\mathbf{y}}_{k,c_n} \, \xi^*_{k,n}\} \ , \qquad (37)$$

where $c_n = 11n_c - 5 \ \ ; n_c = 1, 2, \cdots, (\frac{n}{11})$ and

$$\boldsymbol{\lambda}_{c_n} = [\lambda_{c_1} \ \cdots \ \lambda_{c_n}] \ . \qquad (38)$$

Therefore, the updated tap-weight PTEQ vector $\hat{\mathbf{p}}_{k,n}$ using the per-group forgetting-factor $\boldsymbol{\lambda}_{c_n}$ for each group and the adaptive iQR-RLS algorithm for each tone is presented with the method of modified signal flow graph (SFG) of proposed PGFF-AFiQRRLS PTEQs shown in Fig. 3. The basic building blocks of modified SFG is depicted in Fig. 8. The summary of proposed PGFF-AFiQRRLS PTEQs for modified SFG is presented in Table 4.

5. COMPUTATIONAL COMPLEXITY

In this section, we investigate the complexity of the proposed PTEQs based on an iQR-RLS approach measured in number of real multiplication [9]. We consider that a multiplication of two complex numbers is counted as 4-real multiplications and 2-real additions. A multiplication of a real number with a complex number is computed by 2-real multiplications.

Therefore, the computational complexity of the proposed AF-iQRRLS and PGFF-AFiQRRLS algorithms for PTEQs are given in Table 1, where $\mathbb{T}$ is the number of taps of PTEQ. It is shown that the proposed PGFF scheme can reduce significantly compared with AF-iQRRLS approach when each group is combined with M tones.

Table 1: *The computational complexity per symbol.*

Algorithm	Number of multiplications
PGFF-AFiQRRLS	$(4\mathbb{T}^2 + 10\mathbb{T} + 6) + M(2.5\mathbb{T}^2 - 15.5\mathbb{T} + 3)$
AF-iQRRLS	$M(6.5\mathbb{T}^2 - 5.5\mathbb{T} + 9)$
iQR-RLS [9]	$M(2.5\mathbb{T}^2 - 15.5\mathbb{T} + 3)$

6. SIMULATION RESULTS

We performed ADSL downstream transmission simulations that comprises 512 coefficients of channel impulse response to compare the proposed AF-iQRRLS algorithm with complex-valued conventional complex RLS [26] PTEQs on the parameters shown in Table 3. The carrier serving area (CSA) loop no. 2 was a representative of simulations with all 8 CSA loops [27].

The CSA#2 is a representative loop of 26 and 24 gauge loop of length of 3000 and 700 ft., with 26 gauge bridged taps of length of 700 ft. at 3700 ft. and of 24 and 26 gauge loop of length of 350 and 3000 ft. with 26 gauge bridged taps of length of 650 ft. at 7050 ft. detailed in [19].

Other parameters of proposed PGFF-AFiQRRLS algorithm were $p_g = 11$, $\delta = 0.03$ and $\Delta = 28$. The adaptation parameter α of forgetting-factor parameters λ_{k,c_n} was fixed at 5.25×10^{-5} and $\lambda(0) = 0.95$ for all active tones N_d. This proposed algorithm for $\hat{\mathbf{p}}_{k,n}$ can be calculated with the soft-constrained initialisation starting at tone 38 to 255 for downstream ADSL standard and the NEXT from 24 ADSL disturbers was included.

Fig.5 depicts the sum of squared error curves of the proposed PGFF-AFiQRRLS algorithm for the samples of all active tones as 40, 120, 200 and 250, respectively. It is noted that they are converged to the MMSE.

Fig.7 shows that the trajectories of forgetting-factors λ_{k,c_n} of center tones of each group can converge to their values for each individual tone. Approximately 100 symbols are appeared to converge to their steady-state conditions for the proposed AF-iQRRLS PTEQs with the method of modified adaptive forgetting-factor approach.

Fig.6 illustrates the bit rate learning curves of the proposed PGFF-AFiQRRLS and AF-iQRRLS algorithms as compared to the complex conventional RLS [26] algorithm. At approximate 120 and 140 symbols, the proposed PGFF-AFiQRRLS and AF-iQRRLS PTEQs converge to steady-state, respectively. It is shown that the good performance is obtained with the proposed PGFF-AFiQRRLS algorithm with lower complexity. We notice that the RLS algorithm converges rapidly to steady-state lower bit rate performance than the proposed algorithms.

