Adaptive Neural Network Decentralized Backstepping Output-Feedback Control for Nonlinear Large-Scale Systems with Time Delays

Shao Cheng Tong, Yong Ming Li, and Hua-Guang Zhang

***Abstract*— In this paper, two adaptive neural network (NN) decentralized output feedback control approaches are proposed for a class of uncertain nonlinear large-scale systems with immeasurable states and unknown time delays. Using NNs to approximate the unknown nonlinear functions, an NN state observer is designed to estimate the immeasurable states. By combining the adaptive backstepping technique with decentralized control design principle, an adaptive NN decentralized output feedback control approach is developed. In order to overcome the problem of "explosion of complexity" inherent in the proposed control approach, the dynamic surface control (DSC) technique is introduced into the first adaptive NN decentralized control scheme, and a simplified adaptive NN decentralized output feedback DSC approach is developed. It is proved that the two proposed control approaches can guarantee that all the signals of the closed-loop system are semi-globally uniformly ultimately bounded, and the observer errors and the tracking errors converge to a small neighborhood of the origin. Simulation results are provided to show the effectiveness of the proposed approaches.**

***Index Terms*— Adaptive decentralized control, backstepping technique, neural network, nonlinear large-scale systems, stability analysis, state observer.**

I. Introduction

IN THE past decades, there has been increased interest in the development theories and decentralized control for large-scale systems. Decentralized control issues naturally arise from controlling many complex systems found in the power industry, aerospace and chemical engineering applications, telecommunication network, and so on. The main characteristics of decentralized control are that they can alleviate the computational burden associated with a centralized control and enhance the robustness and reliability against interacting operation failures. Earlier works on adaptive decentralized control were mainly focused on large-scale linear systems or nonlinear systems with the matching condition [1]–[5]. Since backstepping technique was proposed [6], many adaptive decentralized backstepping control schemes have been developed for large-scale nonlinear systems without satisfying the matching condition. Decentralized adaptive backstepping state feedback control schemes were proposed by [7] and [8] for a class of large-scale nonlinear systems. By designing state observers and state estimation filters, several adaptive decentralized output feedback control approaches were developed in [9]–[11] for large-scale nonlinear systems with only output measurements. However, these approaches require that the controlled nonlinear dynamics models be known exactly or the unknown nonlinear functions can be linearly parameterized. If this information is not available *a priori*, these adaptive backstepping controllers cannot be applied.

In the recent years, approximation-based adaptive decentralized control approaches have been developed to uncertain nonlinear large-scale systems via neural networks (NNs) or fuzzy logic systems approximators, (see [12]–[17]). Although these approaches do not require nonlinear dynamics models to be known exactly or the unknown nonlinear functions to be linearly parameterized and still can achieve a good control performances, they are applied only to a relatively simple class of nonlinear systems. A major restriction in the aforementioned schemes is that the nonlinear uncertainties and interconnections must satisfy the strict matching condition. Therefore, global stabilization for the large-scale nonlinear systems with mismatched uncertainties is not possible using these schemes.

With the development of adaptive NN (fuzzy) control and the backstepping technique, many adaptive NN and fuzzy backstepping control approaches have been investigated for uncertain nonlinear systems without satisfying the matching condition (see [18]–[22] for single-input–single-output (SISO) nonlinear systems, [23]–[25] for multiple-input and multiple-output (MIMO) nonlinear systems, and [26] for nonlinear large-scale systems). Adaptive NN or fuzzy backstepping control in general provides a systematic methodology to solve tracking or regulation control problems of unknown nonlinear systems, where NNs or FLSs are used to approximate unknown nonlinear functions. Typically, adaptive NN or fuzzy controllers are constructed recursively in the framework of backstepping design technique. The main features of these adaptive approaches are: 1) they can deal with those nonlinear systems without satisfying the matching conditions, and 2) they do not require the unknown nonlinear functions being linearly parameterized.

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S. C. Tong and Y. M. Li are with the Department of Mathematics, Liaoning University of Technology, Jinzhou 121001, China (e-mail: jztsc@sohu.com; l_y_m_2004@163.com).

H.-G. Zhang is with School of Information Science and Engineering, Northeastern University, Liaoning 116024, China (e-mail: hgzhang@ieee.org).

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Although the adaptive NN or fuzzy backstepping control has become one of the most popular design approaches to a large class of nonlinear systems, the existing adaptive backstepping control methods suffer from two major limitations. The first is that the proposed adaptive control approaches are based on an assumption that the states of the systems are measured directly. As noted in [9]–[11], in practice, state variables are often immeasurable for many nonlinear systems. In such cases, observer-based control schemes should be applied. The second limitation is the so-called the problem of "explosion of complexity" with the existing adaptive NN and fuzzy backstepping controllers, which is caused by repeated differentiations of certain nonlinear functions such as virtual controls, and thus inevitably leads to a complicated algorithm with heavy computation burden [27]–[29]. To solve the first limitation, recently the authors in [30]–[32] have proposed observer-based adaptive NN and fuzzy backstepping output feedback controllers for SISO or MIMO nonlinear systems. The authors of [33] and [34] have developed adaptive fuzzy decentralized output feedback controllers by using k-filters for a class of the uncertain large-scale systems. Although the above-mentioned approaches can solve the problem of the states unmeasured, the explosion of complexity is inherent in them. To overcome the problem of the explosion of complexity, an adaptive NN backstepping control approach was first proposed by [35] for a class of SISO uncertain nonlinear systems based on the so-called dynamic surface control (DSC) technique. Since then, several adaptive NN and fuzzy backstepping DSC control schemes have been developed. For example, [36] and [37] propose adaptive NN backstepping DSC controllers for a class of SISO uncertain nonlinear systems without and with a dead zone, respectively. In [38], adaptive NN backstepping DSC control of nonlinear systems with periodic disturbances is dealt with, whereas [39] presents a robust adaptive fuzzy tracking control for a class of uncertain MIMO nonlinear systems, and [40] and [41] propose the adaptive NN decentralized backstepping DSC designs for a class of nonlinear large-scale systems. Despite these efforts using the DSC technique, the aforementioned adaptive NN or fuzzy backstepping DSC methods still require that the states of the controlled systems be available for measurements.

In this paper, two observer-based adaptive NN decentralized backstepping control approaches are proposed for a class of nonlinear large-scale systems with immeasurable states and unknown time delays. Using NNs to approximate the unknown nonlinear functions, an NN state observer is designed to estimate the immeasurable states. Combining the backstepping technique, DSC technique, and decentralized design principle, two adaptive decentralized backstepping control approaches are developed. It is proved that both these control approaches can guarantee that all the signals of the closed-loop system are semi-globally uniformly ultimately bounded (SUUB), and the observer errors and the tracking errors converge to a small neighborhood of the origin. Compared to the existing results, the main contributions of this paper are as follows: 1) by designing an NN state observer, the proposed NN decentralized control methods do not require that all the states are available for measurements, which is assumed in the existing adaptive NN decentralized backstepping controllers [26], [40], [41]; 2) the proposed second NN decentralized control method can overcome the explosion of complexity inherent in the NN and fuzzy decentralized backstepping controllers [26], [30]–[34]; and 3) the proposed decentralized control schemes can solve the problem of nonlinear large-scale systems with time delays.

II. Problem Formulation and Some Preliminaries

A. System Description

Consider a class of large-scale nonlinear systems that is composed of N subsystems interconnected by their outputs. The ith subsystem Σ_i, $(i = 1, \dots, N)$ is given as

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} + f_{i,1}(y_1, \dots, y_N) \\ \qquad + h_{i,1}(y_1(t - \tau_{i,1,1}(t)), \dots, \\ \qquad\quad y_N(t - \tau_{i,1,N}(t))) + \Delta_{i,1}(y_1, \dots, y_N) \\ \dot{x}_{i,2} = x_{i,3} + f_{i,2}(y_1, \dots, y_N) \\ \qquad + h_{i,2}(y_1(t - \tau_{i,2,1}(t)), \dots, \\ \qquad\quad y_N(t - \tau_{i,2,N}(t))) + \Delta_{i,2}(y_1, \dots, y_N) \\ \quad \vdots \\ \dot{x}_{i,n_i-1} = x_{i,n_i} + f_{i,n_i-1}(y_1, \dots, y_N) \\ \qquad + h_{i,n_i-1}(y_1(t - \tau_{i,n_i-1,1}(t)), \dots, \\ \qquad\quad y_N(t - \tau_{i,n_i-1,N}(t))) \\ \qquad + \Delta_{i,n_i-1}(y_1, \dots, y_N) \\ \dot{x}_{i,n_i} = u_i + f_{i,n_i}(y_1, \dots, y_N) \\ \qquad + h_{i,n_i}(y_1(t - \tau_{i,n_i,1}(t)), \dots, \\ \qquad\quad y_N(t - \tau_{i,n_i,N}(t))) + \Delta_{i,n_i}(y_1, \dots, y_N) \\ y_i = x_{i,1} \end{cases} \tag{1}$$

where $x_i = [x_{i,1}, \dots, x_{i,n_i}]^T \in R^{n_i}$, $u_i \in R$ and $y_i \in R$ are the state, control input and the output of the ith subsystem, respectively. $f_{i,j}(\cdot)$ and $\Delta_{i,j}(\cdot)$ $(1 \le i \le N,\ 1 \le j \le n_i)$ are unknown smooth functions representing the nonlinearities in the ith subsystem and the interconnection effects between the ith subsystem and other subsystems. $h_{i,j}(y_1(t - \tau_{i,j,1}(t)), \dots, y_N(t - \tau_{i,j,N}(t)))$ is an unknown smooth nonlinear function, and $\tau_{i,j,k}(t)$ is an unknown time delay satisfying $\dot{\tau}_{i,j,k}(t) \le \tau^* \le 1$ $(1 \le i \le N,\ 1 \le j \le n_i,\ 1 \le k \le N)$. In this paper, it is assumed that only output y_i is available for measurement. Without loss of generality, denote $\tau_{i,j,k}(t) = \tau_{i,j}(t)$.

Remark 1: It can be used to describe many state space models of practical nonlinear systems with time delays such that interconnected recycled storage tanks, interconnected wind tunnels and interconnected cold-rolling mills, and robotic systems with transmission delays [29]. In particular, most of the telemanipulation systems include interconnected delays in their state equations. On the other hand, if no time-delay terms are included in (1), i.e., $h_{i,j}(.) = 0$, then (1) becomes the interconnected nonlinear strict-feedback systems studied widely, see [7], [9], [10], [33], and [34].

Throughout this paper, the following assumptions are made on (1).

Assumption 1: Nonlinear function $\Delta_{i,j}(\cdot)$ satisfies $\|\Delta_{i,j}(\cdot)\| \le \sum_{k=1}^{p_{i,j}} \sum_{l=1}^{N} q_{i,j,l}^k \|y_l\|^k$, where $q_{i,j,l}^k$ is an unknown constant and $p = \max\{p_{i,j} | 1 \le i \le N, 1 \le j \le n_i\}$ is a known integer.

Remark 2: Note that $\Delta_{i,j}(\cdot)$ may be nonlinearly parameterized in the unknown parameters, provided that Assumption 1 is satisfied. Also, the quantity $p = \max\{p_{i,j} | 1 \leq i \leq N, 1 \leq j \leq n_i\}$ needs to be known. The Assumption 1 is common in existing literature dealing with the similar problems, see [2], [3], [7], and [9].

Assumption 2: Nonlinear function $h_{i,j}(\cdot)$ satisfies the following inequality:

$$\left|h_{i,j}(y_1(t), \ldots y_N(t)\right|^2 \leq z_{i,1}(t)\bar{H}_{i,j}(z_{i,1}(t)) + \bar{h}_{i,j}(y_{i,r}(t)) + \varpi_{i,j}$$

where $\bar{H}_{i,j}(\cdot)$ is a known function, $\bar{h}_{i,j}(\cdot)$ is a bounded function with $\bar{h}_{i,j}(0) = 0$, and $\varpi_{i,j}$ is a positive scalar and $z_{i,1}(t) = y_{i,1}(t) - y_{i,r}(t)$.

Remark 3: The effects of the nonlinear time-delay functions $h_{i,j}(.)$ from other subsystems to a local subsystem are bounded by functions of the output of this subsystem. With this condition, it is possible to design local controller to stabilize the interconnected systems with the time delays [30], the same or similar assumptions can be found in existing literature, (see [31], [32], [42]).

