Adaptive Multi-objective Particle Swarm Optimization Algorithm

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Abstract— **In this article we describe a novel Particle Swarm Optimization (PSO) approach to Multi-objective Optimization (MOO) called Adaptive Multi-objective Particle Swarm Optimization (AMOPSO). AMOPSO algorithm's novelty lies in its adaptive nature, that is attained by incorporating inertia and the acceleration coefficient as control variables with usual optimization variables, and evolving these through the swarming procedure. A new diversity parameter has been used to ensure sufficient diversity amongst the solutions of the non dominated front. AMOPSO has been compared with some recently developed multi-objective PSO techniques and evolutionary algorithms for nine function optimization problems, using different performance measures.**

I. Introduction

Evolutionary algorithms have been found to be very efficient in dealing with multi-objective optimization (MOO) problems [1] due to their population based nature.

Particle Swarm Optimization (PSO), has been relatively recently proposed in 1995 [2]. It is inspired by the flocking behavior of birds, which is very simple to simulate. The simplicity and efficiency of PSO [3], [4] in single objective optimization motivated the researchers to apply it to the MOO [5]–[12].

In MOO, two important goals to attain are the convergence to the Pareto-optimal front and the even spread of the solutions on the front. In the present article we describe a multi-objective PSO, called adaptive multi-objective particle swarm optimization (AMOPSO). In AMOPSO the vital parameters of the PSO i.e., inertia and acceleration coefficients are adapted with the iterations, making it capable of effectively handling optimization problems of different characteristics. Here these vital parameters are treated as control parameters and are also subjected to evolution along with the other optimization variables. To overcome premature convergence, the mutation operator from [13] has been incorporated in AMOPSO.

In order to improve the diversity in the Pareto-optimal solutions, a novel method exploiting the nearest neighbor concept is used. This method for measuring diversity has an advantage that it needs no parameter specification, unlike the one in [11]. Note that diversity has earlier been incorporated in PSO using different approaches, namely the hyper-grid approach [11], σ-method with clustering [9] and NSGA-II based approach [10]. Both the hyper-grid and clustering based approaches for diversity are found to take significant computational time. In the former, the size of the hyper-grid needs to be specified a priori, and the performance depends on its proper choice. The measure adopted in this article is similar to the one in [10], though the way of computing the distance to the nearest neighbor is different. Comparative study of AMOPSO with other multi-objective PSOs viz., σ–MOPSO [9], NSPSO (Non-dominated sorting PSO) [10] and MOPSO [11] and also other multi-objective evolutionary methods viz., NSGA-II [14] and PESA-II [15] has been conducted to establish its effectiveness for nine test problems using qualitative measures and also visual displays of the Pareto front.

II. Basic Principles

A. Multi-objective Optimization (MOO)

A general minimization problem of M objectives can be mathematically stated as: Given $\vec{x} = [x_1, x_2, \ldots, x_d]$, where d is the dimension of the decision variable space,

- Minimize : $\vec{f}(\vec{x}) = [f_i(\vec{x}), i = 1, \ldots, M]$, subject to : $g_j(\vec{x}) \leq 0, \quad j = 1, 2, \ldots, J$, and $h_k(\vec{x}) = 0, \quad k = 1, 2, \ldots, K$, where $f_i(\vec{x})$ is the i^{th} objective function, $g_j(\vec{x})$ is the j^{th} inequality constraint, and $h_k(\vec{x})$ is the k^{th} equality constraint.

A solution is said to dominate another solution if it is not worse than that solution in all the objectives and is strictly better than that in at least one objective. The solutions over the entire solution space that are not dominated by any other solution are called *Pareto-optimal solutions*.

B. Particle Swarm Optimization (PSO)

The PSO is a population based optimization algorithm, inspired by the flocking behavior of birds [2]. The population of the potential solutions is called a *swarm* and each individual solution within the *swarm*, is called a *particle*. Particles in PSO fly in the search domain guided by their individual experience and the experience of the swarm.

Considering a d-dimensional search space, the i^{th} particle is associated with the position attribute $\vec{X}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,d})$, the velocity attribute $\vec{V}_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,d})$ and the individual experience attribute $\vec{P}_i = (p_{i,1}, p_{i,2}, p_{i,d})$. The position attribute ($\vec{X}_i$) signifies the position of the particle in the search space, whereas the velocity attribute ($\vec{V}_i$) is responsible for imparting motion to it. The $\vec{P}_i$ parameter stores the position (coordinates) corresponding to the particle's best individual performance. Similarly the experience of whole of the swarm is captured in the index g, which corresponds to the particle with the best overall performance in the swarm. The movement of the particle towards the optimum solution is governed by updating its position and velocity attributes.