Table 2: *Summary of the proposed AF-iQRRLS PTEQs for modified signal flow graph (SFG).*

- Starting with soft-constrained initialisation as :

$$\hat{\mathbf{p}}(0) = \mathbf{0}; \Phi^{-1}(0) = \delta^{-1}\mathbf{I}; \mathbf{S}(0) = \mathbf{I}; \mathbf{\Psi}(0) = \mathbf{0}.$$

- Do for $\quad n = 1, 2, \ldots, N_d.$

for $\quad k = 1, 2, \ldots, K.$

$$\begin{bmatrix} 1 & \lambda_{k,n}^{-\frac{1}{2}}\tilde{\mathbf{y}}_{k,n}^H\mathbf{\Phi}_{k-1,n}^{-\frac{1}{2}} \\ \mathbf{0} & \lambda_{k,n}^{-\frac{1}{2}}\mathbf{\Phi}_{k-1,n}^{-\frac{1}{2}} \end{bmatrix}\Theta = \begin{bmatrix} \gamma_{k,n}^{\frac{1}{2}} & \mathbf{0}^T \\ \tilde{\mathbf{k}}_{k,n}\gamma_{k,n}^{\frac{1}{2}} & \mathbf{\Phi}_{k,n}^{-\frac{1}{2}} \end{bmatrix},$$

$$\begin{aligned}
\mathbf{\Phi}_{k,n}^{-1} &= \mathbf{\Phi}_{k,n}^{-\frac{1}{2}}\mathbf{\Phi}_{k,n}^{-\frac{H}{2}}, \\
\xi_{k,n} &= \tilde{x}_{k,n} - \hat{\mathbf{p}}_{k-1,n}^H\tilde{\mathbf{y}}_{k,n}, \\
\hat{\mathbf{p}}_{k,n} &= \hat{\mathbf{p}}_{k,n} + \left(\tilde{\mathbf{k}}_{k,n}\gamma_{k,n}^{\frac{1}{2}}\right)\left(\gamma_{k,n}^{-\frac{1}{2}}\right)\xi_{k,n}^*, \\
\mathbf{S}_{k,n} &= \lambda_{k,n}\mathbf{\Phi}_{k,n}^{-1}\mathbf{\Phi}_{k-1,n}\mathbf{S}_{k-1,n}\mathbf{\Phi}_{k-1,n}^{-H}\mathbf{\Phi}_{k-1,n}^H \\
&\quad +\lambda_{k,n}^{-1}\mathbf{k}_{k,n}\mathbf{k}_{k,n}^H - \lambda_{k,n}^{-1}\mathbf{\Phi}_{k,n}^{-1}, \\
\mathbf{\Psi}_{k,n} &= \lambda_{k,n}\mathbf{\Phi}_{k,n}^{-1}\mathbf{\Phi}_{k-1,n}\mathbf{\Psi}_{k-1,n} \\
&\quad +\mathbf{S}_{k,n}\tilde{\mathbf{y}}_{k,n}\xi_{k,n}^*, \\
\lambda_{k,n} &= \lambda_{k-1,n} + \alpha\, \Re\{\mathbf{\Psi}_{k,n}^H\tilde{\mathbf{y}}_{k,n}\xi_{k,n}^*\}.
\end{aligned}$$

