Adaptive Fuzzy Controller for the Nonlinear System with Unknown Sign of the Input Gain

Jang-Hyun Park, Seong-Hwan Kim, and Chae-Joo Moon

Abstract: We propose and analyze a robust adaptive fuzzy controller for nonlinear systems without *a priori* knowledge of the sign of the input gain function. No assumptions are made about the type of nonlinearities of the system, except that such nonlinearities are smooth. The uncertain nonlinearities are captured by the fuzzy systems that have been proven to be universal approximators. The proposed control scheme completely overcomes the singularity problem that occurs in the indirect adaptive feedback linearizing control. Projection in the estimated parameters and switching in the control input are both not required. The stability of the closed-loop system is guaranteed in the Lyapunov viewpoint.

Keywords: Adaptive fuzzy control, nonlinear system, unknown input gain sign.

1. INTRODUCTION

The theory of explicitly linearizing the input-output response of nonlinear systems to linear systems using the state feedback has received great attention. However, it relies on an exact cancellation of nonlinear terms to obtain linear input-output behavior. For many nonlinear dynamical systems, which are highly nonlinear, it is generally difficult to develop accurate mathematical models, i.e., there are inevitable uncertainties in the constructed models. Therefore, the design of a robust controller that can deal with model uncertainties is very important.

Universal function approximators (UFAs) such as fuzzy logic systems and artificial neural networks have been successfully applied to many control problems because they need no accurate mathematical models of the system under control. It is a well-known fact that they can approximate certain classes of functions to a given accuracy and furthermore the output of the system can be represented by a linear combination of basis functions such as fuzzy basis functions or radial basis functions [1-4]. Based on this property many researchers have presented adaptive control architecture for uncertain nonlinear systems [5-15].

However, all the previous results need the assumptions that the input gain, which is the function of the system states in general, is away from zero and its sign is known *a priori*. The sign, called control direction, represents motion direction of the system under any control, and knowledge of this sign makes adaptive control design much easier. In this paper, we eliminate these assumptions. In addition, no assumptions are made about the type of nonlinearities of the system, except that such nonlinearities are smooth.

In the area of adaptive control, by incorporating Nussbaum gain technique [16] into backstepping design, adaptive control schemes have been developed for parametric-strict-feedback nonlinear systems [17, 18] and output-feedback nonlinear systems [19] with unknown control direction. An alternative method, the so-called correction vector approach [20] has been applied to first-order nonlinear systems [21]. The above mentioned papers consider nonlinear systems whose nonlinearities are linear in the unknown parameters and, as far as we know, no research results have been presented on controlling general nonlinear systems whose nonlinearities are functions of system state variables.

We propose an indirect adaptive fuzzy controller and new learning algorithms such that all the signals involved are stable in the Lyapunov viewpoint. In the indirect adaptive control of the feedback linearizable system, the fuzzy systems are employed to estimate the plant dynamics and these estimates are used to generate the control input that achieves tracking of a desired output. Since we have no information on the input gain, its estimate may be zero at some instances. Thus, the conventional adaptive input-output linearizing control scheme where the control law's denominator is the estimate of the input gain cannot

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be employed in this situation. We also propose a new singular-free control scheme. No projection as in [5-7] and no switching in the control as in [8] are needed. The stability of the closed-loop system is guaranteed in the Lyapunov standpoint.

The outline of the paper is as follows. Section 2 presents a brief description of the fuzzy system and universal approximation theorem. In Section 3, a new control law and adaptive algorithms are proposed and stability analysis is given. Simulation examples are illustrated in Section 4. The conclusions are finally given in Section 5.

2. DESCRIPTION OF FUZZY SYSTEMS

In this paper, the fuzzy systems are used to capture the unknown nonlinearities of the system. In general, the output of the multi-input single-output fuzzy system with singleton fuzzifier, product inference, centroid defuzzifier and Gaussian membership functions is described by

$$\hat{h}(\mathbf{x}\,|\,\theta_h) = \theta_h^T \xi_h(\mathbf{x}), \tag{1}$$

where $\mathbf{x} \in R^n$ is the input vector to the fuzzy system, $\hat{h} \in R$ is the output, $\theta_h \in R^L$ is the adjustable parameter vector with its elements being the point at which membership function in the consequent part of a fuzzy rule achieves its maximum, L denotes the number of the fuzzy rules, $\xi_h(\cdot): R^n \to R^L$ is a nonlinear vector function with its element being the normalized firing strength represented by

$$\xi_i(\mathbf{x}) = \frac{\prod_{j=1}^{n} \mu_{A_j^i}(x_j)}{\sum_{i=1}^{L} \prod_{j=1}^{n} \mu_{A_j^i}(x_j)},\ i = 1, \cdots, L \tag{2}$$

in which $\mu_{A_j^i}(x_j)$ are the Gaussian membership functions of x_j associated with the ith fuzzy rule. Function (2) is called the fuzzy basis function (FBF) in [1].

The key advantage of the fuzzy system described above is that it has the capability to approximate nonlinear mappings to any degree of accuracy, which is summarized in the following theorem.

Theorem 1 (Universal Approximation Theorem)**:** For any given real continuous function h on a compact set $\Omega_x \in R^n$ and an arbitrary $\varepsilon_h > 0$, there exists a fuzzy system $\hat{h}$ in the form of (1) and optimal parameter vector θ_h^* such that

$$\sup_{x \in \Omega_x} \left| h(\mathbf{x}) - \hat{h}(x\,|\,\theta_h^*) \right| < \varepsilon_h. \tag{3}$$

A proof of this theorem is given in [1,2]. Note that the reconstruction error arises as a result of the inadequacy of the fuzzy systems to match exactly an uncertain nonlinear function even if optimal weights are selected. However, we can make ε_h arbitrarily small by highly increasing the number of fuzzy rules.