B. RBF Neural Networks

In this paper, the following radial basis function (RBF) NN is used to approximate the continuous function $h(X)$: $R^q \to R$

$$h_{nn}(X) = W^T \varphi(X) \tag{2}$$

where the input vector $X \in D \subset R^q$, weight vector $W = [W_1, \ldots, W_l]^T \in R^l$, the NN node number $l > 1$, and $\varphi(X) = \begin{bmatrix} \varphi_1(X) & \ldots & \varphi_l(X) \end{bmatrix}^T$, with $\varphi_i(X)$ being chosen as the commonly used Gaussian functions, which have the form

$$\varphi_i(X) = \exp\left[\frac{-(X-\mu_i)^T(X-\mu_i)}{\eta^2}\right], \quad i = 1, 2, \ldots, l$$

where $\mu_i = [\mu_{i1}, \ldots, \mu_{iq}]^T$ is the center of the receptive field and η is the width of the Gaussian function.

Let

$$h(X) = W^{*T}\varphi(X) + \varepsilon(X) \qquad \forall X \in D \tag{3}$$

where W^* is an ideal contant weight, and $\varepsilon(X)$ is the approximation error. It is assumed that W^* is bounded and defined as

$$W^* = \arg\min_{W\in\Omega}\left\{\sup_{X\in D}\left|h(X) - W^T\varphi(X)\right|\right\}. \tag{4}$$

The control design presented in this paper employs RBF NNs to approximate the nonlinear function $f_{i,j}(\cdot)$ in (1); assume that

$$f_{i,j}(y_1, \ldots, y_N) = W_{i,j}^{*T}\varphi_{i,j}(y_1, \ldots, y_N) + \varepsilon_{i,j}(y_1, \ldots, y_N) \tag{5}$$

where $W_{i,j}^*$ is the ideal constant weight, and $\varepsilon_{i,j}(\cdot)$ is the approximation error. It is usually assumed that $\left|\varepsilon_{i,j}(\cdot)\right| \leq \varepsilon_{i,j,0}$, where $\varepsilon_{i,j,0}$ is a known constant.

Assumption 3: $\left\|\Lambda_{i,j}\right\| = \left\|W_{i,j}^{*T}\varphi_{i,j}(\bar{y}) - W_{i,j}^{*T}\varphi_{i,j}(\bar{y}_r)\right\| \leq \sum_{k=1}^N s_{i,j,k}\left\|y_k - y_{k,r}\right\|$, where $s_{i,j,k}$ is an unknown constant, $\bar{y} = (y_1, \ldots, y_N)$, and $\bar{y}_r = (y_{1,r}, \ldots, y_{N,r})$.

Remark 4: As far as the Gaussian function $s_i(X) = \exp[-(X-\mu_i)^T(X-\mu_i)/\eta_i^2]$ is concerned, it satisfies the global Lipschitz condition [33], i.e., Assumption 3 is true.

By substituting (5) into (1), the system (1) can be presented in the following form:

$$\begin{cases} \dot{x}_i = A_i x_i + K_i y_i + \sum_{j=1}^{n_i} B_{i,j}W_{i,j}^{*T}\varphi_{i,j}(\bar{y}_r) \\ \qquad + B_{i,n_i}u_i + h_i + \Delta_i + \Lambda_i + \varepsilon_i \\ y_i = C_i^T x_i \end{cases} \tag{6}$$

where $A_i = \begin{bmatrix} -k_{i,1} & \\ \vdots & I_{n_i-1} \\ -k_{i,n_i} & \cdots \; 0 \end{bmatrix}$, $h_i = [h_{i,1}, \ldots, h_{i,N}]^T$, $K_i = [k_{i,1}, \ldots, k_{i,n_i}]^T$, $\Delta_i = [\Delta_{i,1}, \ldots, \Delta_{i,n_i}]^T$, $\Lambda_i = [\Lambda_{i,1}, \ldots, \Lambda_{i,n_i}]^T$, $\varepsilon_i = [\varepsilon_{i,1}, \ldots, \varepsilon_{i,n_i}]^T$, $B_{i,j} = [\underbrace{0, \ldots, 0, 1}_{j}, 0, \ldots, 0]^T$ and $C_i = [1, 0, \ldots, 0]^T$, and vector K_i is chosen such that A_i is a strict Hurwitz matrix. Thus, given a $Q_i = Q_i^T > 0$, there exists a $P_i = P_i^T > 0$ such that

$$A_i^T P_i + P_i A_i = -Q_i. \tag{7}$$

Since the state variables are not available, a local state observer for the ith subsystem is designed as

$$\begin{cases} \dot{\hat{x}}_i = A_i\hat{x}_i + K_i y_i \\ \qquad + \sum_{j=1}^{n_i} B_{i,j}W_{i,j}^T\varphi_{i,j}(\bar{y}_r) + B_{i,n_i}u_i \\ \hat{y}_i = C_i^T\hat{x}_i. \end{cases} \tag{8}$$

Let $e_i = x_i - \hat{x}_i$ be observer error, then from (6) and (8), one can obtain the observer errors equation

$$\begin{aligned} \dot{e}_i &= A_i e_i + \sum_{j=1}^{n_i} B_{i,j}\left[W_{i,j}^{*T}\varphi_{i,j}(\bar{y}_r) - W_{i,j}^T\varphi_{i,j}(\bar{y}_r)\right] \\ &\quad + h_i + \Delta_i + \Lambda_i + \varepsilon_i \\ &= A_i e_i + \delta_i + h_i + \Delta_i + \Lambda_i + \varepsilon_i \end{aligned} \tag{9}$$

where $\delta_i = [\delta_{i,1}, \ldots, \delta_{i,j}, \ldots, \delta_{i,n_i}]^T$ and $\delta_{i,j} = W_{i,j}^{*T}\varphi_{i,j}(\bar{y}_r) - W_{i,j}^T\varphi_{i,j}(\bar{y}_r)$.

Assumption 4: There exists a known constant $\delta_{i,j,0} > 0$, such that $\left|\delta_{i,j}\right| \leq \delta_{i,j,0}$.

Remark 5: The Assumption on $\delta_{i,j}$ is reasonable. Notice that

$$\delta_{i,j} = \left(W_{i,j}^* - W_{i,j}\right)^T\varphi_{i,j}(\bar{y}_r). \tag{10}$$

As noted in many references (see [18], [19], [22]) the ideal constant weight $W_{i,j}^*$ is bounded, the adjusted weight $W_{i,j} \in \Omega_{i,j}$, with $\Omega_{i,j}$ is a bounded compact set, and $\varphi_{i,j}(\bar{y}_r)$ is a bounded radial function. Therefore, by (10), one can assume that $\delta_{i,j}$ is bounded by a known constant $\delta_{i,j,0}$.

Consider the following Lyapunov candidate for the error system (9):

$$V_0 = \sum_{i=1}^N\left(V_{i,0} + \bar{V}_{i,0}\right) \tag{11}$$

with $V_{i,0} = e_i^T P_i e_i$ and $\bar{V}_{i,0} = e^{\tau_i}/(1-\tau^*)\|P_i\|^2 e^{-r_i t}\sum_{j=1}^N\int_{t-\tau_{i,j}(t)}^t e^{r_i s}z_{i,1}(s)(\bar{H}_{i,j}(z_{i,1}(s)))ds$, where r_i is a design constant, and $\tau_i \geq \max\{\tau_{i,1}(t), \ldots, \tau_{i,n_i}(t)\}$.

The time derivative of V_0 along (9) is

$$\begin{aligned}\dot{V}_0 &= \sum_{i=1}^{N}\Big\{e_i^T\left(P_iA_i + A_i^TP_i\right)e_i + 2e_i^TP_i\big(h_i+\Delta_i\\ &\quad +\Lambda_i+\delta_i'\big)+\dot{\bar{V}}_{i,0}\Big\}\\ &\le \sum_{i=1}^{N}\Big\{-e_i^TQ_ie_i + 2e_i^TP_i\big(h_i+\Delta_i+\Lambda_i\\ &\quad +\delta_i'\big)+\dot{\bar{V}}_{i,0}\Big\} \end{aligned} \tag{12}$$

where $\delta_i' = [\delta_{i,1}+\varepsilon_{i,1},\ldots,\delta_{i,n_i}+\varepsilon_{i,n_i}]^T$. By Young's inequality $a^Tb \le 1/2\lambda a^Ta + \lambda/2b^Tb$, $(\lambda > 0)$, one has

$$2e_i^TP_i\delta_i' \le e_i^Te_i + \|P_i\|^2\left\|\delta'_i\right\|^2 \le e_i^Te_i + \delta_{i0}'^2\|P_i\|^2 \tag{13}$$

where $\delta_{i0}'^2 = \{[\sum_{j=1}^{n_i}\varepsilon_{i,j,0}^2]^{1/2} + [\sum_{j=1}^{n_i}\delta_{i,j,0}^2]^{1/2}\}^2$.

Using the Cauchy–Schwartz inequality $(\sum_{k=1}^{p}a_kb_k)^2 \le (\sum_{k=1}^{p}a_k^2)(\sum_{k=1}^{p}b_k^2)$, Young's inequality, and Assumptions 1–3, the following inequalities are obtained:

$$\begin{aligned}2\sum_{i=1}^{N}e_i^TP_i\Delta_i &\le \sum_{i=1}^{N}e_i^Te_i + \sum_{i=1}^{N}\|P_i\|^2\|\Delta_i\|^2\\ &\le \sum_{i=1}^{N}e_i^Te_i + \sum_{i=1}^{N}\sum_{k=1}^{p}2^{2k}q_{i,k}\left(\left\|y_{i,r}\right\|^{2k}+\left\|z_{i,1}\right\|^{2k}\right)\end{aligned} \tag{14}$$

$$2\sum_{i=1}^{N}e_i^TP_i\Lambda_i \le \sum_{i=1}^{N}e_i^Te_i + \sum_{i=1}^{N}\sum_{k=1}^{N}L_{i,k}\left\|y_k - y_{k,r}\right\|^2 \tag{15}$$

where $q_{i,k} = pN\sum_{l=1}^{N}\|P_l\|^2\sum_{j=1}^{n_l}(q_{l,j,i}^k)^2$ and $L_{i,k} = N\|P_i\|^2\sum_{j=1}^{n_i}s_{i,j,k}^2$

$$\begin{aligned}2e_i^TP_ih_i &\le e_i^Te_i + \|P_i\|^2|h_i|^2\\ &\le e_i^Te_i + \sum_{j=1}^{N}d_{i,j}^*\\ &\quad + \|P_i\|^2\sum_{j=1}^{N}z_{i,1}(t-\tau_{i,j}(t))\bar{H}_{i,j}(z_{i,1}(t-\tau_{i,j}(t)))\end{aligned} \tag{16}$$

where $d_{i,j}^*$ is a constant satisfying $d_{i,j}^* > \|P_i\|^2(\bar{h}_{i,j}(y_{i,r}(t-\tau_{i,j}(t))) + \varpi_{i,j})$.

It is noticed that

$$\begin{aligned}\dot{\bar{V}}_{i,0} &= -r_i\frac{e^{\tau_i}}{(1-\tau^*)}\|P_i\|^2\\ &\quad\times e^{-r_it}\sum_{j=1}^{N}\int_{t-\tau_{i,j}(t)}^{t}e^{r_is}z_{i,1}(s)(\bar{H}_{i,j}(z_{i,1}(s)))ds\\ &\quad+\frac{e^{\tau_i}}{(1-\tau^*)}\|P_i\|^2e^{-r_it}\sum_{j=1}^{N}[e^{r_it}z_{i,1}(t)(\bar{H}_{i,j}(z_{i,1}(t)))\\ &\quad- e^{r_i(t-\tau_{i,j}(t))}z_{i,1}(t-\tau_{i,j}(t))\\ &\quad\times(\bar{H}_{i,j}(z_{i,1}(t-\tau_{i,j}(t))))(1-\dot{\tau}_{i,j}(t))]\\ &\le -r_i\bar{V}_{i,0} + \frac{e^{\tau_i}}{(1-\tau^*)}\|P_i\|^2\sum_{j=1}^{N}z_{i,1}(t)(\bar{H}_{i,j}(z_{i,1}(t)))\\ &\quad-\|P_i\|^2\sum_{j=1}^{N}z_{i,1}(t-\tau_{i,j}(t))(\bar{H}_{i,j}(z_{i,1}(t-\tau_{i,j}(t)))).\end{aligned} \tag{17}$$

Substituting (13)–(17) into (12), one has

$$\begin{aligned}\dot{V}_0 &\le -\sum_{i=1}^{N}\Big\{e_i^T(Q_i-4I)e_i + \|P_i\|^2\delta_{i0}'^2\\ &\quad+\sum_{k=1}^{p}2^{2k}q_{i,k}\left(\left\|y_{i,r}\right\|^{2k}+\left\|z_{i,1}\right\|^{2k}\right)\\ &\quad+\sum_{k=1}^{N}L_{i,k}\left\|y_k-y_{k,r}\right\|^2+\sum_{j=1}^{N}d_{i,j}^* - r_i\bar{V}_{i,0}\\ &\quad+\frac{e^{\tau_i}}{(1-\tau^*)}\|P_i\|^2\sum_{j=1}^{N}z_{i,1}(\bar{H}_{i,j}(z_{i,1}))\Big\}.\end{aligned} \tag{18}$$

Remark 6: It can be seen from (18) that the state observer (8) cannot guarantee the convergences of the observer errors, and thus it is necessary to design a suitable controller to achieve this objective, which will be discussed in the next section.