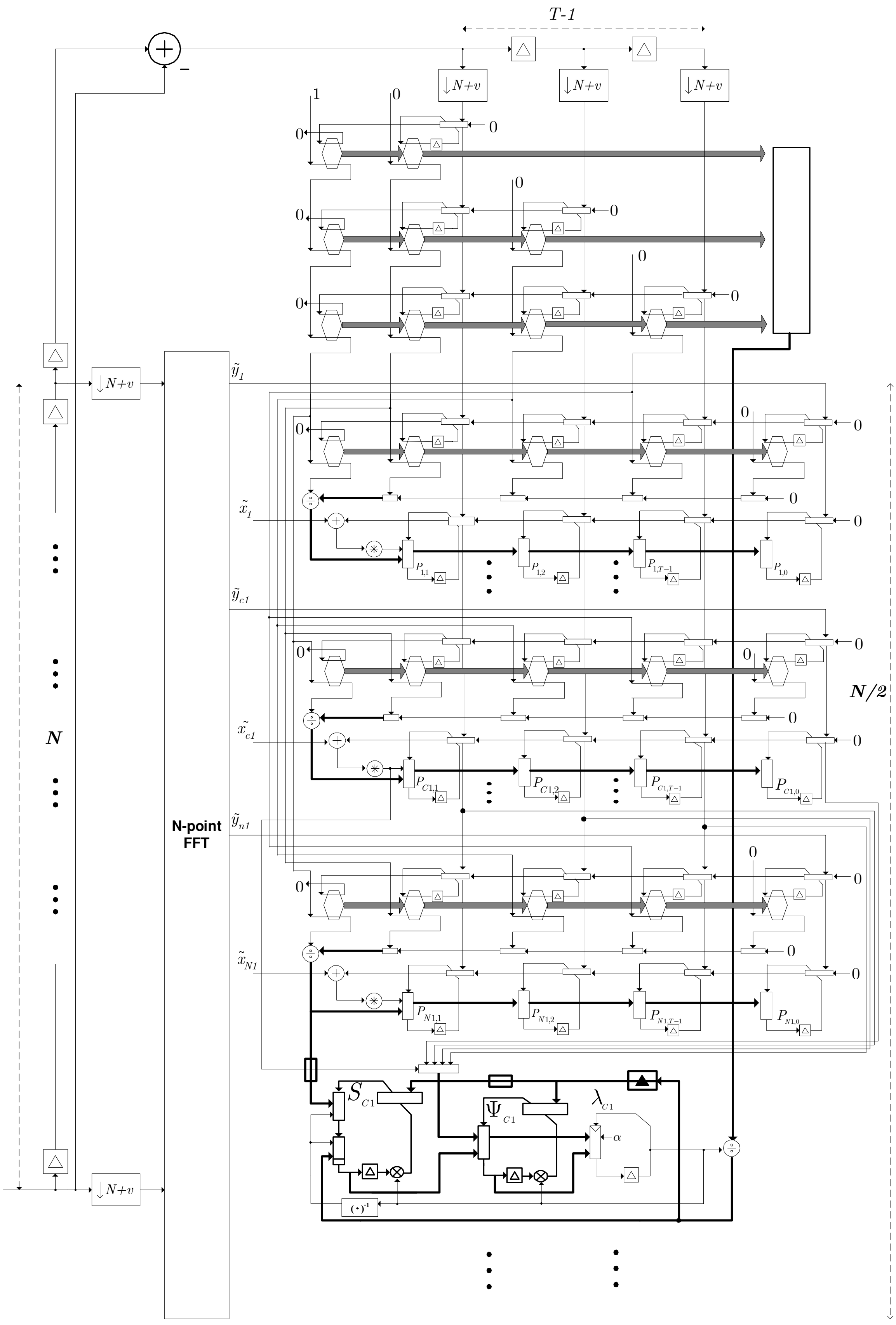

Fig.3: *Modified SFG for the proposed PGFF-AFiQRRLS PTEQs.*

Table 3: *The standard ADSL system for simulation.*

Asymmetric Digital Subscriber Line (ADSL) Specifications			
number of tap ($\mathbb{T}$)	32	CP (ν)	32
Input power	19.83 dBm	f_s	2.208 MHz
FFT size (N)	512	Noise margin	6 dB
TX-DMT block (M)	400	Coding gain	4.2 dB
TX sequence	$M \times N$	SNR gap (Γ)	9.8 dB
Input impedance	100 Ω	AWGN power	-140 dBm/Hz

Table 4: *Summary of the proposed per-group forgetting-factor AFiQR-RLS (PGFF-AFiQRRLS) PTEQs for modified SFG.*

- Starting with soft-constrained initialisation as :

$\hat{\mathbf{p}}(0) = \mathbf{0}; \mathbf{\Phi}^{-1}(0) = \delta^{-1}\mathbf{I}; \mathbf{S}(0) = \mathbf{I};$
$\Psi(0) = \mathbf{0};$

- For $n = 1, 2, \ldots, \frac{N_d}{p_g}$.