3. CONTROLLER DESIGN AND STABILITY ANALYSIS

3.1. Problem formulation

In this section, we first set up control objectives, and then show how to design an adaptive controller based on the fuzzy system to achieve the objectives.

Consider the nth-order nonlinear systems of the form

$$\begin{aligned} x^{(n)} &= f_n(\mathbf{x}) + g(\mathbf{x})u, \\ y &= x, \end{aligned} \tag{4}$$

where f_n and g are unknown smooth functions, $u \in R$ and $y \in R$ are the input and output of the system, respectively, and $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T = [x, \dot{x}, \cdots, x^{(n-1)}]^T \in R^n$ is the state vector of the system that is assumed to be measurable. Note that the previous research results [5-8,22] require conditions on $g(\mathbf{x})$ in which its sign and lower bound are known. These conditions are required not only for affine systems but nonaffine systems [11,23]. Such conditions on $g(\mathbf{x})$ are not needed in this paper. The control objective is to force the output $y(t)$ to track a given bounded reference signal $y_d(t)$, under the constraint that all signals involved must be bounded.

Before preceding, let us rewrite (4) as

$$\dot{x}_n = f(\mathbf{x}) + g(\mathbf{x})u - \mathbf{k}^T\mathbf{x}, \tag{5}$$

where

$$f(\mathbf{x}) = f_n(\mathbf{x}) + \mathbf{k}^T\mathbf{x}$$

and $\mathbf{k} = [k_1 \cdots k_n]^T$ is determined such that the polynomial $h(s) = s^n + k_n s^{n-1} + \cdots + k_1$ is Hurwitz.

We estimate the functions $f(\mathbf{x})$ and $g(\mathbf{x})$ using two fuzzy systems and denote them as $\hat{f}(\mathbf{x})$ and $\hat{g}(\mathbf{x})$ respectively.

3.2. Controller design and stability proof

Typical adaptive input-output linearizing controllers are of the form [5,6,22]:

$$\begin{aligned} u &= \frac{1}{\hat{g}(\mathbf{x})}\left(-\hat{f}(\mathbf{x}) + \alpha + \beta\right), \\ \alpha &= y_d^{(n)} + \mathbf{k}^T\mathbf{x}_d, \end{aligned} \tag{6}$$

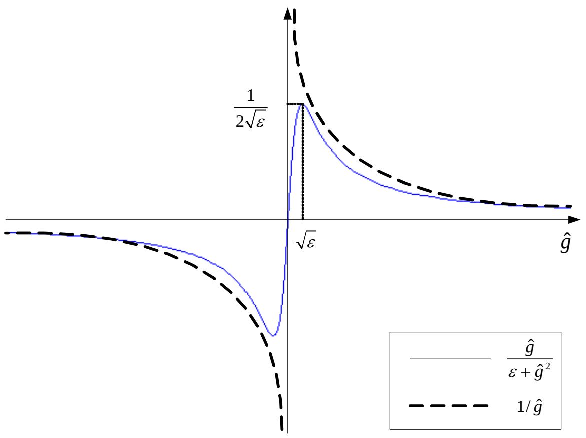

Fig. 1. Plots of $\dfrac{\hat{g}}{\varepsilon+\hat{g}^2}$ and $\dfrac{1}{\hat{g}}$.

where $\mathbf{x}_d = [y_d, \dot{y}_d \cdots y_d^{(n-1)}]^T$, β is an additional robustness term. However, since the numerator of the control input (6) is $\hat{g}(\mathbf{x})$, it must not be zero. In the situation that the sign of $g(\mathbf{x})$ is unknown, its estimate, $\hat{g}(\mathbf{x})$, may cross the zero line to search for its correct sign, which causes a singularity problem. Thus, a new control law that is free form input-singularity is proposed as

$$u = \frac{\hat{g}(\mathbf{x})}{\varepsilon + \hat{g}(\mathbf{x})^2}\left(-\hat{f}(\mathbf{x}) + \alpha + \beta\right), \qquad (7)$$
$$\alpha = y_d^{(n)} + \mathbf{k}^T \mathbf{x}_d,$$

where $\varepsilon > 0$ is a design constant and β is a robustness term defined later. It can easily be observed that if ε approaches zero or if $\hat{g}(\mathbf{x})$ becomes much larger than ε, the modified term $\hat{g}/(\varepsilon+\hat{g}^2)$ approaches $1/\hat{g}$. See Fig. 1.

