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Adaptive Hybrid Intelligent Control for Uncertain Nonlinear Dynamical Systems

Chi-Hsu Wang, *Senior Member, IEEE*, Tsung-Chih Lin, Tsu-Tian Lee, *Fellow, IEEE*, and Han-Leih Liu

***Abstract*—A new hybrid direct/indirect adaptive fuzzy neural network (FNN) controller with state observer and supervisory controller for a class of uncertain nonlinear dynamic systems is developed in this paper. The hybrid adaptive FNN controller, the free parameters of which can be tuned on-line by observer-based output feedback control law and adaptive law, is a combination of direct and indirect adaptive FNN controllers. A weighting factor, which can be adjusted by the tradeoff between plant knowledge and control knowledge, is adopted to sum together the control efforts from indirect adaptive FNN controller and direct adaptive FNN controller. Furthermore, a supervisory controller is appended into the FNN controller to force the state to be within the constraint set. Therefore, if the FNN controller cannot maintain the stability, the supervisory controller starts working to guarantee stability. On the other hand, if the FNN controller works well, the supervisory controller will be deactivated. The overall adaptive scheme guarantees the global stability of the resulting closed-loop system in the sense that all signals involved are uniformly bounded. Two nonlinear systems, namely, inverted pendulum system and Chua's chaotic circuit, are fully illustrated to track sinusoidal signals. The resulting hybrid direct/indirect FNN control systems show better performances, i.e., tracking error and control effort can be made smaller and it is more flexible during the design process.**

***Index Terms*—Adaptive control, fuzzy neural networks (FNNs), nonlinear systems, state observer, supervisory control.**

I. INTRODUCTION

MOST current techniques for designing control systems are based on a good understanding of the plant under consideration and its environment. However, in a number of instances, the plant to be controlled is too complex and the basic physical processes in it are not fully understood. Hence, control design methods need to be augmented with an identification technique aimed at obtaining a progressively better understanding of the plant to be controlled. Adaptive control is a technique of applying some system identification techniques to obtain a model of the process and its environment from input/output experiment and using this model to design a controller. The adaptive control for feedback linearizable nonlinear systems is an approach to nonlinear control design that has attracted a great deal of interest in the nonlinear control community for at least a quarter of a century. By using feedback linearization [1]–[3], the nonlinear adaptive control problem is transformed into a linear adaptive control problem, then the linear control methods can be applied to acquire the desired performance. The adaptive control methodologies include direct adaptive control (DAC) and indirect adaptive control (IAC) algorithms [4]–[9].

Recently, an important adaptive fuzzy neural network (FNN) control system [4]–[14] has been proposed to incorporate with the expert information systematically, and the stability can be guaranteed by universal approximation theorem [15]. For systems with a high degree of nonlinear uncertainty, such as chemical process, aircraft, etc., they are very difficult to control using the conventional control theory. However, human operators can often successfully control them. Based on the fact that FNN logic systems are capable of uniformly approximating a nonlinear function over a compact set to any degree of accuracy, a globally stable adaptive FNN controller is defined as an FNN logic system equipped with an adaptation algorithm. Moreover, FNN is constructed from a collect of fuzzy IF–THEN rules using fuzzy logic principles, and the adaptation algorithm adjusts the free parameters of the FNN based on the numerical experiment data. Like the conventional adaptive control, the adaptive FNN control has direct and indirect FNN adaptive control categories [7], [8]. Direct adaptive FNN control has been discussed in [4] and [7], in which the adaptive FNN controller uses fuzzy logic systems as controller. Hence, linguistic fuzzy control rules can be directly incorporated into the controller. Also, indirect adaptive FNN control has been proposed in [4] and [7], in which the indirect FNN controller uses fuzzy descriptions to model the plant. Hence, fuzzy IF–THEN rules describing the plant can be directly incorporated into the indirect FNN controller.

Can these two adaptive FNN controllers be combined together to yield stable and robust adaptive control laws with supervisory controller? The answer is "yes." A hybrid direct/indirect adaptive FNN controller can be constructed by incorporating both fuzzy description and fuzzy control rules using a weighting factor α to sum together the control efforts from indirect adaptive FNN controller and direct adaptive FNN controller. The weighting factor $\alpha \in [0, 1]$ can be adjusted by the tradeoff between plant knowledge and control knowledge. We let $\alpha = 1$ if pure indirect adaptive FNN controller is required and $\alpha = 0$ when pure direct adaptive FNN controller is chosen. If fuzzy control rules are more important and reliable than fuzzy

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descriptions of the plant, choose smaller α; otherwise choose larger α. In [4], [7], and [8], the full state must be assumed to be available for measurement. This assumption may not hold in practice because either the state variables are not accessible for direct connection or because sensing devices or transducers are not available. In this paper, our main objective is to create a technique for designing a state observer-based [12] hybrid direct/indirect adaptive FNN control for a class of uncertain nonlinear systems in which only the system output is measurable. Based on the Lyapunov synthesis approach, the free parameters of hybrid direct/indirect adaptive FNN controller can be tuned on-line by an observer-based output feedback control law and adaptive law. Also, a supervisory controller is designed to cascade with FNN controller. If the nonlinear system tends to unstable by the FNN controller, especially in the transient period, the supervisory controller will be activated to work with the FNN controller to stabilize the whole system. On the other hand, if the FNN controller works well, the supervisory controller will be deactivated. This will result in a smaller control effort (energy). Therefore, the overall adaptive scheme guarantees that the global stability of the resulting closed-loop system in the sense that all signals involved are uniformly bounded. We have successfully designed the FNN adaptive controllers with supervisory control to control the inverted pendulum and Chua's chaotic circuit [16] to track reference sinusoidal signals. The resulting hybrid direct/indirect FNN control systems show better performances, i.e., both tracking error and control effort can be made smaller.

This paper is organized as follows. Problem formulation is described in Section II. A brief description of the T–S FNN is presented in Section III. The observer-based hybrid direct/indirect FNN controller appended with a supervisory controller is constructed in Section IV. Simulation examples to demonstrate the performances of the proposed method are provided in Section V. Section VI lists the conclusions of the advocated design methodology.

II. Problem Formulation

Consider the nth-order nonlinear dynamical system of the form [1], [17]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\cdots \\ \dot{x}_n &= f(x_1, x_2, \ldots, x_n) + g(x_1, x_2, \ldots, x_n)u + d \\ y &= x_1 \end{aligned} \tag{1}$$

or equivalently the form

$$x^{(n)} = f\left(x, \dot{x}, \ldots, x^{(n-1)}\right) + g\left(x, \dot{x}, \ldots, x^{(n-1)}\right)u + d, \qquad y = x \tag{2}$$

where

f and g	unknown but bounded functions;
$u \in R$ and $y \in R$	control input and output of the system, respectively;
d	external bounded disturbance.

Equation (1) [or (2)] is actually the Isidori–Byrnes canonical form [1], [17] for certain nonlinear systems. We consider only the nonlinear systems which can be represented by (1) or (2). The state space representation of (2) is expressed as

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x} + B\left[f(\underline{x}) + g(\underline{x})u + d\right] \\ y &= C^T\underline{x} \end{aligned} \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

and $\underline{x} = [x_1, x_2, \ldots, x_n]^T = [x, \dot{x}, \ldots, x^{(n-1)}]^T \in R^n$ is a state vector where not all x_i are assumed to be available for measurement. Only the system output y is assumed to be measurable. In order for (2) to be controllable, it is required that $g(\underline{x}) \neq 0$ for $\underline{x}$ in a certain controllability region $U_c \subset R^n$. Without loss of generality, we assume that $0 < g(\underline{x}) < \infty$ for $\underline{x} \in U_c$. The control objective is to force the system output y to follow a given bounded reference signal y_r, under the constraint that all signals involved must be bounded.

To begin with, the reference signal vector $\underline{y}_r$, the tracking error vector $\underline{e}$, and estimation error vector $\hat{\underline{e}}$ will be defined as

$$\begin{aligned} \underline{y}_r &= \left[y_r, \dot{y}_r, \ldots, y_r^{(n-1)}\right]^T \in R^n \\ \underline{e} &= \underline{y}_r - \underline{x} = \left[e, \dot{e}, \ldots, e^{(n-1)}\right]^T \\ &\in R^n (e = y_r - x = y_r - y \in R) \\ \hat{\underline{e}} &= \underline{y}_r - \hat{\underline{x}} = \left[\hat{e}, \dot{\hat{e}}, \ldots, \hat{e}^{(n-1)}\right]^T \\ &\in R^n (\hat{e} = y_r - \hat{x} \in R) \end{aligned}$$

where $\hat{\underline{x}}$ and $\hat{\underline{e}}$ denote the estimates of $\underline{x}$ and $\underline{e}$, respectively.

If the functions $f(\underline{x})$ and $g(\underline{x})$ are known and the system is free of external disturbance d, then we can choose the controller u^* to cancel the nonlinearity and design controller. In particular, let $\underline{k}_c = [k_1^c, k_2^c, \ldots, k_n^c]^T \in R^n$ be chosen such that all roots of the polynomial $p(s) = s^n + k_n^c s^{n-1} + \cdots + k_1^c$ are in the open-left half-plane and control law of the certainty equivalent controller is obtained as [8]

$$u^* = \frac{1}{g(\underline{x})}\left[-f(\underline{x}) + y_r^{(n)} + \underline{k}_c^T \underline{e}\right]. \tag{5}$$

Substituting (5) into (2), we obtain the closed-loop system governed by

$$e^{(n)} + k_n^c e^{(n-1)} + \cdots + k_1^c e = 0$$

where the main objective of the control is $\lim_{t\to\infty} e(t) = 0$. However, $f(\underline{x})$ and $g(\underline{x})$ are unknown, the ideal controller (5) cannot be implemented, and not all system states $\underline{x}$ can be measured. We have to design an observer to estimate the state vector $\underline{x}$ in the following context.

A. Observer-Based Hybrid Direct/Indirect FNN Controller With Supervisory Control Scheme

Here, we will develop the observer-based hybrid direct/indirect FNN controller with supervisory control scheme. The overall control law is constructed as

$$u = \alpha\, u_I(\hat{\underline{x}}) + (1-\alpha)\, u_D(\hat{\underline{x}}\,|\,\underline{\theta}_D) + u_S(\hat{\underline{x}}) \tag{6}$$

where

- u_I indirect FNN controller [see (8)];
- u_D output of the Takagi–Sugeno (T–S)-based DAC FNN controller (described in Sections III and IV);
- u_S supervisory control (described in Section IV) to force the state within the constraint set;
- $\alpha \in [0,\, 1]$ weighting factor.

If the plant knowledge is more important and reliable than the control knowledge, we should choose a larger α; otherwise, a smaller α should be chosen. Since $\underline{x}$ cannot be available and $f(\underline{x})$ and $g(\underline{x})$ are unknown, we replace the functions $f(\underline{x})$, $g(\underline{x})$, and error vector $\underline{e}$ in (5) by estimation functions $\hat{f}(\hat{\underline{x}})$ and $\hat{g}(\hat{\underline{x}})$ (described in Section III), and $\hat{\underline{e}}$. The certainty equivalent controller can be rewritten as

$$u^* = \frac{1}{g(\underline{x})}\left[-f(\underline{x}) + y_r^{(n)} + \underline{k}_c^T\hat{\underline{e}}\right]. \tag{7}$$

The indirect control law is written as

$$u_I = \frac{1}{\hat{g}(\hat{\underline{x}})}\left[-\hat{f}(\hat{\underline{x}}) + y_r^{(n)} + \underline{k}_c^T\hat{\underline{e}}\right]. \tag{8}$$

Applying (6) and (7) to (3), and after some simple manipulations, we can obtain the error dynamic equation

$$\begin{aligned}\dot{\underline{e}} &= A\underline{e} - B\underline{k}_c^T\hat{\underline{e}} + B\Big\{\alpha\left[\hat{f}(\hat{\underline{x}}) - f(\underline{x}) + (\hat{g}(\hat{\underline{x}}) - g(\underline{x}))u_I\right] \\ &\qquad + (1-\alpha)g(\underline{x})(u^* - u_D) - g(\underline{x})u_S - d\Big\} \\ e_1 &= C^T\underline{e}\end{aligned} \tag{9}$$

where $e_1 = y_r - y = y_r - x_1$.

From (9), the following observer that estimates the state error vector $\underline{e}$ in (9)

$$\begin{aligned}\dot{\hat{\underline{e}}} &= A\hat{\underline{e}} - B\underline{k}_c^T\,\hat{\underline{e}} + \underline{k}_o(e_1 - \hat{e}_1) \\ \hat{e}_1 &= C^T\hat{\underline{e}}\end{aligned} \tag{10}$$

where $\underline{k}_o = \left[k_n^o,\, k_{n-1}^o,\, \ldots,\, k_1^o\right]^T \in R^n$ is the observer gain vector.