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The velocity and position update equations are given as [4]:

$$v_{i,j} = wv_{i,j} + c_1r_1(p_{i,j} - x_{i,j}) + c_2r_2(p_{g,j} - x_{i,j}) \quad (1)$$

$$x_{i,j} = x_{i,j} + v_{i,j} \quad (2)$$

where $j = 1, \ldots, d$ and w, c_1, $c_2 \geq 0$. w is the inertia weight, c_1 and c_2 the acceleration coefficients, and r_1 and r_2 are random numbers, generated uniformly in the range $[0, 1]$, responsible for imparting randomness to the flight of the swarm. The c_1 and c_2 values allow the particle to tune the cognition and the social terms respectively in the velocity update equation (Equation 1). A larger value of c_1 allows exploration, while a larger value of c_2 encourages exploitation.

III. Adaptive Multi-objective Particle Swarm Optimization: AMOPSO

The AMOPSO algorithm proposed in this article is described below.

A. Initialization

In the initialization phase of AMOPSO, the individuals of the swarm are assigned random values for the coordinates, from the respective domains, for each dimension. Similarly the velocity is initialized to zero in each dimension. The Step 1 takes care of the initialization of AMOPSO. This algorithm maintains an archive for storing the best non-dominated solutions found in the flight of the particles. The size of the archive l_t at each iteration is allowed to attain a maximum value of N_a. Archive is initialized to contain the non-dominated solutions from the swarm.