Let $e = y - y_d$, $\mathbf{e} = \left[e, \dot{e}, \cdots . e^{(n-1)}\right]$. Substituting (7) into (5) can yield the following error dynamics.

$$\begin{aligned} e^{(n)} &= y^{(n)} - y_d^{(n)} \\ &= f(\mathbf{x}) - g(\mathbf{x})u + \frac{\varepsilon + \hat{g}(\mathbf{x})^2}{\hat{g}(\mathbf{x})}u \\ &\quad - \frac{\varepsilon + \hat{g}(\mathbf{x})^2}{\hat{g}(\mathbf{x})}u - y_d^{(n)} - \mathbf{k}^T\mathbf{x} \\ &= -\mathbf{k}^T\mathbf{e} + \left(f(\mathbf{x}) - \hat{f}(\mathbf{x})\right) \\ &\quad + \left(g(\mathbf{x}) - \hat{g}(\mathbf{x})\right)u - \frac{\varepsilon}{\hat{g}}u + \beta. \end{aligned} \qquad (8)$$

The above equation can be rewritten as

$$\begin{aligned} \dot{\mathbf{e}} &= A\mathbf{e} + b\Big[-\left(\hat{f}(\mathbf{x}) - \hat{f}^*(\mathbf{x})\right) \\ &\quad + \left(f(\mathbf{x}) - \hat{f}^*(\mathbf{x})\right) - \left(\hat{g}(\mathbf{x}) - \hat{g}^*(\mathbf{x})\right)u \\ &\quad + \left(g(\mathbf{x}) - \hat{g}^*(\mathbf{x})\right)u - \frac{\varepsilon}{\hat{g}}u + \beta\Big] \\ &= A\mathbf{e} + b\Big[-\tilde{\theta}_f^T\xi_f(\mathbf{x}) + \delta_f - \tilde{\theta}_g^T\xi_g(\mathbf{x})u \\ &\quad + \delta_g u - \frac{\varepsilon}{\varepsilon+\hat{g}^2}\left(-\hat{f} + \alpha + \beta\right) + \beta\Big], \end{aligned} \qquad (9)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \vdots & \\ -k_n & -k_{n-1} & -k_{n-2} & \cdots & -k_1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \qquad (10)$$

and $\tilde{\theta}_f = \theta_f - \theta_f^*, \tilde{\theta}_g = \theta_g - \theta_g^*$, $\hat{f}^*, \hat{g}^*$ are shortened denotations of $\hat{f}(\mathbf{x}, \theta_f^*), \hat{g}(\mathbf{x}, \theta_g^*)$, respectively. θ_f^* and θ_g^* are optimal approximation parameters and they are assumed to exist according to the universal approximation theorem such that $\hat{f}^*$, $\hat{g}^*$ can approximate f, g as best as possible. $\delta_f = f - \hat{f}^*$ and $\delta_g = g - \hat{g}^*$ denote the corresponding minimum approximation errors.

Since A is a stable matrix, there exists the positive symmetric matrix P and positive constant q satisfying

$$A^T P + PA = -qI. \qquad (11)$$

Remark: The term $\varepsilon u/\hat{g}$ in (9) is generated because of the ε-term in the denominator of the control law, i.e., if $\varepsilon = 0$, the term also becomes zero. This newly-introduced disturbance will later be compensated by additional conditions on the design constants.

According to Theorem 1, there exist unknown constants $\varepsilon_f, \varepsilon_g > 0$ such that

$$\begin{aligned} |\delta_f(\mathbf{x})| &\le \varepsilon_f \\ |\delta_g(\mathbf{x})| &\le \varepsilon_g \end{aligned} \qquad (12)$$

for all $\mathbf{x} \in \Omega_x$. For the stability proof, we need the following assumption.

Assumption 1: There exists a constant γ such that

$$0 < \frac{\varepsilon_g}{2\sqrt{\varepsilon}} \le \gamma < 1 \qquad (13)$$

for all $\mathbf{x} \in \Omega_x$.

Assumption 1 is reasonable since, as already mentioned in Section 2, ε_g can be made arbitrarily small according to the universal approximation theorem and, moreover, ε is the positive design constant. That is, there are two ways to make γ as small as desired: highly increasing the structure (e.g., number of the membership functions and rules) of the fuzzy system and choosing ε sufficiently large.

Before presenting the main theorem, we define the following constants:

$$\begin{aligned} c_1 &= |P\mathbf{b}|, \\ c_2 &= \sup_{\mathbf{x}\in\Omega_x} |\xi_f(\mathbf{x})|, \\ c_3 &= \sup_t |\alpha(t)|, \\ \psi^* &= \frac{\varepsilon_f + \gamma c_3}{1-\gamma}. \end{aligned} \tag{14}$$

Note that ψ^* is a lumped uncertain constant related to ε_f and ε_g, which are assumed to be unknown. We adopt the estimation scheme for ψ^* and denote its estimate as ψ.

Theorem 2: Consider system (4) with the control input (7). If we choose the update laws for fuzzy parameters θ_f, θ_g and estimate of bounding parameter ψ as

$$\dot{\theta}_f = \gamma_f \left(\mathbf{e}^T P \mathbf{b} \xi_f - \sigma_f \theta_f \right), \tag{15}$$

$$\begin{aligned} \dot{\theta}_g = \gamma_g \big(& \mathbf{e}^T P \mathbf{b} \xi_g u \\ & - \sigma_g \left(\theta_g - h(\hat{g}) \theta \exp(-c|\hat{g}|) \right) \big), \end{aligned} \tag{16}$$

$$\dot{\psi} = \gamma_\psi \left({}^T P \mathbf{b} \cdot \tanh\left(\frac{\mathbf{e}^T P \mathbf{b}}{\varepsilon'} \right) - \sigma_\psi \psi \right), \tag{17}$$

where γ_f, γ_g, γ_ψ are adaptation rates, $h(\cdot)$ is a hysteresis switching function defined as

$$h(\hat{g}(t)) = \begin{cases} 1 & \text{if } \hat{g}(t) = -\mu_1 \text{ and } \dfrac{d}{dt}\hat{g}(t) > 0 \\ 1 & \text{if } \hat{g}(t) = \mu_2 \text{ and } \dfrac{d}{dt}\hat{g}(t) > 0 \\ -1 & \text{if } \hat{g}(t) = \mu_1 \text{ and } \dfrac{d}{dt}\hat{g}(t) < 0 \\ -1 & \text{if } \hat{g}(t) = -\mu_2 \text{ and } \dfrac{d}{dt}\hat{g}(t) < 0 \end{cases} \tag{18}$$