III. Decentralized Control Design and Stability Analysis

In this section, an adaptive NN decentralized controller and parameter adaptive laws are to be developed in the framework of the backstepping technique, so that all the signals in the closed-loop system are SUUB and the tracking errors $z_{i,1} = y_i - y_{i,r}$ and observer error vectors e_i are as small as desired.

The n_i-step adaptive NN decentralized output feedback backstepping design is based on the change of coordinates

$$z_{i,1} = y_i - y_{i,r} \tag{19}$$

$$z_{i,j} = \hat{x}_{i,j} - \alpha_{i,j-1}, \qquad j = 2,\ldots,n_i \tag{20}$$

where $\alpha_{i,j-1}$ is an intermediate control, and $u_i(t)$ is designated in the last step. The detailed design procedures will be given based on the above change of coordinates.

Step i, 1: The time derivative of $z_{i,1}$ along (1) is

$$\begin{aligned}\dot{z}_{i,1} &= \dot{y}_i - \dot{y}_{i,r}\\ &= \hat{x}_{i,2} + W_{i,1}^T\varphi_{i,1}(\bar{y}_r) + \tilde{W}_{i,1}^T\varphi_{i,1}(\bar{y}_r) + h_{i,1}\\ &\quad + \Delta_{i,1}(\bar{y}) - \dot{y}_{i,r} + \Lambda_{i,1} + \varepsilon_{i,1} + e_{i,2}\end{aligned} \tag{21}$$

where $\tilde{W}_{i,j} = W_{i,j}^* - W_{i,j}$ is the weight error vector.

Consider the Lyapunov function candidate $\sum_{i=1}^{N}V_{i,1}$ as

$$\begin{aligned}\sum_{i=1}^{N}V_{i,1} &= \sum_{i=1}^{N}\Big\{V_{i,0} + \frac{1}{2}z_{i,1}^2 + \frac{1}{2}\tilde{W}_{i,1}^T\Gamma_{i,1}^{-1}\tilde{W}_{i,1}\\ &\quad+\frac{1}{2\gamma_{i,1}}\tilde{\beta}_i^2 + \frac{1}{2\gamma_{i,2}}\tilde{\pi}_i^2 + \bar{V}_{i,1}\Big\}\end{aligned} \tag{22}$$

with $\bar{V}_{i,1} = e^{\tau_i}/(2(1-\tau^*))e^{-r_it}\int_{t-\tau_{i,1}(t)}^{t}e^{r_is}z_{i,1}(s)(\bar{H}_{i,1}(z_{i,1}(s)))ds$; where $\Gamma_{i,1} = \Gamma_{i,1}^T > 0$; $\gamma_{i,1}$ and $\gamma_{i,2}$

are the design constants. $\tilde{\beta}_i = \beta_i - \hat{\beta}_i$ and $\tilde{\pi}_i = \pi_i - \hat{\pi}_i$. $\hat{\beta}_i$ and $\hat{\pi}_i$ are the estimates of β_i and π_i, respectively. In this paper, $\beta_i = \sum_{k=1}^{N}(L_{k,i} + n_i(N/2)s_{k,1,i}^2)$ and $\pi_i = \max_{1\le k\le p}\{q_{i,k} + n_i q_{1,i,k}\}$ are only used for the purpose of stability analysis.

The time derivative of $\sum_{i=1}^{N} V_{i,1}$ along (18), (20), and (21) is

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_{i,1} &= \dot{V}_0 + \sum_{i=1}^{N}\Big\{z_{i,1}\dot{z}_{i,1} - \tilde{W}_{i,1}^T\Gamma_{i,1}^{-1}\dot{W}_{i,1}\\ &\qquad \frac{1}{\gamma_{i,1}}\tilde{\beta}_i\dot{\hat{\beta}}_i - \frac{1}{\gamma_{i,2}}\tilde{\pi}_i\dot{\hat{\pi}}_i + \dot{\bar{V}}_{i,1}\Big\}\\ &\le \dot{V}_0 + \sum_{i=1}^{N}\Big\{z_{i,1}[z_{i,2} + \alpha_{i,1} + W_{i,1}^T\varphi_{i,1}(\bar{y}_r)\\ &\quad + \tilde{W}_{i,1}^T\varphi_{i,1}(\bar{y}_r) - \dot{y}_{i,r}] + |z_{i,1}\Delta_{i,1}(\bar{y})| + |z_{i,1}h_{i,1}|\\ &\quad + |z_{i,1}\Lambda_{i,1}| + |z_{i,1}\varepsilon_{i,1}| + |z_{i,1}e_{i,2}| - \tilde{W}_{i,1}^T\Gamma_{i,1}^{-1}\dot{W}_{i,1}\\ &\quad - \frac{1}{\gamma_{i,1}}\tilde{\beta}_i\dot{\hat{\beta}}_i - \frac{1}{\gamma_{i,2}}\tilde{\pi}_i\dot{\hat{\pi}}_i + \dot{\bar{V}}_{i,1}\Big\}. \end{aligned} \tag{23}$$

Using Young's inequality, Cauchy–Schwartz inequality, and Assumptions 1–3, one can obtain

$$|z_{i,1}e_{i,2}| \le \frac{1}{4}e_i^Te_i + z_{i,1}^2 \tag{24}$$

$$\begin{aligned}\sum_{i=1}^{N}|z_{i,1}\Delta_{i,1} \le (\bar{y})| &\le \sum_{i=1}^{N} z_{i,1}^2\\ &\quad + \sum_{i=1}^{N}\sum_{k=1}^{p}2^{2k}q_{1,i,k}\left(\|y_{i,r}\|^{2k} + \|z_{i,1}\|^{2k}\right)\end{aligned} \tag{25}$$

$$\sum_{i=1}^{N}|z_{i,1}\Lambda_{i,1}| \le \sum_{i=1}^{N} z_{i,1}^2 + \sum_{i=1}^{N}\sum_{k=1}^{N}\frac{N}{2}s_{i,1,k}^2 z_{k,1}^2 \tag{26}$$

$$\begin{aligned}|z_{i,1}h_{i,1}| &\le \frac{1}{2}z_{i,1}^2\\ &\quad + \frac{1}{2}z_{i,1}(t-\tau_{i,1}(t))\bar{H}_{i,1}(z_{i,1}(t-\tau_{i,1}(t))) + \bar{d}_{i,1}\end{aligned} \tag{27}$$

where $q_{1,i,k} = Np\sum_{l=1}^{N}(q_{l,1,i}^k)^2$ and $\bar{d}_{i,1} = 1/(2\|P_i\|^2)d_{i,1}^*$.

Similar to the procedures in (17), one has

$$\begin{aligned}\dot{\bar{V}}_{i,1} &\le -r_i\bar{V}_{i,1} + \frac{e^{\tau_i}}{2(1-\tau^*)}z_{i,1}(t)(\bar{H}_{i,1}(z_{i,1}(t)))\\ &\quad - \frac{1}{2}z_{i,1}(t-\tau_{i,1}(t))(\bar{H}_{i,1}(z_{i,1}(t-\tau_{i,1}(t)))).\end{aligned} \tag{28}$$

Using inequality $|z_{i,1}\varepsilon_{i,1,0}| - z_{i,1}\varepsilon_{i,1,0}\tanh(z_{i,1}\varepsilon_{i,1,0}/\varsigma) \le 0.2785\varsigma = \varsigma'$, $(\forall\varsigma > 0)$ and from (18), (24)–(28), can be rewritten as

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_{i,1} &\le -\sum_{i=1}^{N}\Big\{e_i^T(Q_i - 5I)e_i + \delta_{i0}'^2\|P_i\|^2\\ &\quad + \sum_{k=1}^{p}2^{2k}(q_{i,k} + q_{1,i,k})(\|y_{i,r}\|^{2k} + \|z_{i,1}\|^{2k})\\ &\quad + \sum_{k=1}^{N}(L_{i,k} + \frac{N}{2}s_{i,1,k}^2)z_{k,1}^2 + \sum_{j=1}^{N}d_{i,j}^* - r_i\bar{V}_{i,0}\\ &\quad + z_{i,1}\Big[\frac{1}{2}z_{i,1} + z_{i,2} + \alpha_{i,1} + \hat{\beta}_i z_{i,1}\\ &\quad + \hat{\pi}_i\sum_{k=1}^{p}2^{2k}z_{i,1}^{2k} + \varepsilon_{i,1,0}\tanh\left(\frac{z_{i,1}\varepsilon_{i,1,0}}{\varsigma}\right)\\ &\quad + W_{i,1}^T\varphi_{i,1}(\bar{y}_r) - \dot{y}_{i,r}\\ &\quad + \frac{e^{\tau_i}}{(1-\tau^*)}\|P_i\|^2\sum_{j=1}^{N}(\bar{H}_{i,j}(z_{i,1}))\\ &\quad + \frac{e^{\tau_i}}{2(1-\tau^*)}(\bar{H}_{i,1}(z_{i,1}))\Big] + 3z_{i,1}^2 + |z_{i,1}\varepsilon_{i,1,0}|\\ &\quad - z_{i,1}\varepsilon_{i,1,0}\tanh\left(\frac{z_{i,1}\varepsilon_{i,1,0}}{\varsigma}\right)\\ &\quad + \tilde{W}_{i,1}^T(z_{i,1}\varphi_{i,1}(\bar{y}_r) - \Gamma_{i,1}^{-1}\dot{W}_{i,1}) + \tilde{\beta}_i\left(z_{i,1}^2 - \frac{1}{\gamma_{i,1}}\dot{\hat{\beta}}_i\right)\\ &\quad + \tilde{\pi}_i\left(\sum_{k=1}^{p}2^{2k}z_{i,1}^{2k} - \frac{1}{\gamma_{i,2}}\dot{\hat{\pi}}_i\right)\\ &\quad - \beta_i z_{i,1}^2 - \pi_i\sum_{k=1}^{p}2^{2k}z_{i,1}^{2k} + \bar{d}_{i,1} - r_i\bar{V}_{i,1}\Big\}.\end{aligned} \tag{29}$$

Design intermediate control function $\alpha_{i,1}$ and the adaptation functions $W_{i,1}$, $\hat{\beta}_i$, and $\hat{\pi}_i$ as

$$\begin{aligned}\alpha_{i,1} &= -\frac{1}{2}z_{i,1} - (c_{i,1}+3)z_{i,1} - \hat{\beta}_i z_{i,1}\\ &\quad - \hat{\pi}_i\sum_{k=1}^{p}2^{2k}z_{i,1}^{2k-1} - W_{i,1}^T\varphi_{i,1}(\bar{y}_r) + \dot{y}_{i,r}\\ &\quad - \varepsilon_{i,1,0}\tanh\left(\frac{z_{i,1}\varepsilon_{i,1,0}}{\varsigma}\right)\\ &\quad - \frac{e^{\tau_i}}{(1-\tau^*)}\|P_i\|^2\sum_{j=1}^{N}(\bar{H}_{i,j}(z_{i,1}))\\ &\quad - \frac{n_ie^{\tau_i}}{2(1-\tau^*)}(\bar{H}_{i,1}(z_{i,1}(t)))\end{aligned} \tag{30}$$

$$\dot{W}_{i,1} = \Gamma_{i,1}\left(z_{i,1}\varphi_{i,1}(\bar{y}_r) - \sigma_{i,1}W_{i,1}\right) \tag{31}$$

$$\dot{\hat{\beta}}_i = \gamma_{i,1}\left(z_{i,1}^2 - \sigma_{i,2}\hat{\beta}_i\right) \tag{32}$$

$$\dot{\hat{\pi}}_i = \gamma_{i,2}\left(\sum_{k=1}^{p}2^{2k}z_{i,1}^{2k} - \sigma_{i,3}\hat{\pi}_i\right). \tag{33}$$

