Adaptive Interference Avoidance for Dynamic Wireless Systems: A Game Theoretic Approach

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***Abstract*—In this paper, we present an adaptive algorithm for interference avoidance, which can be applied in a distributed manner by active users in a CDMA wireless system to obtain optimal codewords and powers for specified target signal-to-interference plus noise ratio (SINR). The algorithm is derived using a game-theoretic approach in which separable cost functions with respect to codeword and power are defined, and joint codeword and power adaptation is formulated as a separable game with two corresponding subgames: the noncooperative codeword adaptation game, and the noncooperative power control game. Codeword and power adaptation use incremental updates in the direction of the best strategy, which are desirable in practical implementations since they allow the receiver to follow transmitter changes with corresponding incremental changes of the receiver filter and continue detection of transmitted symbols with high accuracy. The algorithm can also track variable target SINRs or variable number of active users in the system, and is therefore useful for dynamic wireless systems with varying quality of service (QoS) requirements. We illustrate the proposed algorithm with numerical examples obtained from simulations that show convergence and tracking properties of the algorithm for different scenarios.**

***Index Terms*—Code division multiple access (CDMA), interference avoidance, noncooperative game theory, transmitter optimization.**

I. Introduction

INTERFERENCE avoidance has emerged in the literature as a new technique by which transmitters in a wireless communication system are optimized in response to changing patterns of interference to better suit the environment in which they operate under specified QoS [18]. Currently, interference avoidance algorithms are static in the number of users, and do not allow variable QoS [4], [16], [19], [23], [31], and each time these change the algorithms must be reiterated in order to determine the socially optimal solution for the new number of users and/or QoS. We note that other related algorithms for CDMA codeword adaptation [6], [9], [10], [29], [30], [33]–[35] have the same characteristic, and are not adaptable to changing numbers of active users and/or QoS requirements in the system. In order to overcome this limitation, recent research [7] proposes using Grassmannian signatures in dynamic systems with variable number of users in the system. These are designed to support a maximum number of active users in the system subject to a given interference level, and have the nice property that interference among users does not change when less users are active in the system. As noted in [7] the disadvantage associated with equiangular Grassmannian signatures is that they may not exist for any desired system configuration specified by given number of users and processing gain value.

In this paper, we present an alternative approach to dealing with variable number of active users and/or QoS requirements in the uplink of a CDMA system. More specifically, we propose an adaptive algorithm that moves the system incrementally from an optimal configuration with a given number of active users and/or QoS, to a new optimal configuration with a different number of active users and/or QoS. The transition between the two optimal configurations is based on an adaptive interference avoidance procedure: when a change in the system status occurs this translates in a change of the SINR of active users which will employ incremental updates for codeword and power that decrease individual user cost functions subject to constraints on the codewords and SINR. This procedure works similar to the way adaptive equalization tracks changes in time-varying channels by gradient-based techniques for minimizing the channel estimation error.

The proposed algorithm is derived using a game-theoretic approach in which joint CDMA codeword and power adaptation is modeled as a noncooperative game. Noncooperative game theory has been extensively used by economists for a long time to study how rational individuals (players) that do not cooperate interact to reach their goals. Lately, noncooperative game theory has been applied to communications systems [13], and alternative game-theoretic approaches to CDMA codeword and power adaptation in CDMA systems are discussed in [8], [27], [28]. In [8], CDMA codeword adaptation based on interference avoidance is approached from the perspective of potential game theory, while [27] uses noncooperative game theory to approach codeword adaptation in asynchronous systems. In [28] joint CDMA codeword and power adaptation is formulated as a separable game, with two corresponding subgames: the power control game and the codeword (sequence) control game.

Inspired by the approach in [28], in our paper we formulate also two separate games for codeword and power adaptation. However, we define new cost functions for users in the system, which are different than those in [28], and show that these cost functions are convex. Unlike [28], we then use the theory of convex noncooperative games to investigate best re-

Manuscript received August 29, 2006; revised February 14, 2007. This work was supported in part by the Texas Higher Education Coordinating Board (THECB) under Advanced Research Program (ARP) Grant 010115-0013-2006. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Antonia Papandreou-Suppappola.

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Digital Object Identifier 10.1109/JSTSP.2007.897060

sponse strategies as well as existence of Nash equilibrium solutions implied by these cost functions.

The paper is organized as follows. We describe the system model and problem statement in Section II, followed by presentation of the game-theoretic framework for codeword and power optimization in Section III where we discuss best response strategies for users and investigate the Nash equilibrium solution of the game. In Section IV we discuss incremental codeword and power updates in the direction of best response strategy, and formally state the adaptive algorithm for interference avoidance in Section V. We present and discuss numerical examples obtained from simulations in Section VI, and final conclusions in Section VII.

II. System Model and Problem Statement

We consider the uplink of a synchronous CDMA system with K active users in a signal space of dimension N implied by finite common bandwidth and signaling interval constraints [11]. The received signal at the base station is given by:

$$\mathbf{r} = \sum_{k=1}^{K} b_k \sqrt{p_k} \mathbf{s}_k + \mathbf{n} \tag{1}$$

where $\mathbf{s}_k$ is the unit-norm codeword corresponding to user k, b_k is the information symbol transmitted by user k, p_k is the received power at the base station for user k, and $\mathbf{n}$ is the additive white Gaussian noise (AWGN) that corrupts the received signal with zero-mean and covariance matrix equal to identity $\mathbf{W} = E[\mathbf{n}\mathbf{n}^\top] = \sigma^2 \mathbf{I}_N$.

Formally, we note that all user codewords take values in the N-dimensional sphere with radius 1

$$\mathcal{S}_k = \{\mathbf{s}_k \,|\, \mathbf{s}_k \in \mathbb{R}^N, \|\mathbf{s}_k\| = 1\} \quad \forall k = 1, \ldots, K \tag{2}$$

while powers take values in the set defined by the real interval $(0, P_{\text{sup}}]$

$$\mathcal{P}_k = \{p_k \,|\, p_k \in (0, P_{\text{sup}}]\} \quad \forall k = 1, \ldots, K \tag{3}$$

where P_{sup} is the maximum power value.

By defining the $N \times K$ codeword matrix $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_k, \ldots, \mathbf{s}_K]$ having as columns the user codewords, and the $K \times K$ diagonal matrix of received user powers $\mathbf{P} = \text{diag}[p_1, \ldots, p_k, \ldots, p_K]$ we rewrite the received signal in a compact matrix-vector form as

$$\mathbf{r} = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n} \tag{4}$$

where $\mathbf{b} = [b_1 \ldots b_K]^\top$ is the vector containing the information symbols sent by users.

At the receiver a unit norm linear filter, $\mathbf{c}_k$, is used to obtain the decision variable d_k for user k

$$d_k = \mathbf{c}_k^\top \mathbf{r} = \underbrace{b_k \sqrt{p_k} \mathbf{c}_k^\top \mathbf{s}_k}_{\text{desired signal}} + \underbrace{\mathbf{c}_k^\top \left(\sum_{\ell=1, \ell \neq k}^{K} b_\ell \sqrt{p_\ell} \mathbf{s}_\ell + \mathbf{n} \right)}_{\text{interference+noise}}. \tag{5}$$

The SINR for user k is the ratio of the desired signal corresponding to user k at the receiver to the power of interference and noise that affects user k's signal at the receiver, and is expressed as

$$\gamma_k = \frac{p_k \left(\mathbf{c}_k^\top \mathbf{s}_k\right)^2}{\sum_{\ell=1, \ell \neq k}^{K} p_\ell \left(\mathbf{c}_k^\top \mathbf{s}_\ell\right)^2 + E\left[(\mathbf{c}_k^\top \mathbf{n})^2\right]}. \tag{6}$$

We formally define the denominator of the SINR as the user interference function

$$\begin{aligned} i_k &= \sum_{\ell=1, \ell \neq k}^{K} p_\ell \left(\mathbf{c}_k^\top \mathbf{s}_\ell\right)^2 + E\left[\left(\mathbf{c}_k^\top \mathbf{n}\right)^2\right] \\ &= \mathbf{c}_k^\top \underbrace{\left(\sum_{\ell=1, \ell \neq k}^{K} p_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top + \mathbf{W} \right)}_{\mathbf{R}_k} \mathbf{c}_k = \mathbf{c}_k^\top \mathbf{R}_k \mathbf{c}_k \end{aligned} \tag{7}$$

where

$$\begin{aligned} \mathbf{R}_k &= \sum_{\ell=1, \ell \neq k}^{K} p_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top + \mathbf{W} \\ &= \underbrace{\mathbf{S}\mathbf{P}\mathbf{S}^\top + \mathbf{W}}_{\mathbf{R}} - p_k \mathbf{s}_k \mathbf{s}_k^\top \\ &= \mathbf{R} - p_k \mathbf{s}_k \mathbf{s}_k^\top \end{aligned} \tag{8}$$

is the correlation matrix of the interference+noise seen by user k and

$$\mathbf{R} = \mathbf{S}\mathbf{P}\mathbf{S}^\top + \mathbf{W} \tag{9}$$

is the correlation matrix of the received signal in (4). We note that the interference function for a given user k depends explicitly on user k receiver filter $\mathbf{c}_k$ as well as on all the other users codewords and powers $\mathbf{s}_\ell, p_\ell, \forall \ell \neq k$. The interference function does not depend on user k's power, and depends implicitly on user k's codeword since the receiver filter $\mathbf{c}_k$ depends usually on $\mathbf{s}_k$. We also note that interference functions have been defined in previous work on power control [36] as well as power and codeword adaptation [27], [28].