  $c_n = (p_g \cdot n) - (\frac{p_g - 1}{2})$.
  for $\quad k = 1, 2, \ldots, K$.

$$\begin{bmatrix} 1 & \lambda_{k,n}^{-\frac{1}{2}} \tilde{\mathbf{y}}_{k,n}^H \mathbf{\Phi}_{k-1,n}^{-\frac{1}{2}} \\ \mathbf{0} & \lambda_{k,n}^{-\frac{1}{2}} \mathbf{\Phi}_{k-1,n}^{-\frac{1}{2}} \end{bmatrix} \Theta = \begin{bmatrix} \gamma_{k,n}^{\frac{1}{2}} & \mathbf{0}^T \\ \tilde{\mathbf{k}}_{k,n} \gamma_{k,n}^{\frac{1}{2}} & \mathbf{\Phi}_{k,n}^{-\frac{1}{2}} \end{bmatrix},$$

$$\begin{aligned}
\mathbf{\Phi}_{k,n}^{-1} &= \mathbf{\Phi}_{k,n}^{-\frac{1}{2}} \mathbf{\Phi}_{k,n}^{-\frac{H}{2}}, \\
\xi_{k,n} &= \tilde{x}_{k,n} - \hat{\mathbf{p}}_{k-1,n}^H \tilde{\mathbf{y}}_{k,n}, \\
\hat{\mathbf{p}}_{k,n} &= \hat{\mathbf{p}}_{k,n} + \left(\tilde{\mathbf{k}}_{k,n} \gamma_{k,n}^{\frac{1}{2}}\right)\left(\gamma_{k,n}^{-\frac{1}{2}}\right) \xi_{k,n}^*, \\
\mathbf{S}_{k,n} &= \lambda_{k,n} \mathbf{\Phi}_{k,n}^{-1} \mathbf{\Phi}_{k-1,n} \mathbf{S}_{k-1,n} \mathbf{\Phi}_{k-1,n}^{-H} \mathbf{\Phi}_{k-1,n}^H \\
&\quad + \lambda_{k,n}^{-1} \tilde{\mathbf{k}}_{k,n} \tilde{\mathbf{k}}_{k,n}^H - \lambda_{k,n}^{-1} \mathbf{\Phi}_{k,n}^{-1}, \\
\mathbf{\Psi}_{k,n} &= \lambda_{k,n} \mathbf{\Phi}_{k,n}^{-1} \mathbf{\Phi}_{k-1,n} \mathbf{\Psi}_{k-1,n} \\
&\quad + \mathbf{S}_{k,n} \tilde{\mathbf{y}}_{k,n} \xi_{k,n}^*, \\
\lambda_{k,n} &= \lambda_{k-1,n} + \alpha\, \Re\{\Psi_{k,n}^H \tilde{\mathbf{y}}_{k,n} \xi_{k,n}^*\}.
\end{aligned}$$

end

for $n_g = 1, 2, \ldots, p_g$.
for $\quad k = 1, 2, \ldots, K$.

$$\begin{bmatrix} 1 & \lambda_{c_n}^{-\frac{1}{2}} \tilde{\mathbf{y}}_{k,n}^H \mathbf{\Phi}_{k-1,n}^{-\frac{1}{2}} \\ \mathbf{0} & \lambda_{c_n}^{-\frac{1}{2}} \mathbf{\Phi}_{k-1,n}^{-\frac{1}{2}} \end{bmatrix} \Theta = \begin{bmatrix} \gamma_{k,n}^{\frac{1}{2}} & \mathbf{0}^T \\ \tilde{\mathbf{k}}_{k,n} \gamma_{k,n}^{\frac{1}{2}} & \mathbf{\Phi}_{k,n}^{-\frac{1}{2}} \end{bmatrix},$$

$$\hat{\mathbf{p}}_{k,n} = \hat{\mathbf{p}}_{k,n} + \left(\tilde{\mathbf{k}}_{k,n} \gamma_{k,n}^{\frac{1}{2}}\right)\left(\gamma_{k,n}^{-\frac{1}{2}}\right) \xi_{k,n}^*.$$