Substituting (30)–(33) into (29) results in

$$\sum_{i=1}^{N}\dot{V}_{i,1} \le -\sum_{i=1}^{N}\Big\{e_i^T(Q_i-5I)e_i+\delta_{i0}'^{\,2}\|P_i\|^2 + \sum_{k=1}^{p}2^{2k}(q_{i,k}+q_{1,i,k})(\|y_{i,r}\|^{2k}+\|z_{i,1}\|^{2k}) + \sum_{k=1}^{N}\Big(L_{i,k}+\frac{N}{2}s_{i,1,k}^2\Big)z_{k,1}^2+z_{i,1}z_{i,2} - c_{i,1}z_{i,1}^2-\frac{(n_i-1)e^{\tau_i}}{2(1-\tau^*)}(\bar{H}_{i,1}(z_{i,1}))z_{i,1} + \varsigma'+\sigma_{i,1}\tilde{W}_{i,1}^TW_{i,1}+\sigma_{i,2}\tilde{\beta}_i\hat{\beta}_i+\sigma_{i,3}\tilde{\pi}_i\hat{\pi}_i - \beta_iz_{i,1}^2-\pi_i\sum_{k=1}^{p}2^{2k}z_{i,1}^{2k}+\sum_{j=1}^{N}d_{i,j}^*+\bar{d}_{i,1} - r_i\bar{V}_{i,0}-r_i\bar{V}_{i,1}\Big\}. \tag{34}$$

Step $i, j(j=2,\ldots,n_i-1)$: Differentiating $z_{i,j}$ along (8) and (20) yields

$$\begin{aligned}\dot{z}_{i,j} &= \dot{\hat{x}}_{i,j}-\dot{\alpha}_{i,j-1}\\ &= \hat{x}_{i,j+1}-k_{i,j}\hat{x}_{i,1}+k_{i,j}y_i+H_{i,j} - \frac{\partial\alpha_{i,j-1}}{\partial y_i}\big[\tilde{W}_{i,1}^T\varphi_{i,1}(\bar{y}_r)+h_{i,1} + \Delta_{i,1}(\bar{y})+\Lambda_{i,1}+\varepsilon_{i,1}+e_{i,2}\big]\end{aligned} \tag{35}$$

where

$$H_{i,j} = W_{i,j}^T\varphi_{i,j}(\bar{y}_r)-\frac{\partial\alpha_{i,j-1}}{\partial W_{i,j-1}}\dot{W}_{i,j-1}-\frac{\partial\alpha_{i,j-1}}{\partial\hat{\beta}_i}\dot{\hat{\beta}}_i - \frac{\partial\alpha_{i,j-1}}{\partial\hat{\pi}_i}\dot{\hat{\pi}}_i-\frac{\partial\alpha_{i,j-1}}{\partial\bar{y}_r}\dot{\bar{y}}_r-\frac{\partial\alpha_{i,j-1}}{\partial y_i}\Big[\hat{x}_{i,2} + W_{i,1}^T\varphi_{i,1}(\bar{y}_r)\Big]-\sum_{k=2}^{j}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{i,k-1}}\dot{\hat{x}}_{i,k-1}.$$

Consider the following Lyapunov function candidate as

$$\sum_{i=1}^{N}V_{i,j}=\sum_{i=1}^{N}V_{i,j-1}+\sum_{i=1}^{N}\Big(\tfrac{1}{2}z_{i,j}^2+\tfrac{1}{2}\tilde{W}_{i,j}^T\Gamma_{i,j}^{-1}\tilde{W}_{i,j}+\bar{V}_{i,1}\Big) \tag{36}$$

where $\Gamma_{i,j}=\Gamma_{i,j}^T>0$.

The time derivative of $\sum_{i=1}^{N}V_{i,j}$ is

$$\sum_{i=1}^{N}\dot{V}_{i,j}=\sum_{i=1}^{N}\dot{V}_{i,j-1}+\sum_{i=1}^{N}\big(z_{i,j}\dot{z}_{i,j}-\tilde{W}_{i,j}^T\Gamma_{i,j}^{-1}\dot{W}_{i,j}+\dot{\bar{V}}_{i,1}\big). \tag{37}$$

From (35) and (37), one has

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_{i,j} &\le \sum_{i=1}^{N}\dot{V}_{i,j-1}+\sum_{i=1}^{N}\Big(z_{i,j}(\hat{x}_{i,j+1}-k_{i,j}\hat{x}_{i,1} + k_{i,j}y_i+H_{i,j}-\frac{\partial\alpha_{i,j-1}}{\partial y_i}[h_{i,1}+\Delta_{i,1}(\bar{y})+\Lambda_{i,1} + \varepsilon_{i,1}+e_{i,2}])+\Big|z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial y_i}\tilde{W}_{i,1}^T\varphi_{i,1}(\bar{y}_r)\Big| - \tilde{W}_{i,j}^T\Gamma_{i,j}^{-1}\dot{W}_{i,j}+\dot{\bar{V}}_{i,1}\Big)\\ &\le \sum_{i=1}^{N}\dot{V}_{i,j-1}+\sum_{i=1}^{N}\Big(z_{i,j}(\hat{x}_{i,j+1}-k_{i,j}\hat{x}_{i,1}+k_{i,j}y_i + H_{i,j}-\frac{\partial\alpha_{i,j-1}}{\partial y_i}[h_{i,1}+\Delta_{i,1}(\bar{y})+\Lambda_{i,1}+\varepsilon_{i,1} + e_{i,2}])+\Big|z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial y_i}\delta_{i,1}\Big| - \tilde{W}_{i,j}^T\Gamma_{i,j}^{-1}\dot{W}_{i,j}+\dot{\bar{V}}_{i,1}\Big).\end{aligned} \tag{38}$$

Using Young's inequality, Cauchy–Schwartz inequality, and Assumptions 1–3, we have the following inequalities:

$$z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial y_i}\varepsilon_{i,1} \le \frac{1}{2}\varepsilon_{i,1,0}^2+\frac{1}{2}\Big(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\Big)^2z_{i,j}^2 \tag{39}$$

$$z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial y_i}e_{i,2} \le \frac{1}{2}e_i^Te_i+\frac{1}{2}\Big(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\Big)^2z_{i,j}^2 \tag{40}$$

$$\Big|z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial y_i}\delta_{i,1}\Big| \le \frac{1}{2}\delta_{i,1,0}^2+\frac{1}{2}\Big(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\Big)^2z_{i,j}^2 \tag{41}$$

$$\sum_{i=1}^{N}z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial y_i}\Delta_{i,1}(\bar{y}) \le \sum_{i=1}^{N}\Big(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\Big)^2z_{i,j}^2 + \sum_{i=1}^{N}\sum_{k=1}^{p}2^{2k}q_{1,i,k}\big(\|y_{i,r}\|^{2k}+\|z_{i,1}\|^{2k}\big) \tag{42}$$

$$\sum_{i=1}^{N}z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial y_i}\Lambda_{i,1} \le \sum_{i=1}^{N}\Big(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\Big)^2z_{i,j}^2+\sum_{i=1}^{N}\sum_{k=1}^{N}\frac{N}{2}s_{i,1,k}^2z_{k,1}^2 \tag{43}$$

$$\Big|z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial y_i}h_{i,1}\Big| \le \frac{1}{2}z_{i,j}^2\Big(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\Big)^2 + \frac{1}{2}z_{i,1}(t-\tau_{i,1}(t))\bar{H}_{i,1}(z_{i,1}(t-\tau_{i,1}(t)))+\bar{d}_{i,1}. \tag{44}$$

Substituting (35), (39)–(44) into (38) results in

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_{i,j} \leq & -\sum_{i=1}^{N}\{e_i^T(Q_i-(4+j)I)e_i \\ & + \|P_i\|^2{\delta_{i0}'}^2+\sum_{k=1}^{p}2^{2k}(q_{i,k}+jq_{1,i,k}) \\ & \times \left(\|y_{i,r}\|^{2k}+\|z_{i,1}\|^{2k}\right) \\ & + \sum_{k=1}^{N}(L_{i,k}+j\frac{N}{2}s_{i,1,k}^2)z_{k,1}^2+z_{i,j-1}z_{i,j}+\frac{j-1}{2}\delta_{i,1,0}^2 \\ & + \frac{j-1}{2}\varepsilon_{i,1,0}^2+\sum_{j=1}^{N}d_{i,j}^*+j\bar{d}_{i,1}-r_i\bar{V}_{i,0}-jr_i\bar{V}_{i,1} \\ & - \sum_{k=1}^{j-1}c_{i,k}z_{i,k}^2-\frac{(n_i-j)e^{\tau_i}}{2(1-\tau^*)}(\bar{H}_{i,1}(z_{i,1}))z_{i,1} \\ & + (j-1)\varsigma'+\sum_{k=1}^{j-1}\sigma_{i,k}\tilde{W}_{i,k}^TW_{i,k}+\sigma_{i,2}\tilde{\beta}_i\hat{\beta}_i \\ & + \sigma_{i,3}\tilde{\pi}_i\hat{\pi}_i-\beta_iz_{i,1}^2-\pi_i\sum_{k=1}^{p}2^{2k}z_{i,1}^{2k} \\ & + z_{i,j}[z_{i,j+1}+\alpha_{i,j}-k_{i,j}\hat{x}_{i,1}+k_{i,j}y_i+H_{i,j} \\ & + 4(\frac{\partial\alpha_{i,j-1}}{\partial y_i})^2z_{i,j}]-\tilde{W}_{i,j}^T\Gamma_{i,2}^{-1}\dot{W}_{i,j}\}. \end{aligned} \quad (45)$$

Design the intermediate control function $\alpha_{i,j}$ and the adaptation function $W_{i,j}$ as

$$\begin{aligned}\alpha_{i,j} = & -c_{i,j}z_{i,j}-z_{i,j-1}+k_{i,j}\hat{x}_{i,1}-k_{i,j}y_i-H_{i,j} \\ & - 4\left(\frac{\partial\alpha_{i,1}}{\partial y_i}\right)^2z_{i,j}-\delta_{i,j,0}\tanh\left(\frac{\delta_{i,j,0}z_{i,j}}{\varsigma}\right)\end{aligned} \quad (46)$$

$$\dot{W}_{i,j} = \Gamma_{i,j}(z_{i,j}\varphi_{i,j}(\bar{y}_r)-\sigma_{i,j}W_{i,j}). \quad (47)$$

Substituting (46) and (47) into (45) results in

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_{i,j} \leq & -\sum_{i=1}^{N}\Big\{e_i^T(Q_i-(4+j)I)e_i \\ & + \|P_i\|^2{\delta_{i0}'}^2+\sum_{k=1}^{p}2^{2k}(q_{i,k}+jq_{1,i,k}) \\ & \times \left(\|y_{i,r}\|^{2k}+\|z_{i,1}\|^{2k}\right)+\sum_{k=1}^{N}\left(L_{i,k}+j\frac{N}{2}s_{i,1,k}^2\right)z_{k,1}^2 \\ & - \sum_{k=1}^{j}c_{i,k}z_{i,k}^2+j\varsigma'+\sum_{k=1}^{j}\sigma_{i,k}\tilde{W}_{i,k}^TW_{i,k} \\ & + \sigma_{i,2}\tilde{\beta}_i\hat{\beta}_i+\sigma_{i,3}\tilde{\pi}_i\hat{\pi}_i-\beta_iz_{i,1}^2-\pi_i\sum_{k=1}^{p}2^{2k}z_{i,1}^{2k} \\ & + z_{i,j}z_{i,j+1}+\frac{j-1}{2}\delta_{i,1,0}^2+\frac{j-1}{2}\varepsilon_{i,1,0}^2 \\ & + \sum_{j=1}^{N}d_{i,j}^*+j\bar{d}_{i,1}-r_i\bar{V}_{i,0}-jr_i\bar{V}_{i,1} \\ & - \frac{(n_i-j)e^{\tau_i}}{2(1-\tau^*)}(\bar{H}_{i,1}(z_{i,1}))z_{i,1}\Big\}. \end{aligned} \quad (48)$$