Algorithm AMOPSO: O_f= AMOPSO(N_s,N_a,C,d)
/* N_s: size of the swarm, N_a: size of the archive, C: maximum number of iterations, d: the dimensions of the search space, O_f: the final output */

```
1) $t = 0$, randomly initialize $S_0$,
   /*$S_t$: swarm at iteration $t$ */
   • initialize $x_{i,j}$ ,$\forall i, \quad i \in \{1, \ldots, N_s\}$ and $\forall j$,
     $j \in \{1, \ldots, d\}$
     /* $x_{i,j}$: the $j^{th}$ coordinate of the $i^{th}$ particle */
   • initialize $v_{i,j}$, $\forall i, \quad i \in \{1, \ldots, N_s\}$ and $\forall j$,
     $j \in \{1, \ldots, d\}$
     /* $v_{i,j}$: velocity of $i^{th}$ particle in $j^{th}$ dimension */
   • $Pb_{i,j} \leftarrow x_{i,j}$ ,$\forall i, \quad i \in \{1, \ldots, N_s\}$ and $\forall j$,
     $j \in \{1, \ldots, d\}$
     /* $Pb_{i,j}$: the $j^{th}$ coordinate of the personal best of
     the $i^{th}$ particle */
   • $A_0 \leftarrow non\_dominated(S_0)$, $l_0 = |A_0|$
     /* returns the non-dominated solutions from the swarm*/
     /* $A_t$: archive at iteration $t$ */
2) for $t = 1$ to $t = C$,
   • for $i = 1$ to $i = N_s$   /* update the swarm $S_t$ */
     – /* updating the velocity of each particle */
       * $Gb \leftarrow get\_gbest()$
         /* returns the global best */
       * $Pb_i \leftarrow get\_pbest()$
         /* returns the personal best */
       * $adjust\_parameters(w_i, c_1^i, c_2^i)$
         /* adjusts the parameters, $w_i$: the inertia coefficient, $c_1^i$: the local acceleration coefficient,
         and $c_2^i$: the global acceleration coefficient */
         $v_{i,j} = w_iv_{i,j} + c_1^ir_1(Pb_{i,j} - x_{i,j}) + c_2^ir_2(Gb_j - x_{i,j})$
         $\forall j, \quad j \in \{1, \ldots, d\}$
     – /* updating coordinates */
       $x_{i,j} = x_{i,j} + v_{i,j}$
       $\forall j, \quad j \in \{1, \ldots, d\}$
   • /* updating the archive */
     – $A_t \leftarrow non\_dominated(S_t \cup A_t)$
     – if ($l_t > N_a$) $truncate\_archive()$
       /* $l_t$: size of the archive */
   • mutate ($S_t$)   /* mutating the swarm */
3) $O_f \leftarrow A_t$ and stop.   /* returns the Pareto optimal front */
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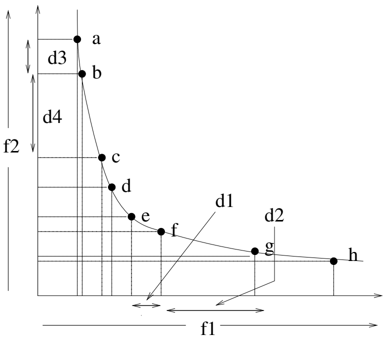

Fig. 1. AMOPSO Diversity computation

B. Update

The Step 2 of AMOPSO deals with the flight of the particles within the swarm through the search space. The flight, given by Equations 1 and 2, is influenced by many vital parameters, which are explained below:

1) Personal Best Performance (pbest): In multi-objective PSO, the *pbest* stores the best non-dominated solution attained by the individual particle. In AMOPSO the present solution is compared with the *pbest* solution, and it replaces the *pbest* solution only if it dominates that solution.