In the case of matched filter (MF) receivers $\mathbf{c}_k = \mathbf{s}_k$ the interference function (7) has the expression

$$i_k^{\text{MF}} = \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \tag{10}$$

and the SINR expression (6) becomes

$$\gamma_k^{\text{MF}} = \frac{p_k}{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k} \tag{11}$$

The MF performs single user detection, and is the simplest receiver filter that one can use for decoding user k. The MF is optimal in the presence of AWGN and maximizes the signal-to-noise ratio, and is suboptimal in the presence of interference caused by the other users.

The minimum mean squared error (MMSE) receiver filter is the linear receiver that minimizes the mean squared error (MSE) between the transmitted symbol and its corresponding decision variable, and is the optimal linear multiuser detector that maximizes also the SINR [14], [32]. The expression of the normalized MMSE receiver is [14]

$$\mathbf{c}_k = \frac{1}{\sqrt{\mathbf{s}_k^\top \mathbf{R}_k^{-2} \mathbf{s}_k}} \mathbf{R}_k^{-1} \mathbf{s}_k \tag{12}$$

for which the interference function expression (7) becomes

$$i_k^{\mathrm{MMSE}} = \frac{\mathbf{s}_k^\top \mathbf{R}_k^{-1} \mathbf{s}_k}{\mathbf{s}_k^\top \mathbf{R}_k^{-2} \mathbf{s}_k} \tag{13}$$

and the corresponding SINR expression (6) is

$$\gamma_k^{\mathrm{MMSE}} = p_k \mathbf{s}_k^\top \mathbf{R}_k^{-1} \mathbf{s}_k. \tag{14}$$

We note that when user codewords and powers are Generalized Welch Bound Equality (GWBE) sequences, MF are equivalent to MMSE receivers [33]–[35].

Our goal is to derive a distributed algorithm in which users individually adjust codewords and powers to meet a set of specified target SINRs $\{\gamma_1^*, \ldots, \gamma_k^*, \ldots, \gamma_K^*\}$. We note that K users with specified SINR requirements, are admissible in the uplink of a CDMA system with signal space dimension N if and only if the sum of their effective bandwidths

$$e(\gamma_k^*) = \frac{\gamma_k^*}{1+\gamma_k^*} \quad k = 1, \ldots, K \tag{15}$$

is less than the dimension of the signal space [34], [35], that is

$$\sum_{k=1}^{K} e(\gamma_k^*) < N. \tag{16}$$

In the case of an overloaded system, with $K > N$, the desired stopping point for the algorithm is a set of GWBE user codewords and powers with eventual oversized users[1] for which the sum of allocated powers among all valid power allocations for the given target SINRs is minimum [34], [35]. According to [34] and [35], with specified target SINRs for users, optimal codewords and powers satisfy the following properties.

- The ℓ oversized users have orthogonal codewords, and the optimal power for oversized user $i = 1, \ldots, \ell$ with target SINR γ_i is $p_i = \sigma^2 \gamma_i^*$. The number of oversized users ℓ in the system is implied by the relation [35]

$$(N-\ell) e(\gamma_\ell^*) > \sum_{j=\ell+1}^{K} e(\gamma_j^*) \geq (N-\ell) e(\gamma_{\ell+1}^*) \tag{17}$$

  which assumes that the target SINRs are specified in decreasing order, that is $\gamma_1^* \geq \gamma_2^* \geq \ldots \geq \gamma_K^*$.
- The $K - \ell$ nonoversized users will share a subspace of dimension $N - \ell$ that is not occupied by oversized users, and will use GWBE user codewords that satisfy

$$\sum_{j=\ell+1}^{K} p_j \mathbf{s}_j \mathbf{s}_j^\top = \frac{c \sum_{j=\ell+1}^{K} e(\gamma_j^*)}{N-\ell} \mathbf{I}_{N-\ell} \tag{18}$$

[1]A user is said to be oversized if the effective bandwidth implied by its target SINR is large relative to the effective bandwidths implied by the other user's target SINRs [34], [35].

with user powers

$$p_j = c e(\gamma_j^*) \quad j = \ell+1, \ldots, K \tag{19}$$

where

$$c = \frac{(N-\ell)\sigma^2}{N - \ell - \sum_{j=\ell+1}^{K} e(\gamma_j^*)}. \tag{20}$$

In the case of an underloaded system, with $K \leq N$, the desired stopping point is one where user codewords are orthogonal, since in this case users do not interfere with each other.

The proposed algorithm is based on an adaptive interference avoidance procedure [26], and will be derived using the noncooperative game theory framework presented in Section III.

III. Formulation as a Noncooperative Game

A noncooperative game is formally defined by the set of players, the sets of strategies (or actions) that each player may take, and the individual player utility or cost functions [15]. The game is noncooperative in the sense that a given player is interested only in minimization of its individual cost function, without paying attention to how its actions affect the other players. For the wireless system described in Section II the players are the active users in the system, and their corresponding strategies are adaptation of codeword and power with strategy spaces formally defined by (2) and (3).

The user cost function is associated with the use of system resources and the satisfaction experienced by users as a result of their actions. Cost functions for wireless systems depend usually on the transmitted power as well as on the QoS desired by a given user in the system. In general, it is desired that user cost functions satisfy the following properties [5]:

- The user cost function increases with increasing user power. The reason for this is that power is a valuable commodity for a mobile terminal, and transmitting at the lowest required power to achieve specified QoS will extend the terminal's battery life and contribute to increasing its level of satisfaction. In addition, transmitting at lowest power for the specified QoS will contribute to minimizing the amount of interference experienced by the other users in the system.
- The cost function decreases with decreasing interference. The reason for this is that for fixed user power decreasing interference will increase the user SINR, and implicitly its satisfaction level.

In our paper we consider the user cost function of a given user to be the product between its power and its corresponding interference function

$$u_k = p_k i_k \quad \forall k = 1, \ldots, K. \tag{21}$$

We note that this function satisfies the two general properties mentioned above. We consider that MF are used at the receiver, which are simple and widely used in practice, so that the actual expression of the cost function becomes

$$u_k = p_k i_k^{\mathrm{MF}} = p_k \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \quad \forall k = 1, \ldots, K. \tag{22}$$

As will become apparent later on, this is not a restriction since our goal is to reach a GWBE set of codewords and powers in which case MF and MMSE receivers are equivalent.

The cost function in (22) is separable with respect to the two parameters that define the user strategy, the corresponding codeword and power, since the interference function does not depend on the user power p_k although it depends on the user codeword $\mathbf{s}_k$ [28]. As noted in [28], this property leads to a separable game with two separate subgames: one for codeword adaptation, and one for power control. We will investigate these two subgames in the following sections, and will discuss best response strategies in terms of codeword and power updates that will minimize the user cost functions. We will also investigate the existence of Nash equilibria for the two subgames, and will use the result in [28] to show that the joint codeword and power adaptation game has a Nash equilibrium provided that Nash equilibria for the two subgames exist.

A. *Noncooperative Codeword Adaptation Game (NCG)*

In this game, user powers are fixed, and individual users adjust only their codewords in their corresponding strategy spaces (2) in order to minimize their corresponding cost function. The NCG is formally defined as

$$\mathrm{NCG} = \langle \mathcal{K}, \{\mathcal{S}_k\}_{k\in\mathcal{K}}, \{u_k(\cdot)\}_{k\in\mathcal{K}} \rangle \tag{23}$$

where the three components of the game are as follows.

1) $\mathcal{K} = \{1, \dots, K\}$ is the set of players which are the active users in the system.
2) $\mathcal{S}_k$ is the set of strategies for player k defined in (2)
3) $u_k : \mathcal{S} \longrightarrow (0, \infty)$ is kth user cost function that maps the joint strategy space $\mathcal{S} = \mathcal{S}_1 \times \ldots \times \mathcal{S}_K$ to the set of positive real numbers.

Individual users select their strategies to minimize their corresponding cost functions for a given set of powers, that is

$$\min_{\mathbf{s}_k} u_k \,|_{\mathbf{P}=\mathrm{fixed}} \quad \forall k = 1, \dots, K. \tag{24}$$

In order to investigate the existence of Nash equilibrium for NCG we state the following formal definitions from game theory in the context of our problem.

