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Adaptive observer-based fault estimation for a class of Lipschitz nonlinear systems

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Fault input channels represent a major challenge for observer design for fault estimation. Most works in this field assume that faults enter in such a way that the transfer functions between these faults and a number of measured outputs are strictly positive real (SPR), that is, the observer matching condition is satisfied. This paper presents a systematic approach to adaptive observer design for joint estimation of the state and faults when the SPR requirement is not verified. The proposed method deals with a class of Lipschitz nonlinear systems subjected to piecewise constant multiplicative faults. The novelty of the proposed approach is that it uses a rank condition similar to the observer matching condition to construct the adaptation law used to obtain fault estimates. The problem of finding the adaptive observer matrices is formulated as a Linear Matrix Inequality (LMI) optimization problem. The proposed scheme is tested on the nonlinear model of a single link flexible joint robot system.

Key words: nonlinear adaptive observer, fault estimation, strictly positive real, Lipschitz systems, observer matching condition, LMI.

1. Introduction

The growing complexity and automation degree of modern technical processes increase the possibility of system failures. Faults in sensors, actuators or process components may lead to the degradation of the overall system performance and could cause serious damage. Therefore, to improve system's reliability and avoid performance deterioration or system shutdown, faults have to be detected and localized timely while the system is still operating in a controllable region. In the community of the control researchers, this has stimulated over the last decades an intense interest in the development of model-based fault detection and identification (FDI) methods (see [5, 7]).

One of the mostly used schemes in this area is the observer-based FDI technique. The basic idea behind this scheme is to use the observer as a system model running parallel to the real system to generate optimal estimates of the system outputs. Errors between the

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estimated outputs and the measured outputs are used to generate fault indicator signals called residuals. By comparing a residual with a threshold function, one can determine if the system is suffering from some faults or not.

During the last two decades, for the purpose of active fault tolerant control, one can notice an increasing interest in the observer-based fault reconstruction and estimation (FRE). Instead of generating and evaluating residuals, the observer-based FRE generates estimates of faults, which means immediate fault detection and identification. The main advantage of this method over FDI techniques is that the used observer for FRE can also be used for state feedback control because it is designed to preserve accurate state estimation even in the faulty case. Various observer-based FRE design approaches have been reported in the literature, mainly based on sliding mode observers and adaptive observers. The first sliding mode observer-based fault reconstruction scheme was proposed in [6] where actuator faults are reconstructed using the equivalent output injection signal. Afterward, several papers have been published on this subject for linear and nonlinear systems [2, 12, 16, 17, 20, 23]. When faults are modeled in terms of parameter changes, adaptive observers can be employed to estimate these faults. One of the benefits of using this technique is that, based on persistent excitation condition, it enables the estimation of any number of faults, regardless the number of measured outputs. Adaptive observers have been exploited for fault estimation by many authors [1, 8, 10, 13, 18, 21]. Nevertheless, they assume that the transfer functions between faults and the number of measured outputs are strictly positive real (SPR), which is not the case for many practical systems.

Few solutions have been developed to remove the SPR requirement. In [11], an adaptive observer for actuator and sensor fault estimation was developed. The proposed scheme circumvents the SPR condition at the expense of requiring the existence of a positive definite solution to a certain matrix inequality. A different technique was provided in [19] where a high gain adaptive observer is used to estimate the fault vector for a class of single-output nonlinear systems.

Motivated by the works in [15] and [24], in this paper, an adaptive observer design technique for fault estimation for a class of Lipschitz nonlinear systems that do not satisfy the SPR requirement is proposed. The main contribution of this work is the proposal of a new adaptation law augmented by a switching leakage term for fault estimation. Unlike adaptation laws in [15] and [24], which relay, respectively, on solving a partial differential equation and a set of algebraic matrix equations, the proposed solution relay on a rank condition similar to the well-known observer matching condition. This makes it possible to use some conventional adaptive observer design tools.

The paper is organized as follows: System description and some background results are presented in section 2. In section 3, the design of the adaptive observer for fault estimation and the analysis of the stability of the error dynamics are provided. An illustrative example with simulation results is provided to verify the efficiency of the proposed scheme in section 4, followed by some concluding remarks in section 5.

Notations

Throughout the paper, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ the set of real $n-$vectors and $\mathbb{R}^{n\times m}$ the set of real $n\times m$ matrices. I_n and $0_{n\times m}$ denote, respectively, the $n\times n$ identity matrix and $n\times m$ zero matrix. The Euclidian norm of a vector x is denoted by $\|x\|$ and the induced norm of a matrix $A\in\mathbb{R}^{n\times m}$ is denoted by $\|A\|$. $\lambda_{min}(A)$ and $\lambda_{max}(A)$ denote, respectively, the minimum and the maximum eigenvalues of the matrix A.