The observation errors are defined as: $\tilde{\underline{e}} = \underline{e} - \hat{\underline{e}}$ and $\tilde{e}_1 = e_1 - \hat{e}_1$.

Subtracting (10) from (9), we can obtain the error dynamics

$$\begin{aligned}\dot{\tilde{\underline{e}}} &= \Lambda_o\tilde{\underline{e}} + B\Big\{\alpha\left[\hat{f}(\hat{\underline{x}}) - f(\underline{x}) + (\hat{g}(\hat{\underline{x}}) - g(\underline{x}))u_I\right] \\ &\qquad + (1-\alpha)g(\underline{x})(u^* - u_D)\Big\} - Bg(\underline{x})u_S - B\,d \\ \tilde{e}_1 &= C^T\tilde{\underline{e}}\end{aligned} \tag{11}$$

where $\Lambda_o = A - \underline{k}_o C^T$. Since $(C,\, \Lambda_o)$ pair is observable, the observer gain vector $\underline{k}_o$ can be chosen such that the characteristic polynomial of Λ_o is strictly Hurwitz (i.e., the roots of the closed-loop system are in the open-left half-plane) and we know that there exists a positive definite symmetric $n \times n$ matrix P which satisfies the Lyapunov equation

$$\Lambda_o^T P + P\Lambda_o = -Q \tag{12}$$

where Q is an arbitrary $n \times n$ positive definite matrix.

Let us rewrite (10) as

$$\dot{\hat{\underline{e}}} = \hat{A}\hat{\underline{e}} + \underline{k}_o C^T\tilde{\underline{e}} \tag{13}$$

where $\hat{A} = A - B\underline{k}_c^T$ is a strictly Hurwitz matrix. Therefore, there exists a positive definite symmetric $n \times n$ matrix $\hat{P}$ which satisfies the Lyapunov equation

$$\hat{A}^T\hat{P} + \hat{P}\hat{A} = -\hat{Q} \tag{14}$$

where $\hat{Q}$ is an arbitrary $n \times n$ positive definite matrix. Let $V_{\hat{\underline{e}}} = (1/2)\hat{\underline{e}}^T\hat{P}\hat{\underline{e}}$, then by using (13) and (14), we have

$$\begin{aligned}\dot{V}_{\hat{\underline{e}}} &= \tfrac{1}{2}\,\dot{\hat{\underline{e}}}^T\hat{P}\hat{\underline{e}} + \tfrac{1}{2}\,\hat{\underline{e}}^T\hat{P}\dot{\hat{\underline{e}}} \\ &= \tfrac{1}{2}\left\{\hat{A}\hat{\underline{e}} + \underline{k}_o C^T\tilde{\underline{e}}\right\}^T\hat{P}\hat{\underline{e}} + \tfrac{1}{2}\,\hat{\underline{e}}^T\hat{P}\left\{\hat{A}\hat{\underline{e}} + \underline{k}_o C^T\tilde{\underline{e}}\right\} \\ &= -\tfrac{1}{2}\,\hat{\underline{e}}^T\hat{Q}\hat{\underline{e}} + \hat{\underline{e}}^T\hat{P}\underline{k}_o C^T\tilde{\underline{e}}.\end{aligned} \tag{15}$$

Since $\hat{Q}$ and $\underline{k}_o$ are determined by the designer, we can choose $\hat{Q}$ and $\underline{k}_o$, such that $\dot{V}_{\hat{\underline{e}}} \le 0$. Hence, $V_{\hat{\underline{e}}}$ is a bounded function and there exists a constant value $\overline{V}_{\hat{\underline{e}}}$, such that $V_{\hat{\underline{e}}} \le \overline{V}_{\hat{\underline{e}}}$.

III. The Takagi–Sugeno (T–S) FNN Systems

Fuzzy logic systems address the imprecision of the input and output variables directly by defining them with fuzzy numbers (and fuzzy sets) that can be expressed in linguistic terms (e.g., *small*, *medium*, and *large*). The basic configuration the T–S FNN system [18]–[22] includes a fuzzy rule base, which consists of a collection of fuzzy IF–THEN rules in the following form:

$$\begin{aligned}&R^{(l)}\text{: IF } x_1 \text{ is } F_1^l, \text{ and } \ldots, \text{ and } x_n \text{ is } F_n^l, \\ &\quad\text{THEN } y_l = q_0^l + q_1^l x_1 + \cdots + q_n^l x_n = \underline{\theta}_l^T[1\;\underline{x}^T]^T\end{aligned} \tag{16}$$

where F_i^l are fuzzy sets and $\underline{\theta}_l^T = [q_0^l,\, q_1^l,\, \ldots,\, q_n^l]$ is a vector of the adjustable factors of the consequence part of the fuzzy rule. Furthermore, y_l is a linguistic variable, and a fuzzy inference engine to combine the fuzzy IF–THEN rules in the fuzzy rule base into a mapping from an input linguistic vector $\underline{x}^T = [x_1,\, x_2,\, \ldots,\, x_n] \in R^n$ to an output variable $y \in R$. Let M be the number of the fuzzy IF–THEN rules. The output of the fuzzy logic systems with central average defuzzifier, product inference, and singleton fuzzifier can be expressed as

$$y(\underline{x}) = \frac{\sum_{l=1}^{M} v^l \cdot y_l}{\sum_{l=1}^{M} v^l} = \frac{\sum_{l=1}^{M} v^l \cdot \underline{\theta}_l^T[1\;\underline{x}^T]}{\sum_{l=1}^{M} v^l} \tag{17}$$

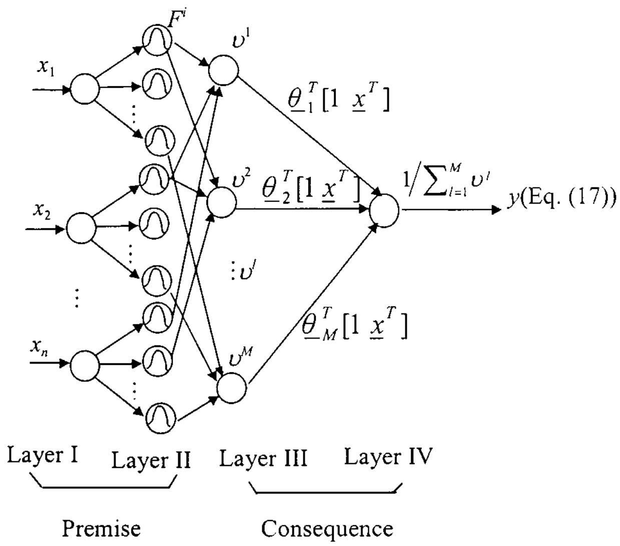

Fig. 1. Configuration of the T–S FNNs.

where $\mu_{F_i^l}(x_i)$ is the membership function value of the fuzzy variable x_i and $v^l = \prod_{i=1}^n \mu_{F_i^l}(x_i)$ is the true value of the lth implication. Equation (17) can be rewritten as

$$y(\underline{x}) = \underline{\theta}^T \underline{\psi}(\underline{x}) \tag{18}$$

where $\underline{\theta}^T = [\underline{\theta}_1^T\, \underline{\theta}_2^T \cdots \underline{\theta}_M^T]$ is an adjustable parameter vector and $\underline{\psi}^T(\underline{x}) = [\psi^1(\underline{x}), \psi^2(\underline{x}), \ldots, \psi^M(\underline{x})]$ is a fuzzy basis function vector defined as

$$\psi^l(\underline{x}) = \frac{v^l[1\, \underline{x}^T]}{\sum\limits_{l=1}^{M} v^l}. \tag{19}$$

When the inputs are fed into the T–S FNN, the true value v^l of the lth implication is computed. Applying the common defuzzification strategy, the output of the NNs expressed as (17) is pumped out. The overall configuration of the T–S FNN is shown in Fig. 1.

Based on the universal approximation theorem [15], the aforementioned fuzzy logic system is capable of uniformly approximating any well-defined nonlinear function over a compact set U_c to any degree of accuracy. It is also straightforward to show that a multi-output system can always be approximated by a group of single-output approximation systems.

IV. Hybrid Direct/Indirect Adaptive FNN Controller With Observer and Supervisory Controller

An adaptive fuzzy system is a fuzzy logic system equipped with a training algorithm to maintain a consistent performance under plant uncertainties. The most important advantage of the adaptive FNN control over conventional adaptive control is that adaptive FNN controllers are capable of incorporating linguistic fuzzy information from a human operator, whereas the conventional adaptive controller is not. The adaptive FNN control is divided into two categories. One is called the *indirect adaptive FNN control* and the other is called the *direct adaptive FNN control* [7], [8]. An adaptive FNN controller that uses fuzzy logic systems as a model of the plant is an indirect adaptive FNN controller. An adaptive FNN controller that directly uses fuzzy logic systems as controller is a direct adaptive FNN controller. Therefore, the indirect adaptive FNN controller can incorporate fuzzy descriptions but cannot incorporate fuzzy control rules. On the other hand, the direct adaptive FNN controller can incorporate fuzzy control rules but cannot incorporate fuzzy descriptions. In this section, we will develop the hybrid direct/indirect adaptive FNN controller that can incorporate linguistic information and design an adaptive law for the adjustable parameters in the controller, such that the closed loop output $y(t)$ follows the reference output $y_r(t)$.

Let us replace $\hat{f}(\underline{\hat{x}})$, $\hat{g}(\underline{\hat{x}})$, and $u_D(\underline{\hat{x}})$ in (11) by the fuzzy logic system $\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f)$, $\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g)$, and $u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D)$, respectively. Therefore, the error dynamics (11) can be rewritten as

$$\begin{aligned}\dot{\underline{\tilde{e}}} =\,& \Lambda_o \underline{\tilde{e}} + B\Big\{\alpha\Big[\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f) - f(\underline{x}) + (\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g) - g(\underline{x}))u_I\Big] \\ &\qquad + (1-\alpha)g(\underline{x})(u^* - u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D))\Big\} \\ &- Bg(\underline{x})u_S - Bd. \end{aligned} \tag{20}$$

Let $V_{\underline{\tilde{e}}} = (1/2)\underline{\tilde{e}}^T P \underline{\tilde{e}}$, then using (12) and (20) we have

$$\begin{aligned}\dot{V}_{\underline{\tilde{e}}} &= \tfrac{1}{2}\dot{\underline{\tilde{e}}}^T P \underline{\tilde{e}} + \tfrac{1}{2}\underline{\tilde{e}}^T P \dot{\underline{\tilde{e}}} \\ &= -\tfrac{1}{2}\underline{\tilde{e}}^T Q \underline{\tilde{e}} + \alpha \underline{\tilde{e}}^T PB \\ &\quad \times \Big[\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f) - f(\underline{x}) + (\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g) - g(\underline{x}))u_I\Big] \\ &\quad + \underline{\tilde{e}}^T PB(1-\alpha)g(\underline{x})(u^* - u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D)) \\ &\quad - \underline{\tilde{e}}^T PBg(\underline{x})u_S - \underline{\tilde{e}}^T PB\, d \\ &\le -\tfrac{1}{2}\underline{\tilde{e}}^T Q \underline{\tilde{e}} + \left|\underline{\tilde{e}}^T PB\right| \\ &\quad \times \Big\{\alpha\Big[\Big|\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f)\Big| + |f(\underline{x})| + |\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g)u_I| + |g(\underline{x})u_I|\Big] \\ &\qquad + (1-\alpha)|g(\underline{x})u^*| + |g(\underline{x})u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D)| + |d|\Big\} \\ &\quad - \underline{\tilde{e}}^T PBg(\underline{x})u_S. \end{aligned} \tag{21}$$

In order to design u_S such that $\dot{V}_{\underline{\tilde{e}}} \le 0$, we need the following assumption.

Assumption I: We can determine functions $f^U(\underline{x})$, $g^U(\underline{x})$, and $g_L(\underline{x})$ such that $|f(\underline{x})| \le f^U(\underline{x}) \approx f^U(\underline{\hat{x}})$ and $g_L(\underline{\hat{x}}) \approx g_L(\underline{x}) \le g(\underline{x}) \le g^U(\underline{x}) \approx g^U(\underline{\hat{x}})$ for $\underline{x} \in U_c$, where $f^U(\underline{x}) \approx f^U(\underline{\hat{x}}) < \infty$, $g^U(\underline{x}) \approx g^U(\underline{\hat{x}}) < \infty$, and $g_L(\underline{x}) \approx g_L(\underline{\hat{x}}) > 0$ for $\underline{x} \in U_c$. This is due to the fact that we can choose $\underline{k}_o$ in (10) to let $\underline{x} \approx \underline{\hat{x}}$. Furthermore, external disturbance is bounded, i.e., $|d| \le d_m$ where d_m is the upper bound of noise d.