Definition 1 (Nash Equilibrium for NCG): The codeword matrix $\mathbf{S} = [\mathbf{s}_1, \dots, \mathbf{s}_{k-1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \dots, \mathbf{s}_K]$ is a Nash equilibrium of the NCG if for every user $k \in \mathcal{K}$ we have that

$$\begin{aligned} &u_k(\mathbf{s}_1, \dots, \mathbf{s}_{k-1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \dots, \mathbf{s}_K) \\ &\quad \le u_k(\mathbf{s}_1, \dots, \mathbf{s}_{k-1}, \mathbf{s}'_k, \mathbf{s}_{k+1}, \dots, \mathbf{s}_K) \\ &\qquad\qquad\qquad\qquad\qquad \forall \mathbf{s}'_k \in \mathcal{S}_k. \end{aligned}$$

Definition 2 (Best Response for NCG): The best response function of user k to the other users' strategies is the set

$$\begin{aligned} B_k^{\mathbf{s}} = \{\mathbf{s}_k \in \mathcal{S}_k \,|\, &u_k(\mathbf{s}_1, \dots, \mathbf{s}_{k-1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \dots, \mathbf{s}_K) \\ &\le u_k(\mathbf{s}_1, \dots, \mathbf{s}_{k-1}, \mathbf{s}'_k, \mathbf{s}_{k+1}, \dots, \mathbf{s}_K) \quad \forall \mathbf{s}'_k \in \mathcal{S}_k\}. \end{aligned}$$

Definition 3 (Convex Game): A game is convex for a closed, convex, and bounded joint strategy space $\mathcal{S}$, if the cost function of each user u_k is convex in $\mathbf{s}_k$ for every fixed $\mathbf{s}_\ell$, such that $\ell \neq k$.

We note that the user cost function in (22) is a quadratic form in the user codeword $\mathbf{s}_k$, which implies that is twice differentiable, and differentiating it twice with respect to $\mathbf{s}_k$ we get

$$\frac{\partial^2 u_k}{\partial \mathbf{s}_k^2} = 2p_k \mathbf{R}_k. \tag{25}$$

Since $\mathbf{R}_k$ is a symmetric positive definite matrix, we have that the user cost function is convex, which implies also that NCG is a convex game. We also note that the result proved in Theorem 1 in [24] for concave games can be extended in a straightforward way to prove existence of a Nash equilibrium point for convex games. As a consequence, we get that a Nash equilibrium point for NCG exists.

In order to find a Nash equilibrium for NCG one could use the approach suggested in many game theory textbooks (see for example [15]) in which the best response function B_k^s of each user is identified, followed by selection of an appropriate strategy in B_k^s. Alternatively, in order to identify properties of the Nash equilibrium, one can look at the minimization of individual user cost functions. This is a constrained optimization problem for which the differential form and the Kuhn–Tucker (KT) conditions can be used to identify necessary and sufficient conditions. Thus, the best response in terms of codeword updates can be found by solving the constrained optimization problem of minimizing the user cost function subject to unit norm constraints on the user codewords

$$\min_{\mathbf{s}_k} u_k \quad \text{subject to} \quad \mathbf{s}_k^\top \mathbf{s}_k = 1. \tag{26}$$

In order to solve this problem we define user k's Lagrangian function

$$\begin{aligned} L_k^{\mathbf{s}}(\mathbf{s}_k, \lambda_k) &= u_k + \lambda_k \left(\mathbf{s}_k^\top \mathbf{s}_k - 1\right) \\ &= p_k \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k + \lambda_k \left(\mathbf{s}_k^\top \mathbf{s}_k - 1\right) \end{aligned} \tag{27}$$

where λ_k is the Lagrange multiplier associated with the constraints on the norm of $\mathbf{s}_k$ in (26).

The necessary conditions for minimizing the Lagrangian in (27) are obtained by differentiating with respect to $\mathbf{s}_k$ and λ_k, and equating the corresponding partial derivatives to zero. Differentiating with respect to the codeword $\mathbf{s}_k$ leads to the eigenvalue/eigenvector equation

$$\frac{\partial L_k^{\mathbf{s}}(\mathbf{s}_k, \lambda_k)}{\partial \mathbf{s}_k} = 0 \quad \Longrightarrow \quad \mathbf{R}_k \mathbf{s}_k = \nu_k \mathbf{s}_k \tag{28}$$

with $\nu_k = -\lambda_k / p_k$. We note that $\lambda_k < 0$, and that for any eigenvector of $\mathbf{R}_k$ we have that $\partial L_k^{\mathbf{s}}(\mathbf{s}_k, \lambda_k)/\partial \mathbf{s}_k = 0$ which satisfies the necessary condition in (28). In this context, the best response for user k is the eigenvector $\mathbf{x}_k$ corresponding to the minimum eigenvalue[2] ν_k^* of $\mathbf{R}_k$, since this choice minimizes the effective interference corrupting user k's signal at the receiver. Thus, at a Nash equilibrium all user codewords will be the minimum eigenvectors of their corresponding interference-plus-noise correlation matrices. We note that the best response strategy based on minimum eigenvector replacement defines a greedy interference avoidance procedure for replacing user codewords [18]. We also note that the second necessary condition implied by

[2]This is also referred to as the minimum eigenvector of $\mathbf{R}_k$.

$\partial L_k^{\mathbf{s}}(\mathbf{s}_k, \lambda_k)/\partial \lambda_k = 0$, is not relevant since our model assumes that users are assigned only unit norm codewords.

In order to investigate whether the minimum eigenvector strategy is also optimal with respect to the constrained minimization of user k cost function, we use the approach in [2, Ch. 3], which involves expanding the Lagrangian function in Taylor series around the point satisfying the necessary KT conditions. In this expansion, the term containing the first derivative disappears since the derivative is equal to zero due to KT conditions, the higher order terms are neglected, and the expansion is essentially left with a differential quadratic form. For the Lagrangian expression in (27) the second order term in the Taylor expansion is

$$D_k^{\mathbf{s}} = (-1) \begin{vmatrix} \frac{\partial^2 L_k^{\mathbf{s}}(\mathbf{s}_k, \lambda_k)}{\partial \mathbf{s}_k^2} & \frac{\partial^2 L_k^{\mathbf{s}}(\mathbf{s}_k, \lambda_k)}{\partial \lambda_k \partial \mathbf{s}_k} \\ \left(\frac{\partial^2 L_k^{\mathbf{s}}(\mathbf{s}_k, \lambda_k)}{\partial \lambda_k \partial \mathbf{s}_k}\right)^\top & \frac{\partial^2 L_k^{\mathbf{s}}(\mathbf{s}_k, \lambda_k)}{\partial \lambda_k^2} \end{vmatrix}$$
$$\text{for } \mathbf{s}_k, \lambda_k = \text{best response}, \quad k = 1, \ldots, K \quad (29)$$

and is positive if the point that satisfies the KT conditions is also the constrained minimum of the Lagrangian. As already mentioned, the best response implies that $\mathbf{s}_k$ is the minimum eigenvector of $\mathbf{R}_k$, for which the value of λ_k that achieves the specified target SINR γ_k^* for user k is $\lambda_k = -\gamma_k^* p_k$. Thus, if a Nash equilibrium point of the NCG satisfies

$$D_k^{\mathbf{s}} = (-1) \begin{vmatrix} 2p_k(\mathbf{R}_k - \gamma_k^* \mathbf{I}_N) & 2\mathbf{s}_k \\ 2\mathbf{s}_k^\top & 0 \end{vmatrix} > 0$$
$$k = 1, \ldots, K \quad (30)$$

it is also an optimal equilibrium point with respect to the constrained minimization of the user cost function.

We conclude this section by noting that a given Nash equilibrium point for the NCG does not correspond to a single codeword matrix $\mathbf{S}$, but rather to an entire class of matrices that can be related by unitary transformations which preserve the spectrum of the cross-correlation matrix $\mathbf{S}\mathbf{S}^\top$.

B. Noncooperative Power Control Game (NPG)

In this game, user codewords are fixed, and individual users adjust only their powers in their corresponding strategy spaces (3) in order to minimize their corresponding cost function. The NPG is formally defined as

$$\text{NPG} = \langle \mathcal{K}, \{\mathcal{P}_k\}_{k \in \mathcal{K}}, \{u_k(\cdot)\}_{k \in \mathcal{K}} \rangle \quad (31)$$

where the three components of the game are as follows.

1) $\mathcal{K} = \{1, \ldots, K\}$ denotes the set of players which are the active users in the system as in the case of NCG.
2) $\mathcal{P}_k$ is the set of strategies for player k defined in (3).
3) $u_k : \mathcal{P} \longrightarrow (0, \infty)$ is kth user cost function that maps the joint strategy space $\mathcal{P} = \mathcal{P}_1 \times \ldots \times \mathcal{P}_K$ to the set of positive real numbers.

Individual users select their strategies to minimize their corresponding cost functions for a given set of user codewords, that is

$$\min_{\mathbf{p}_k} u_k \,|_{\mathbf{S}=\text{fixed}} \quad \forall k = 1, \ldots, K. \quad (32)$$

Similar to the previous section, we make some formal definitions before investigating the existence of a Nash equilibrium for NPG.