2. Problem Statement

2.1. System description

Consider the continuous-time nonlinear system described by the following equations:

$$\begin{cases} \dot{x} &= Ax+B\Phi(x,u)+Ef(x) \\ y &= Cx \end{cases} \tag{1}$$

where $x\in\mathbb{R}^n$ is the state vector, $u\in\mathbb{R}^m$ is the control input vector, $y\in\mathbb{R}^p$ is the output vector. $\Phi(x,u):\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^s$ is a known nonlinear function vector and $f(x):\mathbb{R}^n\to\mathbb{R}^r$ is an unknown state-dependent function vector which is due to the faulty behavior of the system. It is assumed that the control signals ensure that x remains bounded even when faults occur. A, C, B and E are known constant matrices of appropriate dimensions with (A,C) being an observable pair. without loss of generality we assume that $rank(E)=r$ and $rank(C)=p$.

In this paper, we intend to use an adaptive observer to estimate component faults. For this purpose, the following multiplicative fault model is adopted:

$$f(x)=\Psi(x)\theta \tag{2}$$

where $\theta\in\mathbb{R}^q$ is a vector of unknown time-dependent functions which reflect the source and the size of component faults and $\Psi(x):\mathbb{R}^n\to\mathbb{R}^{r\times q}$ is a known function matrix representing the functional structure of faults.

Consider the following assumptions which are typically required in the literature on adaptive observer design for nonlinear systems.

Assumption 1 *The fault vector θ is piecewise constant and bounded in the following sense:*

$$\|\theta\|\leqslant\|\theta_m\|=\rho \tag{3}$$

where $\theta_m\in\mathbb{R}^q$ is a known constant vector and ρ is a known positive constant.

Assumption 2 *The nonlinear function vector $\Phi(x,u)$ and the matrix $\Psi(x)$ satisfy the following Lipschitz conditions:*

$$\|\Phi(x,u)-\Phi(\hat{x},u)\|\leqslant\gamma_1\|(x-\hat{x})\| \tag{4}$$

$$\|\Psi(x)-\Psi(\hat{x})\| \leqslant \gamma_2\|(x-\hat{x})\| \tag{5}$$

$\forall x,\hat{x}\in\mathbb{R}^n$, *where* γ_1 *and* γ_2 *are positive constants.*

Assumption 3 *The matrix* $E\Psi(x)$ *is persistently exciting, i.e. there exist positive constants* T, k_1 *and* k_2 *such that for all* $t \geqslant 0$

$$I_n k_1 \geqslant \int_t^{t+T} E\Psi(x)\Psi(x)^T E^T dt \geqslant I_n k_2. \tag{6}$$

Remark 1 It should be pointed out that the multiplicative fault vector θ may not have the same dimension as the additive fault vector $f(x)$. In an observer-based FRE scheme, for fault observability reasons, the number of additive faults should not exceed the number of the measured outputs. The persistency of excitation condition in Assumption 3 ensures the observability of the multiplicative fault vector θ regardless of the number of measured outputs. This fact can be understood through the Gramian observability matrix of the system obtained by appending θ into the state vector x [19].

2.2. Conventional nonlinear adaptive observer design for fault estimation

Most results on adaptive observer design for nonlinear systems follow [3] where a class of Lispschitz nonlinear systems with a regression matrix that can depend on the whole state is considered. According to [3], under Assumptions 1–3, an adaptive observer for joint estimation of state and faults for system (1) is given by the following equations:

$$\dot{\hat{x}} = A\hat{x} + B\Phi(\hat{x},u) + E\Psi(\hat{x})\hat{\theta} + L(y-C\hat{x}) \tag{7}$$

$$\dot{\hat{\theta}} = \Gamma\Psi(\hat{x})^T F(y-C\hat{x}) \tag{8}$$

where $\hat{x}$ is the state estimate, $\hat{\theta}$ is the fault vector estimate, $\Gamma=\Gamma^T\in\mathbb{R}^{q\times q}$ is learning rate matrix and $L\in\mathbb{R}^{n\times p}$ and $F\in\mathbb{R}^{r\times p}$ are such that

$$(A-LC)^T P + P(A-LC) = -Q \tag{9}$$

$$E^T P = FC \tag{10}$$

for matrices $P=P^T>0$ and $Q=Q^T>0$ satisfying the following inequality:

$$\gamma_1\|B\| + \gamma_2\rho\|E\| < \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)}. \tag{11}$$

The existence of a positive definite matrix P to satisfy the condition (10) is guaranteed when the transfer functions between at least r outputs and the unknown parameters are dissipative or SPR [3]. It is usual to verify the feasibility of (10) using the following lemma.

Lemma 1 [4, 12] *There exist matrices $P = P^T > 0$ and F verifying equality* (10)*, if and only if the observer matching condition*

$$rank(CE) = rank(E) \tag{12}$$

is satisfied.

One can see that the observer matching condition means that $r \leqslant p$ and the relative degrees from the unknown parameters to at least r measured outputs are all equal to one, i.e. the fault input channels are in the measured dynamics. For many physical systems modeled by (1), the observer matching condition is not satisfied. For instance, for mechanical systems with only measured positions, none of possible faults can be found in the measured dynamics. The main purpose of this paper is to design an adaptive observer for system (1) for joint estimation of the state and the fault vector when the observer matching condition is not satisfied.