2) Global Best Performance (gbest) : In multi-objective PSO, often the multiple objectives involved in MOO problems are conflicting in nature thus making the choice of a single optimal solution difficult. To resolve this problem the concept of non-dominance is used. Therefore instead of having just one individual solution as the global best a set of all the non-dominated solutions is maintained in the form of an *archive* [11]. Selecting a single particle from the archive as the $gbest$ is a vital issue. There has been a number of attempts to address this issue, some of which may be found in [16]. In the concepts mentioned in [16], authors have considered non-dominance in the selection of the $gbest$ solution. As the attainment of proper diversity is the second objective of MOO, it has been used in AMOPSO to select the most sparsely populated solution from the archive as the $gbest$. In AMOPSO the diversity measurement has been done using a novel concept. This concept is similar to the *crowding-distance* measure in [14]. The parameter (d_i) is computed as the distance of each solution to its immediate next neighbor summed over each of the M objectives. An example in Figure 1 illustrates the computation of d_i parameter. Density for all the solutions in the archive is obtained. Based on the density values as fitness, roulette wheel selection is done to select a solution as the *gbest*.

3) Control Parameters (w): The performance of PSO to a large extent depends on its inertia weight (w) and the acceleration coefficients (c_1 and c_2). In this article these are called *control parameters*. These parameters in AMOPSO have been adjusted using the function $adjust\_parameters(w_i, c_1^i, c_2^i)$.

Some values have been suggested for these parameters in the literature [4]. In most of the cases the values of these parameters were found to be problem specific, signifying the use of adaptive parameters [3], [4], [17]. In this article a novel concept for adapting the control parameters has been proposed. These control parameters have been subjected to optimization through swarming, in parallel with that of the normal optimization variables. The intuition behind this concept is to evolve the control parameters also so that the appropriate values of these parameters may be obtained for a specific problem. Here the control parameters have been initially assigned some random values in the range suggested in [4]. They are then updated using the following equations:

$$v_{i,j}^c = w_i v_{i,j}^c + c_1^i r_1 (p_{i,j}^c - x_{i,j}^c) + c_2^i r_2 (p_{g,j}^c - x_{i,j}^c) \quad (3)$$

$$x_{i,j}^c = x_{i,j}^c + v_{i,j}^c \quad (4)$$

Here, $x_{i,j}^c$ is the value of the j^{th} control parameter with the i^{th} particle, whereas $v_{i,j}^c$, $p_{i,j}^c$ are the velocity and personal best for the j^{th} control variable with i^{th} particle. $p_{g,j}^c$ is the global best for the j^{th} control parameters. The previous iteration values of the control parameters have been used for the corresponding values of w_i, c_1^i and c_2^i. It is to be noted that in the above equations the values of j equal to, 1, 2 and 3, corresponds to the control parameters inertia weight, cognition acceleration coefficient and the global acceleration coefficient respectively. Note that the control parameters for the flight of a particle are taken from its own copy of control variables.