Step i, n_i: In the final design step, the actual control input u_i will appear. Consider the overall Lyapunov function candidate as

$$\begin{aligned}V = \sum_{i=1}^{N}V_{i,n_i} = & \sum_{i=1}^{N}V_{i,n_i-1} \\ & + \sum_{i=1}^{N}\left(\frac{1}{2}z_{i,n_i}^2+\frac{1}{2}\tilde{W}_{i,n_i}^T\Gamma_{i,n_i}^{-1}\tilde{W}_{i,n_i}+\bar{V}_{i,1}\right). \end{aligned} \quad (49)$$

Similar to the procedures in Step i, j, one can obtain

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{N}\Big\{e_i^T\bar{Q}_ie_i+\|P_i\|^2\|\delta'_{i0}\|^2 \\ & + \sum_{k=1}^{p}2^{2k}(q_{i,k}+n_iq_{1,i,k})(\|y_{i,r}\|^{2k}+\|z_{i,1}\|^{2k}) \\ & + \sum_{k=1}^{N}\left(L_{i,k}+j\frac{N}{2}s_{i,1,k}^2\right)z_{k,1}^2+z_{i,n_i-1}z_{i,n_i} \\ & + \delta_{i,1,0}^2+\frac{1}{2}\varepsilon_{i,1,0}^2+\sum_{j=1}^{N}d_{i,j}^*+n_i\bar{d}_{i,1}-r_i\bar{V}_{i,0} \\ & - n_ir_i\bar{V}_{i,1}-\sum_{k=1}^{n_i-1}c_{i,k}z_{i,k}^2+(n_i-1)\varsigma' \\ & + \sum_{k=1}^{n_i-1}\sigma_{i,k}\tilde{W}_{i,k}^TW_{i,k}+\sigma_{i,2}\tilde{\beta}_i\hat{\beta}_i+\sigma_{i,3}\tilde{\pi}_i\hat{\pi}_i \\ & - \beta_iz_{i,1}^2-\pi_i\sum_{k=1}^{p}2^{2k}z_{i,1}^{2k}+z_{i,n_i}[u_i-k_{i,n_i}\hat{x}_{i,1} \\ & + k_{i,n_i}y_i+H_{i,n_i}+4\left(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i}\right)^2z_{i,n_i}] \\ & - \tilde{W}_{i,n_i}^T\Gamma_{i,2}^{-1}\dot{W}_{i,n_i}\Big\} \end{aligned} \quad (50)$$

where $\bar{Q}_i = Q_i-(4+n_i)I) > 0$.

At the end of the recursive procedure, the last stabilizing function $\alpha_{n_i} = u_i$ and adaptation function W_{n_i} are chosen as

$$\begin{aligned}u_i = & -c_{i,n_i}z_{i,n_i}-z_{i,n_i-1}+k_{i,n_i}\hat{x}_{i,1} \\ & - k_{i,n_i}y_i-H_{i,n_i}-4\left(\frac{\partial\alpha_{i,1}}{\partial y_i}\right)^2z_{i,n_i} \\ & - \delta_{i,n_i,0}\tanh\left(\frac{\delta_{i,n_i,0}z_{i,n_i}}{\varsigma}\right)\end{aligned} \quad (51)$$

$$\dot{W}_{i,n_i} = \Gamma_{i,n_i}(z_{i,n_i}\varphi_{i,n_i}(\bar{y}_r)-\sigma_{i,n_i}W_{i,n_i}) \quad (52)$$

where

$$H_{i,n_i} = W_{i,n_i}^T \varphi_{i,n_i}(\bar{y}_r) - \frac{\partial \alpha_{i,n_i-1}}{\partial W_{i,n_i-1}} \dot{W}_{i,n_i-1} - \frac{\partial \alpha_{i,n_i-1}}{\partial \hat{\beta}_i} \dot{\hat{\beta}}_i - \frac{\partial \alpha_{i,n_i-1}}{\partial \hat{\pi}_i} \dot{\hat{\pi}}_i - \frac{\partial \alpha_{i,n_i-1}}{\partial \bar{y}_r} \dot{\bar{y}}_r - \frac{\partial \alpha_{i,n_i-1}}{\partial y_i} [\hat{x}_{i,2} + W_{i,1}^T \varphi_{i,1}(\bar{y}_r)] - \sum_{k=2}^{n_i} \frac{\partial \alpha_{i,n_i-1}}{\partial \hat{x}_{i,k-1}} \dot{\hat{x}}_{i,k-1}.$$

By the completing squares

$$\tilde{W}_{i,k}^T W_{i,k} \le \tilde{W}_{i,k}^T (W_{i,k}^* - \tilde{W}_{i,k}) \le -\frac{1}{2} \tilde{W}_{i,k}^T \tilde{W}_{i,k} + \frac{1}{2} W_{i,k}^{*T} W_{i,k}^* \tag{53}$$

$$\tilde{\beta}_i \hat{\beta}_i \le -\frac{1}{2} \tilde{\beta}_i^2 + \frac{1}{2} \beta_i^2 \tag{54}$$

$$\tilde{\pi}_i \hat{\pi}_i \le -\frac{1}{2} \tilde{\pi}_i^2 + \frac{1}{2} \pi_i^2. \tag{55}$$

Substituting (51)–(55) into (50), one obtains

$$\dot{V} \le -CV + D \tag{56}$$

where $C = \min\{C_1, \ldots, C_{n_i}\}$, with $C_i = \{\min(\lambda_{\min}(Q_i - (4 + n_i)I)), 2c_{i,k}, 2\sigma_{i,1}/\lambda_{\max}(\Gamma_{i,1}), 2\gamma_{i,1}\sigma_{i,2}, 2\gamma_{i,2}\sigma_{i,3}, r_i\}$

$$D = \sum_{i=1}^{N} \|P_i\|^2 {\delta'_{i0}}^2 + \sum_{k=1}^{p} 2^{2k} (q_{i,k} + n_i q_{1,i,k})(\max \|y_{i,r}\|^{2k}) + n_i \varsigma' + \frac{1}{2} \sum_{k=1}^{n_i} \sigma_{i,k} W_{i,k}^{*T} W_{i,k}^* + \frac{1}{2} \sigma_{i,2} \beta_i^2 + \frac{1}{2} \sigma_{i,3} \pi_i^2 + \sum_{j=1}^{N} d_{i,j}^* + n_i \bar{d}_{i,1} + \frac{n_i - 1}{2} \delta_{i,1,0}^2 + \frac{n_i - 1}{2} \varepsilon_{i,1,0}^2\}$$

and $\max \|y_{i,r}\|^{2k} = \max_{\forall t \in (0,\infty)} \|y_{i,r}\|^{2k}$.

By (56), and using the same arguments as in [20], [22], [25], [29], and [41], it can be proved that for each $j = 1, \ldots, n_i$, all the signals in the closed-loop system are SUUB. Moreover, the tracking errors $z_{i,1} = y_i - y_{i,r}$ and observer error vector e_i, $i = 1, \ldots, N$, can be made arbitrarily small by adjusting the design parameters appropriately.

The above design procedures and stable analysis are summarized in the following theorem:

Theorem 1: For the nonlinear large-scale system (1), under Assumptions 1–4, the decentralized controller (50) with the state observer (8), the intermediate control (30), (46) and parameter laws (31)–(33), (47), and (52) can guarantee that all the signals in the closed-loop system is SUUB. Moreover, the tracking errors and the observer errors can be made arbitrarily small by adjusting the design parameters appropriately.

IV. Simplified Decentralized DSC Design and Stability Analysis

In the previous section, an observer-based adaptive NN decentralized output feedback control approach was developed. It was proved that the proposed approach can guarantee the stability of the closed-loop system. However, to observe the stabilizing functions $\alpha_{i,j}$ in (46) and the control u_i in (51), one can find that an obvious drawback with this approach is that $\alpha_{i,j}$ and u_i contain many derivatives about the variables, which is the so-called the problem of explosion of complexity in [35]–[39]. In order to overcome the problem of explosion of complexity, this section will incorporate the DSC technique proposed in [35] and [36] into the above adaptive NN decentralized control design scheme, and develop a simplified observer-based adaptive NN decentralized DSC approach.

Before starting the adaptive NN control design, the following assumption on reference signal $y_{i,r}(t)$ is given.

Assumption 5 ([35], [40]): The reference signal $y_{i,r}(t)$ is a sufficiently smooth function of t, $y_{i,r}$, $\dot{y}_{i,r}$ and $\ddot{y}_{i,r}$ are bounded, i.e., there exists a positive constant B_{i0} such that

$$\Pi_{i0} := \{(y_{i,r}, \dot{y}_{i,r}, \ddot{y}_{i,r}) : y_{i,r}^2 + \dot{y}_{i,r}^2 + \ddot{y}_{i,r}^2 \le B_{i0}\}.$$

The n_i-step adaptive NN output feedback backstepping design is based on the following change of coordinates

$$z_{i,1} = y_{i,1} - y_{i,r} \tag{57}$$

$$z_{i,j} = \hat{x}_{i,j} - \xi_{i,j}, \quad i = 1, \ldots, N, \; j = 2, \ldots, n_i \tag{58}$$

$$\chi_{i,j} = \xi_{i,j} - \alpha_{i,j-1}, \quad i = 1, \ldots, N, \; j = 2, \ldots, n_i \tag{59}$$

where $z_{i,j}$ is called the error surface, $\xi_{i,j}$ is a variable, which is obtained through a first-order filter on intermediate function $\alpha_{i,j-1}$, and $\chi_{i,j}$ is called the output error of the first-order filter.

Step i, 1: The time derivative of $z_{i,1}$ along (1) is

$$\dot{z}_{i,1} = \hat{x}_{i,2} + W_{i,j}^T \varphi_{i,j}(\bar{y}_r) + \tilde{W}_{i,j}^T \varphi_{i,j}(\bar{y}_r) + h_{i,1} + \Delta_{i,1}(\bar{y}) - \dot{y}_{i,r} + \Lambda_{i,1} + \varepsilon_{i,1} + e_{i,2}. \tag{60}$$

Consider the Lyapunov function candidate $\sum_{i=1}^{N} V_{i,1}$ as

$$\sum_{i=1}^{N} V_{i,1} = \sum_{i=1}^{N} \left\{ V_{i,0} + \frac{1}{2} z_{i,1}^2 + \frac{1}{2} \tilde{W}_{i,1}^T \Gamma_{i,1}^{-1} \tilde{W}_{i,1} + \frac{1}{2\gamma_{i,1}} \tilde{\beta}_i^2 + \frac{1}{2\gamma_{i,2}} \tilde{\pi}_i^2 + \bar{V}_{i,1} \right\} \tag{61}$$

where $\beta_i = \sum_{k=1}^{N} (L_{k,i} + (N/2) s_{k,1,i}^2)$ and $\pi_i = \max_{1 \le k \le p} \{q_{i,k} + q_{1,i,k}\}$.

The intermediate control function $\alpha_{i,1}$ and the adaptation functions $W_{i,1}$, $\hat{\beta}_i$ and $\hat{\pi}_i$ are chosen as

$$\alpha_{i,1} = -\frac{1}{2} z_{i,1} - (c_{i,1} + 3) z_{i,1} - \hat{\beta}_i z_{i,1} - \hat{\pi}_i \sum_{k=1}^{p} 2^{2k} z_{i,1}^{2k-1} - W_{i,1}^T \varphi_{i,1}(\bar{y}_r) + \dot{y}_{i,r} - \varepsilon_{i,1,0} \tanh\left(\frac{z_{i,1} \varepsilon_{i,1,0}}{\varsigma}\right) - \frac{e^{\tau_i}}{(1 - \tau^*)} \|P_i\|^2 \sum_{j=1}^{N} (\bar{H}_{i,j}(z_{i,1})) - \frac{e^{\tau_i}}{2(1 - \tau^*)} (\bar{H}_{i,1}(z_{i,1})) \tag{62}$$

$$\dot{W}_{i,1} = \Gamma_{i,1} (z_{i,1} \varphi_{i,1}(\bar{y}_r) - \sigma_{i,1} W_{i,1}) \tag{63}$$

$$\dot{\hat{\beta}}_i = \gamma_{i,1} (z_{i,1}^2 - \sigma_{i,2} \hat{\beta}_i) \tag{64}$$

$$\dot{\hat{\pi}}_i = \gamma_{i,2}\left(\sum_{k=1}^{p} 2^{2k} z_{i,1}^{2k} - \sigma_{i,3}\hat{\pi}_i\right) \tag{65}$$

where $W_{i,1}(0) = 0$, $\hat{\beta}_i(0) = 0$ and $\hat{\pi}_i(0) = 0$.