$\sigma_f, \sigma_g, \sigma_\psi, c, \mu_1, \mu_2$ are positive design constants

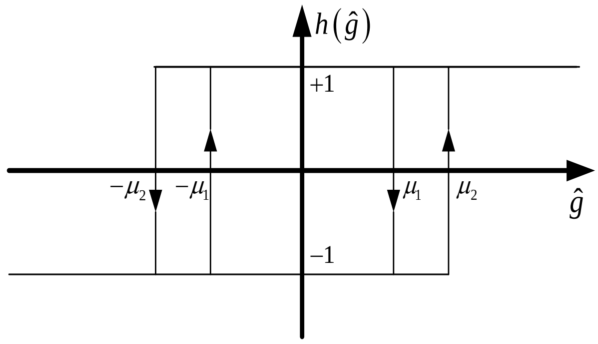

Fig. 2. The hysteresis function $h(\hat{g})$.

and $\theta \in R^L$ is a constant vector determined such that $\theta^T \xi(\mathbf{x}) > 0$ for all $\mathbf{x} \in \Omega_x$. The robustifying term β is chosen as

$$\beta = -\psi \tanh\left(\frac{\mathbf{e}^T P \mathbf{b}}{\varepsilon'} \right) \tag{19}$$

with $\varepsilon' \geq 0$ being a design constant. Then, we can guarantee that all the signals $\mathbf{e}$, θ_f, θ_g, ψ are uniformly ultimately bounded.

Note that the update law (17) guarantees $\psi(t) \geq 0$ for all $t > 0$ if $\psi(0) \geq 0$ since $\dot{\psi} \geq 0$ at $\psi = 0$.

The definition of the function $h(\cdot)$ can easily be understood by looking at Fig. 2.

Before the proof process, we briefly explain why the hysteresis function is introduced in the update law for θ_g (16). If we set $h(\hat{g}) = 0$ for all $\hat{g}$, i.e., we do not perform any modification for $\dot{\theta}_g$, the adaptive system cannot escape $\theta_g = 0$ point. This is because $\dot{\theta}_g = 0$ at $\theta_g = 0$ since $u = 0$. The hysteresis functions are employed to go around the $\theta_g = 0$ point. Moreover, the exponential term has its maximum value at $\hat{g} = 0$, which means it has its maximum effect at $\theta_g = 0$ and has less effect if $\hat{g}$ goes away from 0. The modification of the gain of control input (7) and employing hysteresis function in the adaptive law for θ_g (16) are the key ideas for controlling without control direction.

Proof: Consider the Lyapunov function

$$V = \frac{1}{2}\mathbf{e}^T P \mathbf{e} + \frac{1}{2\gamma_f}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_g}\tilde{\theta}_g^T \tilde{\theta}_g + \frac{1-\gamma}{2\gamma_\psi}\tilde{\psi}^2. \tag{20}$$

Differentiating V along the solution of (9), we obtain

$$\dot{V} = -\frac{1}{2}q|\mathbf{e}|^2 + \mathbf{e}^T P\mathbf{b}\Big[-\tilde{\theta}_f^T\xi_f - \tilde{\theta}_g^T\xi_g u + \delta_f + \delta_g u - \frac{\varepsilon}{\varepsilon+\hat{g}^2}\left(-\hat{f}+\alpha+\beta\right)+\beta\Big] + \frac{1}{\gamma_f}\tilde{\theta}_f^T\dot{\theta}_f + \frac{1}{\gamma_g}\tilde{\theta}_g^T\dot{\theta}_g + \frac{1-\gamma}{\gamma_\psi}\tilde{\psi}\dot{\psi}. \quad (21)$$

Substituting (15) and (16) into (21), we have that

$$\dot{V} = -\frac{1}{2}q|\mathbf{e}|^2 - \sigma_f\tilde{\theta}_f^T\theta_f - \sigma_g\tilde{\theta}_g^T\left(\theta_g - h(\hat{g})\theta\exp(-c|\hat{g}|)\right) - \mathbf{e}^T P\mathbf{b}\left(\frac{\varepsilon}{\varepsilon+\hat{g}^2}\left(-\hat{f}+\alpha+\beta\right)\right)+\Lambda, \quad (22)$$

where

$$\Lambda = \mathbf{e}^T P\mathbf{b}\left(\delta_f+\delta_g u\right)+\mathbf{e}^T P\mathbf{b}\beta + \frac{1-\gamma}{\gamma_\psi}\tilde{\psi}\dot{\psi}. \quad (23)$$