3. Adaptive observer design for unmatched fault estimation

3.1. A canonical form for adaptive observer design

First, let us define the structure of the system considered in this paper using the following assumption.

Assumption 4 *Matrices A, B, E and C satisfy*

$$CB = 0_{p\times s} \tag{13}$$
$$CE = 0_{p\times r} \tag{14}$$
$$rank(CAE) = rank(E). \tag{15}$$

Remark 2 In Assumption 4, equalities (14) and (15) imply that the relative degrees from the unknown parameters to at least r measured outputs are all equal to two and $r \leqslant p$. Although this assumption looks to be restrictive, it can be satisfied by many physical systems like mechanical systems, the drilling system [15], satellite attitude control system [9], some hydraulic systems [22] and many others engineering systems.

Assumption 5 *The first time derivative of $\Psi(x)$ is continuous and bounded provided x is bounded .*

Now, decompose C and E into bloc matrices as follows:

$$C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{16}$$

where $C_1 \in \mathbb{R}^{p\times p}$ and $E_1 \in \mathbb{R}^{p\times r}$. Without loss of generality, it can be assumed that the outputs of the system have been reordered so that the matrix C_1 is full rank. Define new coordinates as $z = Tx$ where

$$T = \begin{bmatrix} C_1 & C_2 \\ 0_{(n-p)\times p} & I_{(n-p)} \end{bmatrix}. \tag{17}$$

Then, using (13) and (14), in the new coordinate system, the original system (1) have the following form:

$$\begin{cases} \dot{z} &= \bar{A}z + \bar{B}\Phi(T^{-1}z,u) + \bar{E}\Psi(T^{-1}z)\theta \\ y &= \bar{C}z \end{cases} \tag{18}$$

where

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, z_1 = y, z_2 \in \mathbb{R}^{(n-p)}, T^{-1} = \begin{bmatrix} C_1^{-1} & -C_1^{-1}C_2 \\ 0_{(n-p)p} & I_{(n-p)} \end{bmatrix},$$
$$\bar{A} = TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \bar{B} = TB = \begin{bmatrix} 0_{p\times s} \\ B_2 \end{bmatrix}, \bar{E} = TE = \begin{bmatrix} 0_{p\times r} \\ E_2 \end{bmatrix},$$
$$\bar{C} = CT^{-1} = \begin{bmatrix} I_{p\times p} & 0_{p\times(n-p)} \end{bmatrix}.$$

Considering the structure of $\bar{E}$, we conclude that $rank(E_2) = r$. Applying equality (15) to system (18) yields

$$rank(A_{12}E_2) = rank(E_2). \tag{19}$$

Equality (19) has the form of the observer matching condition (12). So, by applying Lemma 1, we conclude that (19) holds if and only if there exist matrices $P_3 = P_3^T > 0$ and F such that

$$E_2^T P_3 = FA_{12}. \tag{20}$$

3.2. Main result

Based on the transformed system (18), we propose the following adaptive observer:

$$\dot{\hat{z}} = \bar{A}\hat{z} + \bar{B}\hat{\Phi} + \bar{E}\hat{\Psi}\hat{\theta} + \bar{L}(y - \hat{z}_1) \tag{21}$$

$$\dot{W} = -\Gamma\frac{d\hat{\Psi}^T}{dt}Fy - \Gamma\hat{\Psi}^T(FA_{11}y + FA_{12}\hat{z}_2 - E_2^T P_2^T(y - \hat{z}_1)) - \Gamma\beta(\hat{\theta} - \theta_m) \tag{22}$$

$$\hat{\theta} = W + \Gamma\hat{\Psi}^T Fy \tag{23}$$

where $\hat{z}$ is the state estimate, $\hat{\theta}$ is the fault vector estimate, $\bar{L}$ is the observer gain matrix, $\Gamma = \Gamma^T > 0$ is the learning rate matrix, P_2 and F are matrices to be designed later, $\hat{\Phi} = \Phi(T^{-1}\hat{z},u)$, $\hat{\Psi} = \Psi(T^{-1}\hat{z})$, and β is a switching leakage term defined as

$$\beta = \begin{cases} 0 & \text{if} \quad ||\hat{\theta} - \theta_m|| \leqslant 2\rho, \\ \kappa & \text{if} \quad ||\hat{\theta} - \theta_m|| > 2\rho, \end{cases} \tag{24}$$

which forces the estimated fault vector $\hat{\theta}$ to remain within the region bounded by 2ρ, with κ is a positive constant. Obviously, the term $\frac{d\hat{\Psi}^T}{dt}$ is implemented analytically using the chain rule $\frac{\partial \hat{\Psi}^T}{\partial \hat{z}}\dot{\hat{z}}$, since $\dot{\hat{z}}$ is available.