end
end
end

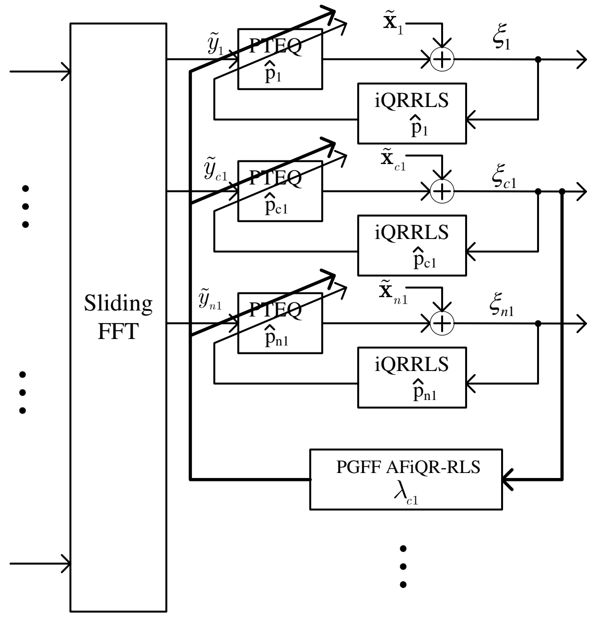

Fig.4: *Block diagram of proposed per-group forgetting-factor AF-iQRRLS (PGFF-AFiQRRLS) PTEQs.*

7. CONCLUSION

In this paper, we have introduced the modified corresponding SFG for proposed AF-iQRRLS and PGFF-AFiQRRLS PTEQs for DMT-based systems. We have described concisely how to define the updated tap-weight PTEQ $\hat{\mathbf{p}}_{k,n}$ vector and per-group forgetting-factor scheme with the method of AFiQR-RLS algorithm. The trajectories of adaptive per-group forgetting-factor parameters are also shown to be aligned adaptively for each individual tone. The learning curves of sum of squared error of proposed AFiQR-RLS algorithm are shown to be converged slowly to MMSE at high tone bins. The bit rate performance of proposed PGFF-AFiQRRLS algorithm can be improved in comparison with the RLS algorithm.

The proposed PGFF-AFiQRRLS algorithm has also been introduced in forms of modified SFG. The adaptive forgetting-factor of the center tone of each group is selected as the representative. After convergence, it is fixed for remaining tones of the whole group for PTEQs. These promising results suggest how the proposed algorithm can be designed with reduced computational complexity.

References

[1] T.Pollet, M.Peeters, M.Moonen, and L.Vandendorpe, "Equalization for DMT-based

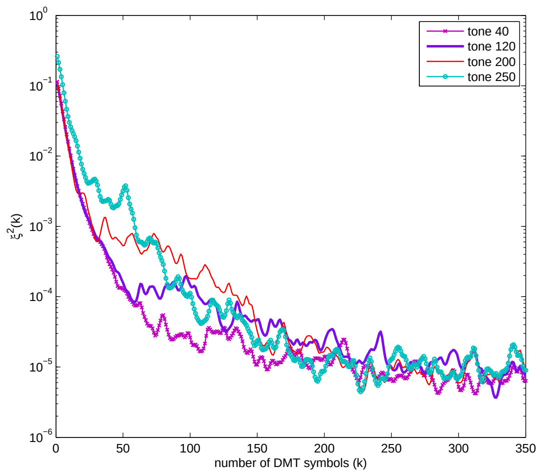

Fig.5: *Learning curves of sum of squared error of the proposed PGFF-AFiQRRLS algorithm for modified SFG.*

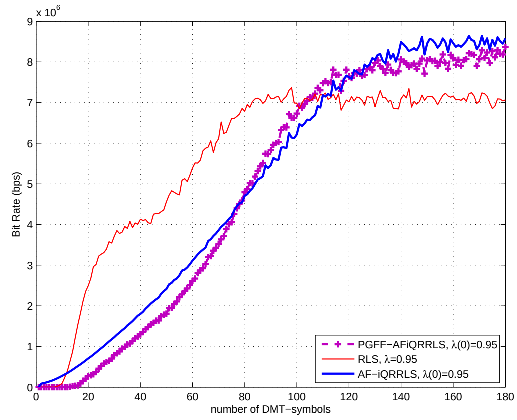

Fig.6: *Learning curves of bit rate convergence of the proposed PGFF-AFiQRRLS, AF-iQRRLS and RLS [26] algorithms.*