Using (12), (13) and (14), and determining the robustifying term β as (19) yields

$$\begin{aligned}\Lambda &= \mathbf{e}^T P\mathbf{b}\left(\delta_f + \frac{\delta_g\hat{g}}{\varepsilon+\hat{g}^2}\left(-\hat{f}+\alpha\right)\right) + \mathbf{e}^T P\mathbf{b}\left(1+\frac{\delta_g\hat{g}}{\varepsilon+\hat{g}^2}\right)\beta + \frac{1-\gamma}{\gamma_\psi}\tilde{\psi}\dot{\psi}\\ &\le |\mathbf{e}^T P\mathbf{b}|\left(\varepsilon_f + \frac{\varepsilon_g}{2\sqrt{\varepsilon}}\left(c_2|\theta_f|+c_3\right)\right) + \mathbf{e}^T P\mathbf{b}(1-\gamma)\beta + \frac{1-\gamma}{\gamma_\psi}\tilde{\psi}\dot{\psi}\\ &\le |\mathbf{e}^T P\mathbf{b}|\left(\varepsilon_f+\gamma c_2|\theta_f|+\gamma c_3\right) - \mathbf{e}^T P\mathbf{b}(1-\gamma)\psi\tanh\left(\frac{\mathbf{e}^T P\mathbf{b}}{\varepsilon'}\right) + \frac{1-\gamma}{\gamma_\psi}\tilde{\psi}\dot{\psi}\\ &= (1-\gamma)|\mathbf{e}^T P\mathbf{b}|\psi^* - (1-\gamma)\mathbf{e}^T P\mathbf{b}\tanh\left(\frac{\mathbf{e}^T P\mathbf{b}}{\varepsilon'}\right)\psi^* + \gamma c_1 c_2|\mathbf{e}||\theta_f| - (1-\gamma)\mathbf{e}^T P\mathbf{b}\tanh\left(\frac{\mathbf{e}^T P\mathbf{b}}{\varepsilon'}\right)\tilde{\psi} + \frac{1-\gamma}{\gamma_\psi}\tilde{\psi}\dot{\psi}.\end{aligned} \quad (24)$$

We have used $\mathbf{e}^T P\mathbf{b}\beta \le 0$ which leads to $\mathbf{e}^T P\mathbf{b}\left(1+\delta_f\hat{g}/(\varepsilon+\hat{g}^2)\right)\beta \le \mathbf{e}^T P\mathbf{b}(1-\gamma)\beta$ in the second line of (24). Choosing the update law for adjusting ψ as (17) and using $0 \le |\eta| - \eta\tanh(\eta/\varepsilon') \le \kappa\varepsilon'$ with $\kappa = 0.2785$, we have that

$$\Lambda \le (1-\gamma)|\kappa\varepsilon'\psi^* - (1-\gamma)\sigma_\psi\tilde{\psi}\psi + \gamma c_1 c_2|\mathbf{e}||\theta_f|. \quad (25)$$

Combining this result with (22) and using the following relations

$$\begin{aligned}&\tilde{\theta}_f\theta_f = \frac{1}{2}|\tilde{\theta}_f|^2 + \frac{1}{2}|\theta_f|^2 - \frac{1}{2}|\theta_f^*|^2\\ &\tilde{\theta}_g\left(\theta_g - h(\hat{g})\theta\exp(-c|\hat{g}|)\right) \ge \frac{1}{2}|\tilde{\theta}_g|^2 - \frac{1}{2}\left|\theta_g^* - h(\hat{g})\theta\exp(-c|\hat{g}|)\right|^2\\ &\tilde{\psi}\psi = \frac{1}{2}|\tilde{\psi}|^2 + \frac{1}{2}|\psi|^2 - \frac{1}{2}|\psi^*|^2\end{aligned} \quad (26)$$

yields

$$\begin{aligned}\dot{V} &\le -\frac{1}{2}q|\mathbf{e}|^2 - \mathbf{e}^T P\mathbf{b}\left(\frac{\varepsilon}{\varepsilon+\hat{g}^2}\left(-\hat{f}+\alpha+\beta\right)\right) - \sigma_f\left(\frac{1}{2}|\tilde{\theta}_f|^2+\frac{1}{2}|\theta_f|^2-\frac{1}{2}|\theta_f^*|^2\right)\\ &\quad - \sigma_g\left(\frac{1}{2}|\tilde{\theta}_g|^2 - \frac{1}{2}\left|\theta_g^* - h(\hat{g})\theta\exp(-c|\hat{g}|)\right|^2\right) - (1-\gamma)\sigma_\psi\left(\frac{1}{2}|\tilde{\psi}|^2+\frac{1}{2}|\psi|^2-\frac{1}{2}|\psi^*|^2\right)\\ &\quad + (1-\gamma)\kappa\varepsilon'\psi^* + \gamma c_1 c_2|\mathbf{e}||\theta_f|\\ &\le -\frac{1}{2}q|\mathbf{e}|^2 + c_1|\mathbf{e}|\left(|\hat{f}|+|\alpha|+|\beta|\right) - \sigma_f\left(\frac{1}{2}|\tilde{\theta}_f|^2+\frac{1}{2}|\theta_f|^2-\frac{1}{2}|\theta_f^*|^2\right)\\ &\quad - \sigma_g\left(\frac{1}{2}|\tilde{\theta}_g|^2 - \frac{1}{2}\left|\theta_g^* - h(\hat{g})\theta\exp(-c|\hat{g}|)\right|^2\right) - (1-\gamma)\sigma_\psi\left(\frac{1}{2}|\tilde{\psi}|^2+\frac{1}{2}|\psi|^2-\frac{1}{2}|\psi^*|^2\right)\\ &\quad + (1-\gamma)\kappa\varepsilon'\psi^* + \gamma c_1 c_2|\mathbf{e}||\theta_f|\\ &\le -\frac{1}{2}\left(q-\frac{1}{2}\right)|\mathbf{e}|^2 - \frac{\sigma_f}{2}|\tilde{\theta}_f|^2 - \frac{\sigma_g}{2}|\tilde{\theta}_g|^2 - \frac{(1-\gamma)\sigma_\psi}{2}|\tilde{\psi}|^2 + (1+\gamma)c_1 c_2|\mathbf{e}||\theta_f|\\ &\quad - \frac{\sigma_f}{2}|\theta_f|^2 + c_1|\mathbf{e}||\psi| - \frac{(1-\gamma)\sigma_\psi}{2}|\psi|^2\\ &\quad + \frac{\sigma_f}{2}|\theta_f^*|^2 + \frac{\sigma_g}{2}|\theta_g^*+\bar{h}\theta|^2 + \frac{(1-\gamma)\sigma_\psi}{2}|\psi^*|^2 + (1-\gamma)|\kappa\varepsilon'\psi^* + c_1^2c_3^2,\end{aligned} \quad (27)$$