Defining the observation error and the fault estimation error as $\widetilde{z} = z - \hat{z}$ and $\widetilde{\theta} = \theta - \hat{\theta}$ respectively, their dynamics can then be represented as follows:

$$\dot{\widetilde{z}} = (\bar{A} - \bar{L}\bar{C})\widetilde{z} + \bar{B}\widetilde{\Phi} + \bar{E}(\widetilde{\Psi}\theta + \hat{\Psi}\widetilde{\theta}) \tag{25}$$

$$\dot{\widetilde{\theta}} = -\Gamma\hat{\Psi}^T \begin{bmatrix} E_2^T P_2^T & FA_{12} \end{bmatrix} \widetilde{z} + \Gamma\beta(\hat{\theta} - \theta_m) \tag{26}$$

where $\widetilde{\Phi} = \Phi(T^{-1}z, u) - \hat{\Phi}$ and $\widetilde{\Psi} = \Psi(T^{-1}z) - \hat{\Psi}$.

Theorem 1 *Under Assumptions* 1, 2, 4 *and* 5*, the observation error* $\widetilde{z}$ *determined by* (25) *is asymptotically stable while the fault estimate error* $\widetilde{\theta}$ *determined by* (26) *remains bounded, if there exist positive real numbers* ε_1, ε_2 *and matrices* $P = P^T \in \mathbb{R}^{n\times n}$, $F \in \mathbb{R}^{r\times p}$ *and* $M = M^T \in \mathbb{R}^{n\times n}$*, such that*

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} > 0 \tag{27}$$

$$\begin{bmatrix} \Lambda & P\bar{B} & P\bar{E} \\ \bar{B}^T P & -\varepsilon_1 I_s & 0_{s\times r} \\ \bar{E}^T P & 0_{r\times s} & -\varepsilon_2 I_r \end{bmatrix} < 0 \tag{28}$$

$$E_2^T P_3 = FA_{12} \tag{29}$$

where $\Lambda = \bar{A}^T P + P\bar{A} - \bar{C}^T\bar{C}M - M\bar{C}^T\bar{C} + (\gamma_1^2\varepsilon_1 + \varepsilon_2\gamma_2^2\rho^2)(T^{-1})^T T^{-1}$*. Once the above conditions are satisfied, the observer gain* $\bar{L}$ *is chosen as*

$$\bar{L} = P^{-1}M\bar{C}^T. \tag{30}$$

Moreover, if the persistency excitation condition in Assumption 3 *holds, then the vector* $\widetilde{\theta}$ *converges to zero.*

Proof Consider the following Lyapunov function:

$$V = \widetilde{x}^T P\widetilde{x} + \widetilde{\theta}^T\Gamma^{-1}\widetilde{\theta}. \tag{31}$$

The derivative of V along with the trajectories of error dynamic systems (25),(26) is

$$\begin{aligned} \dot{V} = {} & \widetilde{z}^T[(\bar{A} - L\bar{C})^T P + P(\bar{A} - L\bar{C})]\widetilde{z} + 2\widetilde{z}^T P\bar{B}\widetilde{\Phi} + 2\widetilde{z}^T P\bar{E}\widetilde{\Psi}\theta + 2\widetilde{z}^T P\bar{E}\hat{\Psi}\widetilde{\theta} \\ & - 2\widetilde{\theta}^T\hat{\Psi}^T \begin{bmatrix} E_2^T P_2^T & FA_{12} \end{bmatrix} \widetilde{z} + 2\widetilde{\theta}^T\beta(\hat{\theta} - \theta_m). \end{aligned} \tag{32}$$

Using relation (24), we get

$$\begin{aligned}\widetilde{\theta}^T\beta(\hat{\theta}-\theta_m) &= (\theta-\theta_m)^T\beta(\hat{\theta}-\theta_m)-(\hat{\theta}-\theta_m)^T\beta(\hat{\theta}-\theta_m)\\ &= (\theta-\theta_m)^T\beta(\hat{\theta}-\theta_m)-\beta||\hat{\theta}-\theta_m||^2\\ &\leqslant \beta||\hat{\theta}-\theta_m||(2\rho-||\hat{\theta}-\theta_m||)\leqslant 0.\end{aligned} \tag{33}$$

It follows that

$$\begin{aligned}\dot{V} \leqslant{}& \widetilde{z}^T[(\bar{A}-L\bar{C})^TP+P(\bar{A}-L\bar{C})]\widetilde{z}+2\widetilde{z}^TP\bar{B}\widetilde{\Phi}+2\widetilde{z}^TP\bar{E}\widetilde{\Psi}\theta+2\widetilde{z}^TP\bar{E}\hat{\Psi}\widetilde{\theta}\\ &-2\widetilde{\theta}^T\hat{\Psi}^T\begin{bmatrix}E_2^TP_2^T & FA_{12}\end{bmatrix}\widetilde{z}.\end{aligned} \tag{34}$$