Definition 4 (Nash Equilibrium for NPG): The set of user powers $\mathbf{p} = \{p_1, \ldots, p_{k-1}, p_k, p_{k+1}, \ldots, p_K\}$ is a Nash equilibrium for NPG if for every user $k \in \mathcal{K}$, we have that

$$u_k(p_1, \ldots, p_{k-1}, p_k, p_{k+1}, \ldots, p_K)$$
$$\leq_k (p_1, \ldots, p_{k-1}, p_k', p_{k+1}, \ldots, p_K) \quad \forall p_k' \in \mathcal{P}_k.$$

Definition 5 (Best Response for NPG): The best response function of user k to the other users' strategies is the set

$$B_k^p = \{p_k \in \mathcal{P}_k \,|\, u_k(p_1, \ldots, p_{k-1}, p_k, p_{k+1}, \ldots, p_K)$$
$$\leq u_k(p_1, \ldots, p_{k-1}, p_k', p_{k+1}, \ldots, p_K) \quad \forall p_k' \in \mathcal{S}_k\}.$$

We note that NPG can also be considered a convex game according to Definition 3, since the user cost function is linear in p_k. Thus, existence of a Nash equilibrium is also guaranteed as in the case of NCG. We use a similar line of reasoning as in the previous section to identify properties of the Nash equilibrium, by looking at the constrained minimization of individual user cost functions. In this case the best response in terms of power updates can be found by solving the convex constrained optimization problem of minimizing the user cost function subject to constraints on the user SINR

$$\min_{p_k} u_k \quad \text{subject to} \quad p_k = \gamma_k^* \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k. \quad (33)$$

Similar to the previous section, we define user k's Lagrangian function

$$L_k^p(p_k, \eta_k) = u_k + \eta_k \left(p_k - \gamma_k^* \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k\right)$$
$$= p_k \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k + \eta_k \left(p_k - \gamma_k^* \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k\right) \quad (34)$$

where η_k is the Lagrange multiplier associated with the power constraint in (33). The necessary KT conditions in this case imply that the best response in this case is $p_k = i_k \gamma_k^*$, with $\eta_k = i_k$. This best response strategy for NPG is also optimal in this case since we have that $D_k^p = 1 > 0$ for all $k = 1, \ldots, K$.

C. Noncooperative Codeword Adaptation and Power Control Game (NCPG)

This game of joint codeword adaptation and power control consists of the two separable subgames NCG and NPG. Formally the NCPG is defined as

$$\text{NCPG} = \langle \mathcal{K}, \{\mathcal{S}_k \times \mathcal{P}_k\}_{k \in \mathcal{K}}, \{u_k(\cdot)\}_{k \in \mathcal{K}} \rangle \quad (35)$$

where the three components of the game are as defined in the previous two subsections. Using the result in Theorem 1 in [28] we note that a Nash equilibrium solution for NCPG exists and is defined by codeword matrix $\mathbf{S}$ and power matrix $\mathbf{P}$ if and only if $\mathbf{S}$ represents a Nash equilibrium for NCG and $\mathbf{P}$ represents a Nash equilibrium for NPG. Since we have shown that both NCG and NPG have Nash equilibria, we conclude that a Nash equilibrium for NCPG also exists. This Nash equilibrium will be optimal with respect to constrained minimization of the user cost function if the sufficient conditions for optimality for codewords in (30) are satisfied.

At the optimal Nash equilibrium point of the game each user's strategy is a best response function to the other users' strategies, and all user codewords are minimum eigenvectors of their corresponding interference+noise correlation matrices, that is

$$\mathbf{R}_k\mathbf{s}_k^* = \nu_k\mathbf{s}_k^* \quad \text{for } k = 1, 2, \ldots, K \tag{36}$$

where ν_k is the minimum eigenvalue of $\mathbf{R}_k$ and $\mathbf{s}_k^*$ is the corresponding minimum eigenvector. Using the expression of $\mathbf{R}_k$ in terms of $\mathbf{R}$ in (8) we obtain from (36) that codewords are also eigenvectors of the the received signal correlation matrix $\mathbf{R}$

$$\mathbf{R}\mathbf{s}_k^* = (\nu_k + p_k)\mathbf{s}_k^*, \qquad k = 1, \ldots, K. \tag{37}$$

In addition, because $\mathbf{s}_k^*$ is the minimum eigenvector in (36), it is also the eigenvector of $\mathbf{R}_k^{-1}$ corresponding to maximum eigenvalue $1/\nu_k$. This implies that at the Nash equilibrium MF and MMSE filters are equivalent receivers that yield the same SINR

$$\gamma_k^{\mathrm{MF}} = \frac{p_k}{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k} = \frac{p_k}{\nu_k} = p_k \mathbf{s}_k^\top \mathbf{R}_k^{-1} \mathbf{s}_k = \gamma_k^{\mathrm{MMSE}}. \tag{38}$$

According to Section V-B in [35] this implies that the corresponding user codewords and powers form a GWBE set that satisfies (18) and (19), with eventual oversized users determined according to (17).

IV. Best Response Versus Incremental Strategies

In order to find a Nash equilibrium for NCPG one must play the two subgames, NCG and NPG, by using their corresponding best response strategies. However, we note that these best response strategies may lead to new user codewords that are distant in signal space from the current user codewords, and/or abrupt power changes to meet the target SINRs. This behavior is not desirable in the practical operation of a system since it may lead to increased probability of error at the receiver or even connection loss between the transmitter and the receiver which is not able to adapt to these sudden changes. From a practical perspective, a more desirable approach is to change the user codewords and powers in small increments, with corresponding incremental changes of the receiver filter that follow the transmitter codeword changes. This will allow the receiver to continue detecting transmitted symbols with high accuracy.

In this section, we discuss the use of incremental codeword and power updates. At a given instance t of the NCPG, codeword adaptation for user k is defined by the incremental update

$$\mathbf{s}_k(t+1) = \frac{\mathbf{s}_k(t) + m\beta\mathbf{x}_k(t)}{\|\mathbf{s}_k(t) + m\beta\mathbf{x}_k(t)\|} \tag{39}$$

where $\mathbf{x}_k(t)$ is the minimum eigenvector of corresponding matrix $\mathbf{R}_k$, and is the best response for NCG, $m = \mathrm{sgn}(\mathbf{s}_k^\top \mathbf{x}_k)$, and β is a parameter that limits how far in terms of Euclidian distance the updated codeword can be from the old codeword. This is an incremental interference avoidance codeword update in the direction of the best response of the NCG, which implies a decrease in the interference function i_k since [26]

$$\mathbf{s}_k^\top(t)\mathbf{R}_k\mathbf{s}_k(t) \geq \mathbf{s}_k^\top(t+1)\mathbf{R}_k\mathbf{s}_k(t+1). \tag{40}$$

User power will be adjusted also in small increments using a gradient-based approach

$$p_k(t+1) = p_k(t) - \mu[p_k(t) - \gamma_k^* i_k(t)] \tag{41}$$

with $0 < \mu < 1$, which is also an incremental power update in the direction of the best response of the NPG defined by $p_k = i_k\gamma_k^*$.

We note that the proposed incremental codeword and power update (39) and (41) are descent-based optimization methods [3] that move the system incrementally in the direction of the minimum cost. We also note that interference avoidance algorithms based on incremental updates do not get trapped in suboptimal points as it may happen in the case of regular interference avoidance [22], since incremental adaptation has effects similar to the "noisy MMSE iteration" proposed in [1] to escape suboptimal points.

V. The Adaptive Interference Avoidance Algorithm

The proposed algorithm for adaptive interference avoidance consists of two main steps, which correspond to the two subgames NCG and NPG that make up NCPG, and which are performed sequentially by users in the system. Instead of using the best response functions discussed in Section III the algorithm employs the incremental updates mentioned in Section IV. The algorithm reacts to changes in the system configuration which may occur as a result of various events like for example admitting new active users into the system, dropping idle/inactive users, or changing the target SINRs of active users. We assume that there is no latency associated with a change in the number of active users, and that the algorithm adds/eliminates users to/from the system as soon as changes occur by immediately updating the codeword and power matrices ($\mathbf{S}$ and $\mathbf{P}$) according to the changes. A formal statement of the algorithm is given below:

Adaptive Interference Avoidance Algorithm

Initial Data:
- Codewords, powers, and desired (target) SINRs for active users (matrices $\mathbf{S}$ and $\mathbf{P}$, and values $\gamma_1^*, \ldots, \gamma_K^*$).
- Noise covariance matrix $\mathbf{W}$
- Constants μ, β, and tolerance ϵ.

Triggering Event:
- The SINR of active users with specified codewords and power does not match the target SINRs.

- New active users are admitted: their codewords, powers, and target SINRs are added to those of the active users in the system by augmenting the corresponding matrices and increasing K accordingly.
- Idle/inactive users are dropped: their codewords, powers, and target SINRs are removed from the corresponding matrices, and K is decreased accordingly.

Admissibility check:

- IF admissibility condition in (16) is satisfied GO TO *Adaptation Stage*, ELSE STOP: the system changed to an unfeasible configuration.

Adaptation Stage:

1) IF change in cost function is bigger than ϵ for any user GO TO Step 3, ELSE a Nash equilibrium has been reached.
2) IF optimality condition in (30) is true STOP: an optimal configuration has been reached, ELSE GO TO Step 3.
3) FOR each user $k = 1, \ldots, K$ DO
    a) Compute current $\mathbf{R}_k(i)$ using (8) and determine the minimum eigenvector $\mathbf{x}_k(i)$.
    b) Replace the current codeword $\mathbf{s}_k(i)$ using codeword update (39).
    c) Update user k's power using (41).
4) GO TO Step 1.