where $\bar{h} = \operatorname{sgn}(g(\mathbf{x}))$. We have also used the facts

$$\frac{\varepsilon}{\varepsilon + \hat{g}^2} \le 1, \qquad |\mathbf{e}| c_1 c_3 \le \frac{1}{4} |\mathbf{e}|^2 + c_1^2 c_3^2. \tag{28}$$

Choose the design constant $q > \underline{q}$ where

$$\underline{q} = \frac{1}{2} + \frac{(1+\gamma)^2 c_1^2 c_2^2}{\sigma_f} + \frac{c_1^2}{\sigma_\psi (1-\gamma)}. \tag{29}$$

Then $\dot{V}$ becomes

$$\begin{aligned} \dot{V} \le & -\frac{1}{2}(q - \underline{q}) |\mathbf{e}|^2 - \frac{\sigma_f}{2} |\tilde{\theta}_f|^2 - \frac{\sigma_g}{2} |\tilde{\theta}_g|^2 \\ & - \frac{(1-\gamma)\sigma_\psi}{2} |\tilde{\psi}|^2 + \frac{\sigma_f}{2} |\theta_f^*|^2 \\ & + \frac{\sigma_g}{2} |\theta_g^* + \bar{h}\theta|^2 + \frac{(1-\gamma)\sigma_\psi}{2} |\psi^*|^2 \\ & + (1-\gamma)\kappa\varepsilon' \psi^* + c_1^2 c_3^2. \end{aligned} \tag{30}$$

Let

$$\begin{aligned} c &= \min\left\{ \frac{q - \underline{q}}{\lambda_{\max}(P)}, \gamma_f \sigma_f, \gamma_g \sigma_g, \gamma_\psi \sigma_\psi \right\}, \\ \lambda &= \frac{\sigma_f}{2} |\theta_f^*|^2 + \frac{\sigma_g}{2} |\theta_g^* + \bar{h}\theta|^2 \\ & \quad + \frac{(1-\gamma)\sigma_\psi}{2} |\psi^*|^2 + (1-\gamma)\kappa\varepsilon' \psi^* + c_1^2 c_3^2. \end{aligned} \tag{31}$$

Then (30) can be written as

$$\dot{V} \le -cV + \lambda. \tag{32}$$

From (32), we have $\dot{V} < 0$ provided that $V > \lambda / c$. Thus we can prove the uniform ultimate boundedness of V with respect to the set

$$\nu = \left\{ V(t) : V \le \frac{\lambda}{c} \right\}. \tag{33}$$

This completes the proof. □

Remark 1: Letting $\varepsilon' = 0$ makes β as

$$\beta = -\psi \operatorname{sgn}\left(\mathbf{e}^T P \mathbf{b}\right) \tag{34}$$

and the constant λ is defined by

$$\begin{aligned} \lambda &= \frac{\sigma_f}{2} |\theta_f^*|^2 + \frac{\sigma_g}{2} |\theta_g^* + \bar{h}\theta|^2 \\ & \quad + \frac{(1-\gamma)\sigma_\psi}{2} |\psi^*|^2 + c_1^2 c_3^2. \end{aligned} \tag{35}$$

In this case, although we can acquire better performance, the chattering phenomenon may occur in the control input due to the sign function in (34).

Remark 2: In our previous work [10], the designer must determine the constant γ in order to determine control input, which is not required in this paper. This is due to a slight modification of the Lyapunov function (20).

4. SIMULATION EXAMPLE

To illustrate the control procedure and the performance we apply the proposed robust adaptive controller to control the mass-spring-damper system described by [24]:

$$\dot{x}_1 = x_2, \tag{36}$$

$$\dot{x}_2 = \frac{-f_K(\mathbf{x}) - f_B(\mathbf{x}) + u + d}{M}, \tag{37}$$

where $y = x_1$ represents the deviation of the mass, x_2 represents its velocity, $f_K(\mathbf{x})$ is the spring force due to spring constant K, $f_B(\mathbf{x})$ is the friction force due to friction constant B, M is the body mass, and u is the applied force (control). The overall block diagram is illustrated in Fig. 3.

We chose the nominal parameters as $M_0 = 1kg$, $K_0 = 2$, $B_0 = 2$ and the perturbations of system parameters as $\Delta M = 0.1\sin(x_1)$, $\Delta K = 0.5$, $\Delta B = 0.5$ in the following simulations. Furthermore, the nonlinear spring and friction forces are assumed to be $f_K(\mathbf{x}) = K_0 x_1 + \Delta K x_1^3$ and $f_B(\mathbf{x}) = B_0 x_2 + \Delta B x_2^2$. The disturbance is assumed to be $d = 0.2\sin(2t)e^{-0.1t}$.