Using the decomposed structure (27) of P and $\bar{E}=\begin{bmatrix}0_{p\times r}^T & E_2^T\end{bmatrix}^T$ in the fourth term of (34), we get

$$\begin{aligned}\dot{V} \leqslant{}& \widetilde{z}^T[(\bar{A}-L\bar{C})^TP+P(\bar{A}-L\bar{C})]\widetilde{z}+2\widetilde{z}^TP\bar{B}\widetilde{\Phi}+2\widetilde{z}^TP\bar{E}\widetilde{\Psi}\theta+2\widetilde{z}^T\begin{bmatrix}P_2E_2\\ P_3E_2\end{bmatrix}\hat{\Psi}\widetilde{\theta}\\ &-2\widetilde{\theta}^T\hat{\Psi}^T\begin{bmatrix}E_2^TP_2^T & FA_{12}\end{bmatrix}\widetilde{z}.\end{aligned} \tag{35}$$

Thus, from (20) it follows that

$$\dot{V} \leqslant \widetilde{z}^T[(\bar{A}-L\bar{C})^TP+P(\bar{A}-L\bar{C})]\widetilde{z}+2\widetilde{z}^TP\bar{B}\widetilde{\Phi}+2\widetilde{z}^TP\bar{E}\widetilde{\Psi}\theta. \tag{36}$$

From Lipschitz conditions (4) and (5), we obtain, respectively, the following inequalities:

$$\| \widetilde{\Phi} \|^2 \leqslant \gamma_1^2 \| T^{-1}\widetilde{z} \|^2 \tag{37}$$

$$\| \widetilde{\Psi}\theta \|^2 \leqslant \| \widetilde{\Psi} \|^2 \| \theta \|^2 \leqslant \gamma_2^2\rho^2 \| T^{-1}\widetilde{z} \|^2 \tag{38}$$

which yield respectively

$$\gamma_1^2\varepsilon_1\widetilde{z}^T(T^{-1})^TT^{-1}\widetilde{z}-\varepsilon_1\widetilde{\Phi}^T\widetilde{\Phi}\geqslant 0 \tag{39}$$

$$\varepsilon_2\gamma_2^2\rho^2\widetilde{z}^T(T^{-1})^TT^{-1}\widetilde{z}-\varepsilon_2\theta^T\widetilde{\Psi}^T\widetilde{\Psi}\theta\geqslant 0 \tag{40}$$

where ε_1 and ε_2 are positive constants. Then, by adding (39) and (40) to (36), we obtain

$$\begin{aligned}\dot{V} \leqslant{}& \widetilde{z}^T[(\bar{A}-L\bar{C})^TP+P(\bar{A}-L\bar{C})]\widetilde{z}+2\widetilde{z}^TP\bar{B}\widetilde{\Phi}+2\widetilde{z}^TP\bar{E}\widetilde{\Psi}\theta\\ &+\gamma_1^2\varepsilon_1\widetilde{z}^T(T^{-1})^TT^{-1}\widetilde{z}-\varepsilon_1\widetilde{\Phi}^T\widetilde{\Phi}+\varepsilon_2\gamma_2^2\rho^2\widetilde{z}^T(T^{-1})^TT^{-1}\widetilde{z}-\varepsilon_2\theta^T\widetilde{\Psi}^T\widetilde{\Psi}\theta.\end{aligned} \tag{41}$$

Let $\bar{L}=P^{-1}M\bar{C}^T$. After some simple algebraic manipulations, we get

$$\dot{V} \leqslant -\xi^T\Xi\xi \tag{42}$$

where $\xi = [\ \dot{\tilde{z}}^T \quad \tilde{\Phi}^T \quad \theta^T\tilde{\Psi}^T\ ]^T$ and

$$\Xi = -\begin{bmatrix} \Lambda & P\bar{B} & P\bar{E} \\ \bar{B}^T P & -\varepsilon_1 I_{s\times s} & 0 \\ \bar{E}^T P & 0 & -\varepsilon_2 I_{r\times r} \end{bmatrix} \tag{43}$$

with $\Lambda = \bar{A}^T P + P\bar{A} - \bar{C}^T\bar{C}M - M\bar{C}^T\bar{C} + (\gamma_1^2\varepsilon_1 + \varepsilon_2\gamma_2^2\rho^2)(T^{-1})^T T^{-1}$. So, based on Lyapunov stability theory, we conclude that the equilibrium $(\tilde{z},\tilde{\theta}) = (0,0)$ is stable if the matrix Ξ is positive definite. To prove asymptotic stability of the observer, we integrate both sides of inequality (42) from $t = 0$ to $t = t_f$. It follows that