4) Mutation: The *mutation* operator plays a key role in MOPSOs [11]. In this article a mutation operator of [13] has been used to allow better exploration of the search space.

5) Update Archive: The selection of the *gbest* solution for the velocity update is done from this archive only. In AMOPSO the maximum size of the archive has been fixed to $N_a = 100$. The archive gets updated by the non dominated solutions of the swarm. All the dominated members from the archive are removed. Since the maximum size of the archive has been fixed, the density has been considered as in [11] to truncate the archive to the desired size.

After running AMOPSO for a fixed number of iterations, the archive is returned as the resultant non dominated set.

IV. Experimental Results

The effectiveness of AMOPSO has been demonstrated on various standard test problems, that have known set of Pareto optimal solutions and are characterized to test the algorithms on different aspects of performance. AMOPSO has been compared with some MOEAs and MOPSOs. The MOEAs include NSGA-II and PESA-II, whereas the MOPSOs are MOPSO, σ-MOPSO and NSPSO. The codes for NSGA-II and MOPSO have been obtained from *http://www.iitk.ac.in/kangal/codes.shtml* and *http://www.lania.mx/ ccoello/EMOO/EMOOsoftware.html* respectively. The program for PESA-II has been obtained from the authors, whereas the other algorithms are implemented. The parameters used are: population/swarm size 100 for NSGA-II and NSPSO, 10 for PESA-II (as suggested in [15]), 50 for MOPSO, σ-MOPSO and AMOPSO, archive size 100 for PESA-II, σ-MOPSO, MOPSO and AMOPSO, number of iterations 250 for NSGA-II and NSPSO, 2500 for PESA-II, and 500 for MOPSO, σ-MOPSO and AMOPSO (to keep the number of function evaluations to 25000 for all the algorithms), cross-over probability 0.9 (as suggested in [14]) for NSGA-II and PESA-II, mutation probability inversely proportional to the chromosome length (as suggested in [14]), coding strategy binary for PESA-II (only binary version available) while real encoding is used for NSGA-II, NSPSO, σ-MOPSO, MOPSO and AMOPSO (PSO naturally operates on real numbers). The values of c_1 and c_2 have been used as 1 and 2 for σ-MOPSO and NSPSO, respectively (as suggested in [9] and [10]). The value of w has been used as 0.4 for σ-MOPSO whereas it has been allowed to decrease from 1.0 to 0.4 for NSPSO (as suggested in [9] and [10]). For AMOPSO the initial range of the values for c_1^i and c_2^i is $[0.5, 2.5]$ and that for w_i is $[0.0, 1.0]$ (as suggested in [17]).