We chose the reference signal $y_d(t) = \pi \sin(t)/30$. The compact set Ω_x is chosen to be $|x_j| \le 0.2$ for both $j = 1, 2$. The membership functions for x_1 and

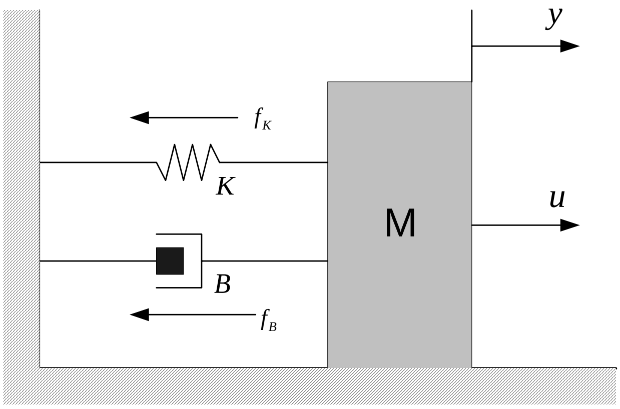

Fig. 3. A mass-spring-damper system.

Table 1. Controller parameters.

Parameter(s)	Value(s)
k_1, k_2	2,1
q	100
$\sigma_f, \sigma_g, \sigma_\psi$	0.1, 0.1, 0.1
$\gamma_f, \gamma_g, \gamma_\psi$	1000, 10, 1
ε	10
ε'	0.001
c	0.1
μ_1, μ_2	0.1, 0.11
$\theta_i, i = 1, \cdots, 25$	1

x_2 chosen as a Gaussian-shaped form are described by

$$A_{i_j}^j(x_j, p_{i_j}, q_{i_j}) = \exp\left[-\frac{(x_j - p_{i_j})^2}{2q_{i_j}^2}\right] \quad (38)$$

with $p_{i_j} = -3 + i_j$ and $2q_{i_j}^2 = 0.0425$ for $i_j = 1, 2, \cdots, N_j$ and $N_j = 5$, $j = 1, 2$. The controller parameters are given in Table 1.

First, we select the initial values as $\theta_{f_i}^0 = 0$ and $\theta_{gi}^0 = 0.5$ for all $i = 1, \cdots, 25$ and $\psi^0 = 0$ where $\theta_{f_i}^0$ and $\theta_{g_i}^0$ denote the i th element of the initial vector of θ_f and θ_g respectively. In this case, $\hat{g}(0) > 0$, i.e., the designer has guessed the correct sign of $g(\mathbf{x})$. The initial state is $\mathbf{x}(0) = [0.1\ 0]^T$. The system output, control input and trajectories of $\hat{g}(\mathbf{x})$, $g(\mathbf{x})$ are illustrated in Fig. 4 through Fig. 6. From the results, it can be inferred that the system output tracks the desired output well by the proposed controller.

Second, we set the simulation parameters as the same values as before except $\theta_{gi}^0 = -0.5$, $i = 1, \cdots, 25$. Note that $\hat{g}(0) < 0$, which means the designer has guessed wrongly the sign of $g(\mathbf{x})$ since it is always positive. The system output, control input and trajectories of $\hat{g}(\mathbf{x})$ and $g(\mathbf{x})$ are illustrated in Fig. 7 through Fig. 9. From the results, it can be inferred that the system output tracks the desired output well by the proposed controller although the control direction is incorrectly guessed. Moreover, as illustrated in Fig. 9, we can observe that even if the trajectory of $\hat{g}$ crosses the zero line, input singularity does not occur.

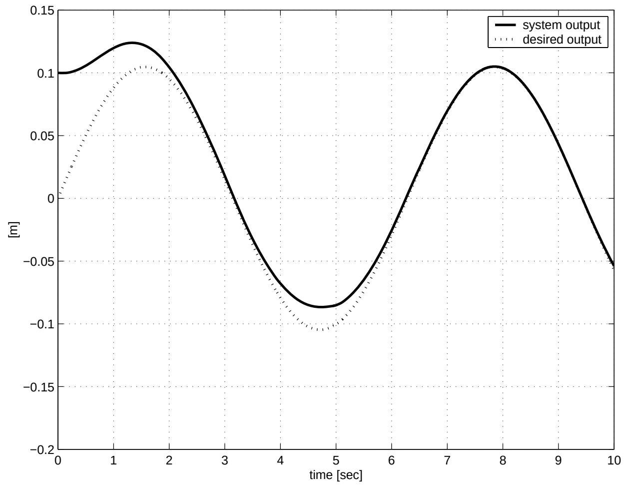

Fig. 4. Reference signal (dotted line) and output of the system (solid line).

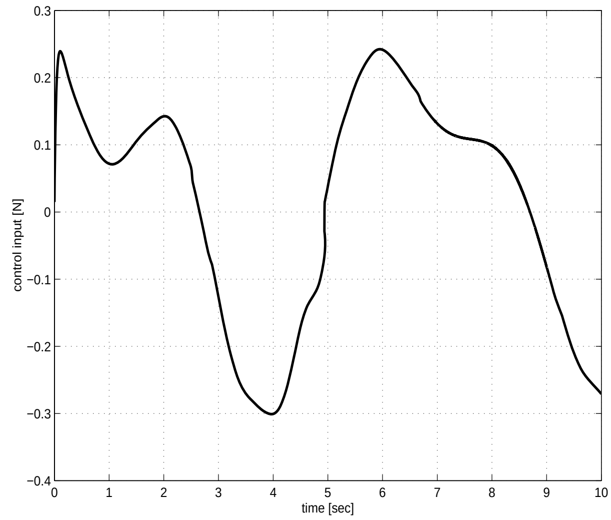

Fig. 5. Control input.