$$V(t_f) \leqslant V(0) - \int_0^{t_f} \xi^T \Xi \xi dt. \tag{44}$$

Since $V > 0$, the above inequality implies that

$$\int_0^{t_f} \xi^T \Xi \xi dt \leqslant V(0). \tag{45}$$

So, for $t_f \to \infty$ the integral in (45) exists and is finite since $V(0)$ is finite. By using (4), (5), (25), (45) and the Barbalat's lemma [14], we ca easily conclude that $\lim_{t\to\infty} \xi^T \Xi \xi = 0$ and thus $\lim_{t\to\infty} \xi = \lim_{t\to\infty} [\ \dot{\tilde{z}}^T \quad \tilde{\Phi}^T \quad \theta^T\tilde{\Psi}^T\ ]^T = 0$. Consequently, it can also be concluded that $\lim_{t\to\infty} \dot{\tilde{z}} = 0$. Hence, considering (25), we conclude that

$$\lim_{t\to\infty} \bar{E}\hat{\Psi}\tilde{\theta} = 0 \tag{46}$$

Thus, if the persistency excitation condition in Assumption 3 holds, the fault estimates converge to their true values. This completes the proof. □

Remark 3 Adaptive observer design by using Theorem 1 involves solving LMIs (27) and (28) under strict equality constraint (29) with respect to P, M and F. To handle this problem using LMI solvers, it is more convenient to transform (29) into an LMI. We can show that equality (29) holds if and only if the following LMI optimization problem has a minimum of $\eta = 0$ [4]:

$$\begin{aligned} &\text{Minimize } \eta \text{ subject to} \\ &\begin{bmatrix} \eta I & FA_{12} - E_2^T P_3 \\ (FA_{12} - E_2^T P_3)^T & \eta I \end{bmatrix} \geqslant 0. \end{aligned} \tag{47}$$

where η is a positive scalar. Therefore, computing L, P and F involves solving (27), (28) and (47) with respect to P_1, P_2, P_3, M, F and η, simultaneously.

4. Illustrative example

Consider the model of a single-link flexible-joint robotic manipulator defined by the following equations [17] :

$$\begin{aligned} &J_m\ddot{q}_m - (k+\theta_1)(q_l - q_m) + (B+\theta_2)\dot{q}_m = k_\tau u, \\ &J_l\ddot{q}_l + (k+\theta_1)(q_l - q_m) + mgh\sin(q_l) = 0, \end{aligned} \tag{48}$$

where q_m and q_l are the angular positions of the motor shaft and the link, respectively, u represents the control torque of the motor, J_m the inertia of the motor, J_l the inertia of the link, k the elastic constant, B the viscous friction coefficient , k_τ the amplifier gain, m the link mass, h the center of mass and g the gravity. θ_1 and θ_2 model the effect of the component faults resulting, respectively, from an elastic deformation of the flexible joint and an abnormal increase of the viscous friction. The angular positions q_m and q_l are assumed as available measurements. The system parameters are: $J_m = 0.037$ Kg m^2, $J_l = 0.093$ Kg m^2, $k = 0.18$ Nm/rad, $B = 0.0083$ Nms/rad, $m = 0.21$ Kg, $g = 9.81$ m/s^2, $h = 0.15$ m, $k_\tau = 0.18$ Nm/V.

Considering the state $x = [\ x_1 \quad x_2 \quad x_3 \quad x_4\ ]^T = [\ q_m \quad q_l \quad \dot{q}_m \quad \dot{q}_l\ ]^T$ and the fault vector $\theta = [\ \theta_1 \quad \theta_2\ ]^T$, we can write the system (48) in the form (1) with

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -48.65 & 48.65 & -2.24 & 0 \\ 19.35 & -19.35 & 0 & 0 \end{bmatrix}, E = B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\Phi(x,u) = \begin{bmatrix} 2.16u \\ -33.32\sin(x_2) \end{bmatrix}, \Psi(x) = \begin{bmatrix} 27.03(x_2 - x_1) & 27.03x_3 \\ -10.75(x_2 - x_1) & 0 \end{bmatrix}.$$

Now, it is easy to check that the observer matching condition (12) is not satisfied and hence no adaptive observer of the form presented in (7), (8) can be used to estimate x and θ. However, Assumption 4 is satisfied, and as a consequence, the adaptation law (23) is feasible. Notice that according to the structure of the matrices C and E, the system is already in the form (18) and therefore no state transformation is needed to design the adaptive observer (21)–(23).

Since $\Psi(x)$ is a matrix, the computation of its Lipschitz constant γ_2 is not a trivial task. In order to do this, we evaluate the Lipschitz constants of the individual rows of $\Psi(x)$. Let $\Psi_1(x) = \begin{bmatrix} 27.03(x_3 - x_1) & 27.03x_2 \end{bmatrix}$ and $\Psi_2(x) = \begin{bmatrix} -10.75(x_3 - x_1) & 0 \end{bmatrix}$, we have

$$\Psi(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Psi_1(x) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Psi_2(x). \tag{49}$$

Thus

$$\begin{aligned}\|\Psi(x)-\Psi(\hat{x})\| &\leqslant \|\Psi_1(x)-\Psi_1(\hat{x})\|+\|\Psi_1(x)-\Psi_1(\hat{x})\| \\ &\leqslant \gamma_{21}\|x-\hat{x}\|+\gamma_{22}\|x-\hat{x}\| \\ &\leqslant \gamma_2\|x-\hat{x}\|\end{aligned}$$

with $\gamma_2=\gamma_{21}+\gamma_{22}$ and γ_{21}, γ_{22} are, respectively the Lipschitz constants of $\Psi_1(x)$ and $\Psi_2(x)$, which can be evaluated, respectively, by computing $max\left(\left\|\frac{\partial\Psi_1}{\partial x}\right\|\right)$ and $max\left(\left\|\frac{\partial\Psi_2}{\partial x}\right\|\right)$. Note that in this example, the nonlinear term $\Phi(x,u)$ is measurable.