We note that Steps 3b and 3c of the algorithm perform incremental codeword and power adaptation in the direction of the best response strategy. We also note that the sum of user cost functions defined as in [24] is decreasing at each iteration, and together with the check of the optimality condition in (30) performed at Step 2 this ensures that the algorithm will converge to the optimal Nash equilibrium point that corresponds to a GWBE set of user codewords and powers with eventual oversized users as mentioned in Section II.

It is also important to note that codeword and power updates of individual users do not require explicit knowledge of the other users' codewords and powers. The only piece of information required by a given user k is its corresponding interference+noise correlation matrix $\mathbf{R}_k$ which can be derived from the correlation matrix of the received signal $\mathbf{R}$ by subtracting the contribution of given user k as implied by (8). Thus, provided that $\mathbf{R}$ is made available to individual users through a feedback channel [12], [25], the algorithm can be implemented in a distributed manner and run independently by active users to adapt to changes in the system configuration as reflected by changes of their SINRs and corresponding cost functions. The distributed implementation of the algorithm which requires knowledge of matrix $\mathbf{R}$ only, implies also that the computational cost of the algorithm for each user will be the same and independent of the number of active users, since the dimension of the eigenvalue/eigenvector problem for matrix $\mathbf{R}_k$ is $N \times N, \forall k = 1, \ldots, K$.

VI. Simulations, Numerical Examples, and Discussions

In order to corroborate our theoretical findings and illustrate the proposed algorithm we performed extensive simulations for various scenarios, which we discuss in detail in this section. More specifically, in Section VI-A we look at the variation of user powers, SINRs, and cost functions after a triggering event occurs, and illustrate it with plots obtained using particular examples. In Section VI-B we investigate the convergence speed of the algorithm using Monte Carlo simulations for a specific scenario with given number of users K and signal dimension N. We also look at convergence speed for increasing K and N such that the ratio K/N remains constant. In Section VI-C we discuss how quantization affects the accuracy of the optimal Nash equilibrium solution of the algorithm, and illustrate quantization effects on a particular example. We conclude with Section VI-D in which we discuss extensions of the algorithm to multiple cell scenarios.

A. Variation of User Powers, SINRs, and Cost Functions After a Triggering Event Occurs

In this section we consider a CDMA system with $K = 5$ users in a signal space of dimension $N = 4$ and AWGN with $\sigma^2 = 0.1$. The algorithm constants are $\mu = 0.1, \beta = 0.2$, and tolerance $\epsilon = 10^{-6}$.

1) SINR of Active Users With Initial Codewords and Powers Does Not Match Target SINRs: The user codeword matrix was initialized randomly and user powers were set up to 0.1. We set different target SINRs for users

$$\gamma^* = \{5, 4, 3, 2.5, 1.5\}$$

such that they satisfy the admissibility condition in (16), and there are no oversized users.

The algorithm yields the codeword matrix

$$\mathbf{S} = \begin{bmatrix} -0.4202 & -0.2009 & 0.3371 & -0.6872 & 0.7331 \\ -0.8887 & 0.4132 & 0.0556 & 0.4023 & -0.1394 \\ 0.1699 & 0.8879 & -0.1876 & -0.5791 & -0.0786 \\ 0.0689 & 0.0230 & 0.9209 & -0.1748 & -0.6610 \end{bmatrix}$$

and power matrix

$$\mathbf{P} = \text{diag}\{1.1024, 1.0583, 0.9921, 0.9449, 0.7937\}$$

These values are within $\mathcal{O}(10^{-6})$ tolerance from the corresponding values implied by (18), and which imply user SINR values within $\mathcal{O}(10^{-6})$ tolerance from the specified target SINRs. The weighted correlation matrix is $\mathbf{SPS}^\top = 1.2228\mathbf{I}_4$, and is within similar tolerance from the corresponding matrix implied by (18).

Next, we present a similar example in which one of the users is oversized. We start with the same initial codeword matrix and powers, and specify the target SINRs as

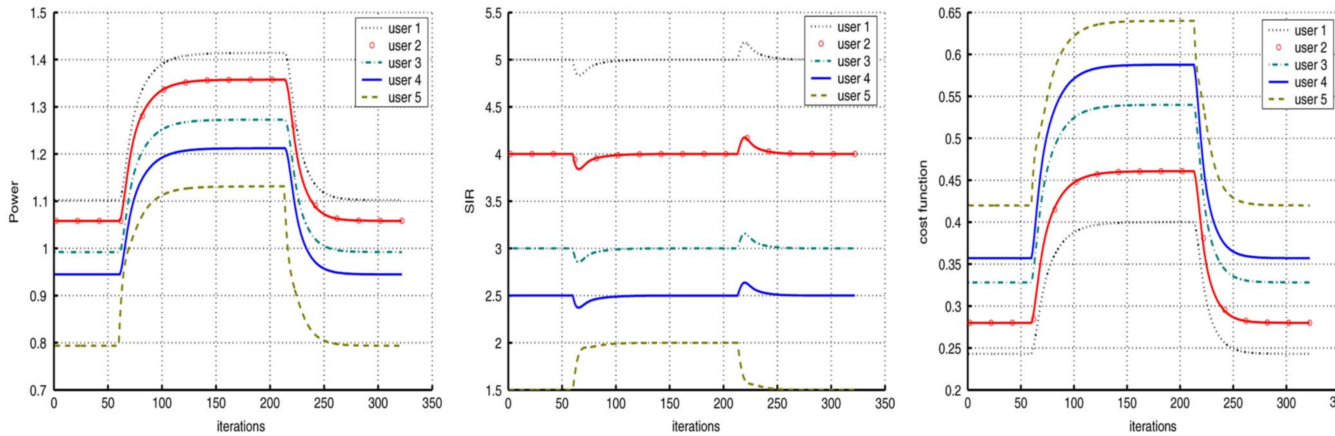
$$\gamma^* = \{9, 2, 2, 2, 2\}$$

which satisfy also the admissibility condition in (16). However, in this case user 1 is oversized according to (17). In addition, for this experiment we choose $\sigma^2 = 1$ in order to compare our results with those in [35]. The algorithm yields the codeword matrix

$$\mathbf{S} = \begin{bmatrix} -0.3310 & -0.1194 & 0.3943 & -0.9223 & 0.4086 \\ -0.7321 & 0.6542 & 0.1967 & 0.3828 & 0.0747 \\ 0.5466 & 0.7230 & 0.1176 & -0.0254 & 0.6308 \\ 0.2362 & 0.1873 & 0.8900 & -0.0472 & -0.6554 \end{bmatrix}$$

and power matrix

$$\mathbf{P} = \text{diag}\{9, 6, 6, 6, 6\}$$



(a) (b) (c)

Fig. 1. Variation of user power, SINR, and cost function for the target SINR tracking example. (a) Power variation. (b) SINR variation. (c) Cost function variation.

which imply user SINRs within the same $\mathcal{O}(10^{-6})$ from specified target SINRs. The weighted correlation matrix is

$$\mathbf{SPS}^\top = \begin{bmatrix} 8.1095 & 0.2423 & -0.1809 & -0.0782 \\ 0.2423 & 8.5359 & -0.4002 & -0.1730 \\ -0.1809 & -0.4002 & 8.2986 & 0.1291 \\ -0.0782 & -0.1730 & 0.1291 & 8.0557 \end{bmatrix}$$

In this case we also look at the matrix of user cross-correlations

$$\mathbf{S}^\top\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.3333 & 0.3333 & 0.3333 \\ 0 & 0.3333 & 1 & -0.3333 & -0.3333 \\ 0 & 0.3333 & -0.3333 & 1 & -0.3333 \\ 0 & 0.3333 & -0.3333 & -0.3333 & 1 \end{bmatrix}$$

This shows that oversized user 1 is orthogonal to the other nonoversized users. We note that user 1 had no *a priori* knowledge of its oversized status in the system, and performed the same updates as the other users which resulted in the right codewords and powers for this system. We also note that although our weighted correlated matrix is not identical to that in the similar example in [35] which is $\mathrm{diag}\{9,8,8,8\}$, they are related by a linear transformation. This can be determined by "aligning" the signal space directions to the oversized user 1. The new user codeword matrix after alignment is

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5047 & 0.2794 & 0.9807 & 0.1966 \\ 0 & 0.8539 & -0.4274 & -0.1934 & -0.6198 \\ 0 & -0.1272 & 0.8598 & -0.0272 & -0.7597 \end{bmatrix}$$

and implies weighted correlation matrix identical to that in [35]

$$\mathbf{SPS}^\top = \mathrm{diag}\{9,8,8,8\}.$$

2) Tracking Variable SINRs With Fixed Number of Active Users: In this section, we illustrate the tracking ability of the proposed algorithm, for fixed number of active users and variable target SINRs. This feature is useful in systems where variable QoS requirements may lead to variable target SINRs. We assume that the system starts with the same initial codewords, powers, and target SINRs as in our first example, and that a steady state configuration (user codeword and powers) is reached. We then change the target SINR for the last user to new value $\gamma_5^* = 2$, let the algorithm reach a new steady state configuration, and then change user 5 SINR back to its old value $\gamma_5^* = 1.5$. The first steady state will have the same codeword and power matrices as in first example of the previous section. After the second stage of the experiment the codeword matrix changes to

$$\mathbf{S} = \begin{bmatrix} -0.4063 & -0.1841 & 0.3652 & -0.6668 & 0.7335 \\ -0.8975 & 0.4140 & 0.0551 & 0.4086 & -0.1297 \\ 0.1633 & 0.8915 & -0.1906 & -0.5923 & -0.0885 \\ 0.0522 & 0.0026 & 0.9095 & -0.1938 & -0.6613 \end{bmatrix}$$

and the power matrix becomes

$$\mathbf{P} = \mathrm{diag}\{1.4141, 1.3576, 1.2727, 1.2121, 1.1313\}$$

implying actual SINR values equal to (new) specified targets, and weighted correlation matrix $\mathbf{SPS}^\top = 1.5969\,\mathbf{I}_4$. After the third stage of the experiment, when the system returns to the original set of target SINRs we obtain essentially the same user codeword and power matrices as in the end of stage 1 of the experiment. Fig. 1 shows that user powers, SINRs, and cost functions vary smoothly during this experiment.