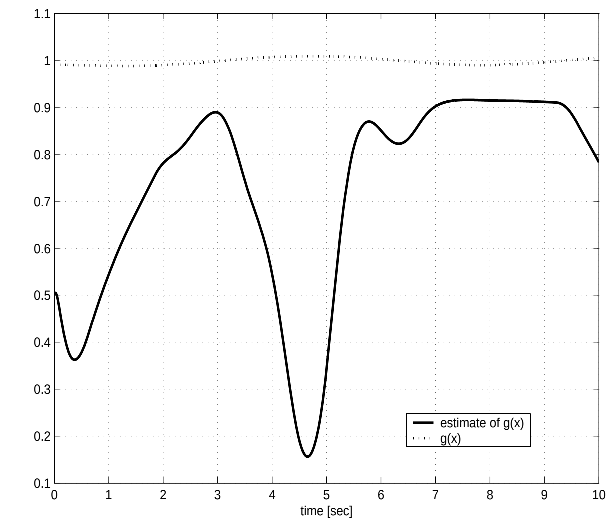

Fig. 6. Trajectory of $g(\mathbf{x})$ (dotted line) and $\hat{g}(\mathbf{x})$ (solid line).

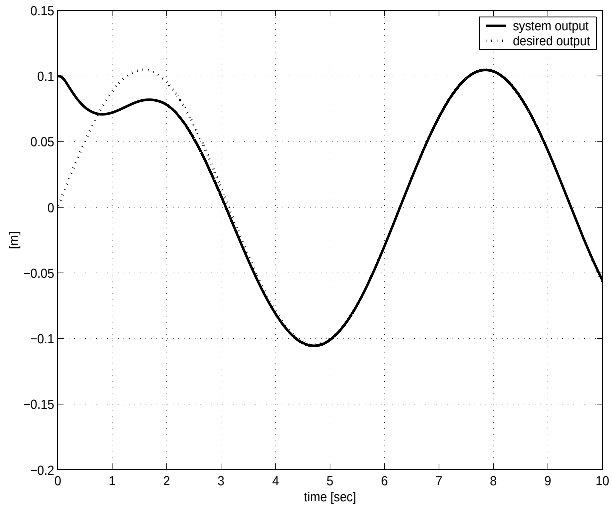

Fig. 7. Reference signal (dotted line) and output of the system (solid line).

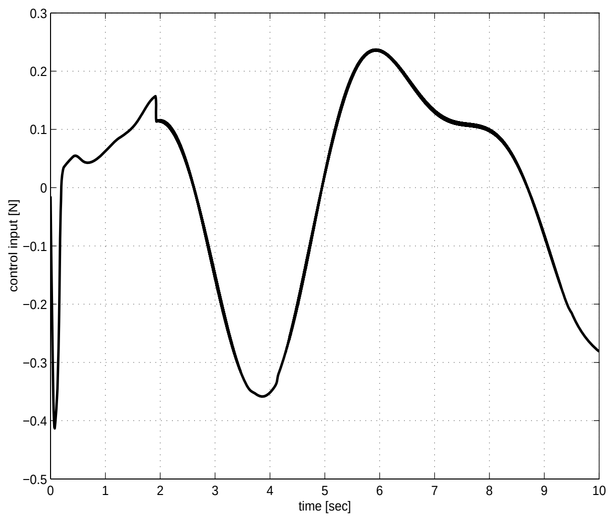

Fig. 8. Control input.

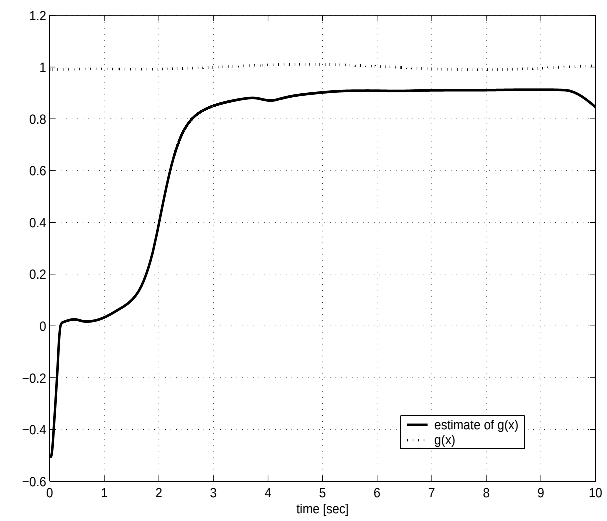

Fig. 9. Trajectory of $g(\mathbf{x})$ (dotted line) and $\hat{g}(\mathbf{x})$ (solid line).

5. CONCLUSION

We have proposed the robust adaptive fuzzy controller for uncertain nonlinear systems with unknown input gain sign. In other words, even though we have no information on *the control direction*, we can robustly control the system using the proposed scheme. Moreover, it completely overcomes the singularity problem that occurs in the indirect adaptive feedback linearizing control scheme. Projection in the estimated parameters and switching in the control input are not needed in order to avoid the input singularity. The stability of the closed-loop system is guaranteed in the Lyapunov viewpoint, and all the signals involved are shown to be uniformly ultimately bounded.

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