The values for the parameters of a particular algorithm have been used keeping in mind the suggestions in the respective literature. Usually a smaller size of the swarm is preferred. The relative performances of NSGA-II, PESA-II, σ-MOPSO, NSPSO, MOPSO and AMOPSO are evaluated on several test problems (i.e., two and three objectives), using some performance measures.

A. Test Problems and Performance Measures

In this article nine standard test problems have been used. Seven of these test problems $SCH1$, $SCH2$, FON [1], $ZDT1$, $ZDT2$, $ZDT3$, $ZDT4$ [18] are of two objectives, while the other two, i.e., $DLTZ2$ and $DLTZ7$ [19], are of three objectives. The performance of the algorithms is evaluated with respect to the convergence measure Υ (viz., distance metric) and the diversity measure Δ [14] (viz., diversity metric). The Υ measure has been used for evaluating the extent of the convergence to the Pareto front, whereas Δ measure has been used for evaluating the diversity of the solutions on the non-dominated set. It should be noted that smaller the value of these parameters, the better will be the performance.

B. Results

The results are reported in terms of the mean and variance of the performance measures over 20 simulations in Tables I and II. It can be seen that AMOPSO has resulted in better convergence on all the test problems in terms of Υ measure except for the $SCH1$ test problem where it is found to be second to the NSGA-II algorithm. PESA-II has also given the best convergence for the $ZDT2$ test problem. Similarly, the values of the Δ measure in Table II show that AMOPSO is able to attain the best distribution of the solutions on the non-dominated front for all the test problems except on FON and $DLTZ2$, where it is second to the NSGA-II and σ-MOPSO respectively.

To demonstrate the distribution of the solutions on the final non-dominated front, FON, $ZDT3$ and $DLTZ2$ test problems have been considered as typical illustrations. Figures 2-4 show the resultant non dominated fronts corresponding to these test problems. Figure 2 provides the non-dominated solutions returned by the six algorithms for the FON test problem. The poor performance of PESA-II and σ-MOPSO is clearly evident. Figure 2(f) shows that σ-MOPSO has resulted in poor diversity amongst the solutions of the non-dominated set. Although, the results in Figure 2(a), Figure 2(d) and Figure 2(e) are better than the aforementioned results, the best result has been obtained by AMOPSO in Figure 2(c), in terms of convergence as well as diversity.