During this experiment, we compared also the efficiency of the MF receiver with that of the MMSE receiver, for which results are presented in Table I. We note that after the first stage of the experiment, when an optimal GWBE set of user codewords and powers is reached, the two receivers are equivalent and imply the same SINR. However, as the SINR of user 5 changes at iteration 62, the MF becomes sub-optimal until the second stage of the experiment is completed at iteration 120, when the new optimal GWBE configuration is reached. The same situation is true for the third stage of the experiment, when the system transitions back to the original set of target SINRs, between 214 and 300. Nevertheless, as shown by the SINR values in Table I, we note that the MF and MMSE receivers imply essentially the same SINR values during the transition periods. As a consequence, we note that the use of MF receiver during system transitions from one optimal configuration to another one implies performance very close to the optimal MMSE performance, and

TABLE I
MF EFFICIENCY DURING ALGORITHM ITERATIONS

Iteration	User1 SINR		User2 SINR		User3 SINR		User4 SINR		User5 SINR	
	MMSE	MF	MMSE	MF	MMSE	MF	MMSE	MF	MMSE	MF
50	5.000	5.0000	4.0000	4.0000	3.0000	3.0000	2.5000	2.5000	1.5000	1.5000
61	5.000	5.0000	4.0000	4.0000	3.0000	3.0000	2.5000	2.5000	1.5000	1.5000
62	4.8926	4.8875	3.8966	3.8920	2.9065	2.9027	2.4149	2.4115	1.6786	1.6786
80	4.9433	4.9433	3.9501	3.9500	2.9554	2.9554	2.4587	2.4587	1.9606	1.9606
100	4.9852	4.9852	3.9866	3.9866	2.9879	2.9879	2.4889	2.4889	1.9898	1.9898
110	4.9922	4.9922	3.9930	3.9930	2.9937	2.9937	2.4942	2.4942	1.9946	1.9946
120	5.000	5.0000	4.0000	4.0000	3.0000	3.0000	2.5000	2.5000	2.0000	2.0000
212	5.000	5.0000	4.0000	4.0000	3.0000	3.0000	2.5000	2.5000	2.0000	2.0000
214	5.0489	5.0475	4.0445	4.0432	3.0386	3.0375	2.5351	2.5341	1.9116	1.9116
225	5.1311	5.1309	4.1202	4.1200	3.1093	3.1091	2.5980	2.5978	1.5629	1.5629
250	5.0138	5.0138	4.0121	4.0121	3.0106	3.0106	2.5096	2.5096	1.5086	1.5086
270	5.021	5.0021	4.0018	4.0018	3.0016	3.0016	2.5015	2.5015	1.5013	1.5013
280	5.000	5.0000	4.0000	4.0000	3.0000	3.0000	2.5000	2.5000	1.5000	1.5000
300	5.000	5.0000	4.0000	4.0000	3.0000	3.0000	2.5000	2.5000	1.5000	1.5000

will not disrupt symbol detection at the receiver and/or cause outages due to increased probability of symbol error.

3) Tracking Variable Number of Active Users: In this section, we illustrate the ability of the proposed algorithm to track variable number of active users in the system. We start with $K = 6$ active users with different target SINRs

$$\gamma^* = \{1.6, 1.5, 1.4, 1.3, 1.2, 1\}$$

that satisfy the admissibility condition in (16). The user codeword matrix is initialized randomly, and the user power matrix is taken $P = 0.1\mathbf{I}_6$, which imply initial SINRs below the specified targets. This triggers the adaptation stage of the algorithm which yields codeword matrix (see the equation at the bottom of the page)

$$\mathbf{P}_1 = \mathrm{diag}\{0.4168, 0.4064, 0.3951, 0.3828, 0.3694, 0.3386\}$$

and power matrix for which the weighted codeword correlation matrix is $\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^\top = 0.5773\,\mathbf{I}_4$, and corresponds to a GWBE set [35]. At this point the last user becomes inactive, and is dropped from the system. Thus, the new number of active users becomes $K = 5$, with codeword matrix containing only the first five columns of $\mathbf{S}_1$ derived previously and diagonal power matrix containing only the first five diagonal elements in $\mathbf{P}_1$ derived previously. We assume that the remaining active users keep the same target SINRs as before

$$\gamma^* = \{1.6, 1.5, 1.4, 1.3, 1.2\}$$

which satisfy the admissibility condition in (16) and implies that the new configuration is feasible. This change in system configuration triggers the adaptation stage of the algorithm which yields new optimal codeword and power matrices

$$\mathbf{S}_2 = \begin{bmatrix} -0.4766 & -0.3566 & 0.4410 & -0.4400 & 0.7271 \\ -0.8305 & 0.1226 & -0.0140 & 0.7155 & -0.0902 \\ -0.2841 & 0.9255 & -0.1094 & -0.5257 & 0.0305 \\ -0.0500 & 0.0352 & 0.8907 & -0.1342 & -0.6799 \end{bmatrix}$$

and

$$\mathbf{P}_2 = \mathrm{diag}\{0.2257, 0.2201, 0.2139, 0.2073, 0.2001\}$$

that imply the specified target SINRs, with the new weighted codeword correlation matrix $\mathbf{S}_2\mathbf{P}_2\mathbf{S}_2^\top = 0.2668\,\mathbf{I}_4$ corresponding to a GWBE configuration for the new number of users. We now add a new active user in the system, such that the new number of active users becomes $K = 6$ again. The new user codeword is chosen randomly and appended as the sixth column to matrix $\mathbf{S}_2$ to form a 4×6 codeword matrix. The new user power $p_6 = 0.1$ is added to the previous user powers to form a 6×6 diagonal matrix, and user 6 target SINR $\gamma_6^* = 1.1$ is added to the previous set of user target SINRs. We note that

$$\mathbf{S}_1 = \begin{bmatrix} -0.4931 & -0.4450 & 0.3958 & -0.4143 & 0.8285 & 0.2053 \\ -0.7941 & 0.3157 & 0.1983 & 0.7698 & 0.0200 & -0.3045 \\ -0.3541 & 0.8363 & -0.2955 & -0.4762 & 0.0258 & 0.5934 \\ 0.0282 & -0.0539 & 0.8466 & 0.0947 & -0.5591 & 0.7162 \end{bmatrix}$$

$$\mathbf{S}_3 = \begin{bmatrix} -0.2531 & -0.1550 & 0.3295 & -0.2189 & 0.6060 & 0.9903 \\ -0.9015 & 0.0994 & -0.0660 & 0.7701 & -0.1546 & 0.0538 \\ -0.3501 & 0.9816 & -0.1255 & -0.5875 & 0.0121 & -0.0305 \\ -0.0271 & 0.0516 & 0.9334 & -0.1183 & -0.7802 & 0.1241 \end{bmatrix}$$

and

$$\mathbf{P}_3 = \mathrm{diag}\{0.4343, 0.4234, 0.4117, 0.3989, 0.3849, 0.3697\}$$

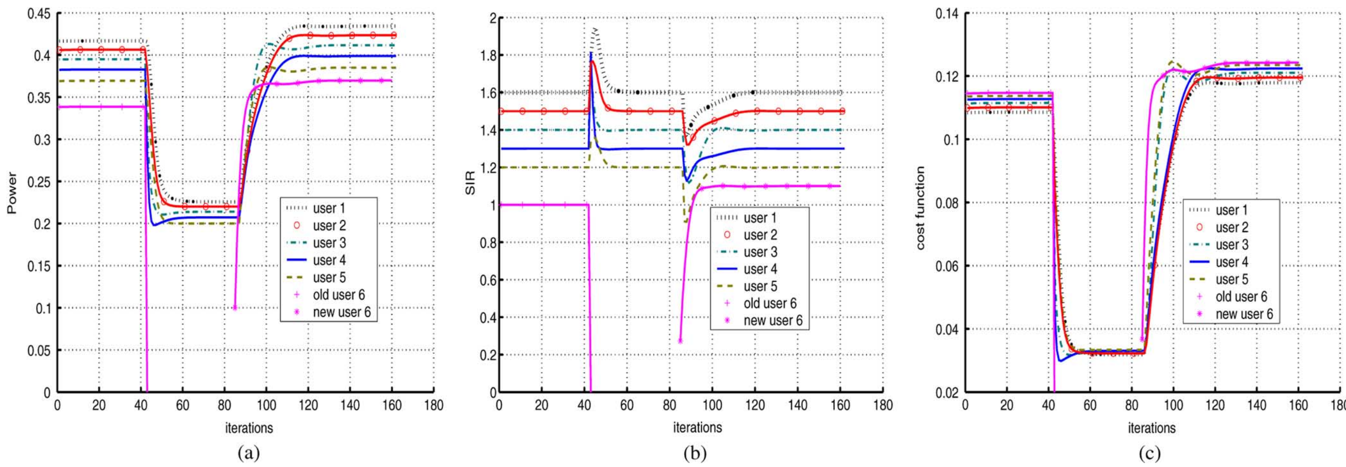


Fig. 2. Variation of user power, SINR, and cost function for variable number of active users. (a) Power variation. (b) SINR variation. (c) Cost function variation.