Let $\varepsilon_1=1/5, \varepsilon_2=1/150, \theta_m=[\ 0.19 \quad 0.06\ ]^T, \rho=0.2, \kappa=250, \gamma_1=0, \gamma_2=53.31$ and $\Gamma=10\times I_2$. By using CVX as a solver, the resolution of the LMI optimization problem defined by (27), (28) and (47) yields:

$$P=\left[\begin{array}{cc|cc} 57.6853 & 14.8663 & -0.9877 & -0.2879 \\ 14.8663 & 64.4713 & -0.3501 & -1.0478 \\ \hline -0.9877 & -0.3501 & 0.0412 & -0.0007 \\ -0.2879 & -1.0478 & -0.0007 & 0.0343 \end{array}\right], F=\begin{bmatrix} 0.0412 & -0.0007 \\ -0.0007 & 0.0343 \end{bmatrix},$$

$$M=\begin{bmatrix} 136.6898 & 24.7649 & 52.4035 & 14.8682 \\ 24.7649 & 111.0212 & 16.4015 & 59.1034 \\ 52.4035 & 16.4015 & 0 & 0 \\ 14.8682 & 59.1034 & 0 & 0 \end{bmatrix}.$$

Hence, using (30), we get

$$L=\begin{bmatrix} 52.5 & 37.1 \\ 37.9 & 74.9 \\ 2889.3 & 1996.1 \\ 2088.6 & 4359.4 \end{bmatrix}$$

A simulation was performed using the input $u=\sin(0.2t)$ and the following faults:

$$\theta_1=\begin{cases} 0 & \text{for } t\leqslant 4 \text{ sec}, \\ -0.06 & \text{for } 4<t\leqslant 8 \text{ sec}, \\ -0.12 & \text{for } t>8 \text{ sec}. \end{cases},$$

$$\theta_2=\begin{cases} 0 & \text{for } t\leqslant 6 \text{ sec}, \\ 0.008 & \text{for } 6<t\leqslant 10 \text{ sec}, \\ 0.02 & \text{for } t>10 \text{ sec}. \end{cases}$$

The used initial conditions were $x=[\ 0.5 \quad 0.5 \quad 0 \quad 0\ ]^T$, $\hat{x}=[\ 0 \quad 0 \quad 0 \quad 0\ ]^T$ and $\hat{\theta}=[\ 0 \quad 0\ ]^T$.

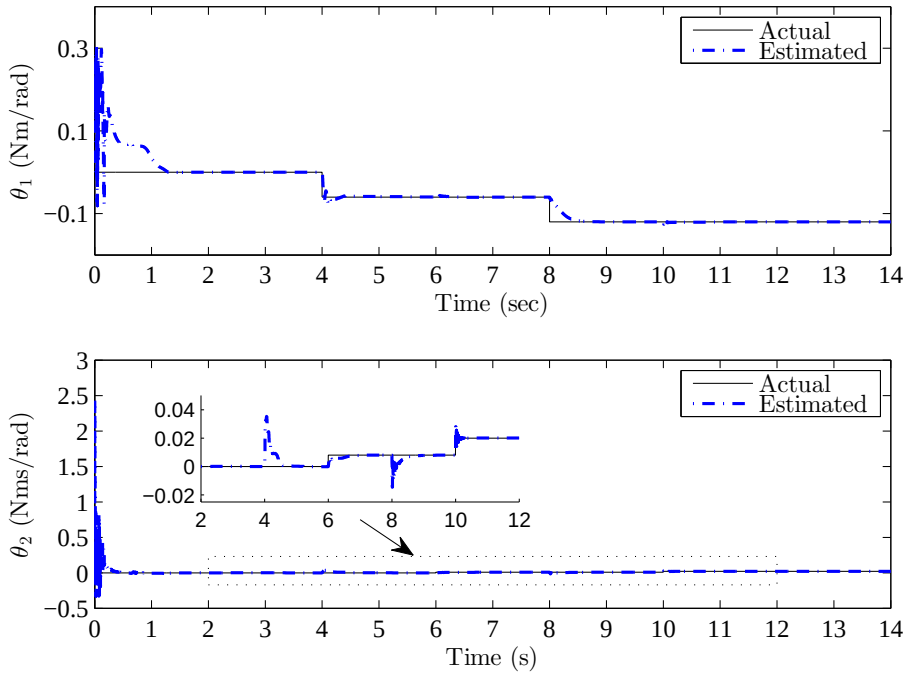

Figure 1. Actual faults and their estimates

Fig. 1 shows that after a transient caused by the initial conditions, the estimated parametric faults converge to their true values accurately before and after the fault occurrence, which means that the persistency of excitation condition is satisfied. Fig. 2 shows that the estimated states track accurately the real ones despite the presence faults.