Similarly, Figure 3 represents the final fronts obtained by the six algorithms for $ZDT3$ function. It can be seen from Figure 3(f) that σ-MOPSO has failed to converge to the true Pareto-front properly. It should be noted from Figure 3(b) that PESA-II has failed to obtain all the five disconnected Pareto-optimal fronts. Although, NSGA-II has been successful in obtaining the five fronts, one of these did not come out properly (Figure 3(a)). MOPSO is found to be better than PESA-II and NSGA-II in this regard, but its solutions have poor spread on the front as is clearly evident from Figure 3(d). Moreover MOPSO is often found to converge to a local optimal front for this test function. Such an instance is shown in Figure 3(g), where MOPSO has been able to obtain only one front (not all the five) because of local optima problem. NSPSO has resulted in very good convergence, as evident from Figure 3(e), but its diversity is not as good as that of AMOPSO. Compared to all these algorithms, AMOPSO in Figure 3(c) has given better convergence and spread of the solutions on this test function. Figure 4 represents the final non-dominated fronts obtained by the algorithms on $DLTZ2$ test problem. From Figure 4(a) it can be seen that NSGA-II has failed considerably in attaining the non-dominated set properly in terms of both convergence as well as diversity. MOPSO (Figure 4(d)) has failed to attain the full non-dominated set. Similarly σ-MOPSO (Figure 4(f)) could not attain the non-dominated set properly. Although NSPSO has resulted in better shape of the Pareto front (Figure 4(e)), its convergence is not as good as that of AMOPSO and PESA-II as shown in Figure 4(c) and Figure 4(b) respectively.

V. Conclusions and Discussion

In the present article, a novel multi-objective PSO algorithm, called AMOPSO, has been presented. AMOPSO is adaptive in nature with respect to its inertia weight and acceleration coefficients. This adaptiveness enables it to attain a good balance between exploration and exploitation of the search space. A mutation operator has been incorporated in AMOPSO to resolve the problem of premature convergence to the local Pareto-optimal front (often observed in multi-objective PSOs). An archive has been maintained to store the non-dominated solutions found during AMOPSO execution. The selection of the *gbest* solution is done from this archive, using the diversity consideration. The method for computing diversity of the solutions is based on the nearest neighbor concept. The performance of AMOPSO is compared with some recently developed multi-objective PSO techniques and evolutionary algorithms, for nine function optimization problems of two and three objectives using some performance measures. AMOPSO is found to be good not only in approximating the Pareto optimal front, but also in terms of diversity of the solutions on the front.

In this article only one version of the adaptation of control parameters has been addressed, where each particle has its own control parameter. This form of adaptation can be achieved at other levels also, like at $pbest$ and $gbest$ level. Further it would be interesting to study the values of these control parameters finally when the non dominated front is obtained.

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TABLE I