From *Assumption I*, and by observing (21), we choose the supervisory control u_S as

$$\begin{aligned} u_S =\,& I^* \operatorname{sgn}(\underline{\tilde{e}}^T PB)\,\frac{1}{g_L(\underline{x})} \\ &\times \Big[\Big|\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f)\Big| + 2f^U(\underline{x}) + \Big|y_r^{(n)}\Big| + \left|\underline{k}_c^T \underline{\hat{e}}\right| \\ &\qquad + g^U(\underline{x})(|u_I| + |u_D|) + |\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g)u_I| + d_m\Big] \end{aligned} \tag{22}$$

where $I^* = 1$ if $V_{\underline{\tilde{e}}} > \overline{V}$ (which is a constant chosen by the designer), $I^* = 0$ if $V_{\underline{\tilde{e}}} \leq \overline{V}$, and $\mathrm{sgn}(\tau) = 1(-1)$ if $\tau \geq 0(<0)$. Considering the case $V_{\underline{\tilde{e}}} > \overline{V}$ and substituting (22) into (21), we obtain

$$\begin{aligned}\dot{V}_{\underline{\tilde{e}}} \leq & -\tfrac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} + \left|\underline{\tilde{e}}^T PB\right| \\ & \times\left\{\alpha\left[\left|\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f)\right| + |f(\underline{x})| + |\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g)u_I| + |g(\underline{x})u_I|\right]\right. \\ & \left. +(1-\alpha)\,|g(\underline{x})u^*| + |g(\underline{x})u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D)| + |d|\right\} \\ & - \underline{\tilde{e}}^T PBg(\underline{x})u_S \\ \leq & -\tfrac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} + \left|\underline{\tilde{e}}^T PB\right|\left\{(1-\alpha)\,|g(\underline{x})u^*|\right. \\ & \left. - \frac{g(\underline{x})}{g_L(\underline{x})}\left(f^U(\underline{x}) + \left|y_r^{(n)}\right| + \left|\underline{k}_c^T\underline{\hat{e}}\right|\right)\right\} \\ \leq & -\tfrac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} + \left|\underline{\tilde{e}}^T PB\right| \\ & \times\left\{(1-\alpha)|g(\underline{x})|\left(|f(\underline{x})| + \left|y_r^{(n)}\right| + \left|\underline{k}_c^T\underline{\hat{e}}\right|\right)\right. \\ & \left. - \frac{g(\underline{x})}{g_L(\underline{x})}\left(f^U(\underline{x}) + \left|y_r^{(n)}\right| + \left|\underline{k}_c^T\underline{\hat{e}}\right|\right)\right\} \\ \leq & -\tfrac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} \leq 0. \end{aligned} \tag{23}$$

Therefore, we always have $V_{\underline{\tilde{e}}} \leq \overline{V}$, by using the supervisory control u_S [see (22)]. Because $P > 0$, the bound of $V_{\underline{\tilde{e}}}$ implies the bound of $\underline{\tilde{e}}$, which in turn implies the bound of $\underline{\hat{e}}$. Moreover, it implies the bound of $\underline{\hat{x}}$. It is obvious that the supervisory control u_S is nonzero when $V_{\underline{\tilde{e}}}$ is greater than a positive value $\overline{V}$. Therefore, if the closed-loop system with the fuzzy controller u as

$$\begin{aligned} u = & \frac{\alpha}{\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g)}\left[-\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f) + y_r^{(n)} + \underline{k}_c^T\underline{\hat{e}}\right] \\ & + (1-\alpha)u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D) + u_S \end{aligned} \tag{24}$$

works well in the sense that the error is not too large, i.e., $V_{\underline{\tilde{e}}} \leq \overline{V}$, then the supervisory control u_s is zero. On the other hand, if the system tends to diverge, i.e., $V_{\underline{\tilde{e}}} > \overline{V}$, then the supervisory control u_S begins to operate to force $V_{\underline{\tilde{e}}} \leq \overline{V}$.

We replace $\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f)$, $\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g)$, and $u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D)$ in specific fuzzy logic systems as (18), i.e.,

$$\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f) = \underline{\xi}^T(\underline{\hat{x}})\underline{\theta}_f \tag{25}$$

$$\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g) = \underline{\xi}^T(\underline{\hat{x}})\underline{\theta}_g \tag{26}$$

$$u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D) = \underline{\eta}^T(\underline{\hat{x}})\underline{\theta}_D \tag{27}$$

where $\underline{\xi}(\underline{\hat{x}})$ is a vector of fuzzy base, and $\underline{\theta}_f$ and $\underline{\theta}_g$ are the corresponding parameters of fuzzy logic systems. Also, $\underline{\eta}(\underline{\hat{x}})$ is a vector of fuzzy base, and $\underline{\theta}_D$ is the corresponding parameters of fuzzy logic systems. In order to adjust the parameters in the fuzzy logic systems, we have to derive adaptive laws. Hence, the optimal parameter estimations $\underline{\theta}_f^*$, $\underline{\theta}_g^*$, and $\underline{\theta}_D^*$ are defined as

$$\underline{\theta}_f^* = \arg\min_{\underline{\theta}_f\in\Omega_f}\left[\sup_{\underline{\hat{x}}\in\Omega_{\underline{\hat{x}}},\underline{x}\in\Omega_{\underline{x}}}\left|\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f) - f(\underline{x})\right|\right] \tag{28}$$

$$\underline{\theta}_g^* = \arg\min_{\underline{\theta}_g\in\Omega_g}\left[\sup_{\underline{\hat{x}}\in\Omega_{\underline{\hat{x}}},\underline{x}\in\Omega_{\underline{x}}}\left|\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g) - g(\underline{x})\right|\right] \tag{29}$$

and

$$\underline{\theta}_D^* = \arg\min_{\underline{\theta}_D\in\Omega_D}\left[\sup_{\underline{\hat{x}}\in\Omega_{\underline{\hat{x}}},\underline{x}\in\Omega_{\underline{x}}}\left|u^*(\underline{x}) - u(\underline{\hat{x}}\,|\,\underline{\theta}_D)\right|\right] \tag{30}$$

where Ω_f, Ω_g, Ω_D, $\Omega_{\underline{\hat{x}}}$, and $\Omega_{\underline{x}}$ are compact sets of suitable bounds on $\underline{\theta}_f$, $\underline{\theta}_g$, $\underline{\theta}_D$, $\underline{\hat{x}}$, and $\underline{x}$, respectively, and they are defined as $\Omega_f = \{\underline{\theta}_f \,|\, |\underline{\theta}_f| \leq M_f\}$, $\Omega_g = \{\underline{\theta}_g \,|\, |\underline{\theta}_g| \leq M_g\}$, $\Omega_D = \{\underline{\theta}_D \,|\, |\underline{\theta}_D| \leq M_D\}$, $\Omega_{\underline{\hat{x}}} = \{\underline{\hat{x}} \,|\, |\underline{\hat{x}}| \leq M_{\underline{\hat{x}}}\}$, and $\Omega_{\underline{x}} = \{\underline{x} \,|\, |\underline{x}| \leq M_{\underline{x}}\}$, where M_f, M_g, M_D, $M_{\underline{\hat{x}}}$, and $M_{\underline{x}}$ are positive constants.

Define the minimum approximation errors as

$$\begin{aligned}\omega = & \alpha\left[\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f^*) - f(\underline{x}) + (\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g^*) - g(\underline{x}))u_I\right] \\ & + (1-\alpha)g(\underline{x})\left(u^* - u_{D(\underline{\hat{x}}\,|\,\underline{\theta}_D^*)}\right) - d. \end{aligned} \tag{31}$$

The error dynamics (20) can be expressed as

$$\begin{aligned}\dot{\underline{\tilde{e}}} = & \Lambda_o\underline{\tilde{e}} - Bg(\underline{x})u_S + B \\ & \times\left\{\alpha\left[\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f) - \hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f^*) + (\hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g) - g(\underline{\hat{x}}\,|\,\underline{\theta}_g^*))u_I\right]\right. \\ & \left. -(1-\alpha)g(\underline{x})(u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D) - u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D^*)) + \omega\right\}. \end{aligned} \tag{32}$$

Substituting (25)–(27) into (32), the above equation can be rewritten as

$$\begin{aligned}\dot{\underline{\tilde{e}}} = & \Lambda_o\underline{\tilde{e}} - Bg(\underline{x})u_S + B\alpha\left[\underline{\xi}^T(\underline{\hat{x}})\underline{\tilde{\theta}}_f + \underline{\xi}^T(\underline{\hat{x}})\underline{\tilde{\theta}}_g u_I\right] \\ & - B(1-\alpha)g(\underline{x})\underline{\eta}^T(\underline{\hat{x}})\underline{\tilde{\theta}}_D + B\omega \end{aligned} \tag{33}$$

where $\underline{\tilde{\theta}}_f = \underline{\theta}_f - \underline{\theta}_f^*$, $\underline{\tilde{\theta}}_g = \underline{\theta}_g - \underline{\theta}_g^*$, and $\underline{\tilde{\theta}}_D = \underline{\theta}_D - \underline{\theta}_D^*$. Now consider the Lyapunov function

$$V = \frac{1}{2}\underline{\tilde{e}}^T P\underline{\tilde{e}} + \frac{\alpha}{2\gamma_1}\underline{\tilde{\theta}}_f^T\underline{\tilde{\theta}}_f + \frac{\alpha}{2\gamma_2}\underline{\tilde{\theta}}_g^T\underline{\tilde{\theta}}_g + \frac{(1-\alpha)}{2\gamma_3}\underline{\tilde{\theta}}_D^T\underline{\tilde{\theta}}_D. \tag{34}$$

The time derivative of V is

$$\begin{aligned}\dot{V} = & \frac{1}{2}\dot{\underline{\tilde{e}}}^T P\underline{\tilde{e}} + \frac{1}{2}\underline{\tilde{e}}^T P\dot{\underline{\tilde{e}}} + \frac{\alpha}{\gamma_1}\dot{\underline{\tilde{\theta}}}_f^T\underline{\tilde{\theta}}_f + \frac{\alpha}{\gamma_2}\dot{\underline{\tilde{\theta}}}_g^T\underline{\tilde{\theta}}_g \\ & + \frac{(1-\alpha)}{\gamma_3}\dot{\underline{\tilde{\theta}}}_D^T\underline{\tilde{\theta}}_D. \end{aligned} \tag{35}$$

Since $\dot{\underline{\tilde{\theta}}}_f = \dot{\underline{\theta}}_f$, $\dot{\underline{\tilde{\theta}}}_g = \dot{\underline{\theta}}_g$, and $\dot{\underline{\tilde{\theta}}}_D = \dot{\underline{\theta}}_D$, and by using (12) and (34), (35) can be rewritten as

$$\begin{aligned}\dot{V} = & \tfrac{1}{2}\left\{\underline{\tilde{e}}^T\Lambda_o^T P\underline{\tilde{e}} - u_S g(\underline{x})B^T P\underline{\tilde{e}} + \alpha\underline{\tilde{\theta}}_f^T\underline{\xi}(\underline{\hat{x}})B^T P\underline{\tilde{e}}\right. \\ & + \alpha u_I\underline{\tilde{\theta}}_g^T\underline{\xi}(\underline{\hat{x}})B^T P\underline{\tilde{e}} - (1-\alpha)\underline{\tilde{\theta}}_D^T\underline{\eta}(\underline{\hat{x}})g(\underline{x})B^T P\underline{\tilde{e}} \\ & + \omega B^T P\underline{\tilde{e}} + \underline{\tilde{e}}^T P\Lambda_o\underline{\tilde{e}} - \underline{\tilde{e}}^T PBg(\underline{x})u_S \\ & + \alpha\underline{\tilde{e}}^T PB\underline{\xi}^T(\underline{\hat{x}})\underline{\tilde{\theta}}_f + \alpha\underline{\tilde{e}}^T PB\underline{\xi}^T(\underline{\hat{x}})\underline{\tilde{\theta}}_g u_I \\ & \left. -(1-\alpha)\underline{\tilde{e}}^T PBg(\underline{x})\underline{\eta}^T(\underline{\hat{x}})\underline{\tilde{\theta}}_D + \underline{\tilde{e}}^T PB\omega\right\} \\ & + \frac{\alpha}{\gamma_1}\dot{\underline{\tilde{\theta}}}_f^T\underline{\tilde{\theta}}_f + \frac{\alpha}{\gamma_2}\dot{\underline{\tilde{\theta}}}_g^T\underline{\tilde{\theta}}_g + \frac{(1-\alpha)}{\gamma_3}\dot{\underline{\tilde{\theta}}}_D^T\underline{\tilde{\theta}}_D \\ = & -\tfrac{1}{2}\underline{\tilde{e}}^T Q\underline{\tilde{e}} - \underline{\tilde{e}}^T PBg(\underline{x})u_S + \underline{\tilde{e}}^T PB\omega \\ & + \frac{\alpha}{\gamma_1}\left(\dot{\underline{\theta}}_f^T + \gamma_1\underline{\tilde{e}}^T PB\underline{\xi}^T(\underline{\hat{x}})\right)\underline{\tilde{\theta}}_f \\ & + \frac{\alpha}{\gamma_2}\left(\dot{\underline{\theta}}_g^T + \gamma_2\underline{\tilde{e}}^T PB\underline{\xi}^T(\underline{\hat{x}})u_I\right)\underline{\tilde{\theta}}_g \\ & + \frac{(1-\alpha)}{\gamma_3}\left(\dot{\underline{\theta}}_D^T - \gamma_3\underline{\tilde{e}}^T PBg(\underline{x})\underline{\eta}^T(\underline{\hat{x}})\right)\underline{\tilde{\theta}}_D. \end{aligned} \tag{36}$$