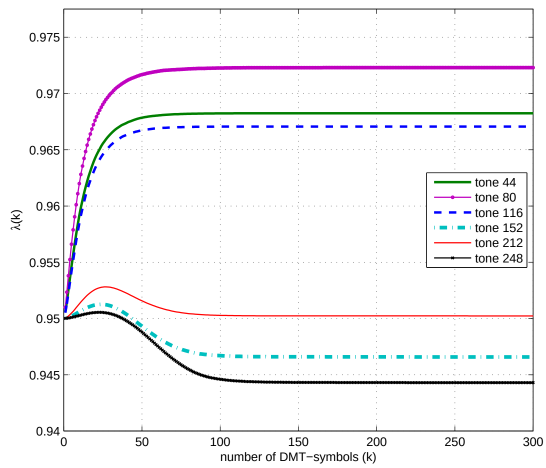

Fig.7: *Trajectories of forgetting-factor parameters λ of the proposed PGFF-AFiQRRLS algorithm using the initial forgetting-factor $\lambda(0) = 0.95$ and the adaptation constant $\alpha = 5.25 \times 10^{-5}$ with different active tones.*

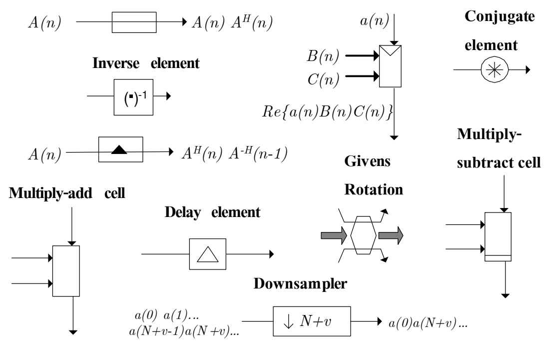

Fig.8: *Basic Building Blocks of modified SFG.*

broadband modems", *IEEE Commununications Magazine*, pp.106-113, May 2000.

[2] R.Baldemair and P.Frenger, "A Time-domain Equalizer Minimizing Intersymbol and Intercarrier Interference in DMT Systems", in *Proc. IEEE Global Telecommunications Conference (GLOBECOM'01)*, vol.1, pp.381-385, 2001.

[3] P.Golden, H.Dedieu, and K.S.Jacobsen, *Fundamentals of DSL Technology*, Auerbach Publications, Taylor&Francis Group, 2006.

[4] B.F-Boroujeny and M.Ding, "Design Methods for Time-Domain Equalizers in DMT Transceivers", *IEEE Transactions on Communications*, vol. 49, no. 3, pp. 554-562, Mar. 2001.

[5] S.Sitjongsataporn and P.Yuvapoositanon, "Adaptive Step-size Order Statistic LMS-based Time-domain Equalisation in Discrete Multitone Systems", *Discrete Time Systems*, Mario Alberto Jordán (Ed.), ISBN: 978-953-307-200-5, InTech, April 2011, Available from: http://www.intechopen.com/articles/show/title/adaptive-step-size-order-statistic-lms-based-time-domain-equalisation-in-discrete-multitone-systems

[6] K.V.Acker, G.Leus, M.Moonen, O.van de Wiel and T.Pollet, "Per Tone Equalization for DMT-Based Systems", *IEEE Transactions on Communications*, vol. 49, no. 1, pp. 109-119, Jan. 2001.

[7] M.H.Verhaegen, "Round-off Error Propagation in Four Generally-Applicable, Recursive, Least-Squares Estimation Schemes", *Automatica*, vol. 25, pp. 437-444, 1989.

[8] S.Haykin, *Adaptive Filter Theory*, Prentice Hall, 1996.

[9] K.V.Acker, G.Leus, M.Moonen and T.Pollet,

"RLS-Based Initialization for Per-Tone Equalizers in DMT Receivers", *IEEE Trans. on Comm.*, vol. 51, no. 6, pp. 885-889, Jun. 2003.

[10] K.V.Acker, G.Leus, M.Moonen and T.Pollet, "Frequency Domain Equalization with Tone Grouping in DMT/ADSL-Receivers",in *Proc. IEEE Asilomar'99*, pp.1067-1070, Oct.1999.

[11] K.Vanbleu, G.Ysebaert, G.Cuypers and M.Moonen, "Bitrate Maximizing Per-Group Equalization for DMT-based Systems",in *Signal Processing*, vol.86, pp.2952-2965, Oct.2006.