MEAN (M) AND VARIANCE (VAR) OF THE Υ MEASURES FOR THE TEST PROBLEMS

Υ Measure									
Algorithm	SCH1	SCH2	FON	ZDT1	ZDT2	ZDT3	ZDT4	DLTZ2	DLTZ7
NSGA-II(M)	**0.00339**	0.01644	0.00193	0.03348	0.07239	0.11450	0.51305	0.72775	0.04976
(VAR)	0.00000	0.00000	0.00000	0.00475	0.03168	0.00794	0.11846	0.20400	0.00000
PESA-II(M)	0.01687	0.00962	0.02002	0.00105	**0.00074**	0.00789	9.98254	0.03219	0.05139
(VAR)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00011	20.13400	0.00000	0.00000
σ-MOPSO(M)	0.01364	0.01516	0.00125	0.01638	0.00584	0.10205	3.83344	0.03167	0.24494
(VAR)	0.00000	0.00000	0.00000	0.00048	0.00000	0.00238	1.87129	0.00000	0.01126
NSPSO(M)	0.01002	0.00854	0.00255	0.00642	0.00951	0.00491	4.95775	0.04938	0.05618
(VAR)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	7.43601	0.00000	0.00019
MOPSO(M)	0.01148	0.01405	0.00122	0.00133	0.00089	0.00418	7.37429	0.82799	0.04986
(VAR)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	5.48286	0.01133	0.00000
AMOPSO(M)	0.00800	**0.00554**	**0.00120**	**0.00099**	**0.00074**	**0.00391**	**0.40311**	**0.02024**	**0.02306**
(VAR)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.01259	0.00000	0.00000

TABLE II

MEAN (M) AND VARIANCE (VAR) OF THE Δ MEASURES FOR THE TEST PROBLEMS

Δ Measure									
Algorithm	SCH1	SCH2	FON	ZDT1	ZDT2	ZDT3	ZDT4	DLTZ2	DLTZ7
NSGA-II(M)	0.47789	1.22823	**0.37806**	0.39031	0.43077	0.73854	0.70261	0.97599	0.89149
(VAR)	0.00347	0.06216	0.00064	0.00187	0.00472	0.01971	0.06465	0.00738	0.00052
PESA-II(M)	0.65025	1.24480	0.85217	0.84816	0.89292	1.22731	1.01136	0.74808	0.74791
(VAR)	0.01298	0.06092	0.00032	0.00287	0.00574	0.02925	0.00072	0.00093	0.00106
σ-MOPSO(M)	0.60937	1.47054	0.85813	0.39856	0.38927	0.76016	0.82842	**0.60405**	0.94113
(VAR)	0.02237	0.03025	0.00172	0.00731	0.00458	0.00349	0.00054	0.00194	0.00640
NSPSO(M)	0.39165	1.07167	0.78024	0.90695	0.92156	0.62072	0.96462	0.72988	0.73812
(VAR)	0.00494	0.04239	0.00013	0.00000	0.00012	0.00069	0.00156	0.00091	0.01002
MOPSO(M)	0.76097	1.43353	0.84943	0.68132	0.63922	0.83195	0.96194	0.74808	0.87375
(VAR)	0.016427	0.03504	0.00016	0.01335	0.00114	0.00892	0.00114	0.00093	0.08186
AMOPSO(M)	**0.32074**	**0.96105**	0.72422	**0.31826**	**0.31996**	**0.53154**	**0.65060**	0.67273	**0.73201**
(VAR)	0.00023	0.00003	0.00003	0.00060	0.00068	0.00036	0.00376	0.00087	0.00134

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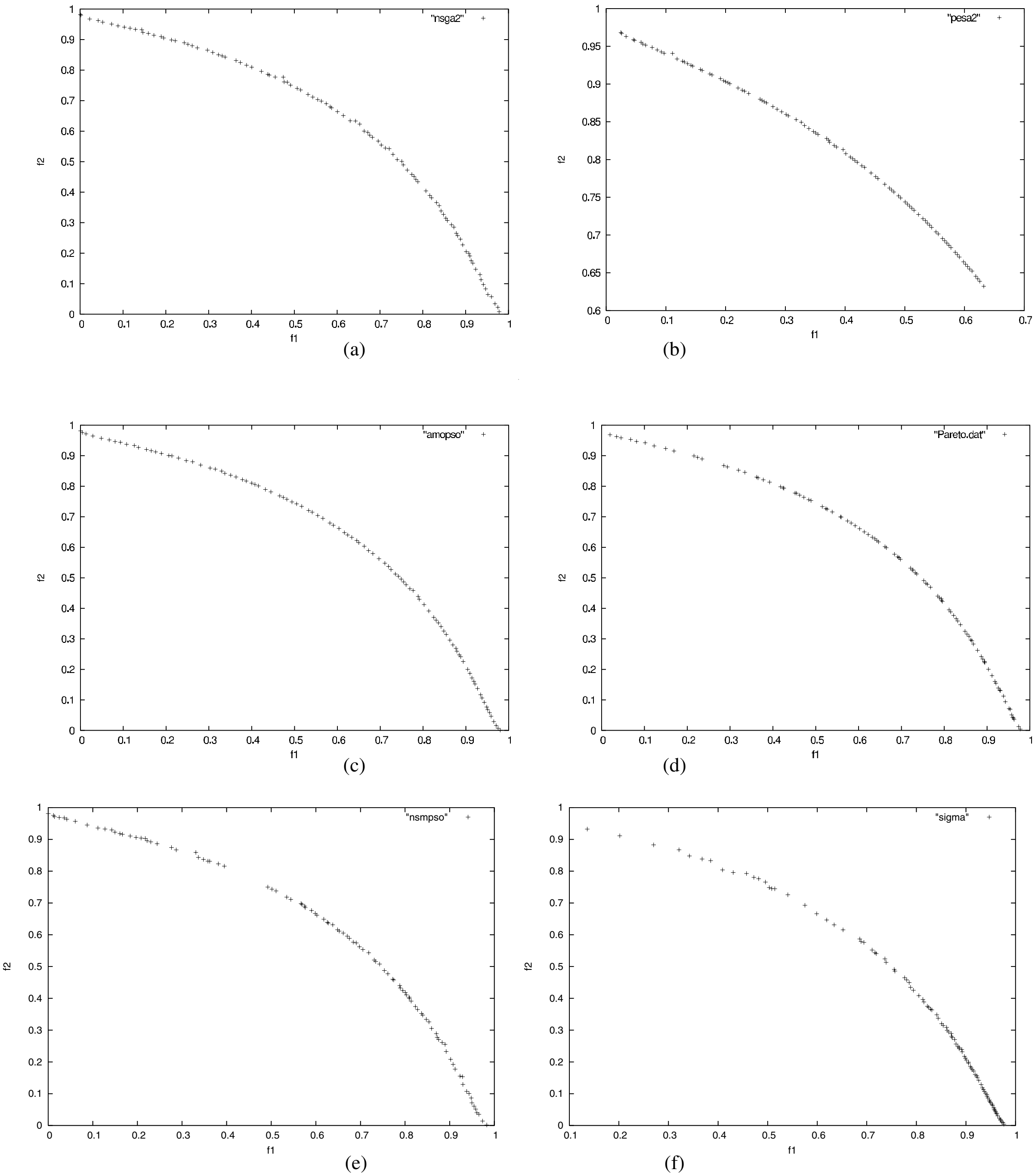


(a)

(b)

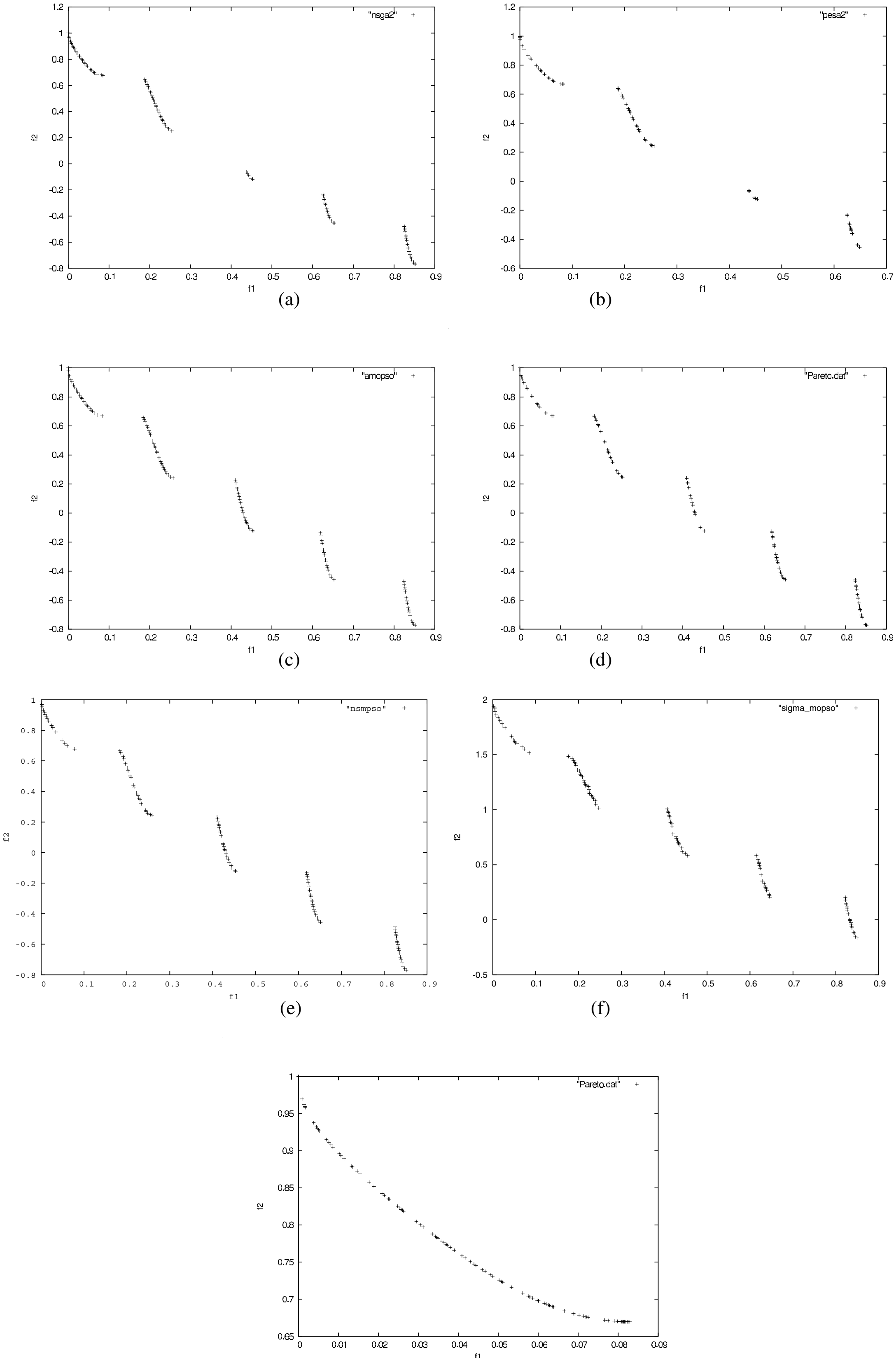
(c)

(d)

(e)

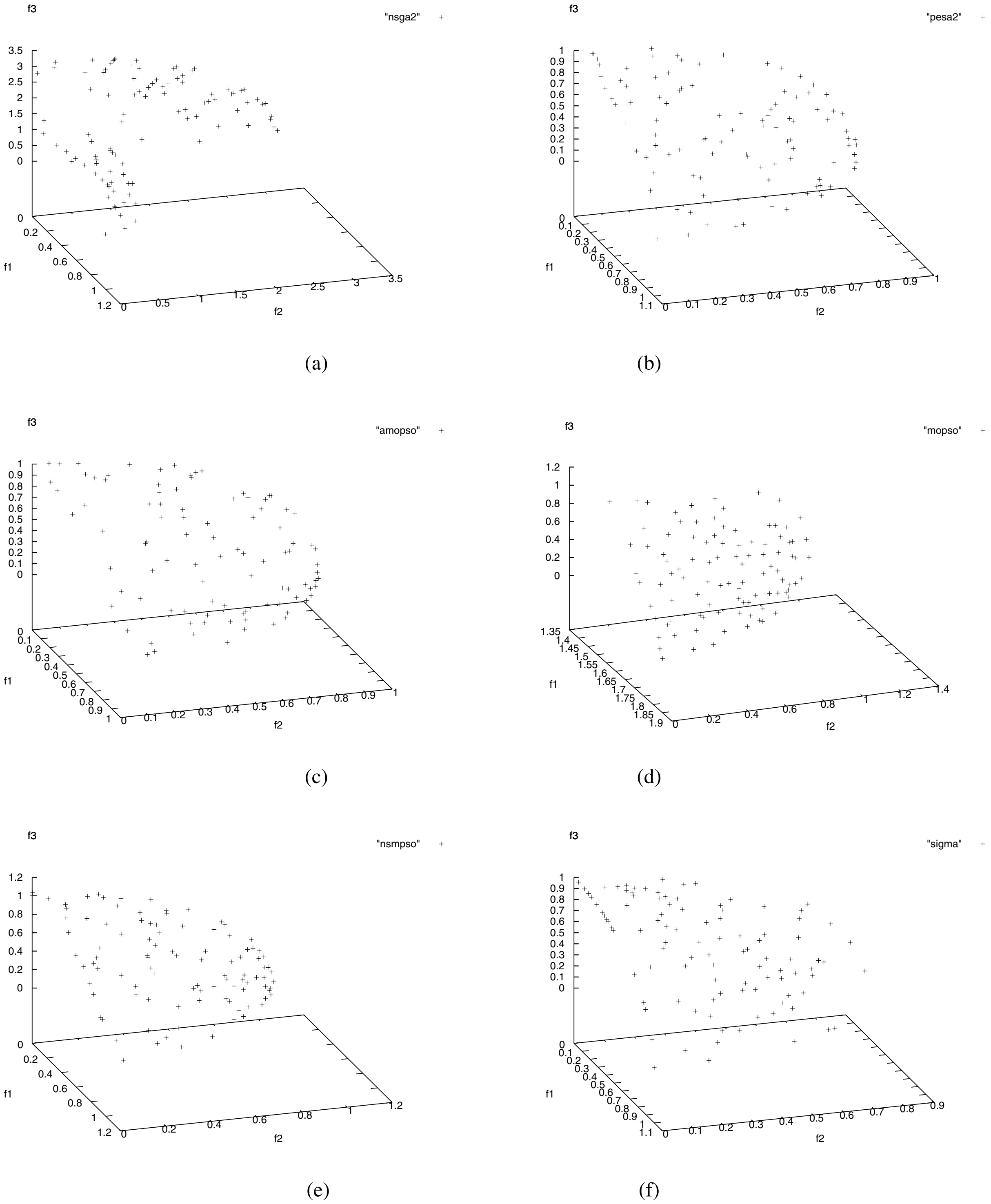
(f)

Fig. 2. Final Fronts of (a): NSGA-II, (b): PESA2, (c): AMOPSO and (d): MOPSO, (e): NSPSO and (f): σ-MOPSO on FON



(a) (b) (c) (d) (e) (f) (g)

Fig. 3. Final Fronts of (a): NSGA-II, (b): PESA2, (c): AMOPSO, (d): MOPSO, (e): NSPSO and (f): σ-MOPSO (g): MOPSO (local convergence) on ZDT3



(a) (b)

(c) (d)

(e) (f)

Fig. 4. Final Fronts of (a): NSGA-II, (b): PESA2, (c): AMOPSO, (d): MOPSO, (e): NSPSO and (f): σ-MOPSO on DLTZ2