Similar to the procedures in Step i,1 in the above design approach, one can obtain

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_{i,1} \le -\sum_{i=1}^{N}\Big\{ & e_i^T(Q_i - 5I)e_i + \delta_{i0}'^2\|P_i\|^2 \\ & + \sum_{k=1}^{p} 2^{2k}(q_{i,k} + q_{1,i,k})\big\|y_{i,r}\big\|^{2k} \\ & + z_{i,1}z_{i,2} - c_{i,1}z_{i,1}^2 + \varsigma' + \sigma_{i,1}\tilde{W}_{i,1}^T W_{i,1} \\ & + \sigma_{i,2}\tilde{\beta}_i\hat{\beta}_i + \sigma_{i,3}\tilde{\pi}_i\hat{\pi}_i - \beta_i z_{i,1}^2 \\ & + \sum_{j=1}^{N} d_{i,j}^* + \bar{d}_{i,1} - r_i\bar{V}_{i,0} - r_i\bar{V}_{i,1}\Big\}. \end{aligned} \tag{66}$$

Step $i, j (j = 2, \ldots, n_i - 1)$: From (58) and (59), the time derivative of $z_{i,j}$ is

$$\begin{aligned}\dot{z}_{i,j} &= \dot{\hat{x}}_{i,j} - \dot{\xi}_{i,j} \\ &= \hat{x}_{i,j+1} - k_{i,j}\hat{x}_{i,1} + k_{i,j}y_i \\ &\quad + W_{i,j}^T\varphi_{i,j}(\bar{y}_r) - \dot{\xi}_{i,j} \\ &= z_{i,j+1} + \chi_{i,j+1} + \alpha_{i,j} - k_{i,j}\hat{x}_{i,1} \\ &\quad + k_{i,j}y_i + W_{i,j}^T\varphi_{i,j}(\bar{y}_r) - \dot{\xi}_{i,j}. \end{aligned} \tag{67}$$

Consider the following Lyapunov function candidate

$$\begin{aligned}\sum_{i=1}^{N} V_{i,j} &= \sum_{i=1}^{N} V_{i,j-1} \\ &\quad + \sum_{i=1}^{N}\Big(\frac{1}{2}z_{i,j}^2 + \frac{1}{2}\chi_{i,j}^2 + \frac{1}{2}\tilde{W}_{i,j}^T\Gamma_{i,j}^{-1}\tilde{W}_{i,j}\Big). \end{aligned} \tag{68}$$

The time derivative of $\sum_{i=1}^{N} V_{i,j}$ along (67) is

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_{i,j} &= \sum_{i=1}^{N}\dot{V}_{i,j-1} + \sum_{i=1}^{N}(z_{i,j}(z_{i,j+1} + \chi_{i,j+1} \\ &\quad - k_{i,j}\hat{x}_{i,1} + k_{i,j}y_i + \alpha_{i,j} + W_{i,j}^T\varphi_{i,j}(\bar{y}_r) \\ &\quad - \dot{g}_{i,j}) + \chi_{i,j}\dot{\chi}_{i,j} - \tilde{W}_{i,j}^T\Gamma_{i,j}^{-1}\dot{W}_{i,j}). \end{aligned} \tag{69}$$

Choose the intermediate control function $\alpha_{i,j}$ and the adaptation function $W_{i,j}$ as

$$\begin{aligned}\alpha_{i,j} &= -c_{i,j}z_{i,j} - z_{i,j-1} + k_{i,j}\hat{x}_{i,1} \\ &\quad - k_{i,j}y_i - W_{i,j}^T\varphi_{i,j}(\bar{y}_r) \\ &\quad - \delta_{i,j,0}\tanh(\frac{\delta_{i,j,0}z_{i,j}}{\varsigma}) + \dot{\xi}_{i,j} \end{aligned} \tag{70}$$

$$\dot{W}_{i,j} = \Gamma_{i,j}(z_{i,j}\varphi_{i,j}(\bar{y}_r) - \sigma_{i,j}W_{i,j}) \tag{71}$$

where $W_{i,j}(0) = 0$.

To avoid repeatedly differentiating $\alpha_{i,j}$ in the traditional backstepping design, which leads to the so-called explosion of complexity, in the sequel, one can introduce a new state variable $\xi_{i,j+1}$ and let $\alpha_{i,j}$ pass through a first-order filter with the constant $\lambda_{i,j+1}$ to obtain $\xi_{i,j+1}$

$$\lambda_{i,j+1}\dot{\xi}_{i,j+1} + \xi_{i,j+1} = \alpha_{i,j}, \xi_{i,j+1}(0) = \alpha_{i,j}(0). \tag{72}$$

By the definition of $\chi_{i,j+1} = \xi_{i,j+1} - \alpha_{i,j}$, it yields $\dot{\xi}_{i,j+1} = -\chi_{i,j+1}/\lambda_{i,j+1}$ and

$$\begin{aligned}\dot{\chi}_{i,j+1} &= -\frac{\chi_{i,j+1}}{\lambda_{i,j+1}} + B_{i,j+1}(z_{i,1}, \ldots, z_{i,j+1}, \\ &\quad \chi_{i,2}, \ldots, \chi_{i,j+1}, W_{i,1}, \ldots, W_{i,j}, y_{1,r}, \\ &\quad \dot{y}_{1,r}, \ddot{y}_{1,r}, \ldots, y_{N,r}, \dot{y}_{N,r}, \ddot{y}_{N,r}) \end{aligned} \tag{73}$$

where

$$\begin{aligned}B_{i,j+1}(\cdot) &= c_{i,j}\dot{z}_{i,j} + \dot{z}_{i,j-1} - k_{i,j}\dot{\hat{x}}_{i,1} \\ &\quad + k_{i,j}\dot{y}_i + \dot{W}_{i,j}^T\varphi_{i,j}(\bar{y}_r) + \frac{W_{i,j}^T\partial\varphi_{i,j}(\bar{y}_r)}{\partial\bar{y}_r}\dot{\bar{y}}_r \\ &\quad + \frac{d\delta_{i,j,0}\tanh(\frac{\delta_{i,j,0}z_{i,j}}{\varsigma})}{dz_{i,j}}\dot{z}_{i,j} - \frac{\dot{\chi}_{i,j}}{\lambda_{i,j}}. \end{aligned} \tag{74}$$

From (70)–(74), one has

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_{i,j} \le -\sum_{i=1}^{N}\{ & e_i^T(Q_i - 4I)e_i \\ & + \|P_i\|^2\delta_{i0}'^2 + \sum_{k=1}^{p}2^{2k}(q_{i,k} + q_{1,i,k})\big\|y_{i,r}\big\|^{2k} \\ & - \sum_{k=1}^{j}c_{i,k}z_{i,k}^2 + j\varsigma' + \sum_{k=1}^{j}\sigma_{i,k}\tilde{W}_{i,k}^T W_{i,k} \\ & + \sum_{l=1}^{j}z_{i,l}\chi_{i,l+1} + \sigma_{i,2}\tilde{\beta}_i\hat{\beta}_i + \sigma_{i,3}\tilde{\pi}_i\hat{\pi}_i \\ & + z_{i,j}z_{i,j+1} - \sum_{k=1}^{j-1}\Big(\frac{\chi_{i,k+1}^2}{\lambda_{i,k+1}} - B_{i,k+1}(\cdot)\chi_{i,k+1}\Big) \\ & + \sum_{j=1}^{N}d_{i,j}^* + \bar{d}_{i,1} - r_i\bar{V}_{i,0} - r_i\bar{V}_{i,1}\}. \end{aligned} \tag{75}$$

Step i, n_i: In the final design step, the actual control input u_i will appear. The time derivative of z_{i,n_i} is

$$\begin{aligned}\dot{z}_{i,n_i} &= \dot{\hat{x}}_{i,n_i} - \dot{\xi}_{i,n_i} \\ &= u_i - k_{i,n_i}\hat{x}_{i,1} + k_{i,n_i}y_i \\ &\quad + W_{i,n_i}^T\varphi_{i,n_i}(\bar{y}_r) - \dot{\xi}_{i,n_i}. \end{aligned} \tag{76}$$

Consider the following Lyapunov function candidate

$$\begin{aligned}V &= \sum_{i=1}^{N}V_{i,n_i} = \sum_{i=1}^{N}V_{i,n_i-1} \\ &\quad + \sum_{i=1}^{N}\Big(\frac{1}{2}z_{i,n_i}^2 + \frac{1}{2}\chi_{i,n_i}^2 + \frac{1}{2}\tilde{W}_{i,n_i}^T\Gamma_{i,n_i}^{-1}\tilde{W}_{i,n_i}\Big). \end{aligned} \tag{77}$$

Take the control u_i and the adaptation function W_{i,n_i} as follows:

$$\begin{aligned}u_i &= -c_{i,n_i}z_{i,n_i} - z_{i,n_i-1} + k_{i,n_i}\hat{x}_{i,1} \\ &\quad - k_{i,n_i}y_i - W_{i,n_i}^T\varphi_{i,n_i}(\bar{y}_r) \\ &\quad - \delta_{i,n_i,0}\tanh\Big(\frac{\delta_{i,n_i,0}z_{i,n_i}}{\varsigma}\Big) + \dot{\xi}_{i,n_i} \end{aligned} \tag{78}$$

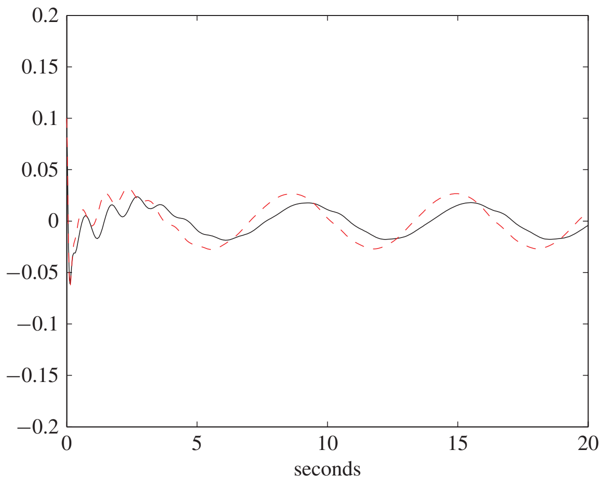

Fig. 1. $z_{1,1}$ "solid line" and $z_{2,1}$ "dash-dotted."

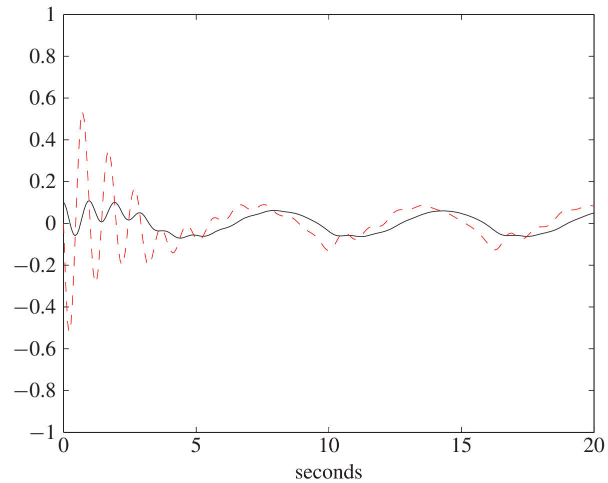

Fig. 2. $e_{1,1}$ "solid line" and $e_{1,2}$ "dash-dotted."

$$\dot{W}_{i,n_i} = \Gamma_{i,n_i}(z_{i,n_i}\varphi_{i,n_i}(\bar{y}_r) - \sigma_{i,n_i}W_{i,n_i}). \quad (79)$$

From (77)–(79), using the similar procedures to Step i, j, one has

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{N}\{e_i^T(Q_i - 4I)e_i + \|P_i\|^2\delta'^2_{i0} \\ & + \sum_{k=1}^{p}2^{2k}(q_{i,k} + q_{1,i,k})\|y_{i,r}\|^{2k} \\ & - \sum_{k=1}^{n_i}c_{i,k}z_{i,k}^2 + n_i\varsigma' + \sum_{k=1}^{n_i}\sigma_{i,k}\tilde{W}_{i,k}^T W_{i,k} \\ & + \sum_{l=1}^{n_i}z_{i,l}\chi_{i,l+1} + \sigma_{i,2}\tilde{\beta}_i\hat{\beta}_i + \sigma_{i,3}\tilde{\pi}_i\hat{\pi}_i \\ & - \sum_{k=1}^{n_i-1}\left(\frac{\chi_{i,k+1}^2}{\lambda_{i,k+1}} - B_{i,k+1}(\cdot)\chi_{i,k+1}\right) + \sum_{j=1}^{N}d_{i,j}^* \\ & + \bar{d}_{i,1} - r_i\bar{V}_{i,0} - r_i\bar{V}_{i,1}\}. \end{aligned} \quad (80)$$

By using Young's inequality, one has

$$\left|B_{i,k+1}\chi_{i,k+1}\right| \leq \frac{\chi_{i,k+1}^2 B_{i,k+1}^2}{2\vartheta_i} + 2\vartheta_i \quad (81)$$

where ϑ_i is a positive design constant.