[12] B.Lindoff, "On the Optimal Choice of the Forgetting Factor in the Recursive Least Squares Estimator ", TFMS-3137, Technical report, Math Stat, LU/LTH, 1997.

[13] S.H.Leung and C.F.So, "Nonlinear RLS Algorithm using Variable Forgetting-factor in Mixture Noise", in *Proc. IEEE International Conference Acoustics, Speech and Signal Processing (ICASSP'01)*, vol.6, pp. 3777-3780, 2001.

[14] S.Sitjongsataporn and P.Yuvapoositanon, "Adaptive Forgetting-factor Gauss-Newton inverse QR-RLS Per-Tone Equalisaiton for Discrete Multitone Systems", in *Pro. IEEE International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON'08)*, Krabi, Thailand, pp. 561-564, May 2008.

[15] F.Yu and M.Bouchard, "Recursive Least-Squares Algorithms with Good Numerical Stability for Multichannel Active Noise Control", in *Proc. IEEE International Conference Acoustics, Speech and Signal Processing (ICASSP'01)*, vol. 5, pp. 3221-3224, May 2001.

[16] M.Moonen and J.G.McWhirter, "Systolic Array for Recursive Least Squares by Inverse Updating", *Electronics Letters*, vol. 29, no. 13, pp. 1217-1218, June 1993.

[17] M.Moonen and I.K.Proudler, "Generating 'Fast QR' Algorithms Using Signal Flow Graph Techniques", in *Proc. IEEE Asilomar'96*, Pacific Grove, CA, vol. 1, pp. 410-414, 1996.

[18] K.V.Acker, G.Leus, M.Moonen, O.van de Wiel and T.Pollet, "Per Tone Equalization for DMT receivers", in *Proc. IEEE Global Telecommunications Conference (GLOBECOM'99)*, pp. 2311-2315, Dec. 1999.

[19] N.Al-Dhahir and J.M.Cioffi, "Optimum Finite-Length Equalization for Multicarrier Transceivers", *IEEE Transactions on Communications*, vol. 44, pp. 56-64, Jan. 1996.

[20] J.S.Chow, J.C.Tu, and J.M.Cioffi, "A discrete multitone transceiver system for HDSL applications",in *IEEE Journal on Selected Area in Communications*, vol. 9, no. 6, pp.895-907, Aug. 1991.

[21] A.H.Sayed and T.Kailath, "A State-Space Approach to Adaptive RLS filtering", *IEEE Signal Processing Magazine*, vol. 11, pp. 18-60, 1994.

[22] S.Sitjongsataporn and P.Yuvapoositanon, "An inverse QR Adaptive Forgetting-factor RLS algorithm for Per-Tone Equalisation", in *Proc.* 31st *Electrical Engineering Conference (EECON-31)*, Nakorn-nayok, Thailand, pp. 1097-1100, Oct. 2008.

[23] S.T.Alexander and A.L.Ghirnikar, "A Method for Recursive Least Squares Filtering Based Upon an Inverse QR Decomposition", *IEEE Transactions on Signal Processing*, vol. 41, no. 1, Jan. 1993.

[24] S.Ljung and L.Ljung, "Error propagation properties of recursive least-squares adaptation algorithms", *Automatica*, vol.21, pp.157-167, Mar. 1985.

[25] B.Yang, "A Note on the Error Propagation Analysis of Recursive Least Squares Algorithms", *IEEE Transactions on Signal Processing*, vol. 42, no. 12, pp. 3523-3525, Dec. 1994.

[26] P.S.R.Diniz, *Adaptive Filtering Algorithms and Practical Implementation*, Springer, Boston, MA, 2008.

[27] International Telecommunications Union (ITU). Recommendation G.996.1, *Test Procedures for Asymmetric Digital Subscriber Line (ADSL) Transceivers*, Feb. 2001.

Suchada Sitjongsataporn received the B.Eng. and D.Eng. degrees of Electrical Engineering from Mahanakorn University of Technology, Bangkok, Thailand in 2002 and 2009, respectively. She has worked as lecturer at department of Electronic Engineering, Mahanakorn University of Technology, since 2002. Her research interests are in the area of adaptive algorithm, adaptive equalisation and adaptive signal processing for wireline and wireless communications.