new sum of effective bandwidths is equal to $3.4332 < N$ which implies that the new system configuration is also feasible. This new change in system configuration triggers again the adaptation stage of the algorithm which yields new optimal codeword and power matrices (see the equation at the bottom of the next page) which imply the specified target SINRs for users, and for which the new weighted codeword correlation matrix $\mathbf{S}_3\mathbf{P}_3\mathbf{S}_3^\top = 0.6057\,\mathbf{I}_4$ corresponds to a new GWBE set. Fig. 2 shows the smooth variations of user powers, SINRs, and cost functions during the tracking of variable number of users. We note that the system remains in a optimal configuration until iteration 38 when user 6 is dropped from the system. This implies an increase of user SINRs above their corresponding targets due to decreasing multiuser interference.

We have also looked at the particular situation when the number of active users in the system becomes smaller than the dimension of the signal space. In this case, as was expected, an orthogonal set of codewords was yielded by the algorithm as shown in the following example. We started with the system having $K = 5$ users in $N = 4$ dimensions, target SINRs $\gamma^* = \{2, 3, 4, 5, 6\}$, and codeword and power matrices

$$\mathbf{S}_1 = \begin{bmatrix} -0.2742 & -0.3159 & 0.4577 & -0.4687 & 0.7646 \\ -0.9512 & 0.3891 & 0.1418 & 0.5292 & 0.1115 \\ -0.1021 & 0.8647 & -0.2284 & -0.6637 & 0.0162 \\ -0.0986 & 0.0336 & 0.8475 & -0.2445 & -0.6345 \end{bmatrix}$$

and

$$\mathbf{P}_1 = \mathrm{diag}\{2.8717, 3.2307, 3.4460, 3.5896, 3.6922\}.$$

We drop user 1 from the system, so that the new number of active users becomes $K = 4$, and we assume that these continue to keep the original target SINRs $\gamma^* = \{3, 4, 5, 6\}$. This change in system configuration triggers the adaptation stage of the algorithm which in this case yields the codeword matrix

$$\mathbf{S}_2 = \begin{bmatrix} -0.5373 & -0.5294 & 0.1951 & -0.6262 \\ -0.8192 & 0.3242 & 0.1063 & 0.4620 \\ -0.0879 & 0.7423 & -0.2300 & -0.6236 \\ 0.1804 & 0.2522 & 0.9475 & -0.0747 \end{bmatrix}$$

which satisfies $\mathbf{S}_2\mathbf{S}_2^\top = \mathbf{S}_2^\top\mathbf{S}_2 = \mathbf{I}_4$, and implies that in the resulting configuration the active users are orthogonal. This is the optimal configuration when the number of users in the system is less or equal to the dimensionality of the signal space, and in this case the user powers in matrix

$$\mathbf{P}_2 = \mathrm{diag}\{0.3000, 0.4000, 0.5000, 0.6000\}$$

are proportional to the corresponding SINRs achieved $\gamma = \{3, 4, 5, 6\}$ which are within $\mathcal{O}(10^{-6})$ tolerance from the specified target SINRs.

B. *Convergence Speed*

In order to investigate the convergence of the proposed algorithm for adaptive interference avoidance in dynamic systems we consider a scenario in which we have initially $K = 6$ active users in $N = 4$ signal dimensions, with target SINRs $\gamma^* = \{1.6, 1.5, 1.4, 1.3, 1.2, 1\}$. For this system we ran 10 000 simulations of the proposed algorithm initialized with randomly generated user codewords and powers, and plot the number of ensemble iterations to reach the optimal Nash equilibrium configuration of user codeword and powers in Fig. 3(a). When the optimal configuration has been reached in each of these 10 000 simulations, we drop the last user from the system and plot the number of ensemble iterations to reach a new optimal configuration with $K = 5, N = 4$, and same target SINRs $\gamma^* = \{1.6, 1.5, 1.4, 1.3, 1.2\}$ for the remaining active users. The number of ensemble iterations needed to reach a new optimal configuration with $K = 5$ and $N = 4$ from $K = 6$ and $N = 4$ is plotted in Fig. 3(b). Once the new optimal configuration with $K = 5$ and $N = 4$ is reached we add a new active user in the system with target SINR equal to 1.1, and with randomly generated codeword and power, and plot the number of ensemble iterations to a new optimal configuration with $K = 6, N = 4$, and target SINRs $\gamma^* = \{1.6, 1.5, 1.4, 1.3, 1.2, 1.1\}$ in Fig. 3(c). From the plots in Fig. 3 we note that in all the considered cases the convergence speed of the algorithm to an optimal configuration of codewords and powers for a given set of target SINRs is well approximated by a Gaussian distribution, with a mean of about

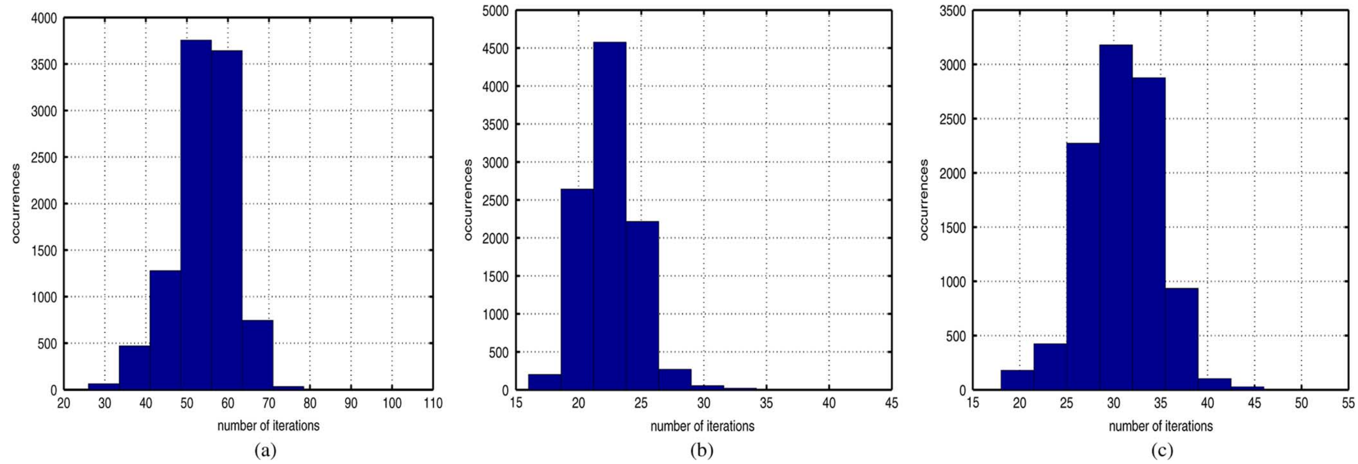


Fig. 3. Monte Carlo simulations to investigate convergence of the proposed algorithm. (a) Random codewords and powers. (b) Removing one active user. (c) Adding a new user.

TABLE II
THE CONVERGENCE SPEED OF THE DYNAMIC ALGORITHM FOR DIFFERENT CONFIGURATIONS

N	**K**	Number of iterations required for convergence for		
		Random user codewords	GWBE codewords and increasing target SINR $2 \nearrow 3$	GWBE codewords and decreasing target SINR $3 \searrow 2$
4	5	35	65	19
16	20	30	65	19
32	40	28	65	19
64	80	37	65	19
128	160	36	65	19

55 iterations when initial codewords and powers are random, 23 iterations when one user is dropped from the system, and approximately 32 iterations when a new user is added to the system.

We have also observed from simulations that the number of ensemble iterations to an optimal configuration depends on the desired target SINR values, and the higher these are, the slower the convergence of the algorithm will be. However, the noise level does not seem to be a critical issue, and this is usually the case in CDMA systems which are interference limited rather than noise limited. In noisy environments when noise level is high and the signal-to-noise ratio (SNR) is low, all users can increase their powers by the same relative amount to boost their SNR without changing the relative interference experienced by users in the system (which is usually measured in terms of the signal-to-interference ratio—SIR). This will only affect the life of users' batteries which will be exhausted faster. Otherwise, if users start with a low initial SNR, then with the same fixed algorithm step μ, the algorithm needs more iterations to reach the same target SINRs.