Substituting (53)–(55) and (81) into (80) results in

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{N}\{e_i^T(Q_i - 4I)e_i - \sum_{k=1}^{n_i}(c_{i,k} - \frac{1}{2})z_{i,k}^2 \\ & - \frac{1}{2}\sum_{k=1}^{n_i}\sigma_{i,k}\tilde{W}_{i,k}^T\tilde{W}_{i,k} - \frac{1}{2}\sigma_{i,2}\tilde{\beta}_i^2 - \frac{1}{2}\sigma_{i,3}\tilde{\pi}_i^2 \\ & - \sum_{k=1}^{n_i-1}\left(\frac{1}{\lambda_{i,k+1}} - \frac{B_{i,k+1}^2}{2\vartheta_i} - \frac{1}{2}\right)\chi_{i,k+1}^2 \\ & - r_i\bar{V}_{i,0} - r_i\bar{V}_{i,1}\} + D \end{aligned} \quad (82)$$

where

$$\begin{aligned} D = & \sum_{i=1}^{N}\Big\{\|P_i\|^2\delta'^2_{i0} + \sum_{k=1}^{p}2^{2k}(q_{i,k} + q_{1,i,k}) \\ & \times(\max\|y_{i,r}\|^{2k}) + n_i\varsigma' - \tfrac{1}{2}\sum_{k=1}^{n_i}\sigma_{i,k}W_{i,k}^{*T}W_{i,k}^* \\ & + \tfrac{1}{2}\sigma_{i,2}\beta_i^2 + \tfrac{1}{2}\sigma_{i,3}\pi_i^2 + \sum_{j=1}^{N}d_{i,j}^* + \bar{d}_{i,1}\Big\}. \end{aligned}$$

Let $A_{i,j} = \{\sum_{k=1}^{j}(z_{i,k}^2 + \tilde{W}_{i,k}^T\Gamma_{i,k}^{-1}\tilde{W}_{i,k}) + \sum_{k=1}^{j-1}\chi_{i,k+1}^2 + (1/\gamma_{i,1})\tilde{\beta}_i^2 + (1/\gamma_{i,2})\tilde{\pi}_i^2 + 2e_i^T P_i e_i + 2\bar{V}_{i,0} + 2\bar{V}_{i,1} \leq 2p_i\}$, $i = 1, \ldots, N$, $j = 1, \ldots, n_i$. Since $A_{i,j}$ is a compact set and $B_{i,k+1}$ is a continuous function, there exists a positive constant $M_{i,k+1}$ such that $\left|B_{i,k+1}\right| \leq M_{i,k+1}$ on $A_{i,j}$; therefore, one has

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{N}\{e_i^T(Q_i - 4I)e_i - \sum_{k=1}^{n_i}(c_{i,k} - \frac{1}{2})z_{i,k}^2 \\ & -\frac{1}{2}\sum_{k=1}^{n_i}\sigma_{i,k}\tilde{W}_{i,k}^T\tilde{W}_{i,k} - \frac{1}{2}\sigma_{i,2}\tilde{\beta}_i^2 - \frac{1}{2}\sigma_{i,3}\tilde{\pi}_i^2 \\ & - \sum_{k=1}^{n_i-1}(\frac{1}{\lambda_{i,k+1}} - \frac{M_{i,k+1}^2}{2\vartheta_i} - \frac{1}{2})\chi_{i,k+1}^2 \\ & -r_i\bar{V}_{i,0} - r_i\bar{V}_{i,1}\} + D. \end{aligned} \quad (83)$$

Choose the design parameters $\lambda_{i,k+1}$ and $c_{i,k}$ such that $c_{i,k} - 1/2 > 0$ and $(1/\lambda_{i,k+1}) - (M_{i,k+1}^2/2\vartheta_i) - (1/2) > 0$, respectively. Define $C = \min\{C_1, \ldots, C_{n_i}\}$, where $C_i = \{\min(\lambda_{\min}(Q) - 4), 2(c_{i,k} - 1/2), 2\sigma_{i,1}/\lambda_{\max}(\Gamma_{i,1}), (2(1/\lambda_{i,k+1}) - (M_{i,k+1}^2/2\vartheta_i) - (1/2)), 2\gamma_{i,1}\sigma_{i,2}, 2\gamma_{i,2}\sigma_{i,3}, r_i\}$. From (83), one can obtain

$$\dot{V} \leq -CV + D. \quad (84)$$

Assumption 6: For a given $p > 0$, there exists $V(0) \leq p$.

Remark 7: It should be pointed out that Assumption 6 means that all initial conditions on the closed-loop system are bounded by the given parameter p, where p can increase or decrease by choosing the initial conditions arbitrarily. That is,

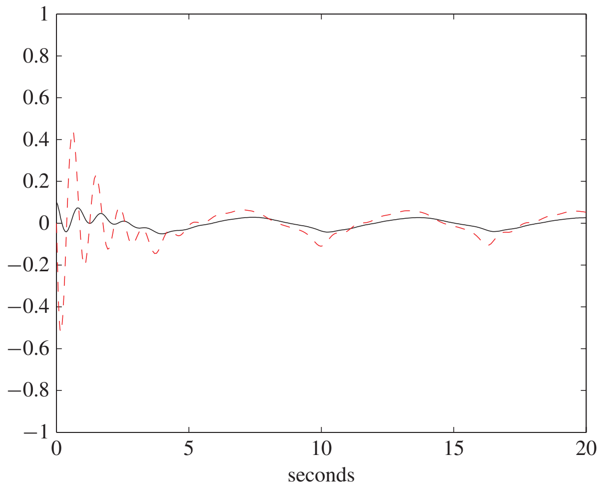

Fig. 3. $e_{2,1}$ "solid line" and $e_{2,2}$ "dash-dotted."

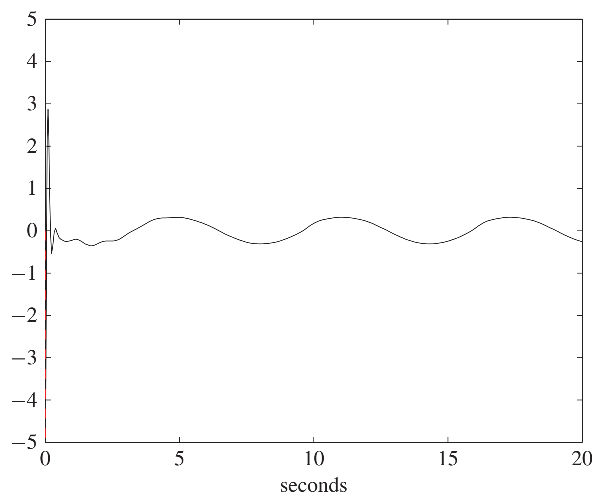

Fig. 4. u_1 "solid line" and u_2 "dash-dotted."

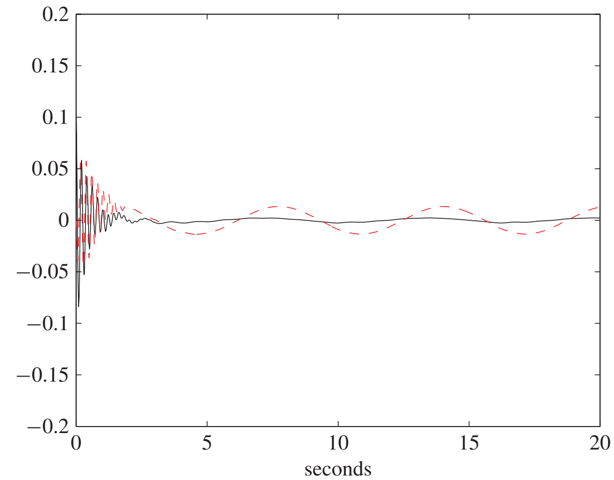

Fig. 5. $z_{1,1}$ "solid line" and $z_{2,1}$ "dash-dotted."

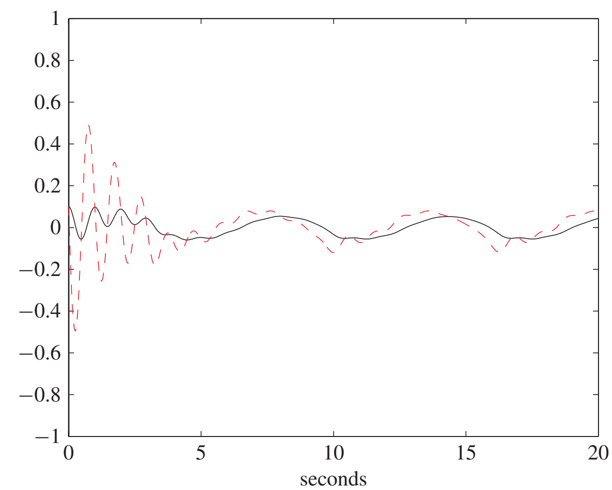

Fig. 6. $e_{1,1}$ "solid line" and $e_{1,2}$ "dash-dotted."

Assumption 6 is presented in terms of the concept of semi-global stability (see [29], [36]–[37], [39]).

Theorem 2: Under Assumption 1–5 and suppose that $W_{i,1}$, $\hat{\beta}_i$, $\hat{\pi}_i$, $W_{i,j}$ and W_{i,n_i} are adapted via adaptation laws (63)–(65), (71), and (79). Then for any initial conditions satisfying Assumption 6, there exist $c_{i,j}$, ς, $\gamma_{i,1}$, $\gamma_{i,2}$, $\Gamma_{i,1}$, $\sigma_{i,1}$, $\sigma_{i,2}$, $\sigma_{i,3}$, $\lambda_{i,j}$, Q_i and $k_{i,j}$ such that all signals of the closed-loop system are SUUB. Moreover, the tracking errors and the observer errors can be made arbitrarily small by choosing appropriate design parameters.

V. Simulation Study

In this section, a simulation example is presented to show the effectiveness of the proposed two adaptive NN decentralized control approaches and make some necessary comparisons between these two control schemes.

Example([12], [33]): Consider the control systems of two inverted pendulums connected by a spring. Each pendulum may be positioned by a torque input u_i applied by a servomotor at its base. It is assumed that θ_i and $\dot{\theta}_i$ are the angular position and rate, θ_i is available to the ith controller for $i = 1, 2$. Let $\theta_1 = x_{1,1}$, $\theta_2 = x_{2,1}$, $\dot{\theta}_1 = x_{1,2}$, and $\dot{\theta}_2 = x_{2,2}$, then inverted pendulum equation can be described as

$$\begin{cases} \dot{x}_{1,1} = x_{1,2} + f_{1,1}(y_1, y_2) + h_{1,1} + \Delta_{1,1}(y_1, y_2) \\ \dot{x}_{1,2} = \frac{u_1}{J_1} + f_{1,2}(y_1, y_2) \\ \qquad + d_1 + \Delta_{1,2}(y_1, y_2) + h_{1,2} \\ y_1 = x_{1,1} \end{cases} \tag{85}$$

$$\begin{cases} \dot{x}_{2,1} = x_{2,2} + f_{2,1}(y_1, y_2) + h_{2,1} + \Delta_{2,1}(y_1, y_2) \\ \dot{x}_{2,2} = \frac{u_2}{J_2} + f_{2,2}(y_1, y_2) + h_{2,2} \\ \qquad + d_2 + \Delta_{2,2}(y_1, y_2) \\ y_2 = x_{2,1} \end{cases} \tag{86}$$

where $f_{1,1} = 0$, $h_{1,1} = 0$, $\Delta_{1,1} = 0$, $f_{1,2} = ((m_1 gr/J_1) - (kr^2/4J_1))\sin(x_{1,1})$, $d_1 = (kr/2J_1)(l - b)$, $\Delta_{1,2} = (kr^2/4J_1)\sin(x_{2,1})$, $h_{1,2} = (x_{1,1}(t - \tau_{1,2}(t)))/(1 + x_{1,1}^2(t - \tau_{1,2}(t)))$, $f_{2,1} = 0$, $h_{2,1} = 0$, $\Delta_{2,1} = 0$, $\Delta_{2,1} = 0$, $f_{2,2} = ((m_2 gr/J_2) - (kr^2/4J_2))\sin(x_{2,1})$, $h_{2,2} = (x_{2,1}(t - \tau_{2,2}(t)))/(1 + x_{2,1}^2(t - \tau_{2,2}(t)))$, $d_2 = (kr/2J_2)(l - b)$, $\Delta_{2,2} = (kr^2/4J_2)\sin(x_{1,1})$ and $\tau_{1,2} = \tau_{2,2} = 0.5(1 + \sin(t))$. $\tau_{1,2}(t)$ and $\tau_{2,2}(t)$ satisfy $\dot{\tau}_{i,2}(t) \leq \tau^* < 1$, $(i = 1, 2)$.