5. Conclusion

This paper has addressed the design of an adaptive observer for joint estimation of the state and multiplicative faults for a class of Lipschitz systems whose structure does not satisfy the SPR requirement. The proposed scheme requires that the transfer function matrix between faults and the measured outputs to be relative degree two. The design procedure has been formulated into an LMI optimization problem and has been illustrated by a numerical example, and efficiency has also been demonstrated by the simulations.

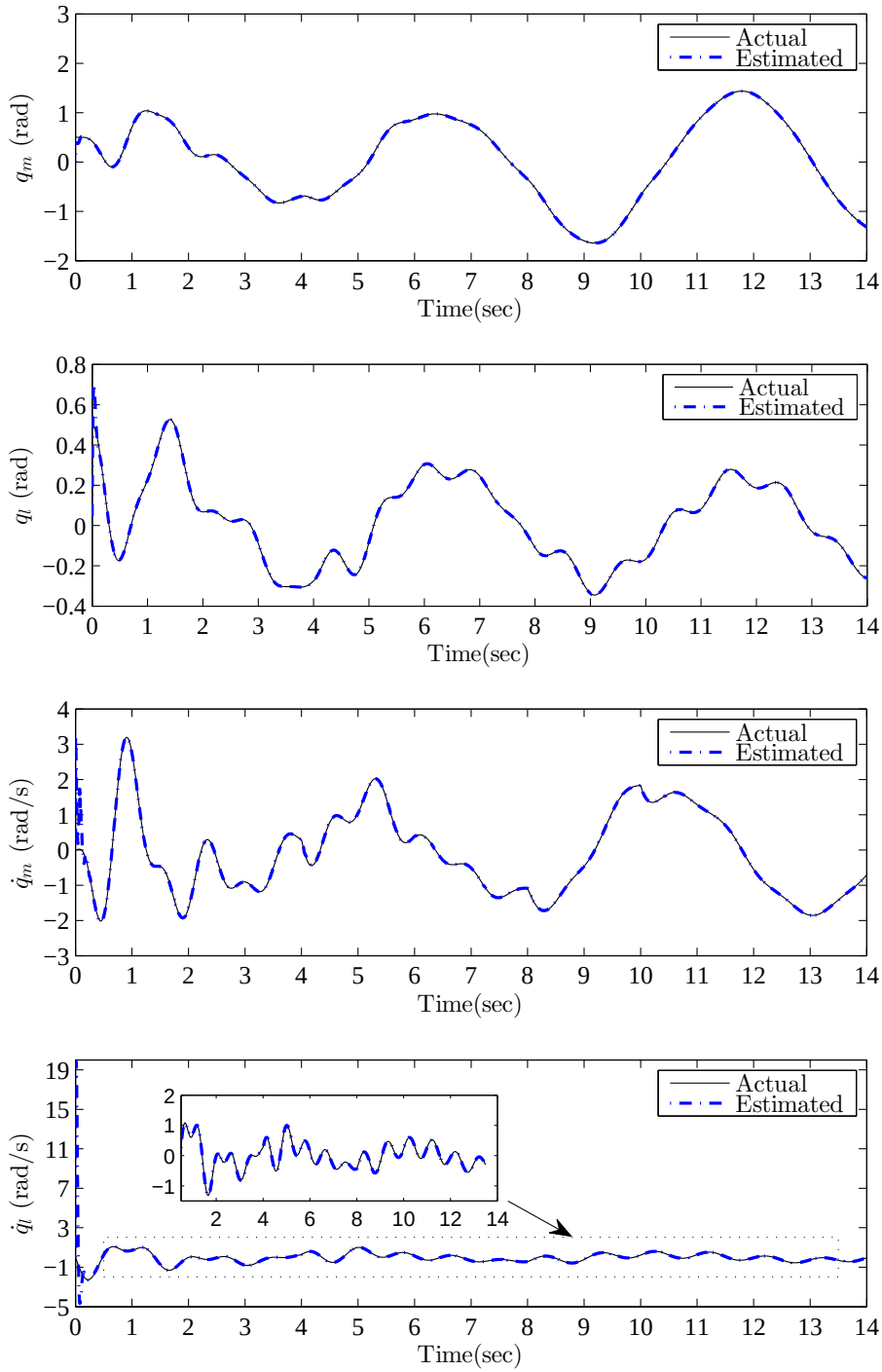

Figure 2. Actual state variables and their estimates

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