According to (22) and $g(\underline{x}) > 0$, we have $g(\underline{x})\underline{\tilde{e}}^T PBu_S \geq 0$. If the adaptive laws are chosen as

$$\dot{\underline{\theta}}_f = -\gamma_1 \underline{\xi}(\hat{\underline{x}}) B^T P \underline{\tilde{e}} \tag{37}$$

$$\dot{\underline{\theta}}_g = -\gamma_2 \underline{\xi}(\hat{\underline{x}}) B^T P \underline{\tilde{e}} u_I \tag{38}$$

$$\dot{\underline{\theta}}_D = \gamma_3 \underline{\eta}(\hat{\underline{x}}) g(\underline{x}) B^T P \underline{\tilde{e}}. \tag{39}$$

Substituting (37)–(39) into (36), we have

$$\dot{V} \leq -\tfrac{1}{2}\, \underline{\tilde{e}}^T Q \underline{\tilde{e}} + \underline{\tilde{e}}^T PB\omega. \tag{40}$$

Since the term $\underline{\tilde{e}}^T PB\omega$ is of the order of the minimum approximation error, this is the best we can hope to obtain. If $\omega = 0$, from (40) we have

$$\dot{V}_{\underline{\tilde{e}}} \leq -\tfrac{1}{2}\, \underline{\tilde{e}}^T Q \underline{\tilde{e}} \leq 0.$$

If ω is not equal to zero, we can expect ω to be small based on the universal approximation theorem. From (28) to (30), the constraint sets Ω_f, Ω_g, and Ω_D of the optimal parameters $\underline{\theta}_f^*$, $\underline{\theta}_g^*$, and $\underline{\theta}_D^*$, respectively, if we can constrain $\underline{\theta}_f$, $\underline{\theta}_g$, and $\underline{\theta}_D$ within the sets, then u in (24) and u_S in (22) will be bounded due to the fact that, in this case, $\hat{f}$, $\hat{g}$, and u_D are bounded, and it should be reminded that $\underline{\tilde{e}}$ is bounded because of the supervisory control u_S. Obviously, the adaptive laws in (37)–(39) are unable to guarantee that $\underline{\theta}_f \in \Omega_f$, $\underline{\theta}_g \in \Omega_g$, and $\underline{\theta}_D \in \Omega_D$. Therefore, all of the adaptive laws have to be modified by using the parameters projection algorithm [4], [8], [12], such that the parameter vectors will remain inside the constraints. The modified adaptive laws are given as follows.

- Use the following adaptive law to adjust the parameter vector $\underline{\theta}_f$:

$$\dot{\underline{\theta}}_f = \begin{cases} -\gamma_1 \underline{\xi}(\hat{\underline{x}}) B^T P \underline{\tilde{e}} & \text{if } (|\underline{\theta}_f| < M_f) \\ & \text{or } (|\underline{\theta}_f| = M_f \\ & \text{and } \underline{\tilde{e}}^T PB \underline{\xi}^T(\hat{\underline{x}}) \underline{\theta}_f \geq 0) \\ Proj\{-\gamma_1 \underline{\xi}(\hat{\underline{x}}) B^T \underline{\tilde{e}}\} & \text{if } (|\underline{\theta}_f| = M_f \\ & \text{and } \underline{\tilde{e}}^T PB \underline{\xi}(\hat{\underline{x}}) \underline{\theta}_f < 0) \end{cases} \tag{41}$$

where the projection operator $Proj\{*\}$ is defined as

$$Proj\left\{-\gamma_1 \underline{\xi}(\hat{\underline{x}}) B^T P \underline{\tilde{e}}\right\} = -\gamma_1 \underline{\xi}(\hat{\underline{x}}) B^T P \underline{\tilde{e}} + \gamma_1 \underline{\tilde{e}}^T PB \frac{\underline{\theta}_f \underline{\theta}_f^T \underline{\xi}^T(\hat{\underline{x}})}{|\underline{\theta}_f|^2}. \tag{42}$$

- Use the following adaptive law to adjust the parameter vector $\underline{\theta}_g$:
  □ Whenever an element q_{gi}^l in (16) of $\underline{\theta}_g = \varepsilon$, use

$$\dot{q}_{gi}^l = \begin{cases} -\gamma_2 \xi^l(\hat{\underline{x}}) B^T P \underline{\tilde{e}} u_I & \text{if } \underline{\tilde{e}}^T PB \xi^l(\hat{\underline{x}}) u_I < 0 \\ 0 & \text{if } \underline{\tilde{e}}^T PB \xi^l(\hat{\underline{x}}) u_I \geq 0 \end{cases} \tag{43}$$

where $\xi^l(\hat{\underline{x}})$ is the lth component of $\underline{\xi}^T(\hat{\underline{x}})$.

□ Otherwise, use

$$\dot{\underline{\theta}}_g = \begin{cases} -\gamma_2 \underline{\xi}(\hat{\underline{x}}) B^T P \underline{\tilde{e}} u_I & \text{if } (|\underline{\theta}_g| < M_g) \\ & \text{or } (|\underline{\theta}_g| = M_g \\ & \text{and } \underline{\tilde{e}}^T PB \underline{\xi}^T(\hat{\underline{x}}) \\ & \underline{\theta}_g u_I \geq 0) \\ Proj\{-\gamma_2 \underline{\xi}(\hat{\underline{x}}) B^T P \underline{\tilde{e}} u_I\} & \text{if } (|\underline{\theta}_g| = M_g \\ & \text{and } \underline{\tilde{e}}^T PB \underline{\xi}^T(\hat{\underline{x}}) \\ & \underline{\theta}_g u_I < 0) \end{cases} \tag{44}$$

where the projection operator $Proj\{*\}$ is defined as

$$Proj\left\{-\gamma_2 \underline{\xi}(\hat{\underline{x}}) B^T P \underline{\tilde{e}} u_I\right\} = -\gamma_2 \underline{\xi}(\hat{\underline{x}}) B^T \underline{\tilde{e}} u_I + \gamma_2 \underline{\tilde{e}}^T PB \frac{\underline{\theta}_g \underline{\theta}_g^T \underline{\xi}^T(\hat{\underline{x}}) u_I}{|\underline{\theta}_g|^2}. \tag{45}$$

- Use the following adaptive law to adjust the parameter vector $\underline{\theta}_D$:

$$\dot{\underline{\theta}}_D = \begin{cases} \gamma_3 \underline{\eta}(\hat{\underline{x}}) g(\underline{x}) B^T P \underline{\tilde{e}} & \text{if } (|\underline{\theta}_D| < M_D) \\ & \text{or } (|\underline{\theta}_D| = M_D \\ & \text{and } \underline{\tilde{e}}^T PBg(\underline{x}) \underline{\eta}^T(\hat{\underline{x}}) \\ & \underline{\theta}_D \geq 0) \\ Proj\{\gamma_3 \underline{\eta}(\hat{\underline{x}}) g(\underline{x}) B^T P \underline{\tilde{e}}\} & \text{if } (|\underline{\theta}_D| = M_D \\ & \text{and } \underline{\tilde{e}}^T PBg(\underline{x}) \underline{\eta}^T(\hat{\underline{x}}) \\ & \underline{\theta}_D < 0) \end{cases} \tag{46}$$

where the projection operator $Proj\{*\}$ is defined as

$$Proj\left\{\gamma_3 \underline{\eta}(\hat{\underline{x}}) g(\underline{x}) B^T P \underline{\tilde{e}}\right\} = \gamma_3 \underline{\tilde{e}}^T PBg(\underline{x}) \underline{\eta}(\hat{\underline{x}}) - \gamma_3 \underline{\tilde{e}}^T PBg(\underline{x}) \frac{\underline{\theta}_D \underline{\theta}_D^T \underline{\eta}^T(\tilde{\underline{x}})}{|\underline{\theta}_D|^2}. \tag{47}$$

Following the preceding consideration, we obtain the following theorem.

Theorem 1: Consider the plant (2) with control (24), where u_I is given by (8) and u_S is given by (22), and the fuzzy logic systems $\hat{f}$, $\hat{g}$, and u_D are represented in (27) form. Let *Assumption I* be true and the parameter vectors $\underline{\theta}_f$, $\underline{\theta}_g$, and $\underline{\theta}_D$ be adjusted by the adaptive laws (41)–(47). Then, the overall observer-based control scheme as shown in Fig. 2 guarantees the following properties:

1) $|\underline{\theta}_f| \leq M_f$, $|\underline{\theta}_g| \leq M_g$, $|\underline{\theta}_D| \leq M_D$, and all of the elements q_{gi}^l in (16) of $\theta_g \geq \varepsilon$

$$|\hat{\underline{x}}(t)| \leq \left[\left|\underline{y}_r\right| + \left(\frac{2\overline{V}_{\underline{\tilde{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2}\right] = M_{\hat{\underline{x}}} \tag{48}$$

and

$$\begin{aligned} |u| \leq \frac{\alpha}{\varepsilon} &\left[M_{f\hat{\underline{x}}} + \left|y_r^{(n)}\right| + |\underline{k}_c| \left(\frac{2\overline{V}_{\underline{\tilde{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2} \right] \\ &+ (1-\alpha) M_{D\hat{\underline{x}}} + \frac{1}{g_L(\underline{x})} \left\{ \left(1 + \frac{g^U(\underline{x}) + M_{g\hat{\underline{x}}}}{\varepsilon}\right) \right. \\ &\times \left[M_{f\hat{\underline{x}}} + \left|y_r^{(n)}\right| + |\underline{k}_c| \left(\frac{2\overline{V}_{\underline{\tilde{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2} \right] \\ &\left. + 2f^U(\underline{x}) + g^U(\underline{x}) M_{D\hat{\underline{x}}} + d_m \right\} \end{aligned} \tag{49}$$

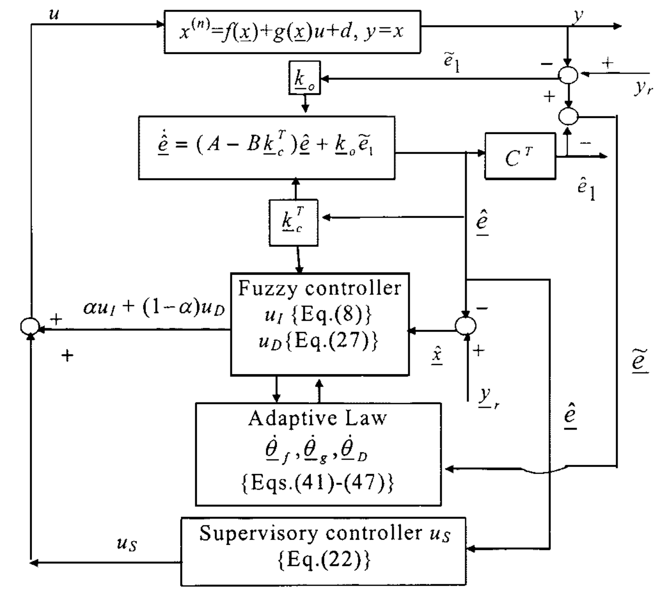

Fig. 2. Overall scheme of the observer-based hybrid direct/indirect adaptive FNN control.

for all $t \geq 0$, where $\lambda_{\hat{P}\min}$ is the minimum eigenvalue of $\hat{P}$, $M_{f\hat{\underline{x}}} = M_f(1+M_{\hat{\underline{x}}})$, $M_{g\hat{\underline{x}}} = M_g(1+M_{\hat{\underline{x}}})$, and $M_{D\hat{\underline{x}}} = M_D(1+M_{\hat{\underline{x}}})$.

2)

$$\int_0^t |\underline{\tilde{e}}(\zeta)|^2 \, d\zeta \leq a + b\int_0^t |\omega(\zeta)|^2 \, d\zeta \tag{50}$$

for all $t \geq 0$, where a and b are constants and ω is the minimum approximation error defined in (31).

3) If ω is squared integrable, i.e., $\int_0^\infty |\omega(t)|^2 \, dt < \infty$, then $\lim_{t\to\infty} |\underline{\tilde{e}}(t)| = 0$.