The parameters in (85) and (86) are chosen as $m_1 = 2$ kg, $m_2 = 2.5$ kg, $J_1 = 5$ kg, $J_2 = 6.25$ kg, $k = 100$ N/m, $r = 0.5$ m, $l = 0.5$ m, $g = 9.81$ m/s^2, and $b = 0.5$ m.

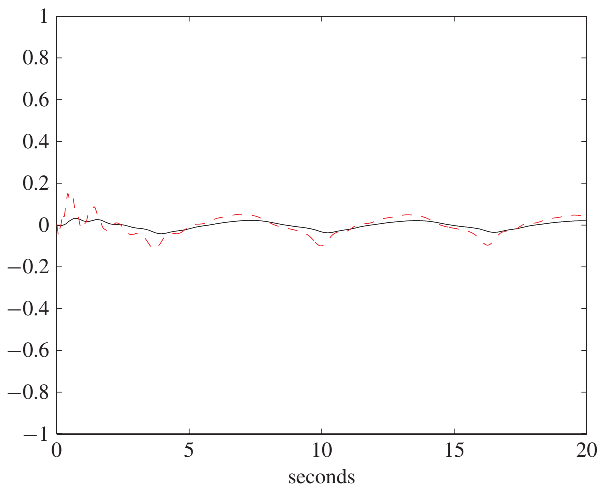

Fig. 7. $e_{2,1}$ "solid line" and $e_{2,2}$ "dash-dotted."

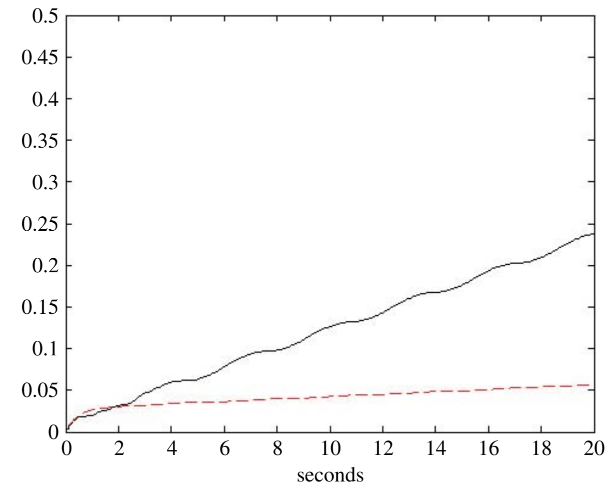

Fig. 9. I_1 in Approach 1 "solid line" and in Approach 2 "dash-dotted."

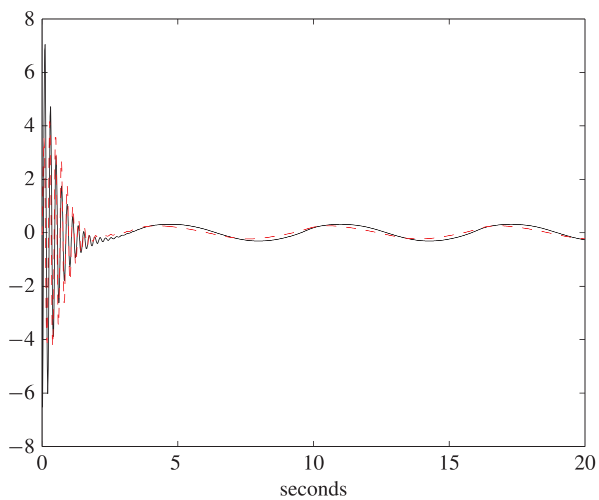

Fig. 8. u_1 "solid line" and u_2 "dash-dotted."

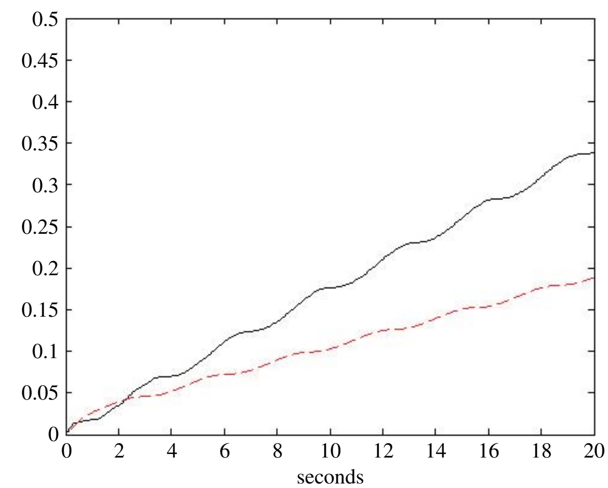

Fig. 10. I_2 in Approach 1 "solid line" and in Approach 2 "dash-dotted."

It is apparent that $\Delta_{i,j}$ satisfies the condition of Assumption 1. By selecting the functions $\bar{H}_{1,2} = 2z_{1,1}$, $\bar{H}_{2,2} = 2z_{2,1}$, $\bar{h}_{1,2} = 2\sin^2(t)$, $\bar{h}_{2,2} = 2\sin^2(t)$, $\varpi_{1,2} = 0$ and $\varpi_{2,2} = 0$, Assumption 2 is satisfied.

Choose $k_{1,1} = 1$, $k_{1,2} = 40$, $k_{2,1} = 2$, $k_{2,2} = 50$, $Q_1 = \text{diag}\{8, 8\}$, and $Q_2 = \text{diag}\{10, 10\}$. By solving Lypunov equation (8), the symmetric positive matrices P_1 and P_2 are obtained as

$$P_1 = \begin{bmatrix} 164.0000 & -4.0000 \\ -4.0000 & 4.0000 \end{bmatrix}, P_2 = \begin{bmatrix} 125.2500 & -5.0000 \\ -5.000 & 2.7050 \end{bmatrix}.$$

All basis functions are chosen as the Gaussian functions, which contain 36 nodes with the centers $\mu_{i,j,k}$ evenly spaced in $[-1, 1] \times [-1, 1]$ and the width $\eta_{i,j} = 0.1$.

Approach 1: Apply the first adaptive NN decentralized control scheme to control the systems (85) and (86).

Choose design parameters in the controller and adaptive laws: $\varepsilon_{1,1,0} = 0.1$, $\varepsilon_{2,1,0} = 0.2$, $c_{1,1} = 5$, $c_{2,1} = 7$, $c_{1,2} = 10$, $c_{2,2} = 6$, $\delta_{1,2,0} = 0.2$, $\delta_{2,2,0} = 0.3$, $\sigma_{1,1} = 0.1$, $\sigma_{2,1} = 0.2$, $\sigma_{1,2} = 0.15$, $\sigma_{2,2} = 0.1$, $\sigma_{3,1} = 0.2$, $\sigma_{3,2} = 0.1$, $\varsigma = 0.05$, $\gamma_{1,1} = 1$, $\gamma_{1,2} = 10$, $\gamma_{2,1} = 12$, $\gamma_{2,2} = 14$, $\Gamma_{1,1} = \text{diag}\{1, 1\}$, $\Gamma_{2,1} = \text{diag}\{2, 2\}$, $\Gamma_{1,2} = \text{diag}\{3, 3\}$, $\Gamma_{2,2} = \text{diag}\{2, 2\}$, $\tau_1 = 2$, $\tau_2 = 2$, $\tau^* = 0.6$, $p = 2$.

The desired trajectories are given as $y_{1,r} = y_{2,r} = \sin(t)$, and the initial conditions are chosen as $x_{1,1}(0) = 0.2$, $x_{1,2}(0) = 0.2$. The others initial conditions are chosen as zeros. The simulation results are shown by Figs. 1–8, where Fig. 1 shows the trajectories of the tracking errors $z_{1,1}$ and $z_{2,1}$. Figs. 2 and 3 show the trajectories of the observer errors $e_{1,1}$, $e_{1,2}$, $e_{2,1}$, and $e_{2,2}$, respectively. Fig. 4 shows the trajectories of the control inputs u_1 and u_2.

Approach 2: Apply the second adaptive NN decentralized control scheme to control the systems (85) and (86). In the simulation, choose parameters $\lambda_{1,2} = \lambda_{2,2} = 0.008$, and the other parameters and the initial conditions are chosen as the same as Approach 1. The simulation results are shown by Figs. 5–8.

Figs.1–8 show that the proposed two adaptive NN decentralized control approaches can guarantee the stability of the resulting closed-loop system. Meanwhile, by integrating the tracking errors over the interval $[0, 20]$, the values of $I_1 = \int_0^{20} |z_{1,1}| dt$ and $I_2 = \int_0^{20} |z_{2,1}| dt$ are depicted by Figs. 9 and 10, respectively. From Figs. 9 and 10, it is

TABLE I
TIME COMPARISONS OF THE TWO APPROACHES

Simulation time (s)	Program execution time of Approach 1 (s)	Program execution time of Approach 2 (s)
10	13.36	4.59
20	26.91	8.19
30	41.13	13.32

apparent that Approach 2 has better tracking performance than Approach 1.

To further compare with the computation complexity between control Approach 1 and Approach 2, we set simulation time 10, 20, and 30 s, and the corresponding program execution times are listed in Table I.

From Table I, we see that the program execution times needed by Approach 2 is less that those needed by Approach 1. Therefore, it can be concluded that Approach 2 can overcome the computation burden inherent in the first approach.

VI. CONCLUSION

In this paper, two observer-based adaptive NN decentralized output feedback control approaches have been proposed for a class of uncertain nonlinear large-scale systems with unknown time delays and unmeasured states. The first one has been designed based on the principle of the adaptive backstepping technique, although it can solve the problem of the unmeasured states and guarantee the stability of the close-loop system; however, the problem of explosion of complexity is inherent in the first control approach. The second one has been designed by combining the adaptive backstepping technique with the dynamic DSC technique, therefore, the second control approach not only has the features of the first one, but also overcomes the drawback existing in the first control approach. Simulation results and simulation comparisons have confirmed the effectiveness of the proposed approaches.

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Shao Cheng Tong received the B.S. degree in mathematics from BoHai University, Jinzhou, China, the M.S. degree in fuzzy mathematics from Dalian Marine University, Dalian, China, and the Ph.D. degree in fuzzy control from Northeastern University, Shenyang, China, in 1982, 1988, and 1997, respectively.

He is currently a Professor with the Department of Basic Mathematics, Liaoning University of Technology, Jinzhou. His current research interests include fuzzy and neural networks, control theory, and nonlinear adaptive control.

Yong Ming Li received the B.S. and M.S. degrees in applied mathematics from Liaoning University of Technology, Jinzhou, China, in 2004 and 2007, respectively.

He is currently a Lecturer in the Department of Basic Mathematics, Liaoning University of Technology. His current research interests include fuzzy and neural networks, control theory, and nonlinear adaptive control.

Hua-Guang Zhang received the B.S. and M.S. degrees in control engineering from Northeastern Electric Power University of China, Jilin, China, in 1982 and 1985, respectively, and the Ph.D. degree in thermal power engineering and automation from Southeast University, Nanjing, China, in 1991.

He joined the Department of Automatic Control, Northeastern University, Shenyang, China, in 1992, where he was a Post-Doctoral Fellow for two years and has been a Professor and the Head of the Institute of Electric Automation, School of Information Science and Engineering, since 1994. He has authored or co-authored more than 200 research papers published in journal and conference proceedings, has written four monographs, and holds 20 patents. His current research interests include fuzzy control, stochastic system control, neural-network-based control, nonlinear control, and their applications.

Dr. Zhang is an Associate Editor of *Automatica* and *Neurocomputing*. He is an Associate Editor of the IEEE TRANSACTIONS ON FUZZY SYSTEMS AND THE IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS-PART B. He was awarded the Outstanding Youth Science Foundation Award from the National Natural Science Foundation Committee of China in 2003. He was named the Cheung Kong Scholar by the Education Ministry of China in 2005.