We conclude this section with an empirical analysis of the convergence speed of the proposed algorithm for increasing numbers of active users in the system and signal space dimensions. In Table II, we present the number of iterations needed to reach convergence within tolerance $\epsilon = 10^{-4}$, for algorithm parameters $\mu = 0.1$ and $\beta = 0.2$, for different numbers of users K and N such that the ratio K/N remains constant and equal to $K/N = 5/4$. We note that while there are minor variations in the number of iterations needed for convergence when user codewords are randomly chosen, this is essentially the same when the system transitions from one GWBE configuration to a different one.

C. *Quantization Effects*

The proposed algorithm for adaptive interference avoidance is designed for jointly convex codeword and power strategy spaces $\mathcal{S}$ and $\mathcal{P}$, and yields real-valued codewords and powers that can take any value in $\mathcal{S}_k$ and $\mathcal{P}_k$. However, practical implementation of the algorithm in hardware employing digital signal processors usually uses a finite number of values corresponding to scalar quantization of user codewords and powers. Nevertheless, provided that the quantization interval q used to quantize each dimension of the user codewords as well as user powers is sufficiently small to ensure a convex perception of the quantized joint codeword and power strategy spaces—$\mathcal{S}^q$ and $\mathcal{P}^q$—by the algorithm, this will converge to a point which is the quantized version of the optimal Nash equilibrium. This is similar to the case of adaptive equalizers, where the equalizer coefficients are obtained as the result of convex optimization problems, but where the optimization signal space is quantized with a sufficiently small quantization step.

To illustrate the effects of scalar quantization on the proposed algorithm we consider that in Steps 3b and 3c of the algorithm user codeword and power are updated with uniform quantized versions of corresponding (39) and (41). We take the same system as in Section VI-A1 with $K = 5$ active users in a signal space of dimension $N = 4$ and AWGN with $\sigma^2 = 0.1$, with target SINRs $\gamma^* = \{5, 4, 3, 2.5, 1.5\}$, and algorithm constants

$\mu = 0.1, \beta = 0.2$, and $\epsilon = 10^{-6}$. With a uniform quantization interval $q = 0.01$ the algorithm yields the following user codeword matrix

$$\mathbf{S}_q = \begin{bmatrix} 0.8700 & -0.2700 & 0.2500 & -0.5000 & 0.0900 \\ -0.4300 & -0.4500 & -0.2800 & -0.8100 & -0.4500 \\ 0.2300 & 0.5500 & -0.8500 & -0.2900 & 0.2000 \\ -0.0800 & -0.6600 & -0.3800 & 0.1600 & 0.8600 \end{bmatrix}$$
and
$$\mathbf{P}_q = \text{diag}\{1.1000, 1.0300, 0.9700, 0.9700, 0.7700\}.$$

We note that although elements of $\mathbf{S}_q$ are quite different than corresponding elements in $\mathbf{S}$ corresponding to optimal Nash equilibrium point obtained in Section VI-A1, user powers are actually quantized versions of the optimal power values obtained in Section VI-A1, and imply the corresponding weighted correlation matrix

$$\mathbf{S}_q\mathbf{P}_q\mathbf{S}_q^\top = \begin{bmatrix} 1.2170 & 0.0074 & 0.0155 & -0.0032 \\ 0.0074 & 1.2804 & 0.0257 & 0.0233 \\ 0.0155 & 0.0257 & 1.1830 & 0.0066 \\ -0.0032 & 0.0233 & 0.0066 & 1.1901 \end{bmatrix}$$

whose elements are within $\mathcal{O}(q)$ tolerance from the corresponding values obtained in Section VI-A1 $\mathbf{SPS}^\top = 1.2228\,\mathbf{I}_4$. We also note that decreasing the quantization interval q will imply a smaller difference between the codewords and powers yielded by the algorithm and the codewords and powers corresponding to the optimal Nash equilibrium. However, coarser quantization implied by larger values of q leads the algorithm to stop further away from the optimal Nash equilibrium, and yield codewords and powers for which the corresponding target SINRs are no longer close to the desired target SINR values. For the particular system considered in our example we have observed that this situation occurs for a quantization interval q of approximately 0.032. This value $(q = 0.032)$ implies 63 intervals for uniform scalar quantization of codeword elements in the set $[-1, 1]$, and requires 6 bits for encoding. The fact that performance of the proposed adaptive algorithm for interference avoidance deteriorates in this case comes in agreement with the empirical studies on codeword quantization for interference avoidance in [17], which show that performance of interference avoidance algorithms degrades when uniform quantizers with 6 bits or less are used for codeword quantization.

D. Extension to Multicell Scenarios

In a multicell scenario we have a set of users and bases that are distributed over a given geographical area, such that subsets of users communicate directly with a given base, while creating interference to the other bases for which their transmission is not intended. One possible approach to the multicell scenario is to assume collaboration among bases [20], [21], [28]. In this case the system can be regarded as a single base system with multiple inputs as multiple outputs (MIMO) [20], [21] which enables application of algorithms joint codeword design and power optimization established for single base systems. However, in this case gains from users to bases must be explicitly considered, and as discussed in [28], there may exist particular link gains which lead to instability of the system in this approach. Instability is not desirable in real systems, and makes implementation of algorithms for joint codeword design and power optimization impractical. In addition, the collaborative approach requires a high-speed backbone network and/or high data rate feedback and control channels which can lead to delays and implementation issues in large geographical areas.

An alternative approach is to consider that there is no collaboration among bases in the multicell scenario. This approach is practical in unlicensed systems where competing service providers with possibly different air interfaces may operate in different cells, making collaboration either impractical (due to competition), or impossible (due to incompatibility of air interfaces), or both. In this case, a given base may regard the intercell interference coming from other cells in the system as a colored Gaussian noise process, and consider that the covariance matrix of the intercell interference-plus-noise seen by the cell's base station receiver is a general symmetric and positive definite matrix $\tilde{\mathbf{W}}$, as opposed to the scaled identity matrix $\mathbf{W}$ that corresponds to the additive white Gaussian noise used in the single-cell case. We note that for any symmetric and positive definite matrix $\tilde{\mathbf{W}}$, the convexity of user cost function, u_k, in both $\mathbf{s}_k$ and p_k is preserved, which implies that for convex and bounded joint codeword and power strategy spaces, $\mathcal{S}$ and $\mathcal{P}$, the NCG and NPG will be convex games for which a Nash equilibrium exists. Furthermore, according to the discussion in Section III-C. the NCPG will also have a Nash equilibrium for which the optimal codewords and powers are according to [34].

We illustrate this approach for the system with $K = 5$ users in a signal space of dimension $N = 4$, and with covariance matrix of the intercell interference plus noise seen by the cell base station receiver $\tilde{\mathbf{W}} = \text{diag}\{0.8501, 0.5167, 0.3471, 0.1467\}$. Assuming the same initial target SINR as in Section VI-A, $\gamma^* = \{5, 4, 3, 2.5, 1.5\}$, the algorithm yields the codeword matrix

$$\mathbf{S} = \begin{bmatrix} -0.1991 & -0.3642 & 0.4354 & -0.5732 & 0.7592 \\ -0.5444 & 0.4253 & 0.3477 & 0.6727 & 0.4292 \\ 0.3082 & 0.8252 & -0.2891 & -0.3985 & 0.4891 \\ 0.7544 & 0.0747 & 0.7784 & 0.2450 & -0.0152 \end{bmatrix}$$

and power matrix

$$\mathbf{P} = \text{diag}\{5.1268, 4.9217, 4.6141, 4.3944, 3.6912\}$$

for which the weighted correlation matrix $\mathbf{SPS}^\top + \tilde{\mathbf{W}} = 6.1522\,\mathbf{I}_4$ is within $\mathcal{O}(10^{-6})$ tolerance from the corresponding GWBE values implied by [34].

VII. Conclusion

In this paper we proposed an adaptive algorithm for interference avoidance for dynamic wireless systems which can be applied in a distributed manner by active users in the systems to derive optimal codewords and powers for given target SINRs. The algorithm is derived using a game-theoretic approach in which separable cost functions with respect to codeword and power are defined, such that joint codeword and power adaptation is formulated as a separable game, with two corresponding subgames: the noncooperative codeword adaptation game NCG, and the noncooperative power control game NPG. The best response strategies of users are then obtained by constrained minimization of the user cost function subject to constraints on user SINR and codeword norm. However, the codeword and power updates used in the algorithm are based on incremental updates in the direction of the best strategy. Such incremental adaptation procedure is desirable in practical implementations since it allows the receiver to follow codeword changes at the transmitter with corresponding incremental changes of the receiver filter that allow continued detection of transmitted symbols with high accuracy.

If the specified target SINRs are admissible then the algorithm always yields GWBE user codewords and powers [34], [35] with eventual oversized users that have codewords which are orthogonal to the other users' codewords. The algorithm can also track variable target SINRs or variable number of active users in the system, and is therefore useful for dynamic wireless systems with varying QoS requirements.

The proposed algorithm is illustrated with numerical examples obtained from simulations which illustrate convergence and tracking properties of the algorithm for different scenarios.

Acknowledgment

The authors are grateful to the anonymous reviewers for their constructive comments on the paper.

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