Proof:

I. i). To prove $|\underline{\theta}_f| \leq M_f$:

A) Let $V_f = (1/2)\underline{\theta}_f^T \underline{\theta}_f$, if the first line of (41) is true, we have either $|\underline{\theta}_f| \leq M_f$ or, $\dot{V}_f = -\gamma_1 \underline{\theta}_f^T \underline{\xi}(\hat{\underline{x}}) B^T P \underline{\tilde{e}} \leq 0$ for $|\underline{\theta}_f| = M_f$, i.e., we always have $|\underline{\theta}_f| \leq M_f$.

B) If the second line of (41), we have $|\underline{\theta}_f| = M_f$, and

$$\dot{V}_f = -\gamma_1 \underline{\theta}_f^T \underline{\xi}(\underline{x}) B^T P \underline{\tilde{e}} + \gamma_1 \underline{\tilde{e}}^T P B \frac{|\underline{\theta}_f|^2 \underline{\theta}_f^T \underline{\xi}(\hat{\underline{x}})}{|\underline{\theta}_f|^2} = 0, \quad \text{i.e., } |\underline{\theta}_f| \leq M_f.$$

Therefore, we prove that $|\underline{\theta}_f| \leq M_f$, $t \geq 0$.

ii) Use the similar method to show that $|\underline{\theta}_g| \leq M_g$, $|\underline{\theta}_D| \leq M_D$, $t \geq 0$.

From (43), we see that if q_{gi}^l in (16) $=\varepsilon$, then $\dot{q}_{gi}^l \geq 0$; that is, we have $q_{gi}^l \geq \varepsilon$ for all elements q_{gi}^l of $\underline{\theta}_g$.

iii) To prove (48).

In the above description, we prove that $V_{\hat{\underline{e}}} \leq \overline{V}_{\hat{\underline{e}}}$; therefore, $(1/2)\lambda_{\hat{P}\min}|\hat{\underline{e}}|^2 \leq (1/2)\hat{\underline{e}}^T \hat{P} \hat{\underline{e}} \leq \overline{V}_{\hat{\underline{e}}}$; i.e.

$$|\hat{\underline{e}}| \leq \left(\frac{2\overline{V}_{\hat{\underline{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2}.$$

Since $\hat{\underline{e}} = \underline{y}_r - \hat{\underline{x}}$, we have

$$|\hat{\underline{x}}| \leq \left[|\underline{y}_r| + \left(\frac{2\overline{V}_{\hat{\underline{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2}\right] = M_{\hat{\underline{x}}}.$$

iv) To prove (49).

Since $\hat{f}(\hat{\underline{x}}\,|\,\underline{\theta}_f)$, $\hat{g}(\hat{\underline{x}}\,|\,\underline{\theta}_g)$ and $u_D(\hat{\underline{x}}\,|\,\underline{\theta}_D)$ are weighted averages of the elements of $\underline{\theta}_f$, $\underline{\theta}_g$, and $\underline{\theta}_D$, respectively, we have

$$\left|\hat{f}(\hat{\underline{x}}\,|\,\underline{\theta}_f)\right| \leq M_f(1+M_{\hat{\underline{x}}}) = M_{f\hat{\underline{x}}} \tag{51}$$

$$|\hat{g}(\hat{\underline{x}}\,|\,\underline{\theta}_g)| \leq M_g(1+M_{\hat{\underline{x}}}) = M_{g\hat{\underline{x}}} \tag{52}$$

$$|u_D(\hat{\underline{x}}\,|\,\underline{\theta}_D)| \leq M_D(1+M_{\hat{\underline{x}}}) = M_{D\hat{\underline{x}}} \tag{53}$$

and $|\hat{g}(\hat{\underline{x}}\,|\,\underline{\theta}_g)| \geq \varepsilon > 0$ [since q_{gi}^l in (16) $\geq \varepsilon$]. Therefore, from (8) we obtain

$$\begin{aligned}
|u_I| &\leq \frac{1}{|\hat{g}(\hat{\underline{x}}\,|\,\underline{\theta}_g)|}\left[\left|\hat{f}(\hat{\underline{x}}\,|\,\underline{\theta}_f)\right| + \left|y_r^{(n)}\right| + |\underline{k}_c| \cdot |\hat{\underline{e}}|\right] \\
&\leq \frac{1}{|\hat{g}(\hat{\underline{x}}\,|\,\underline{\theta}_g)|}\left[\left|\hat{f}(\hat{\underline{x}}\,|\,\underline{\theta}_f)\right| + \left|y_r^{(n)}\right| + |\underline{k}_c|\left(\frac{2\overline{V}_{\hat{\underline{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2}\right] \\
&\leq \frac{1}{\varepsilon}\left[M_{f\hat{\underline{x}}} + \left|y_r^{(n)}\right| + |\underline{k}_c|\left(\frac{2\overline{V}_{\hat{\underline{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2}\right].
\end{aligned} \tag{54}$$

According to (22) and (51)–(54), we manipulate them and have

$$\begin{aligned}
|u_S| &\leq \frac{1}{g_L(\underline{x})}\left[\left|\hat{f}(\hat{\underline{x}}\,|\,\underline{\theta}_f)\right| + 2f^U(\underline{x}) + \left|y_r^{(n)}\right| + \left|\underline{k}_c^T \hat{\underline{e}}\right| \right. \\
&\qquad \left. + g^U(\underline{x})\left(|u_I| + |u_D|\right) + |\hat{g}(\hat{\underline{x}}\,|\,\underline{\theta}_g) u_I| + d_m\right] \\
&\leq \frac{1}{g_L(\underline{x})}\left\{M_{f\hat{\underline{x}}} + \left|y_r^{(n)}\right| + |\underline{k}_c|\left(\frac{2\overline{V}_{\hat{\underline{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2}\right. \\
&\qquad + \left(g^U(\underline{x}) + M_{g\hat{\underline{x}}}\right) \\
&\qquad \times \frac{1}{\varepsilon}\left[M_{f\hat{\underline{x}}} + \left|y_r^{(n)}\right| + |\underline{k}_c|\left(\frac{2\overline{V}_{\hat{\underline{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2}\right] \\
&\qquad \left. + 2f^U(\underline{x}) + g^U(\underline{x}) M_{D\hat{\underline{x}}} + d_m\right\} \\
&= \frac{1}{g_L(\underline{x})}\left\{\left(1 + \frac{g^U(\underline{x}) + M_{g\hat{\underline{x}}}}{\varepsilon}\right)\right. \\
&\qquad \times \left[M_{f\hat{\underline{x}}} + \left|y_r^{(n)}\right| + |\underline{k}_c|\left(\frac{2\overline{V}_{\hat{\underline{e}}}}{\lambda_{\hat{P}\min}}\right)^{1/2}\right] \\
&\qquad \left. + 2f^U(\underline{x}) + g^U(\underline{x}) M_{D\hat{\underline{x}}} + d_m\right\}.
\end{aligned} \tag{55}$$

By combining (53)–(55) and substituting into (6), we can prove (49).

II. From (36), and by using the modified adaptive laws in (41)–(47), we have

$$\dot{V} \leq -\tfrac{1}{2}\,\underline{\tilde{e}}^T Q \underline{\tilde{e}} - g(\underline{x})\underline{\tilde{e}}^T PBu_S + \underline{\tilde{e}}^T PB\omega. \tag{56}$$

Since $g(\underline{x}) > 0$ and from (22), we have $g(\underline{x})\underline{\tilde{e}}^T PBu_S \geq 0$. Hence, (56) can be simplified as

$$\begin{aligned}\dot{V} &\leq -\tfrac{1}{2}\,\underline{\tilde{e}}^T Q \underline{\tilde{e}} + \underline{\tilde{e}}^T PB\omega \\ &\leq -\frac{\lambda_{Q\min}-1}{2}\,|\underline{\tilde{e}}|^2 - \frac{1}{2}\left(|\underline{\tilde{e}}|^2 - 2\underline{\tilde{e}}PB\omega + |PB\omega|^2\right) \\ &\quad + \tfrac{1}{2}\,|PB\omega|^2 \\ &\leq -\frac{\lambda_{Q\min}-1}{2}\,|\underline{\tilde{e}}|^2 + \frac{1}{2}\,PB\omega|^2 \end{aligned} \tag{57}$$

where $\lambda_{Q\min}$ is the minimum eigenvalue of Q. By integrating both sides of (57) and assuming that $\lambda_{Q\min} > 1$ (since Q is specified by the designer, we can choose such a Q), after some simple manipulations, we can obtain

$$\begin{aligned}\int_0^t |\underline{\tilde{e}}(\zeta)|^2\, d\zeta \leq{}& \frac{2}{\lambda_{Q\min}-1}\,(|V(0)+|V(t)|) \\ &+ \frac{1}{\lambda_{Q\min}-1}\,|PB|^2 \int_0^t |\omega(\zeta)|^2\, d\zeta. \end{aligned} \tag{58}$$

Defining $a = (2/(\lambda_{Q\min}-1))(|V(0)+|V(t)|)$ and $b = (1/(\lambda_{Q\min}-1))|PB|^2$, we can prove (50) by substituting a and b into (58).

III. From (50), if $\omega \in L_2$, we have $\underline{\tilde{e}} \in L_2$. We have $\dot{\underline{\tilde{e}}} \in L_\infty$, because we have proven that all variables in the right-hand side of (33) are bounded. Using Barbalat's lemma [23] [if $\underline{\tilde{e}} \in L_2 \cap L_\infty$ and $\dot{\underline{\tilde{e}}} \in L_\infty$, then $\lim_{t\to\infty} |\underline{\tilde{e}}(t)| = 0$] we have $\lim_{t\to\infty} |\underline{\tilde{e}}(t)| = 0$. This completes the proof.

Remark I: It is obvious that we need to know $g(\underline{x})$ beforehand in adaptive law (39), i.e., in the above theorem the adaptive FNN control works under those nonlinear systems of which $g(\underline{x})$ is well known. If the dynamics $g(\underline{x})$ can be split into a well-known nominal part $g_0(\underline{x})$, plus an uncertain part $g_d(\underline{x})$, then $g_d(\underline{x})u$ can be considered as a part of the external disturbance. In the meantime, it can be attenuated by the proposed methodology.

To summarize the above analysis, the design algorithm for observer-based hybrid direct/indirect adaptive FNN control is proposed as follows.

Step 1) Specify the feedback and observer gain vector $\underline{k}_c$ and $\underline{k}_o$, such that the characteristic matrices $A - B\underline{k}_c^T$ and $A - \underline{k}_o C^T$ are strictly Hurwitz matrices, respectively.

Step 2) Specify a positive definite $n \times n$ matrix Q and solve the Lyapunov equation (12) to obtain a positive definite symmetric $n \times n$ matrix P.

Step 3) Solve the state error equation (10) to obtain estimate state vector $\underline{\hat{x}} = \underline{y}_r - \underline{\hat{e}}$.

Step 4) Specify the parameters M_f, M_g, M_D, $M_{\underline{\hat{x}}}$, γ_1, γ_2, γ_3, ε, and $\overline{V}$ based on the practical constraints. Although $\overline{V}$ is any given constant, we let $\overline{V}$ be the same as $\overline{V}_{\underline{\hat{e}}}$ (described at the end of Section II), which can be determined from $M_{\underline{\hat{x}}}$, $|y_r|$ and $\lambda_{\hat{P}\min}$ of $\hat{P}$ in (48). This is to match the magnitude scale of the system so that the designer is free from supplying $\overline{V}$ at random to the system.

Step 5) Define the membership function $\mu_{F_i^l}(\underline{\hat{x}})$ for $i = 1, 2, \ldots, M$ and compute the fuzzy basis functions $\underline{\xi}(\underline{\hat{x}})$. Then, fuzzy logic control systems are constructed as

$$\hat{f}(\underline{\hat{x}}\,|\,\underline{\theta}_f) = \underline{\xi}^T(\underline{\hat{x}})\underline{\theta}_f, \qquad \hat{g}(\underline{\hat{x}}\,|\,\underline{\theta}_g) = \underline{\xi}^T(\underline{\hat{x}})\underline{\theta}_g.$$

Similarly, define the other membership functions and compute $\underline{\eta}(\underline{\hat{x}})$. Then, fuzzy logic control system is constructed as

$$u_D(\underline{\hat{x}}\,|\,\underline{\theta}_D) = \underline{\eta}^T(\underline{\hat{x}})\underline{\theta}_D.$$

Step 6) Obtain the control and apply it to the plant, then compute the adaptive laws (41)–(47) to adjust the parameter vectors $\underline{\theta}_f$, $\underline{\theta}_g$, and $\underline{\theta}_D$. Following *Remark I*, we let the unknown $g(\underline{x})$ be $g_0(\underline{x})$ in (46) and (47).

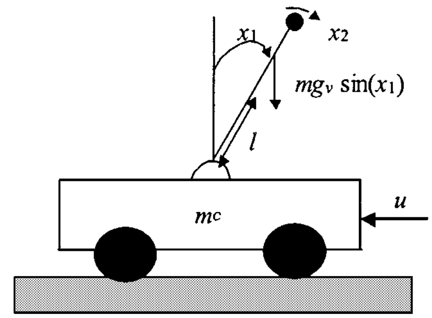

Fig. 3. Inverted pendulum system.

V. EXAMPLES

In this section, we will apply our observer-based hybrid direct/indirect adaptive FNN controller to control inverted pendulum and Chua's chaotic circuit to track a sine-wave trajectory.

Example 1: Consider the inverted pendulum system shown in Fig. 3. Let $x_1 = \theta$ be the angle of the pendulum with respect to the vertical line.

The dynamic equations of the inverted pendulum system [4], [8], [12], [23] are

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} &= \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}(f + gu + d) \\ y &= [1\ \ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}\end{aligned} \tag{59}$$

where

$$f = \frac{g_v \sin x_1 - \frac{mlx_2^2\cos x_1 \sin x_1}{m_c+m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c+m}\right)}; \quad g = \frac{\frac{\cos x_1}{m_c+m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c+m}\right)}$$

and $g_v = 9.8$ m/s^2 is the acceleration due to gravity; m_c is the mass of the cart; l is the half-length of the pole; m is the mass of the pole; and u is the control input. In this example, we assume that $m_c = 1$ kg, $m = 0.1$ kg, and $l = 0.5$ m. It is obvious that $0 < g < \infty$ so that *Assumption I* in Section IV is satisfied. This is due to $\cos x_1/(m_c + m) > 0$ and $m\cos^2 x_1/(m_c + m) < 1$. We also have to determine the bounds f^U, g^U, and g_L as follows:

$$\begin{aligned}|f(x_1, x_2)| &\le \frac{9.8 + (0.025x_2^2)}{(2/3) - (0.05/1.1)} = 15.78 + 0.0366x_2^2\\ &\approx 15.78 + 0.0366\hat{x}_2^2 = f^U(\hat{x}_1, \hat{x}_2)\\ |g(x_1, x_2)| &\le 1.46 = g^U(x_1, x_2) \approx g^U(\hat{x}_1, \hat{x}_2)\end{aligned} \tag{60}$$

and if we require that $|x_1| \le \pi/6$, then

$$|g(x_1, x_2)| \ge 1.12 = g_L(x_1, x_2) \approx g_L(\hat{x}_1, \hat{x}_2). \tag{61}$$

The control object is to control the state x_1 of the system to track the reference trajectory $y_r(t) = (\pi/30)\sin(t)$ if only the system output y is measurable. Also, the external disturbance d is assumed to be a square-wave with amplitude ± 0.1, period 2π, and the parameters are chosen as $\gamma_1 \approx 45.1$, $\gamma_2 \approx 11.2$, $\gamma_3 \approx 15.1$, and step size $h = 0.002\,85$. The choices of γs and h are to improve the convergence rate of the closed-loop system controlled by our proposed controller.

According to the design procedure, the design is given in the following steps.

Step 1) The observer and feedback gain vectors are chosen as $\underline{k}_o^T = [89\ \ 184]$ and $\underline{k}_c^T = [4\ \ 4]$, respectively.

Step 2) We select Q in (12) as $\begin{bmatrix}10 & 13\\ 13 & 28\end{bmatrix}$, then after solving (12), the positive definite symmetric 2×2 matrix P in (12) is $\begin{bmatrix}29 & -14\\ -14 & 7\end{bmatrix}$. The minimum eigenvalue of Q, i.e., $\lambda_{Q\,\min}$ is 3.19, which satisfies the transition from (56) to (57).

Step 3) Solve (10) to obtain $\underline{\hat{x}}$.

Step 4) We select $M_f = 20$, $M_g = 20$, $M_d = 20$, $M_{\underline{\hat{x}}} = \pi/6$, and $\varepsilon = 0.3$, and $\hat{Q}$ in (14) is chosen as $\begin{bmatrix}40 & 25\\ 25 & 30\end{bmatrix}$ and $\hat{A} = \begin{bmatrix}0 & 1\\ -4 & -4\end{bmatrix}$ in (14). Therefore, the positive definite symmetric 2×2 matrix $\hat{P}$ in (14) can be solved as $\begin{bmatrix}15 & 5\\ 5 & 5\end{bmatrix}$. The minimum eigenvalue of value $\hat{P}$, i.e., the $\lambda_{\hat{P}\min}$ in (48), is 2.93. Therefore, we can have $\overline{V}_{\underline{\hat{e}}}$ from (48) as 0.257.

Step 5) The following membership functions for $\hat{x}_i$, $i = 1, 2$ are selected as:

$$\mu_{F_1^i}(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i + \pi/6}{\pi/24}\right)^2\right]$$

$$\mu_{F_2^i}(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i + \pi/12}{\pi/24}\right)^2\right]$$

$$\mu_{F_3^i}(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i}{\pi/24}\right)^2\right]$$

$$\mu_{F_4^i}(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i - \pi/12}{\pi/24}\right)^2\right]$$

$$\mu_{F_5^i}(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i - \pi/6}{\pi/24}\right)^2\right].$$

To cover whole cases, we apply 25 fuzzy rules. For simplification, we let $\underline{\xi}(\underline{\hat{x}}) = \underline{\eta}(\underline{\hat{x}})$.

Hence, u_I and u_D are constructed.

TABLE I
FOUR CASES OF THE INITIAL STATES

Cases	Initial states	Cases	Initial states
Case1	$x(0)=[0.1\ \ 0]^T$ $\hat{x}(0)=[-0.15\ \ 0]^T$	Case 3	$x(0)=[-0.2\ \ 0]^T$ $\hat{x}(0)=[0.15\ \ 0]^T$
Case 2	$x(0)=[0.25\ \ 0.25]^T$ $\hat{x}(0)=[0.1\ \ 0.1]^T$	Case4	$x(0)=[-0.25\ \ -0.25]^T$ $\hat{x}(0)=[0.25\ \ 0.25]^T$

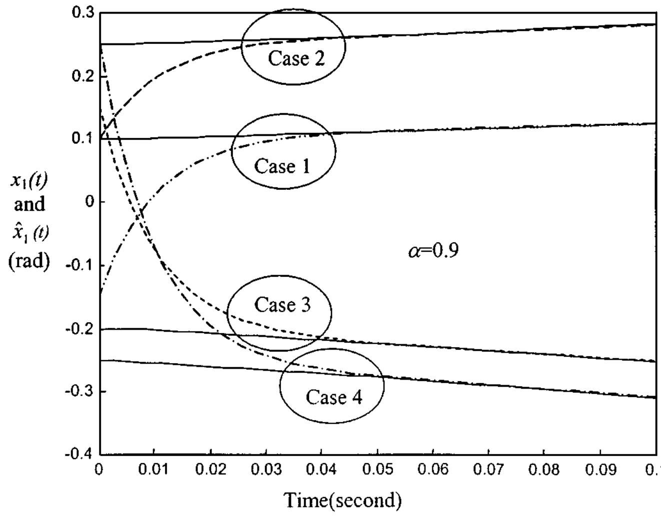

Fig. 4. Trajectories of the states x_1 (solid line) and $\hat{x}_1$ (dashed line) of four cases.

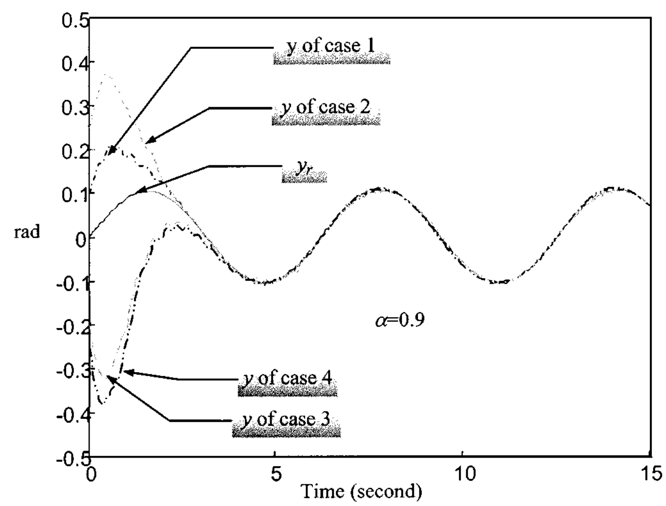

Fig. 5. Output trajectories y of four cases and reference y_r with $\alpha = 0.9$.

Step 6) Compute the adaptive laws (41)–(47). From (60) and (61), we can let $g_0(x_1, x_2) = 1$ to replace the unknown $g(x_1, x_2)$ in (46) and (47). This has been explained in *Remark I*.

According to the initial states, four cases are simulated, as shown in Table I.

Fig. 4 shows the trajectories of the states x_1 and $\hat{x}_1$ of four cases if $\alpha = 0.9$ is chosen and it shows that the estimation state $\hat{x}_1$ takes very short time to catch up to the system state x_1.

The tracking performances of four cases are also very good, as shown in Fig. 5, in which y_r is the reference trajectory and y is the system output trajectory. This result is better, as shown in [4] and [12].

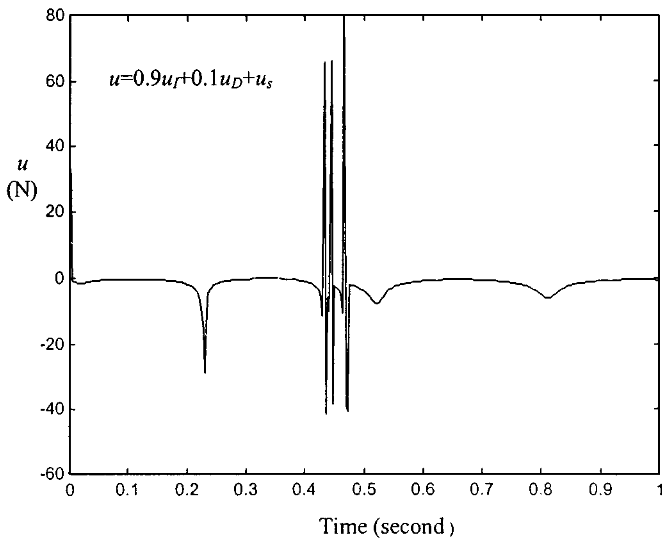

Fig. 6. Trajectory of the control input (include supervisory control) of Case 1 with $\alpha = 0.9$ (time $= 0 \sim 1$ s).

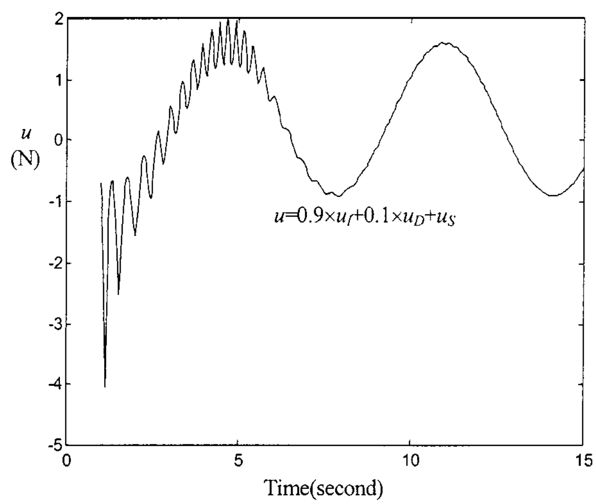

Fig. 7. Trajectory of the control input (include supervisory control) of Case 1 with $\alpha = 0.9$ (time $= 1 \sim 15$ s).

We show the control input $u = \alpha u_I + (1-\alpha)u_D + u_S$ of Case 1 with $\alpha = 0.9$ in Figs. 6 and 7.

Fig. 8 shows the supervisory control u_S, and one can obviously see that it is activated in four periods: [0, 0.0057], [0.4361, 0.4389], [0.4475, 0.4503], and [0.4703, 0.4760]. After time = 0.4760 s, the FNN controller can stabilize the system and the supervisory controller will never be activated again. The spikes in Figs. 6 and 8 are caused by the fact that u_S must maintain a larger initial value to stabilize the system when the system tends to be unstable. Therefore, the adaptive controller can perform successful control and the desired performance can be achieved.

Applying the different weighting factor α, the tracking error performance of Case 1 is shown in Fig. 9.

Example 2: The typical Chua's chaotic circuit in Fig. 10 consists of one linear resistor (R), two capacitors ($C1$, $C2$), one inductor, and one piecewise-linear resistor (λ) [16], [24]. It has been shown to own very rich nonlinear dynamics such as chaos and bifurcations.

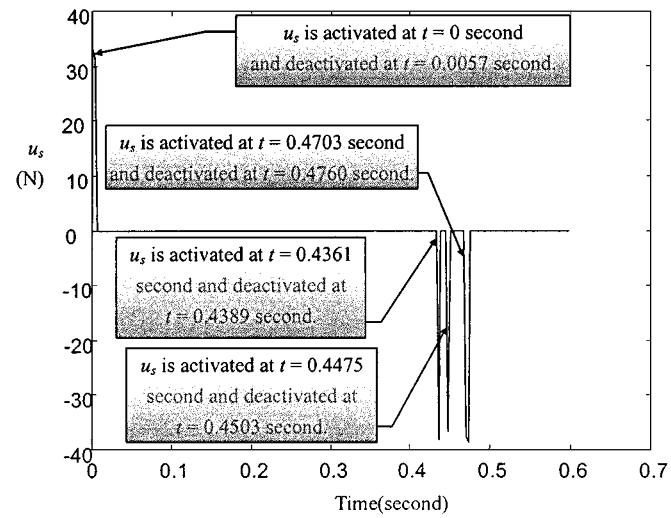

Fig. 8. Trajectory of the supervisory control u_S of Case 1 with $\alpha = 0.9$ (time $= 0 \sim 0.6$ s).

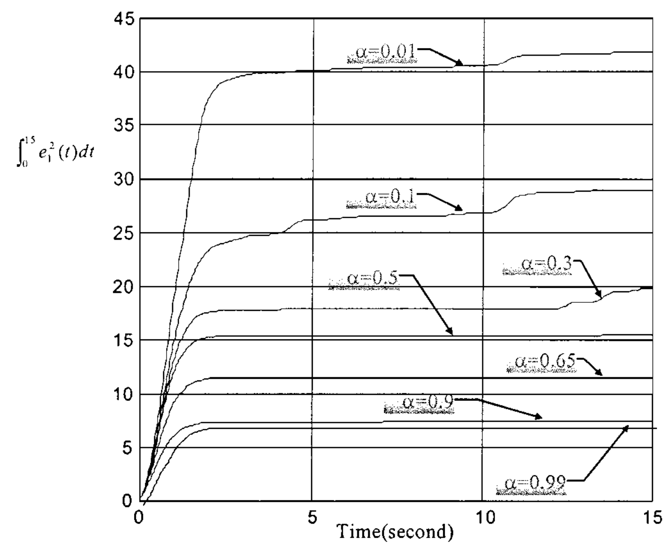

Fig. 9. Tracking performance $\int_0^{15} e_1^2(t)\,dt$ for Case 1 with different α.

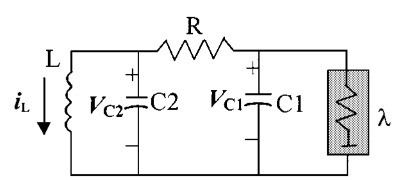

Fig. 10. Chua's chaotic circuit.

The dynamic equations of Chua's chaotic circuit are written as

$$\dot{V}_{C1} = \frac{1}{C1}\left(\frac{1}{R}(V_{C2} - V_{C1}) - \lambda(V_{C1})\right)$$

$$\dot{V}_{C2} = \frac{1}{C2}\left(\frac{1}{R}(V_{C1} - V_{C2}) + i_L\right)$$

$$\dot{i}_L = \frac{1}{L}(-V_{C1} - R_0 i_L) \tag{62}$$

where voltages V_{C1} and V_{C2} and current i_L are state variables; R_0 is a constant; and λ denotes the nonlinear resistor, which is a function of the voltage across the two terminals of $C1$. Here,

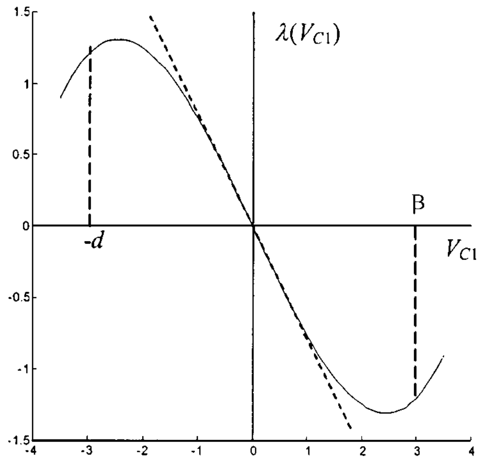

Fig. 11. Nonlinear resistor characteristics.

we define λ as a cubic function as in (63), and its diagram is shown in Fig. 11 [24]

$$\lambda(V_{C1}) = aV_{C1} + cV_{C1}^3 \qquad (a < 0,\, c > 0). \tag{63}$$

The system can be rewritten as

$$\dot{Z}(t) = GZ(t) + H\lambda \tag{64}$$

where $Z = [z_1\ z_2\ z_3]^T = [V_{C1}\ V_{C2}\ i_L]^T$

$$G = \begin{bmatrix} -\dfrac{1}{C_1 R} & \dfrac{1}{C_1 R} & 0 \\ \dfrac{1}{C_2 R} & -\dfrac{1}{C_2 R} & \dfrac{1}{C_2} \\ 0 & -\dfrac{1}{L} & -\dfrac{R_0}{L} \end{bmatrix} \quad \text{and} \quad H = \begin{bmatrix} -\dfrac{1}{C_1} \\ 0 \\ 0 \end{bmatrix}.$$

The above state space equations are not in the standard canonical form defined in (3). Therefore, we need to perform a linear transformation to transform them into the form of (3). Let us define $Z(t) = TZ^*(t)$ or $Z^*(t) = T^{-1}Z(t)$ where T is a transformation matrix. Using the transformation in [25] and [26], the transformed system can be obtained as

$$\dot{Z}^*(t) = T^{-1}GTZ^*(t) + T^{-1}H\lambda = G^*Z^*(t) + H^*\lambda \tag{65}$$

where, as shown in the equation at the bottom of the page, $G^* = T^{-1}GT$, $H^* = T^{-1}H$.

Choose the parameters as follows:

$$R = \tfrac{10}{7}, \quad R_0 = 0, \quad C_1 = 1, \quad C_2 = \tfrac{19}{2}$$
$$L = \tfrac{19}{14}, \quad a = -\tfrac{4}{5}, \quad c = \tfrac{2}{45}, \quad \beta = 3.$$

Therefore, after computation, we get the transformed system as follows:

$$\begin{aligned} &\dot{z}_1^* = z_2^*; \quad \dot{z}_2^* = z_3^* \\ &\dot{z}_3^* = \tfrac{14}{1805}\, z_1^* - \tfrac{168}{9025}\, z_2^* + \tfrac{1}{38}\, z_3^* \\ &\qquad - \tfrac{2}{45}\left(\tfrac{28}{361}\, z_1^* + \tfrac{7}{95}\, z_2^* + z_3^*\right)^3. \end{aligned} \tag{66}$$

For comparison, the simulation results of Chua's chaotic circuit and its transformed system are shown in Fig. 12.

We will design the hybrid FNN adaptive controller to dominate the transformed system to track a reference signal. For convenience, we let $\underline{x}$ replace $\underline{z}^*$ in the above transformed system. Therefore, the closed-loop configuration of (66) can be represented by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (f + gu + d)$$

and

$$y = [1\ \ 0\ \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{67}$$

where

$$\begin{aligned} f = \tfrac{14}{1805}\, x_1 - \tfrac{168}{9025}\, x_2 + \tfrac{1}{38}\, x_3 - \tfrac{2}{45} \\ \times \left(\tfrac{28}{361}\, x_1 + \tfrac{7}{95}\, x_2 + x_3\right)^3, \quad g = 1 \end{aligned}$$

and d is the external disturbance. Although the f above is well defined since the Chua's circuit is well specified, we do not apply it in the adaptive law. However, we can indeed use it to estimate the upper bound of f, which is required in our design

$$T = \begin{bmatrix} -\dfrac{R + R_0}{C_1 C_2 RL} & -\dfrac{RR_0 C_2 + L}{C_1 C_2 RL} & -\dfrac{1}{C_1} \\ -\dfrac{R_0}{C_1 C_2 RL} & -\dfrac{1}{C_1 C_2 R} & 0 \\ \dfrac{1}{C_1 C_2 RL} & 0 & 0 \end{bmatrix}$$

and

$$G^* = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\dfrac{1}{C_1 C_2 RL} & -\dfrac{C_1 R + C_2 R_0 + C_1 R_0}{C_1 C_2 RL} & -\dfrac{C_1 C_2 R R_0 + C_2 L + C_1 L}{C_1 C_2 RL} \end{bmatrix}$$

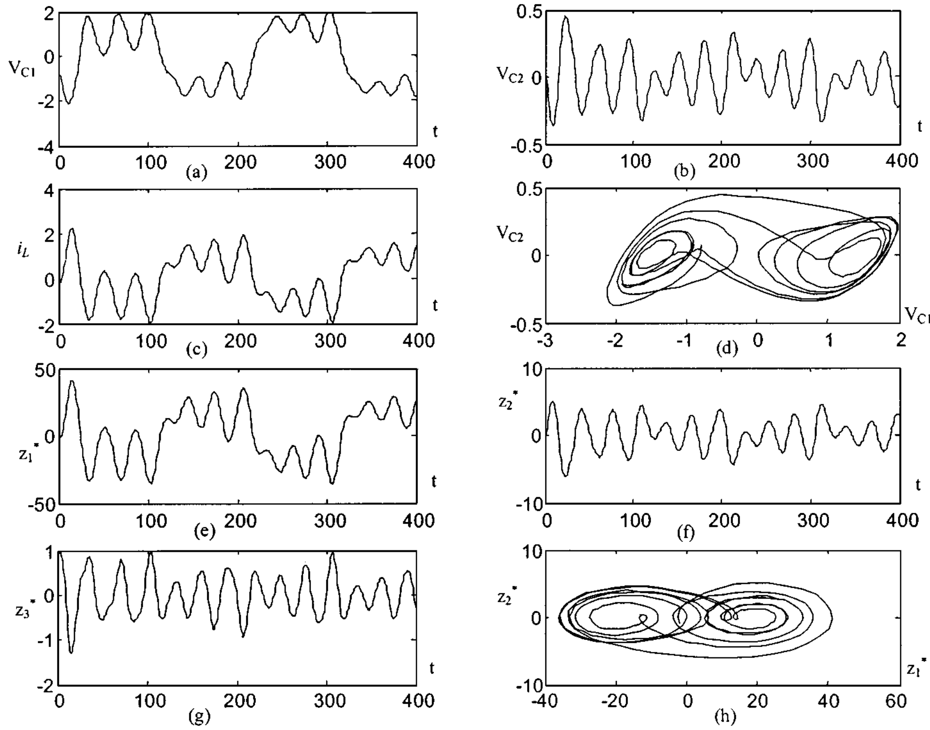

Fig. 12. (a) V_{C1} of Chua's circuit. (b) V_{C2} of Chua's circuit. (c) i_L of Chua's circuit. (d) Phase-plane trajectory of Chua's circuit. (e) z_1^* of transformed system. (f) z_2^* of transformed system. (g) z_3^* of transformed system. (h) Phase-plane trajectory of transformed system.

procedure. The bounds f^U, g^U, and g_L can be estimated as follows:

$$\begin{aligned} &|f(x_1, x_2, x_3)| \\ &\quad \le 14/1805 \cdot |x_1| + 168/9025 \cdot |x_2| + 1/38 \cdot |x_3| \\ &\qquad + 2/45(28/361|x_1| + 7/95|x_2| + |x_3|)^3 \\ &\quad \le 14/1805 \times 50 + 168/9025 \times 10 + 1/38 \times 2 \\ &\qquad + 2/45(28/361 \times 50 + 7/95 \times 10 + 2)^3 \\ &\quad \approx 13.5 \approx 14/1805 \cdot |\hat{x}_1| + 168/9025 \cdot |\hat{x}_2| + 1/38 \cdot |\hat{x}_3| \\ &\qquad + 2/45(28/361|\hat{x}_1| + 7/95|\hat{x}_2| + |\hat{x}_3|)^3 \\ &\quad = f^U(\hat{x}_1, \hat{x}_2, \hat{x}_3). \end{aligned} \tag{68}$$

The above estimation comes from several simulation runs of the uncontrolled and transformed Chua's circuit in (66). Since $g = 1$, we let

$$g_0 = 1, \quad g^U(x_1, x_2, x_3) \approx g^U(\hat{x}_1, \hat{x}_2, \hat{x}_3) = 1 \tag{69}$$

and

$$g_L(x_1, x_2, x_3) \approx g_L(\hat{x}_1, \hat{x}_2, \hat{x}_3) = 1. \tag{70}$$

The control object is to control the state x_1 of the system to track the reference trajectory $y_r(t) = 1.5\sin(t)$ if only the system output y is measurable. Therefore, in the phase plane, this reference trajectory is a circle with radius 1.5: $y_r^2 + \dot{y}_r^2 = 1.5$. Also the external disturbance d is assumed to be a square-wave with amplitude ± 0.5, period 2π and the parameters are chosen as $\gamma_1 \approx 0.077\,71$, $\gamma_2 \approx 0.008\,32$, $\gamma_3 \approx 0.038\,08$, and step size $h = 0.002\,55$. The choices of γs and h are to improve the convergence rate of the closed-loop system controlled by our proposed controller.

According to the design procedure, the design is given in the following steps.

Step 1) The observer and feedback gain vectors are chosen as $\underline{k}_o^T = [5 \quad 237 \quad 3]$ and $\underline{k}_c^T = [12 \quad 13 \quad 3]$, respectively.

Step 2) We select Q in (12) as $6 \times I_{3\times3}$, then after solving (12), the positive definite symmetric 3×3 matrix P in (12) is

$$\begin{bmatrix} 143.2233 & -3 & -0.7056 \\ -3 & 0.7055 & -3 \\ -0.7056 & -3 & 237.1759 \end{bmatrix}.$$

The minimum eigenvalue of Q, i.e., $\lambda_{Q\min}$ is 6, which satisfies the transition from (56) to (57).

Step 3) Solve (10) to obtain $\underline{\hat{x}}$.

Step 4) We select $M_f = 50$, $M_g = 50$, $M_d = 50$, $M_{\underline{\hat{x}}} = 2.5$, and $\varepsilon = 0.3$, and $\hat{Q}$ in (14) is chosen as $10 \times I_{3\times3}$ and $\hat{A}$ in (14) is computed. Therefore, the positive definite symmetric 3×3 $\hat{P}$ in (14) can be solved as

$$\begin{bmatrix} 40.4166 & 32.9166 & 0.4166 \\ 32.9166 & 42.6388 & 2.9166 \\ 0.4166 & 2.9166 & 2.6388 \end{bmatrix}.$$

The minimum eigenvalue of value $\hat{P}$, i.e., the $\lambda_{\hat{P}\min}$ in (48) is 2.1. Therefore, we can have $\overline{V}_{\underline{\hat{e}}}$ from (48) as 1.05.

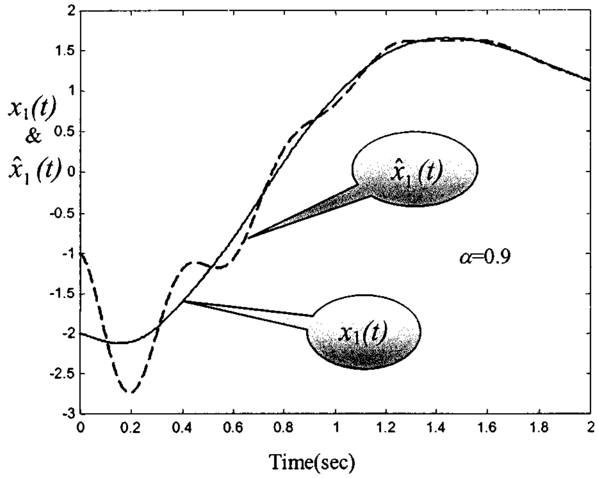

Fig. 13. Trajectories of the states x_1 (solid line) and $\hat{x}_1$ (dashed line).

Step 5) The following membership functions are selected as:

$$\mu_{F_1^i}(\hat{x}_i) = 1/\{1+\exp[5(\hat{x}_i+\tau\cdot\zeta_1)]\},$$
$$\mu_{F_2^i}(\hat{x}_i) = \exp\left[-(\hat{x}_i+\tau\cdot\zeta_2)^2\right]$$
$$\mu_{F_3^i}(\hat{x}_i) = \exp\left[-(\hat{x}_i+\tau)^2\right],$$
$$\mu_{F_4^i}(\hat{x}_i) = \exp\left[-(\hat{x}_i-\tau)^2\right]$$
$$\mu_{F_5^i}(\hat{x}_i) = \exp\left[-(\hat{x}_i-\tau\cdot\zeta_2)^2\right],$$
$$\mu_{F_6^i}(\hat{x}_i) = 1/\{1+\exp\left[-5(\hat{x}_i-\tau\cdot\zeta_1)\right]\}.$$

Let $\tau = 1$, $\zeta_1 = 3.7$, $\zeta_2 = 3.3$, for $\hat{x}_i$, $i = 1, 2$. Set $\tau = 3.8$, $\zeta_1 = 3.3$, $\zeta_2 = 2.6$, for $\hat{x}_3$.

To cover whole cases, we apply 216 fuzzy rules. For simplification, we let $\underline{\xi}(\hat{\underline{x}}) = \underline{\eta}(\hat{\underline{x}})$. Hence, u_I and u_D are constructed.

Step 6) Compute the adaptive laws (41)–(47). From (69) and (70), it is obvious that we can let $g_0(\underline{x}) = 1$ to replace the unknown $g(\underline{x})$ in (46) and (47). This has been explained in the *Remark I.*

Fig. 13 shows the trajectories of the states x_1 and $\hat{x}_1$ if $\alpha = 0.9$ is chosen and it shows that the estimation state $\hat{x}_1$ takes less than 1.4 s to catch up to the system state x_1.

Fig. 14(a)–(c) shows the responses of the transformed Chua's circuit. Fig. 14(d)–(f) shows the responses of the original Chua's circuit by restoring the transformed system to its original states.

Fig. 15 shows the phase plane trajectories of the transformed and original Chua's circuit. Fig. 15 clearly indicates the fact that the tracking performances are guaranteed by our hybrid adaptive FNN controller.

Fig. 16(a) shows the overall control effort u for the first 6 s. Fig. 16(b) extends the time-scale in Fig. 16(a) to 25 s. Obviously, the overall control effort u in the steady state has its maximum magnitude less than 5 NT. Fig. 16(c) shows the supervisory control u_s with its activation and activation periods in the initial 5 s. After 5 s, the u_s is no longer necessary.

Applying a different weighting factor α, the tracking error performance of *Example 2* is shown in Fig. 17.

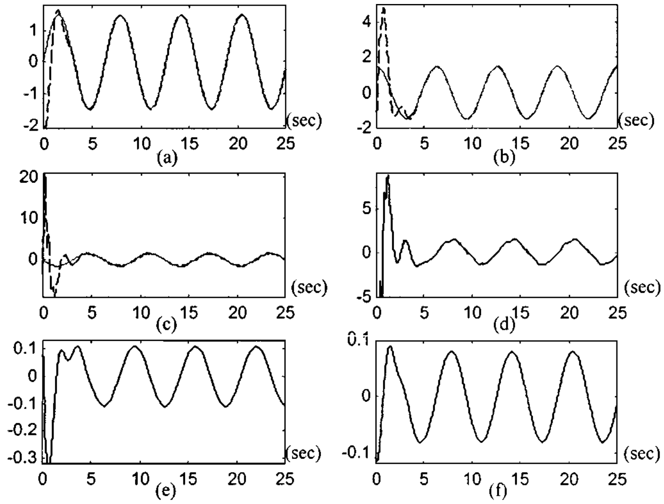

Fig. 14. (a) Output trajectories of y (dashed line) and y_r (solid line) with $\alpha = 0.9$. (b) Output trajectories of $\dot{y}$ (dashed line) and $\dot{y}_r$ (solid line) with $\alpha = 0.9$. (c) Output trajectories of $\ddot{y}$ (dashed line) and $\ddot{y}_r$ (solid line) with $\alpha = 0.9$. (d) Trajectory of V_{C1}. (e) Trajectory of V_{C2}. (f) Trajectory of i_L.

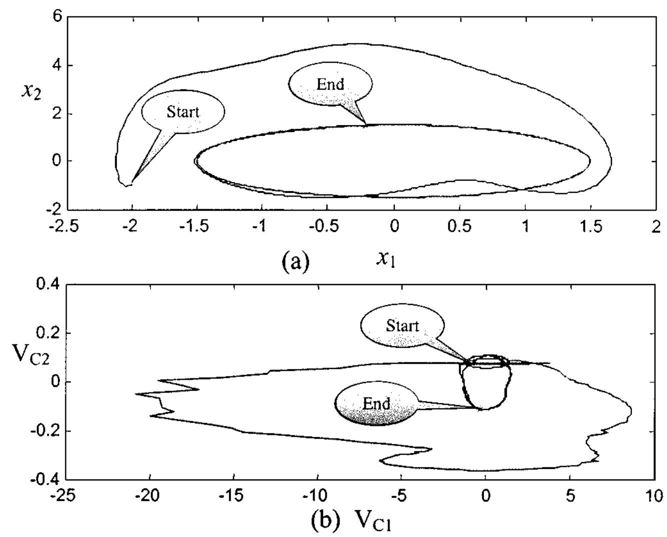

Fig. 15. (a) Phase-plane trajectory of transformed Chua's circuit with $\alpha = 0.9$. (b) Phase-plane trajectory of Chua's circuit.

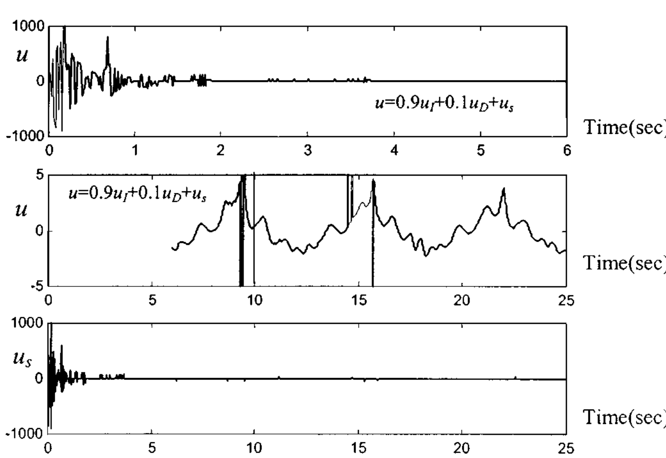

Fig. 16. (a) Trajectory of the control input (including supervisory control) with $\alpha = 0.9$ (time $= 0 \sim 6$ s). (b) Trajectory of the control input (including supervisory control) with $\alpha = 0.9$ (time $= 6 \sim 25$ s). (c) Trajectory of the supervisory control u_S with $\alpha = 0.9$.

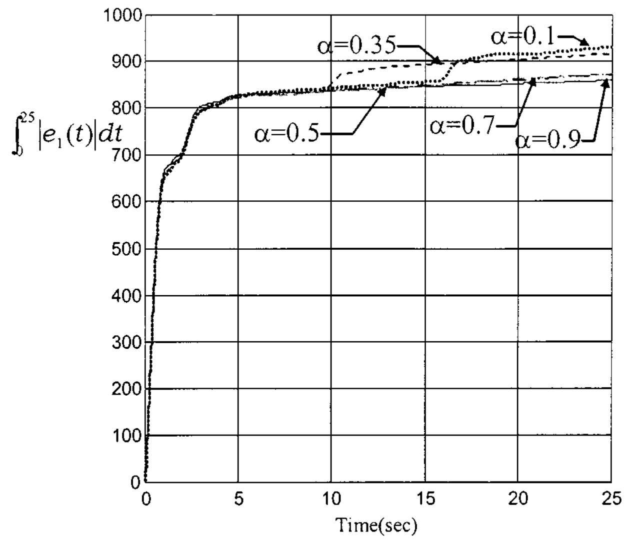

Fig. 17. Tracking performance $\int_0^{25} e_1^2(t)\,dt$ with different α.

VI. Conclusion

An observer-based hybrid direct/indirect adaptive FNN controller appended with a supervisory controller for a class of unknown nonlinear dynamical systems is proposed in this paper. It is a flexible design methodology by the tradeoff between plant knowledge and control knowledge using a weighting factor α adopted to sum together the control effort from indirect adaptive FNN controller and direct adaptive FNN controller. If the fuzzy descriptions of the plant are more important and viable, then choose large α; otherwise, choose small α. Based on the Lyapunov synthesis approach, the free parameters of the adaptive FNN controller can be tuned on-line by an observer-based output feedback control law and adaptive law. Furthermore, it is a valuable idea that the supervisory control is appended into the FNN controller. The supervisory controller will be activated to force the state to be within the constraint set as long as the system tends to be unstable controlled only by the FNN controller. On the other hand, if the FNN controller works well, the supervisory controller will be deactivated. The simulation results show explicitly that the tracking error of larger α is less than smaller α i.e., the plant knowledge is more important and viable, and the supervisory controller only works in the beginning period and after that the FNN controller is a main controller. Two nonlinear systems, namely, inverted pendulum system and Chua's chaotic circuit, are fully illustrated to track sinusoidal signals. Furthermore, it is obvious that the control effort is much less and tracking performance is better than those in previous works.

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